Copyrighted Material

Universitext

Paolo Baldi

Probability

An Introduction Through Theory and Exercises

Springer

Copyrighted Material

Copyrighted Material

Paolo Baldi

Probability

An Introduction Through Theory and Exercises

Springer

Copyrighted Material

Copyrighted Material

Contents

1 Elements of Measure Theory 1
- 1.1 Measurable Spaces, Measurable Functions 1
- 1.2 Real Measurable Functions 4
- 1.3 Measures 7
- 1.4 Integration 12
- 1.5 Important Examples 23
- 1.6 L^p Spaces 26
- 1.7 Product Spaces, Product Measures 28

Exercises 35

2 Probability 41
- 2.1 Random Variables, Laws, Expectation 41
- 2.2 Independence 44
- 2.3 Computation of Laws 53
- 2.4 A Convexity Inequality: Jensen 60
- 2.5 Moments, Variance, Covariance 63
- 2.6 Characteristic Functions 69
- 2.7 The Laplace Transform 82
- 2.8 Multivariate Gaussian Laws 86
- 2.9 Quadratic Functionals of Gaussian r.v.'s, a Bit of Statistics 90

Exercises 98

3 Convergence 115
- 3.1 Convergence of r.v.'s 115
- 3.2 Almost Sure Convergence and the Borel-Cantelli Lemma 117
- 3.3 Strong Laws of Large Numbers 124
- 3.4 Weak Convergence of Measures 129
- 3.5 Convergence in Law 140
- 3.6 Uniform Integrability 144
- 3.7 Convergence in a Gaussian World 148
- 3.8 The Central Limit Theorem 152
- 3.9 Application: Pearson's Theorem, the χ^2 Test 154

Copyrighted Material

Copyrighted Material

3.10 Some Useful Complements 162
Exercises 167

4 Conditioning 177
4.1 Introduction 177
4.2 Conditional Expectation 178
4.3 Conditional Laws 188
4.4 The Conditional Laws of Gaussian Vectors 196
Exercises 198

5 Martingales 205
5.1 Stochastic Processes 205
5.2 Martingales: Definitions and General Facts 206
5.3 Doob's Decomposition 208
5.4 Stopping Times 209
5.5 The Stopping Theorem 213
5.6 Almost Sure Convergence 216
5.7 Doob's Inequality and L^p Convergence, $p > 1$ 222
5.8 L^1 Convergence, Regularity 224
Exercises 231

6 Complements 241
6.1 Random Number Generation, Simulation 241
6.2 Tightness and the Topology of Weak Convergence 252
6.3 Applications 255
Exercises 258

7 Solutions 261

References 385

Index 387

Copyrighted Material