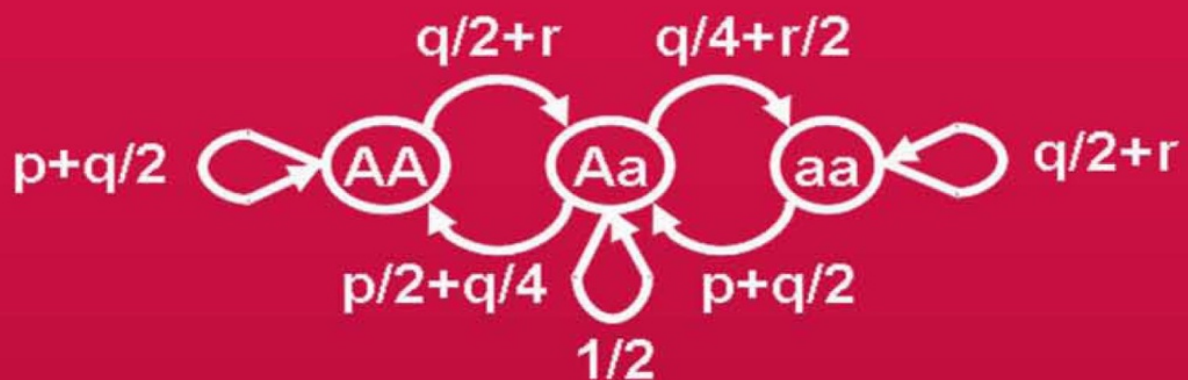


Texts in Statistical Science

Introduction to Probability with R



Kenneth Baclawski



Chapman & Hall/CRC
Taylor & Francis Group

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