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Electroweak standard model renormalization and application to the computation of one loop radiative corrections

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Dedication

I dedicate this work
To my parent, who have provided me with there encouragement, love and passion.
Especially to my mother for her support through my life.
To my brother Hatem for his whole hearted support.
To all my extended family, friends and teachers.
And especially to my soul mate, my fiance Nadia.

Ghoulam Imad Eddine BOUDIBA

Dedication

I dedicate this work to my
Mother and father for their incredible support, to all my family, friends and especially my classmates.

Tayeb OTMANE

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Firstly and before everything. All praises and thanks is due to *Allah* for providing us with the will to keep mouving on the right path. It is never enough thanking the creator of our universe which we try our best to understand it. Praise be to Allah.

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Abstract

In this master thesis, we tackle the problematic of pedagogically illuminating the link between the theoretical framework of quantum field theory and actual experiments in high energy physics (HEP) especially in the electroweak sector. To do that, we briefly review the theoretical structure of the electroweak standard model theory and its quantized Lagrangian, then revisit the ideas behind renormalization with applying the on-shell renormalization scheme to the theory. Application to a leptonic process $e\nu_\mu \to e\nu_\mu$ was made by the use of symbolic computation tools called FeynCalc[26, 37], FeynArts and FormCalc packages of Mathematica software.

ملخص

في هذه الأطروحة، نعالج إشكالية إلقاء الضوء على العلاقة بين الإطار النظري لنظرية الكم والتجارب الفعلية في فيزياء الطاقة العالية (HEP). للقيام بذلك ، نستعرض باختصار الهيكل النظري لنظرية نموذج المعيار الضعيف الكهربائي ثم تكميم اللاجرانجيان ، ثم نعيد النظر في الأفكار الكامنة وراء إعادة الهيكلة مع تطبيق مخطط إعادة الهيكلة على النظرية. تم إجراء التطبيق على عملية لبتونية باستخدام أدوات حساب رمزية تسمى حزم FeynCalc و FeynArts و FormCalc من برنامج Mathematica.

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Introduction

The standard model of elementary particles as we know it, is a result of many years of developments and progress of which a lot of physicists of the past 50 years or so have contributed to its now flourishing state. This model so far, is humanity's best attempt to understand how our universe works at an infinitesimal level. By now, it is a known fact that there are four fundamental forces of nature that governs the dynamics of the few basic structures of matter called fundamental particles. Three of these forces (electromagnetic, strong and weak forces) are embedded in the standard model where it has been tested against experimental measurements and passed with flying colors. This triumph over many experiment made the model known as a well-tested physical theory. This model can be summarized in the following figure

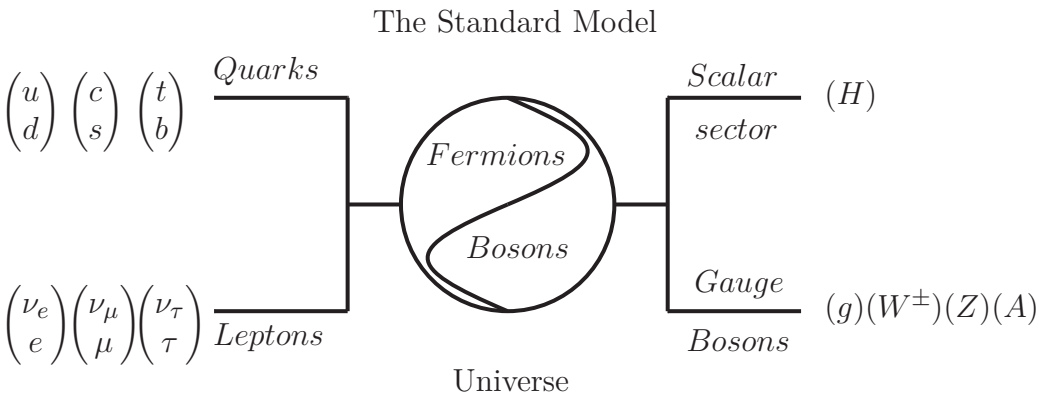


Figure 1: Elementary particles

The root to this piece of work was brought in 1960's by Weinberg[43], Salam[33] and Glashow[13] unifying the electromagnetic and the weak interactions into one. Before then the weak interaction was known to be carried only by two gauge bosons W^+ and W^- while the electromagnetic by the photon, the key idea that made the unification possible is the existence of an additional electrically neutral and massive particle called Z boson. This latter was first confirmed experimentally[19] in 1973 by observing a scattering event between neutrinos and electrons which wouldn't have happened if the Z boson doesn't exist.

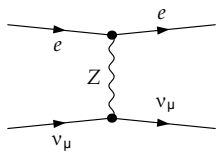


Figure 2: Tree level Feynman diagram of the process $e^-\nu_\mu \to e^-\nu_\mu$

In this thesis, we are interested in the process of $e\nu_\mu \to e\nu_\mu$ for its historical and theoretical importance to achieve the goal of clarifying the connection of the purly theoretical model of $\mathbb{EWSM}$ to real experimental process. Surely, the full treatment of this process was done before by a lot of physicists [34, 5, 22, 44]. We are interested in performing its tree as well as its one-loop radiative corrections calculations only by the modern techniques utilizing symbolic and numerical packages for Mathematica software, FeynArts and FeynCalc [7] for symbolic manipulations, and FormCalc [18] for numerical result. The former is fully automatic compared to the other two. For the radiative corrections, we need to eliminate the divergences that appears from loop diagrams. The procedure of which does the job is renormalization. For a brief historical notes, ultraviolet divergences were first stumbled upon in the 1930's in quantum electrodynamics (QED) when physicists tried to include the contributions of loop diagrams to get a more precise results than the tree level. The absurdity and persistence of these infinities made many physicists believe that there must be something wrong in the fundamental principles of physics and they had to abandon QED, until the late 1940's when Feynman[12], Schwinger[36], Tomonaga[40] and many other physicists proposed a solution that yields to theoretical results which agrees with experiments to a great deal of accuracy in QED. This solution is none other than renormalization. Its basic idea is absorbing the divergences by redefining the physical quantities. At first these redefinition seemed to have no justifications and that they are nothing but an ad-hoc solution just to get rid of the nuisance of infinities. This method however was only justifiable by experiments. This was the case till around 1955, the concept of renormalization was embraced as an important and self-consistent mechanism of physics thanks to the work of Lehmann, Symanzik and Zimmerman[25] on their reduction formalism. Back to the renormalization constants of electroweak theory, the renormalization constant are fixed by the on mass shell conditions ($p^2 = m^2$). Hence the on-shell scheme, also known as the physical scheme. In summary, the plan of this thesis to reach our objective is as follows

- **Chapter 1**
  We briefly review the structure of the electroweak standard model from gauge principle, and introduce masses of particles with the Higgs mechanism. Then we derive the LSZ reduction formula for real scalar fields for the purpose of illustrating the idea behind redefining the physical quantities and introducing the notion of renormalization constants. We finish this chapter by listing some selected Feynman rules for EW theory.

- **Chapter 2**
  We reveal the source of the loop diagrams from the perturbation series of Green's

functions, discuss the different regularization schemes that are used to render the divergent integrals convergent. Then briefly review some of the most commonly used renormalization schemes to fix the counter-terms. Next we apply the on-shell scheme for $\mathbb{EWSM}$. So we finish the second chapter by providing a FeynCalc (and FeynArts) codes for computing some selected $\mathbb{EWSM}$ renormalization constants as an illustrative example.

- **Chapter 3**
  This chapter consists of applying the established knowledge of the first two chapters to a real scattering unpolarized process of $e\nu_\mu \to e\nu_\mu$ by first computing the tree level approximation of the total cross section through the use of FeynCalc and FeynArts. Then include the one-loop radiative corrections with the help of the automatic package FormCalc after knowing and explaining the necessary steps that must be done to obtain the finite and desired result. Idem a result free from any UV or IR divergences.

Finally we draw our conclusion. Some computational details, proofs and examples are deferred as appendices

> "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve"
>
> in the words of Eugene Wigner

Chapter 1

Electroweak standard model theory

The standard model theory of the electroweak interaction is a Yang-Mills gauge theory which means that it is based on a gauge invariance under a certain Lie group of local transformations, in this case it is the $SU_{I_3}(2) \times U_Y(1)$ group[1]. To make the theory realistic the fields must acquire mass and that is when the role of the Higgs mechanism comes into place. In this chapter we will be addressing these notions of the theory along with the Lehmann-Symanzik-Zimmermann reduction formula[25] (LSZ for short) and its consequences. At the end of this chapter, we shall provide some selected Feynman rules of the theory.

1.1 Lagrangian of the electroweak *SM* theory

To construct the lagrangian that describes the electroweak dynamics[1], we start with the gauge principle and then perform the canonical quantization in covariant gauge. By then the constructed lagrangian will be of a massless particles. To adjust this issue we will be introducing the Higgs mechanism and spontaneous electroweak symmetry breaking.

1.1.1 Gauge principle and quantization

It has been known for a long time[11] that the weak theory is describable by a current-current lagrangian of the form:

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} j_\mu^+ j^\mu. \tag{1.1.1}$$

With j_μ being the weak charged current. To keep things simple we shall restrict our argument to one generation of leptons only. For the full generalization incorporating quarks and multi-generations it is a straightforward inclusion process. In this case of inters the current takes the form:

$$j_\mu = \overline{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu, \tag{1.1.2}$$

[1]Note that the full standard model is based on the $SU_C(3) \times SU_{I_3}(2) \times U_Y(1)$ group symmetry but in this work we restrict our selves to only the $SU_{I_3}(2) \times U_Y(1)$ part which correspond to the electroweak interaction (no QCD)

where ψ_e (ψ_ν) represent the electron (electron-neutrino) field. By other side the electromagnetic interaction of charged leptons is expressed by:

$$\mathcal{L}_{EM} = e j_\mu^{em} A^\mu, \tag{1.1.3}$$

With A^μ being the photon field and j_μ^{em} the electromagnetic current of the form:

$$j_\mu^{em} = \overline{\psi}_e \gamma_\mu \psi_e. \tag{1.1.4}$$

Accordingly the weak and electromagnetic interactions are expressible by their respective currents j_μ and j_μ^{em}.
By readjusting the composition of the currents j_μ and j_μ^{em}, one can find that they can be reformulated so they would have an explicit $SU_{I_3}(2) \times U_Y(1)$ transformations properties. Actually we would have:

$$j_\mu = 2(J_\mu^1 - iJ_\mu^2), \tag{1.1.5}$$

$$j_\mu^{em} = J_\mu^0 - J_\mu^3, \tag{1.1.6}$$

Where J_μ^0 and J_μ^n $(n = 1,2,3)$ are given by:

$$J_\mu^0 = \overline{\psi_e} \gamma_\mu \frac{1+\gamma_5}{2} \psi_e + \frac{1}{2} \bar{\psi} \gamma_\mu \frac{1-\gamma_5}{2} \psi, \tag{1.1.7}$$

$$J_\mu^n = \overline{\psi} \gamma_\mu \frac{1-\gamma_5}{2} \frac{\tau^n}{2} \psi, \tag{1.1.8}$$

With $\psi = (\psi_\nu, \psi_e)$ and $\tau^n/2$ the $SU(2)$ generators in the fundamental representation. The currents J_μ^0 and J_μ^n may be associated with the above mentioned $SU_{I_3}(2) \times U_Y(1)$ symmetry. Left-handed fermions $\frac{(1-\gamma_5)}{2}\psi$ are classified in doublet representation under $SU(2)$ symmetry. In another hand right-handed fermions $\frac{(1+\gamma_5)}{2}\psi$ are classified in singlets representation. This remark suggests that weak and electromagnetic interactions upholds the $SU_{I_3}(2) \times U_Y(1)$ symmetry. From now on we shall take the $SU_{I_3}(2) \times U_Y(1)$ symmetry as the main local gauge symmetry of which we will be building the lagrangian upon.

Denoting the $su(2) \times u(1)$ algebra elements as T^n and Y we than can write:

$$\begin{aligned} [T^l, T^m] &= i\epsilon^{lmn} T^n, \\ [T^n, Y] &= 0. \end{aligned} \tag{1.1.9}$$

Here ε^{lmn} is the Levi Civita tensor with $\varepsilon^{123} = +1$ ($\varepsilon^{213} = -1$), and we are making use of *Einstein* summation convention of which repeated indices are summed over. To combine the two equation in 1.1.9 into one, we define Y as:

$$T^0 \equiv Y/2. \tag{1.1.10}$$

So that 1.1.9 becomes:

$$[T^a, T^b] = i f^{abc} T^c. \tag{1.1.11}$$

Where her $f^{0bc} = 0$ and $f^{abc} = \varepsilon^{abc}$ $(a,b,c = 1,2,3)$. The Y eigenvalues are known as weak hypercharge and their values are determined by the relation $Q = T^3 + Y/2$ that often referred to as *Gell-Mann-Nishijima* relation, where $\hat{Q}$ is the electric charge operator

and $\hat{T}^3$ being the isospin with eigenvalues of $+1/2$ $(-1/2)$ for $\psi_{\nu L}$ (ψ_{eL}) and 0 for any right-handed fermion.

Since the lepton fields of e and ν_e are taking with respect to $SU_{I_3}(2)$ to be a doublet in left handed case and a singlet in the right handed case we adopt from now on the following notation:

$$R = \psi_{eR}\,, \quad L = \begin{pmatrix} \psi_{\nu L} \\ \psi_{eL} \end{pmatrix}, \tag{1.1.12}$$

With $\psi_{R,L} = \frac{1\pm\gamma_5}{2}\psi$ being the left-handed and right-handed parts of field ψ.

Now we construct the classical lagrangian based on gauge principle which dictates that the lagrangian must be invariant under the following locale field transformations:

$$\delta\psi(x) = ig^a\theta^a(x)T^a\psi(x), \tag{1.1.13}$$

$$\delta W^a_\mu = \partial_\mu\theta^a - g^a f^{abc}\theta^b W^c_\mu, \tag{1.1.14}$$

Where $\theta(x)^a$'s are the space-time dependent parameters of the gauge transformation, W^a_μ are the $SU_{I_3}(2)\times U_Y(1)$ gauge fields while g^a's are defined as:

$$g^a = \begin{cases} g & for \quad a = 1,2,3, \\ g' & for \quad a = 0. \end{cases} \tag{1.1.15}$$

With g (g') as the $SU(2)$ $(U(1))$ gauge coupling constant. One should bear in mind that these transformations are the infinitesimal versions while their full form is given by:

$$\psi(x) \to \psi'(x) = U\psi(x), \tag{1.1.16}$$

$$W^a_\mu T^a \to W'^a_\mu T^a = UW^a_\mu T^a U^\dagger - i(\partial_\mu U)U^\dagger, \tag{1.1.17}$$

Where $U = e^{ig^{(a)}\theta^a(x)T^a}$ is the transformation operator.

At this stage the classical lagrangian contains only two main parts, the gauge part which is the pure gauge fields lagrangian also known by the *Yang-Mills* part, and the fermionic part that describes the dynamics of the fermion fields:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F, \tag{1.1.18}$$

$$\mathcal{L}_G = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a, \tag{1.1.19}$$

$$\mathcal{L}_F = i\bar{L}\not{D}_L L + i\bar{R}\not{D}_R R. \tag{1.1.20}$$

Where we have used the following expression for:

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g^{(a)} f^{abc} W^b_\mu W^c_\nu, \tag{1.1.21}$$

And made use of the Feynman's slash notation $\not{D} = \gamma^\mu D_\mu$ with:

$$D_{L\mu} = \partial_\mu - ig^{(a)}T^a W^a_\mu, \tag{1.1.22}$$
$$D_{R\mu} = \partial_\mu - ig' T^0 W^0_\mu. \tag{1.1.23}$$

This form of the lagrangian is no where near what the processes obey due to the lack of fermions and bosons mass terms, because these terms spoils the gauge invariance of the lagrangian. For addressing this issue we shall make use of two very important ideas. Spontaneous symmetry breaking and Higgs mechanism. For the time being we will continue the progress through quantization and its consequences. In order to perform a covariant canonical quantization in the operator formalism[1, 23], we are required [41] to introduce a gauge fixing term, and by doing so the local gauge invariance of the lagrangian is no longer preserved. To restore this invariance latter it is necessary to add the well-known (by now) gauge compensating Faddeev-Popov[10] ghost term($\mathcal{L}_{FP} = \mathcal{L}(c, \bar{c})$ where $c(x)$ and $\bar{c}(x)$ are Grassman fields[41, 15]) so we would have:

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \tag{1.1.24}$$

Where in the gauge term we have:

$$\mathcal{L}_{GF} = B^a F^a_i \phi^i + \frac{\alpha}{2}(B^a)^2. \tag{1.1.25}$$

We have used the auxiliary field $B^a(x)$(This is referred to as the Lautrup-Nakanishi formalism[24, 28] [2]) that fulfills the Euler-Lagrange equation of motion:

$$F^a_i \phi^i + \alpha B^a = 0, \tag{1.1.26}$$

With $F^a_i \phi^i$ being the general gauge fixing condition having $F_i = (\partial_\mu, \nabla, \partial_0 ...)$ an operator applied to $\phi^i = W^a_\mu$ the gauge fields[3].
This lagrangian however is not gauge invariant under the classical gauge transformation but it is invariant under the quantum version of these transformation *i.e.*replacing the parameter $\theta^a(x)$ by $\tilde{\lambda} c^a(x)$ for the ordinary fields W^a_μ and $\psi(x)$, where $\tilde{\lambda}$ is a Grassman constant which anti-commutes with $c(x)$, $\bar{c}(x)$ and $\psi(x)$. Adding a transformation properties for $c(x)$, $\bar{c}(x)$ and $B(x)$ that was stated by Becchi, Rouet and Stora[2], $\mathcal{L}'$ is invariant under the following infinitesimal transformations:

$$\delta^{BRS}\psi(x) = i\tilde{\lambda} g^{(a)} c^a T^a \psi(x), \tag{1.1.27}$$
$$\delta^{BRS} W^a_\nu(x) = \tilde{\lambda} D^{ab}_\nu c^b(x), \tag{1.1.28}$$
$$\delta^{BRS}\bar{c}^a = \tilde{\lambda} B^a(x), \tag{1.1.29}$$
$$\delta^{BRS} c^a = -\frac{\tilde{\lambda}}{2} g^{(a)} f^{abc} c^b(x) c^c(x), \tag{1.1.30}$$
$$\delta^{BRS} B^a(x) = 0. \tag{1.1.31}$$

[2]this formalism is seen as a generalization of the Gupta-Bleuler formalism [17, 4]
[3]for example in the QED $F^a_i \phi^i = \partial_\mu A^\mu(x)$

The origin of Faddeev-Popov term was given by Faddeev and Popov using the Feynman path-integral method[10]. Now thanks to the requirement of the total lagrangian $\mathcal{L}$ be invariant under BRS transformation, we can now determine $\mathcal{L}_{FP}$ by:

$$\delta^{BRS}\mathcal{L}' = \delta^{BRS}\mathcal{L} + \delta^{BRS}\mathcal{L}_{GF} + \delta^{BRS}\mathcal{L}_{FP} = 0. \tag{1.1.32}$$

It is obvious that $\delta^{BRS}\mathcal{L} = 0$. Therefore we get:

$$\begin{aligned}
\delta^{BRS}\mathcal{L}' &= \delta^{BRS}\mathcal{L}_{GF} + \delta^{BRS}\mathcal{L}_{FP} \\
&= \tilde{\lambda} B^a \delta^b (F_i^a \phi^i) c^b + \delta^{BRS}\mathcal{L}_{FP} \\
&= \delta^{BRS}(\bar{c}^a)\delta^b (F_i^a \phi^i) c^b + \delta^{BRS}\mathcal{L}_{FP} \\
&= \delta^{BRS}(\bar{c}^a \delta^b (F_i^a \phi^i) c^b) - \bar{c}^a \delta^{BRS}(\delta^b (F_i^a \phi^i) c^b) + \delta^{BRS}\mathcal{L}_{FP}.
\end{aligned}$$

Where we used the following definitions for δ^a, D_μ^{ab} and δ^{BRS}:

$$\delta^a = \frac{\delta}{\tilde{\lambda}\delta c^a}, \tag{1.1.33}$$

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - g^{(a)} f^{abc} W_\mu^c, \tag{1.1.34}$$

$$\delta^{BRS} = \tilde{\lambda} c^a \delta^a. \tag{1.1.35}$$

And because of $\delta^{BRS}(\delta^b (F_i^a \phi^i) c^b) = 0$ (for more details on calculation see appendix). We identify $\mathcal{L}_{FP}$ as:

$$\mathcal{L}_{FP} = -\bar{c}^a \delta^b (F_i^a \phi^i) c^b. \tag{1.1.36}$$

Where in a covariant global gauge symmetry:

$$F_i^a \phi^i = \partial^\mu W_\mu^a. \tag{1.1.37}$$

Performing the integration by parts, we find:

$$\mathcal{L}_{FP} = (\partial^\mu \bar{c}^a) D_\mu^{ab} c^b. \tag{1.1.38}$$

Note that the lagrangian $\mathcal{L}'$ is the proper lagrangian from which the Feynman rules are inferred.

1.1.2 Higgs mechanism and mass terms

To make the theory realistic fermions and weak gauge bosons must acquire finite masses while preserving the gauge invariance, we need to apply a new mechanism, that is the Higgs mechanism[20, 3].

It is necessary to hypothesize the existence of a complex scalar doublet field in $SU_{I_3}(2)$ with $Y = 1$. It interact with the gauge fields is given through the covariant derivative:

$$|D_\mu \Phi|^2 = |(\partial_\mu - i g^a T^a W_\mu^a)\Phi|^2. \tag{1.1.39}$$

We also suppose that this field is subject to a potential of the form:

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4. \tag{1.1.40}$$

The lagrangian of the Higgs part than takes the form:

$$\mathcal{L}_H = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi). \tag{1.1.41}$$

The field Φ can be expressed as:

$$\Phi = \begin{pmatrix} \phi^\dagger(x) \\ \phi^o(x) \end{pmatrix}. \tag{1.1.42}$$

The potential $V(\Phi)$ presents an extremum determined by solving the following criterion:

$$\frac{\partial V(\Phi)}{\partial|\Phi|} = 2(\mu^2|\Phi| + 2\lambda|\Phi|^3) = 0. \tag{1.1.43}$$

This condition is fulfilled when $|\Phi|$ takes the values:

$$|\Phi_1| = 0 \qquad or \qquad |\Phi_2| = \sqrt{\frac{-\mu^2}{2\lambda}}. \tag{1.1.44}$$

We should restrict our self to the case $\mu^2 < 0$ and $\lambda > 0$, $|\Phi_1|$ is a maximum and $|\Phi_2|$ is a minimum, and this minimum is degenerate by a phase β. Hence, let's take:

$$\Phi_o =< 0|\Phi|0 >= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\beta}. \tag{1.1.45}$$

But in reality the field will choose one distinct direction since the electroweak symmetry has to be spontaneously broken.
The theory is then constructed such that the classical ground state of the scalar field satisfies.

$$|\Phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}. \tag{1.1.46}$$

In perturbation calculation we develop the fields around the ground state, to take into account quantum fluctuations with respect to the vacuum. Hence the following expression for Φ:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_2(x) + i\chi_1(x) \\ v + H(x) - \chi_3(x) \end{pmatrix} = \frac{1}{\sqrt{2}} e^{\frac{i\vec{\sigma}.\vec{\chi}}{v}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}. \tag{1.1.47}$$

With bearing in mind that χ_1, χ_2, χ_3 and H are small fluctuation and $\chi_i (i = 1, 2, 3)$ (called the Nambu-Goldstone fields) are non-physical owing to a specific gauge transformation called unitary gauge:

$$e^{-\frac{i\vec{\sigma}.\vec{\chi}}{v}}\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{1.1.48}$$

This gauge makes the Goldstones disappear except for the $H(x)$ field and that's what makes it physical Higgs boson with a mass:

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}v. \tag{1.1.49}$$

The ground state Φ_o breaks the $SU_{I_3}(2)$ and $U_Y(1)$ symmetries as a result of not being invariant under each group transformation separately, however if we would choose the transformation operator as:

$$U = e^{\frac{i(\sigma_3+Y)\theta}{2}}. \tag{1.1.50}$$

The ground state Φ_o would be invariant:

$$e^{\frac{i(\sigma_3+Y)\theta}{2}} \simeq (1 + \frac{i(\sigma_3+Y)\theta}{2})\Phi_o \tag{1.1.51}$$

$$= \begin{pmatrix} 1+i & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \Phi_o. \tag{1.1.52}$$

And since the electric charge is:

$$Q = T^3 + \frac{Y}{2}. \tag{1.1.53}$$

We see that $U(1)_{em}$ is the residual symmetry:

$$SU(2)_I \otimes U(1)_Y \to U(1)_{em}. \tag{1.1.54}$$

Through $\mathcal{L}_H$ the gauge boson mass matrix is generated as follows:

$$\mathcal{L}_{mass} = M_{ab} W^a_\mu W^{b\mu}, \tag{1.1.55}$$

$$M_{ab} = \begin{array}{c|cccc} & W_1 & W_2 & W_3 & W_0 \\ \hline W_1 & g^2 & 0 & 0 & 0 \\ W_2 & 0 & g^2 & 0 & 0 \\ W_3 & 0 & 0 & g^2 & gg' \\ W_0 & 0 & 0 & gg' & g'^2 \end{array} \times \frac{v^2}{8}.$$

Then by redefining the W^a_μ gauge fields as follows:

$$W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2} \tag{1.1.56}$$

$$Z_\mu = (gW^3_\mu - g'W^0_\mu)/\sqrt{g^2+g'^2} \tag{1.1.57}$$

$$A_\mu = (g'W^3_\mu + gW^0_\mu)/\sqrt{g^2+g'^2}, \tag{1.1.58}$$

Where $W^\pm_\mu$ and Z_μ represent the physical weak boson and A_μ the photon.
By doing so the mass matrix is diagonalized and generates the bosons masses:

$$\mathcal{L}_{mass} = M^2_W W^+_\mu W^{-\mu} + \frac{1}{2} M^2_Z Z_\mu Z^\mu + \frac{1}{2}(0)^2 A_\mu A^\mu. \tag{1.1.59}$$

With the below identifications[4][5].

$$M_W = \frac{1}{2}gv \qquad , \qquad M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,.\,v. \tag{1.1.60}$$

Now for the fermion fields mass terms it is provided by the Yukawa interaction:

$$\mathcal{L}_Y = -f\bar{L}\Phi R - f\bar{R}\Phi^\dagger L. \tag{1.1.61}$$

Which generates fermion mass terms:

$$\mathcal{L}_{mass} = -f(v/\sqrt{2}).\bar{\psi}_e\psi_e. \tag{1.1.62}$$

Here f is the Yukawa coupling parameter which has to be related to the mass of the electron as an experimental input.

Now we finally arrive at the full lagrangian from which we will consider later its renormalization:

$$\mathcal{L}' = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_{GF} + \mathcal{L}_{FP} + \mathcal{L}_H + \mathcal{L}_Y, \tag{1.1.63}$$

$$\begin{aligned}
\mathcal{L}_G &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a,\\
\mathcal{L}_F &= i\bar{L}\not{D}_L L + i\bar{R}\not{D}_R R,\\
\mathcal{L}_{GF} &= B^a\partial^\mu W^a_\mu + \frac{\alpha}{2}(B^a)^2,\\
\mathcal{L}_{FP} &= (\partial^\mu\bar{c}^a)D^{ab}_\mu c^b,\\
\mathcal{L}_H &= (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi),\\
\mathcal{L}_Y &= -f\bar{L}\Phi R - f\bar{R}\Phi^+ L.
\end{aligned}$$

It worthwhile to say here that the lagrangian $\mathcal{L}'$ is constructed by first assuming that all introduced fields are associated to isolated point like particles (fermions and bosons) and that the interactions are mediated by gauge bosons through covariant derivative. Also the masses of particles are acquired through Higgs mechanism and spontaneous breaking of electroweak symmetry.

1.2 LSZ reduction formalism

In particle physics the most efficient way to test a theory is by studying the outcome of collisions of particles. With that we can determine a lot of information on the physical

[4]for the electric charge would be identified by

$$e = gg'/\sqrt{g^2+g'^2}$$

[5]there is a relation between M_W and M_Z

$$\cos\theta_W = \frac{M_W}{M_Z} \quad , \quad \sin\theta_W = \sqrt{1-\frac{M_W^2}{M_Z^2}}$$

world and compare it with the predictions of the theory. The most important measurable quantity in these experiments is the cross section which is related to the scattering amplitude of the studied process. The latter encodes the information on how and what the interaction should look like. In order to be able to compare correctly the theoretical results with experimental ones, we need a relation that links the scattering amplitude with the fields of the theory. This relation is non other than the famous LSZ reduction formalism[25].
In this section which is strongly influenced by the Srednicki book[38] and the lecture from *Perimeter Institute of Theoretical Physics web site*[6] we shall derive this relation for the Klein-Gordon real field (scalar field) only. Than we will give the final result for other types of fields due to the similar reasoning in their derivation. Finally we will provide an explanation on why we need renormalization.

1.2.1 Derivation of LSZ formula for real scalar fields

The scattering amplitude are transition amplitudes of some number of particles from initial state to some other number of particles of final state:

$$< f|S|i >_H =< f, +\infty|i, -\infty >_s, \tag{1.2.1}$$

With $|i>$ and $|f>$ as the initial and final particle states respectively while S being the scattering matrix. the subscripts S (H) point out the Schrodinger (Heisenberg) picture. The S-matrix embeds all the information about how the initial states $|i>$ can shift into final states $|f>$. In a free theory, *i.e* no interaction, the S-matrix is just the identity operator, so we can write S as:

$$S = \mathbb{1} + i\mathcal{T}. \tag{1.2.2}$$

Where $\mathcal{T}$ is the transfer matrix, since we are interested in the interacting theory, we only need the transfer matrix part:

$$< f|S - \mathbb{1}|i >= i < f|\mathcal{T}|i > . \tag{1.2.3}$$

All we know about $\mathcal{T}$ is that it should vanish if the total 4-momentum is not conserved. Thus $\mathcal{T}$ is proportional to a momentum conserving delta function:

$$i\mathcal{T} = (2\pi)^4 \delta^4(\Sigma p) i\mathcal{M}. \tag{1.2.4}$$

And therefore, we are interested in computing the matrix element of $\mathcal{M}$.

$$\begin{aligned} < f|S - \mathbb{1}|i > &= (2\pi)^4 \delta^4(\Sigma p) i < f|\mathcal{M}|i > \\ &= (2\pi)^4 \delta^4(\Sigma p) i\mathcal{M}_{i\to f}. \end{aligned} \tag{1.2.5}$$

From now to the end of this section, we shall consider the case of $2 \to 2$, scattering process of scalar particles with identical masses[7].

[6] http://perimeterinstitute.ca/training/perimeter-scholars-international/lectures/2014/2015-psi-lectures
[7] for general case it is a straightforward generalization of the final result

In the free theory the action of a creation operator $a^\dagger(k)$ on the vacuum state results in 1 particle state:

$$|k>= a_k^\dagger(t)|0> . \tag{1.2.6}$$

In the Klein-Gordon theory, we have the following relation[8].

$$a_k^\dagger(t) = -i \int d^3x \, e^{-ikx} \overleftrightarrow{\partial_0} \phi(x). \tag{1.2.7}$$

While the action of the annihilation operator on $|0>$ gives:

$$a_k(t)|0>= 0. \tag{1.2.8}$$

The free theory vacuum stat has the unit norm:

$$<0|0>= 1. \tag{1.2.9}$$

Now we want to create a particle that is localized near k_1 in momentum space and near the origin ($x = 0$) in position space at time $t \to 0$. To do that we define the wave packet:

$$a_1^\dagger \equiv \int d^3k \, f_1(k) a_k^\dagger(t), \tag{1.2.10}$$

With $f_1(k)$ having the Gaussian form with some width σ.

$$f_1(k) \propto e^{-(k-k_1)^2/4\sigma^2}. \tag{1.2.11}$$

So now, we would have the following state:

$$a_1^\dagger(t) a_2^\dagger(t)|0>= |k_1, k_2> \quad , \quad k_1 \neq k_2. \tag{1.2.12}$$

It is a two particles state localized at the origin without interacting with each other since we are on the scope of the free theory.
If we time-evolve this state, the wave packet will spread out and the two particles are now localized far from the origin, this state can be identified as finale and initial states as shows:

$$\begin{aligned} |i> &= \lim_{t\to-\infty} a_1^\dagger(t) a_2^\dagger(t)|0>, \\ |f> &= \lim_{t\to+\infty} a_3^\dagger(t) a_4^\dagger(t)|0> . \end{aligned} \tag{1.2.13}$$

This is true for free theory, but what we are interested in is the interacting theory where all the interesting stuff happens. In order for us to switch to the interacting theory we have to make some assumptions to drive the desired formula, these assumptions at first hand may seem to be unintuitive but we shall proceed with them, and once we derive the formula we will return and discuss under what circumstances these assumptions are valid.

[8] where $\overleftrightarrow{\partial_0}$ is defined by:

$$A \overleftrightarrow{\partial_0} B = A(\partial_0 B) - (\partial_0 A) B$$

The first thing occurs when we are considering the interacting theory is that the vacuum changes due to the change of the Hamiltonian when taking into account the interaction, the ground state changes, so we introduce the new vacuum state:

$$|0> \longrightarrow |\Omega> . \tag{1.2.14}$$

We assume that $a_1^\dagger(\pm\infty)|\Omega>$ are still one particle states, it is an assumption by reason of in interacting theory the number of particles is not conserved so we could have a transition between a one particle state and a multi particle state thus we assume that in the far future or past the particles are far from each other. So we could talk about one particle state. With this assumption in mind the initial and final states can be expressed as:

$$\begin{aligned} |i> &= a_1^\dagger(-\infty)a_2^\dagger(-\infty)|\Omega>, \\ |f> &= a_3^\dagger(+\infty)a_4^\dagger(+\infty)|\Omega> . \end{aligned} \tag{1.2.15}$$

Our aim is to compute the scattering amplitude for the $2 \to 2$ scattering process $< f|S - \mathbb{1}|i >$, for that matter we will make use of the following relation, it is the key identity to derive the desired result:

$$\begin{aligned} a_1^\dagger(+\infty) - a_1^\dagger(-\infty) &= -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{ikx}(\partial^2 + m^2)\phi(x) \\ &\equiv I_1^\dagger \qquad \text{(short hand notation for simplicity).} \end{aligned} \tag{1.2.16}$$

At first glance, we can see the Klein-Gordon operator acting on the field, this would give zero for a free theory but in our case (interacting theory) it is non-zero $(\partial^2 + m^2)\phi = J$. For the full proof of this identity please see appendix.
In the proof of this relation we also made the assumption of the relationship between $a_k^\dagger(t)$ and $\phi(x)$ is the same in the interaction theory as in the free theory:

$$a_k^\dagger(t) = -i \int d^3x e^{ixk} \overleftrightarrow{\partial_0} \phi(x). \tag{1.2.17}$$

What is going to change is the time dependence of the operators.
In the scattering amplitude expression we don't only have the creation operators, but we also have an annihilation operators, for that we are in need for a similar relation for the annihilation operators, to do so we only need to calculate the complex conjugate of 1.2.16, we get:

$$\begin{aligned} a_1(+\infty) - a_1(-\infty) &= i \int d^3k\, f_1(\vec{x}) \int d^4x e^{-ikx}(\partial^2 + m^2)\phi(x) \\ &\equiv I_1. \end{aligned} \tag{1.2.18}$$

Now we are ready to compute our scattering amplitude:

$$< f|S - \mathbb{1}|i > = < \Omega|a_4(+\infty)a_3(+\infty)a_1^\dagger(-\infty)a_2^\dagger(-\infty)|\Omega> . \tag{1.2.19}$$

We notice that the creation and annihilation operators are already time ordered, so we can introduce the time ordering symbol T, where its function is to move the latest operators

to the left:

$$< f|S - \mathbb{1}|i >=< \Omega|\mathrm{T}(a_4(+\infty)a_3(+\infty)a_1^\dagger(-\infty)a_2^\dagger(-\infty))|\Omega > . \tag{1.2.20}$$

At this stage we use 1.2.16 and its complex conjugate:

$$a_1^\dagger(-\infty) = a_1^\dagger(+\infty) - I_1^\dagger \tag{1.2.21}$$
$$a_1(+\infty) = I_1 + a_1(-\infty), \tag{1.2.22}$$

We get:

$$\begin{aligned} < f|S - \mathbb{1}|i > &= < \Omega|\mathrm{T}(a_4(+\infty)a_3(+\infty)a_1^\dagger(-\infty)a_2^\dagger(-\infty))|\Omega > \\ &= < \Omega|\mathrm{T}((I_4 + a_4(-\infty))(I_3 + a_3(-\infty))(a_1^\dagger(+\infty) - I_1^\dagger)(a_2^\dagger(+\infty) - I_2^\dagger))|\Omega > \\ &= < \Omega|\mathrm{T}(a_4(-\infty)a_3(-\infty)a_1^\dagger(+\infty)a_2^\dagger(+\infty))|\Omega > \\ &\quad + \cdots + < \Omega|\mathrm{T}(I_4 I_3 I_1^\dagger I_2^\dagger)|\Omega > . \end{aligned} \tag{1.2.23}$$

Due to the time ordering operator, the annihilation operators will act on $|\Omega >$ the vacuum state and gives zero as a consequence all the terms vanish except for the last one.
Now the wave packet no longer play an important role we take $f_1(\overrightarrow{k}) \to \delta^3(\overrightarrow{k} - \overrightarrow{k_1})$ (we can do that by taking the limit $\sigma \to 0$ in the equation 1.2.11) then I^+ well as I, becomes:

$$I_1^\dagger = -i \int d^4x_1 e^{-ik_1x_1}(\partial_1^2 + m^2)\phi(x) \quad ; \quad \partial_{1\mu} = \frac{\partial}{\partial x_1^\mu} \tag{1.2.24}$$
$$I_1 = i \int d^4x_1 e^{ik_1x_1}(\partial_1^2 + m^2)\phi(x). \tag{1.2.25}$$

Now we finally arrive at the LSZ reduction formula:

$$\begin{aligned} < f|S - \mathbb{1}|i > =& i^{2+2} \prod_{j=1}^{2+2} \left(\int d^4x_j e^{-i\epsilon k_j x_j}(\partial_j^2 + m^2) \right) \\ &\times < \Omega|\mathrm{T}(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4))|\Omega >, \end{aligned} \tag{1.2.26}$$

With,

$$\epsilon = \begin{cases} +1 & for\ final\ state \\ -1 & for\ initial\ state \end{cases} \tag{1.2.27}$$

Now that we have derived the formula for the real Klein-Gordon field we shall provide the corresponding result for the spinor type of fields because in real scattering processes, matter particles are the most types of scattering dealt with in experimental research in particle physics. So for the spinors $\Psi_{\alpha_i}(x_i)$ with spin $s_i \quad (i = 1, 2, 3, 4)$ we have the relation

as follows[9].

$$
\begin{aligned}
< f|S-\mathbb{1}|i > = i^{2+2} \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \\
&\times e^{-ik_3x_3}[\bar{u}_{s_3}(p_3)(-i\,\partial\!\!\!/_3 + m)]_{\alpha_3} \\
&\times e^{-ik_4x_4}[\bar{u}_{s_4}(p_4)(-i\,\partial\!\!\!/_4 + m)]_{\alpha_4} \\
&\times < \Omega|\mathrm{T}(\Psi_{\alpha_4}(x_4)\Psi_{\alpha_3}(x_3)\overline{\Psi}_{\alpha_1}(x_1)\overline{\Psi}_{\alpha_2}(x_2))|\Omega > \\
&\times [(i\overleftarrow{\partial\!\!\!/}_1 + m)u_{s_1}(p_1)]_{\alpha_1} e^{-ik_1x_1} \\
&\times [(i\overleftarrow{\partial\!\!\!/}_2 + m)u_{s_2}(p_2)]_{\alpha_2} e^{-ik_2x_2}.
\end{aligned}
$$

Now that we have reached our goal on deriving the relevant formula, we get back to the main assumption that we have began with. We shall examine whether the state $a_1^\dagger(\pm\infty)|\Omega >$ is indeed a one particle state in the interacting theory, and since we assumed that the relation between the creation (or annihilation) operator and the fields is the same in the interacting theory as in the free theory. Then $\phi(x)|\Omega >$ is also a one particle state (if this is the case). To check that this is the case we want the one-particle state to be orthogonal with the vacuum state (no overlap between the two states), that means it should verify:

$$
\begin{aligned}
< \Omega|\phi(x)|\Omega > &=< \Omega|e^{ipx}\phi(0)e^{-ipx}|\Omega > \\
&=< \Omega|\phi(0)|\Omega > \\
&\stackrel{?}{=} 0.
\end{aligned}
$$

Where in the first equality we expressed $\phi(x)$ by the space-time translation operator acting on $\phi(0)$, and in the second line we use the property of the vacuum state that is if we space-time translate a vacuum state it would still be a vacuum state. In general $< \Omega|\phi(0)|\Omega >$ have no reason be zero because in interacting theory there could be some overlap between one-particle state and vacuum state.
Let's say that it have some constant value v:

$$< \Omega|\phi(0)|\Omega >= v. \tag{1.2.28}$$

This is easily fixed all we have to do is to shift our field and redefine it as:

$$\tilde{\phi}(0) = \phi(0) - v. \tag{1.2.29}$$

By doing so our main assumption is verified:

$$< \Omega|\tilde{\phi}(0)|\Omega >= 0. \tag{1.2.30}$$

Therefore if we want to use the LSZ formula we need to shift our fields. This condition alone is not ebough, we need for the one particle state to be very well normalized, that is:

$$< k|\phi(x)|\Omega > \stackrel{?}{=} e^{ikx}, \tag{1.2.31}$$

[9]we also consider the $2 \to 2$ scattering process of identical masses

Where $|k>$ being a one particle stale with 4-momentum k. This is true for the case of a free theory. We want it to be true also for the interacting theory. In general it is not true, without specifying the theory we can see that it gives:

$$\begin{aligned} <k|\phi(x)|\Omega> &=<k|e^{i\hat{p}x}\phi(0)e^{-i\hat{p}x}|\Omega> \\ &= e^{ikx}\underbrace{<k|\phi(0)|\Omega>}_{extra\ factor}. \end{aligned} \tag{1.2.32}$$

Again we have written the field $\phi(x)$ as a space-time transformation of $\phi(0)$ (it is more like a relativistic generalization of switching between Heisenberg and Schrodinger picture)[10] and use the action of $e^{i\hat{p}x}$ on $|k>$ would give the eigenvalue of the operator $\hat{p}$ which is k. This extra factor is Lorentz invariant number so it is just a constant that can depend on some parameters of the theory[11], we would like for it to be equal to one if we want in interacting field theory $a_1^\dagger(\pm\infty)$ to create a correctly normalized one-particle state. Again we can fix this by rescaling (renormalizing) our field and redefining it as:

$$\tilde{\phi}(x) = Z^{1/2}\phi(x), \tag{1.2.33}$$

Where we identify:

$$Z^{-1/2} \equiv <k|\phi(0)|\Omega>. \tag{1.2.34}$$

Now that we have shifted and rescaled our field they obey, by construction:

$$<\Omega|\tilde{\phi}(x)|\Omega> = 0, \tag{1.2.35}$$

$$<k|\tilde{\phi}(x)|\Omega> = e^{ikx}. \tag{1.2.36}$$

And by that the fields in the LSZ formula are these new defined fields.
Doing that alone is not enough because we would still have to be able to distinguish a one-particle state from a multi-particle state. Going with the naive approach, we would get:

$$\begin{aligned} <p,n|\phi(x)|\Omega> &=<p,n|e^{i\hat{k}x}\phi(0)e^{-i\hat{k}x}|\Omega> \\ &= e^{ipx}\underbrace{<p,n|\phi(0)|\Omega>}_{\neq 0}. \end{aligned} \tag{1.2.37}$$

Where the multi-particle state $<p,n|$ is represented by the total 4-momentum p and for any other (discrete or continue) parameter the label n, again (in general) this has no reason to be zero, unlike the previous times this is not easy due to the many different states, it is not clear what can be done (like shifting or rescaling) to make it vanish. So we shall give this extra-quantity a a specify name:

$$A_n(p) =<p,n|\phi(0)|\Omega>. \tag{1.2.38}$$

In fact making $A_n(p) = 0$ is a too strong of a condition. What we really want is to

[10] Since in relativistic theory we deal with space and time as the same entity

[11] when we look at it we may think it depends on k but since it is Lorentz invariant it can only be a function of k^2 which is equals to a constant m^2

distinguish a one-particle state in the far past and the far future that is:

$$< p, n|a_1^\dagger(\pm\infty)|\Omega > \stackrel{?}{=} 0. \tag{1.2.39}$$

Also we need to test the overlap against normalizable state, in view of the fact that non-normalizable states brings on a lot of conflict as a consequence we define the normalizable multi-particle state as:

$$|\psi >= \sum_n \int d^3p\psi_n|p, n > . \tag{1.2.40}$$

And the symbol $\sum_n$ is just the sum or integral if the parameter is continues over all of the parameters of the state with the $\phi_n(\vec{p})$'s as wave packets for the total momentum $\vec{p}$. Now what we really need to examine is:

$$< \psi|a_1^\dagger(\pm\infty)|\Omega > \stackrel{?}{=} 0. \tag{1.2.41}$$

At first, we refrain from taking the limit $t \to \pm\infty$ and use 1.2.17 for the creation operator, we get:

$$\begin{aligned}
< \psi|a_1^\dagger(t)|\Omega > &= -i\sum_n \int d^3p\psi_n^*(\vec{p}) \int d^3k f_1(\vec{k}) \int d^3x e^{-ikx} \overleftrightarrow{\partial_0} < p, n|\phi|\Omega > \\
&= -i\sum_n \int d^3p\psi_n^*(\vec{p}) \int d^3k f_1(\vec{k}) \int d^3x (e^{-ikx} \overleftrightarrow{\partial_0} e^{ipx}) A_n(p) \\
&= -i\sum_n \int d^3p\psi_n^*(\vec{p}) \int d^3k f_1(\vec{k}) \int d^3x (i(p_0 + k_0)e^{i(p-k)x}) A_n(p) \\
&= \sum_n \int d^3p\psi_n^*(\vec{p}) \int d^3k f_1(\vec{k})(p_0 + k_0)(2\pi)^3\delta^3(\vec{p} - \vec{k}) A_n(p) e^{i(p_0-k_0)t} \\
&= \sum_n \int d^3p\psi_n^*(\vec{p}) f_1(\vec{p})(p_0 + k_0)(2\pi)^3 A_n(p) e^{i(p_0-k_0)t}.
\end{aligned} \tag{1.2.42}$$

Where in the first line we use the $|\psi >$ and $a_1^\dagger$ definitions. In the second line we replaced $< p, n|\phi(x)|\Omega >$ by $e^{ipx}A_n(p)$ and calculated the $\overleftrightarrow{\partial_0}$ on the third line. And we did the x integration in the forth line. Finally, we arrived at an expression that looks like the following form:

$$< \psi|a_1^\dagger(t)|\Omega >= \int d^3p f(p) e^{i(p_0-k0)t}, \tag{1.2.43}$$

Where[12] $p_0 = (\vec{p}^2 + M^2)^{1/2} > k_0 = (\vec{p}^2 + m^2)^{1/2}$. Now we take the limit $t \to \pm\infty$ and according to Riemann-Lebesgue Lemma[13] this integral goes to zero. As a result there is

[12] because of $M \geq 2m > m$ as the mass of the multi-particle state since: $p^2 = M^2$ and $k^2 = m^2$

[13] If $f(x)$ is integrable on $[-\pi, \pi]$, then:

$$\lim_{t\to\infty} \int_{-\pi}^{\pi} f(x)\sin(tx)dx \to 0,$$

And

$$\lim_{t\to\infty} \int_{-\pi}^{\pi} f(x)\cos(tx)dx \to 0.$$

no overlap between the two states just as desired to be:

$$< \psi | a_1^\dagger(t) | \Omega >= 0. \tag{1.2.44}$$

Physically this means that as $t \to \pm\infty$ the spread out of the one-particle wave packet differ from the spread out of the multi-particle state. So at the far future of the far past we can track our particle of study without the contribution of the multi-particle state.

Now that we have examined our assumptions (which are all about what is a one-particle state is an interacting theory) and made the fields satisfying them by shifting and rescaling, as a consequence of that, the lagrangian must also change. To give an example for that let us consider a simple lagrangian of a scalar real ϕ^3 theory of the form:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 + \frac{1}{3!}g_0\phi^3. \tag{1.2.45}$$

In order to apply the LSZ reduction formula, for processes described by this theory, the field must be redefined (to satisfy the assumption that was made) and the parameter (mass and coupling) must be adjusted on the observed ones at asymptotic states. By doing so the lagrangian will change as well giving rise to new coefficients and a new term as follows:

$$\Rightarrow \begin{cases} \mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 + \frac{1}{3!}g_0\phi^3 \\ \phi \to \bar{\phi} - v \\ \mathcal{L} = \frac{1}{2}\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} - \frac{1}{2}m_0^2\bar{\phi}^2 + \frac{1}{3!}g_0\bar{\phi}^3 + Y(v)\bar{\phi} \end{cases}, \tag{1.2.46}$$

$$\tilde{\phi} = Z_\phi^{1/2}\bar{\phi} \quad ; \quad m_0^2 = Z_m^2 m^2 \quad ; \quad g_0 = Z_g g,$$

$$\mathcal{L} = \frac{1}{2}Z_\phi\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}Z_m^2 m^2 Z_\phi\tilde{\phi}^2 + \frac{1}{3!}Z_g g Z_\phi^{3/2}\tilde{\phi}^3 + Y(v)Z_\phi^{1/2}\tilde{\phi}, \tag{1.2.47}$$

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\tilde{\phi}^2 + \frac{1}{3!}g\tilde{\phi}^3 \\ &+ \frac{1}{2}(Z_\phi - 1)\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}(Z_m^2 Z_\phi - 1)m^2\tilde{\phi}^2 + \frac{1}{3!}g(Z_g Z_\phi^{3/2} - 1)\tilde{\phi}^3 \\ &+ Y(v)Z_\phi^{1/2}\tilde{\phi}. \end{aligned} \tag{1.2.48}$$

Let us define:

$$\delta Z_\phi = Z_\phi - 1 \quad ; \quad \delta m^2 = (Z_m^2 Z_\phi - 1)m^2 \quad ; \quad \delta g = g(Z_g Z_\phi^{3/2} - 1) \quad ; \quad \tilde{Y} = Y Z_\phi^{1/2},$$

Hence:

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\tilde{\phi}^2 + \frac{1}{3!}g\tilde{\phi}^3 \\ &+ \frac{1}{2}\delta Z_\phi\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}\delta m^2\tilde{\phi}^2 + \frac{1}{3!}\delta g\tilde{\phi}^3 + \tilde{Y}\tilde{\phi}, \end{aligned} \tag{1.2.49}$$

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT}, \tag{1.2.50}$$

where,

$$\mathcal{L}_R = \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}m^2\tilde{\phi}^2 + \frac{1}{3!}g\tilde{\phi}^3 \tag{1.2.51}$$

$$\mathcal{L}_{CT} = \frac{1}{2}\delta Z_\phi\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{2}\delta m^2\tilde{\phi}^2 + \frac{1}{3!}\delta g\tilde{\phi}^3 + \tilde{Y}\tilde{\phi}. \tag{1.2.52}$$

We notice that $\mathcal{L}$ and $\mathcal{L}_R$ have the same structure, therefore Feynman rules of $\mathcal{L}_R$ would be similar to that of $\mathcal{L}$ with the only change is the parameter of mass (m_0, g_0 for $\mathcal{L}$ and m, g for $\mathcal{L}_R$).

Therefore if we would like to consider studying a real process (interacting theory) we are required to renormalize.

In the next chapter we shall delve deeper into the details of renormalization. The term $\tilde{Y}\tilde{\phi}$ which is the tadpole term is only present in the counter term part, where the bare lagrangian1.2.45 which is the original lagrangian of the theory, does not contain such term, therefore a condition is imposed to eliminate it.

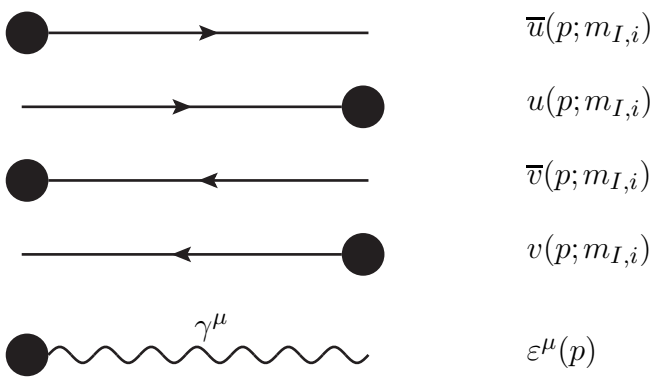
$$Y(v) = 0. \tag{1.2.53}$$

1.3 Selected tree level Feynman rules of the electroweak *SM* theory

In this section and for future use in dealing with scattering process (see chapter 3), we introduce some selected tree level Feynman rules and their corresponding Feynman diagrams[1][35] in the framework of $\mathbb{EWSM}$ theory.

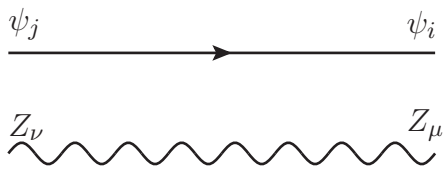
External lines

We begin by presenting the external lines[35]:



Propagators

Some of the propagators of the theory[1].

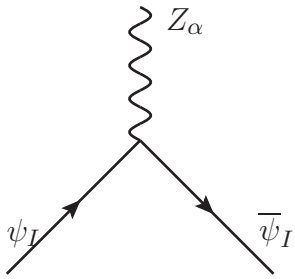


$$\frac{-i}{m-k-i\epsilon} = \frac{m+k}{m^2-k^2-i\varepsilon}$$

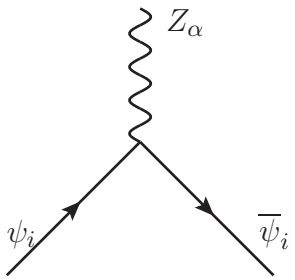
$$\frac{-i}{k^2+i\varepsilon-M_Z^2}$$

Vertices

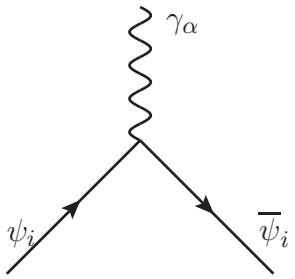
The vertices between the Z-boson and fermionic particles[1] (ψ_I for up fermion flavor and ψ_i for down fermion flavor).



$$\frac{ieM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_\alpha(\tfrac{1}{2}-2Q_I\tfrac{M_Z^2-M_W^2}{M_Z^2}-\tfrac{1}{2}\gamma_5)$$



$$\frac{ieM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_\alpha(-\tfrac{1}{2}-2Q_i\tfrac{M_Z^2-M_W^2}{M_Z^2}+\tfrac{1}{2}\gamma_5)$$



$$ieQ_i\gamma_\alpha$$

Chapter 2

Electroweak On-Shell Renormalization

The development of experimental physics lead to an unprecedented precision in measurement, putting us in need for an equivalent advancement in the theoretical framework. The leading order (LO) in perturbation theory is not enough any more, we need to go a step further and include the next to leading order (NLO) perturbation terms. When we try and do the calculation of the NLO, we encounter an unpleasant infinities. The procedure of which we get rid of these infinities is called renormalization. In this chapter we shall see the origin of these infinities and give a brief introduction of different renormalization schemes and the philosophy behind each one. Then we apply the on-shell scheme to $\mathbb{EWSM}$[1]. Finally, we provide the way to calculate the $\mathbb{EWSM}$ renormalization constants with explicitly giving some results[2].

2.1 Renormalization and regularization schemes

2.1.1 The origin of infinities and regularization schemes

As we have said before, scattering processes are the most commonly used method to probe the physical world, and that gets us to the need of evaluating the n-point functions (Green functions). For a scalar interacting theory we can write:

$$G^{(n)}(x_1, \cdots, x_n) =< \Omega|T(\phi(x_1)\cdots\phi(x_n))|\Omega > \tag{2.1.1}$$

Where ϕ's are interacting fields on which we do not have a detailed knowledge.
It would be better if the Green functions were expressed in terms of free vacuum states and free fields that we understand better and know how to manipulate. Fortunately there is such a relation (for details on its derivation see[41]) and it is as follows[3]:

$$G^{(n)}(x_1, \cdots, x_n) = \frac{< 0|T(\phi_0(x_1)\cdots\phi_0(x_n)e^{-i\int_{-\infty}^{+\infty} d\tau H_{int}(\tau)})|0 >}{< 0|Te^{-i\int_{-\infty}^{+\infty} d\tau H_{int}(\tau)}|0 >} \tag{2.1.2}$$

Where $\phi_0(x_i)$ $(i = 1\cdots n)$ are the asymptotic fields satisfying the free field equation $(\partial^2 + m^2)\phi_0(x_i) = 0$ and H_{int} is the Hamiltonian interaction part expressed in term of

[1] the renormalizability of $\mathbb{EWSM}$ was proved by t'Hooft[21] in 1971
[2] From now on, we adopt the definition of the physical mass as the isolated pole of the exact propagator
[3] we are considering a real scalar fields

asymptotic fields and their canonical conjugate. In the case of the 2-point function,

$$G^{(2)}(x_1,x_2) = \frac{<0|T(\phi_0(x_1)\phi_0(x_2)e^{-i\int_{-\infty}^{+\infty} d\tau H_{int}(\tau)})|0>}{<0|Te^{-i\int_{-\infty}^{+\infty} d\tau H_{int}(\tau)}|0>}. \tag{2.1.3}$$

By inserting the Taylor expression of the exponential function, we obtain the perturbative series of the 2-point function (this holds for any n-point function):

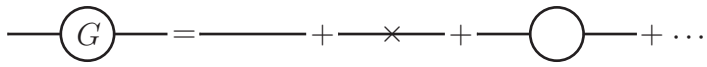
$$G^{(2)}(x_1,x_2) = \mathcal{N}\sum_{k=1}^{\infty}\frac{(-i)^k}{k!}\int d^4y_1\cdots d^4y_k <0|T(\phi_0(x_1)\phi_0(x_2)H_{int}(y_1)\cdots H_{int}(y_k))|0> \tag{2.1.4}$$

Where the normalization factor $\mathcal{N}$ describes the unobservable vacuum fluctuations, therefore it can simply be ignored as long as we don't take into account the disconnected vacuum bubles when evaluating 2.1.4:

$$\mathcal{N} =<0|Te^{-i\int d\tau H_{int}(\tau)}|0>$$

$= 1+$ $+\cdots$ (2.1.5)

And the graphical representation of 2.1.4 is given by:



(2.1.6)

Figure 2.1: Perturbation series of 2 point Green function

The first thing, we can see is that the zeroth order of the perturbation series is non other than the tree level Feynman propagator (the free propagator):

$$G^{(2)}(x_1,x_2) \simeq \int d^4p e^{i(x_1-x_2)p}\frac{1}{p^2-m^2+i\varepsilon} \tag{2.1.7}$$

If we carry on the calculation with this order (which is the LO) of the perturbation, we are going to reproduce the exact tree level approximation that is given by the Lagrangian (for example we continue with the same Lagrangian as in the previous chapter):

$$\mathcal{L}_{tree} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{3!}g\phi^3 \tag{2.1.8}$$

Now when we go further to include the first order of perturbation, we encounter a loop diagrams that we must include its contribution to the scattering amplitude. To show a practical example of this contribution, we position ourselves in a QED vacuum polarization diagram which is the first order correction to the photon propagator

Applying QED Feynman rules, gives us:

$$G^{(1)}_{\mu\nu}(k) = G^{(0)}_{\mu\mu'}(k)i\Pi_{\mu'\nu'}(k)G^{(0)}_{\nu'\nu} \tag{2.1.9}$$

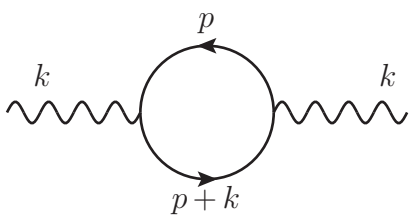
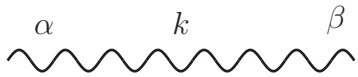


Figure 2.2: Photon self interaction

Where the free propagator $G^{(0)}_{\alpha\beta}(k)$ is as usual:

$$G^{(0)}_{\alpha\beta}(k) = -i\left[\frac{g_{\alpha\beta}}{k^2+i\varepsilon} + (\xi-1)\frac{k_\alpha k_\beta}{k^2+i\varepsilon}\right]$$

$=$ 

(2.1.10)

And $i\Pi_{\alpha\beta}(k)$ called the polarization operator is expressed as follows:

$$\begin{aligned} i\Pi_{\alpha\beta}(k) &= -(ie)^2\int\frac{d^4p}{(2\pi)^4}Tr\left(\gamma_\alpha\frac{i}{\not p - m + i\varepsilon}\gamma_\beta\frac{i}{\not p + \not k - m + i\varepsilon}\right) \\ &= -4e^2\int\frac{d^4p}{(2\pi)^4}\frac{p_\alpha p_\beta + p_\alpha k_\beta + p_\beta k_\alpha - g_{\alpha\beta}(p^2+pk-m^2)}{(p^2-m^2+i\varepsilon)\left((p+k)^2-m^2+i\varepsilon\right)} \end{aligned} \tag{2.1.11}$$

We can see by simple power counting that this integral is divergent, it is of the form:

$$i\Pi_{\alpha\beta} \sim \int d^4p\frac{p^2}{p^4} \sim \int pdp = \infty$$

This type of divergence is ultraviolet divergence as a result of $(p \to \infty)$ adding up the fluctuations of very large momenta scales[4]. This would mean that the theoretical value for the cross section is infinite, while in reality, what we measure is a finite value.

The physical reality is not just the first order of perturbation series or the second one, it is the whole thing, it is all the terms of the perturbation series and it is finite. Thus taking in principle the correction of the first order, it would only result in a more precise prediction. According to this reasoning, the nature of these divergences is nothing but a flaw in our understanding. In fact, it is a consequence of trying to compare an incomparable quantities, the physical quantities that are the observable ones (such as the mass of a dressed electron or its charge) with the ones inaccessible by any experiment (unobservable) and called bare quantities and on which our constructed theories were based upon. In other words, what we actually measure as a mass is in reality a whole package of constantly self interacting particle with a bare mass m_0. With taking that into account, adjustments[5] are made to the Lagrangian of the theory.We can write:

$$\mathcal{L}_0(m_0, g_0, \phi_0, \cdots) = \mathcal{L}_R(m_R, g_R, \phi_R, \cdots) + \mathcal{L}_{CT}(\delta m, \delta Z_g, \delta Z_\phi, \cdots) \tag{2.1.12}$$

[4]this divergence manifests it self in any loop diagrams

[5]we have seen what kind of adjustment we need to perform in the LSZ formalism section

These adjustments are going to be the key ingredients to the process of eliminating the relevant divergences rendering any higher order contribution finite. The renormalization recipe is encapsulated as follows:

- First, we choose a set of independent bare parameters of the relevant theory (*e.g.* $g_0,\ m_0, \cdots$)
- Introduce renormalization constant by re-defining the bare parameters and fields in terms of a renormalized ones (*e.g.* $m_0^2 = m_R^2 + \delta m^2, \phi_0 = Z_\phi^{1/2}\phi_R, \cdots$)
- Choose a suitable renormalization scheme to fix the counter terms (*i.e.* fixing the renormalization conditions)
- Calculate the amplitudes of the relevant loop diagrams with appropriate regularization scheme
- Eliminate all the divergences by the predetermined counter terms for chosen renormalization scheme

At this point, the physical quantities (such as the cross section) are expressed as a function of the renormalized parameters, where their numerical values are not fixed yet, so the next thing would be

- Choosing experimental data to fix the values of the renormalized parameters.

The result of applying this procedure will yield to finite and well defined quantities. We have mentioned in the fourth step of the above procedure that we need to make use of an appropriate regularization scheme, this is for the reason of rendering the relevant loop integrals convergent. There are several types of regularization procedures for that objective, we shall briefly mention some of these methods.

1. Cut-off method

The cut-off method is considered the simplest one, it simply restrict the boundaries of the integral:

$$\int_{-\infty}^{+\infty} d^4k \rightarrow \int_{|k^2|<\Lambda^2} d^4k$$

With that, the source of the divergence is eliminated. This regularization however, does not maintain gauge invariance, hence this method is not suitable for gauge theories.

2. Lattice regularization

This particular method relays on discretizing space-time in a way it would be made of small blocks of size a, the lattice constant.

$$x = a(n_0, n_1, n_2, n_3)$$

Where n_i $(i = 0, 1, 2, 3)$ are integers. According to that, the contribution of short distances[6] is eliminated in the space-time integration, this correspond to cutting off the high momentum scales which result to a convergent momentum integrals. This method preserves gauge invariance, but for the space-time symmetries (Lorentz invariance), it no longer holds true, hence it is not ideal for relativistic theories.

[6] distances that are smaller than the lattice constant a

3. Pauli-Villars regularization

The main idea of this scheme is to substitute the propagator in the divergent integral by:

$$\frac{1}{k^2-m^2} \to \frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2} = \frac{m^2-\Lambda^2}{(k^2-m^2)(k^2-\Lambda^2)}$$

Where Λ is the UV-cutoff, this is equivalent to adding unphysical scalar Grassmann fields with mass Λ to the Lagrangian and choosing the interaction of these fields to make the loop integrals finite. This method is Lorentz invariant and for the case of quantum electrodynamics (QED), it is gauge invariant, that makes it ideal for the whole treatment of QED. It is also applicable to massless non-abelian gauge theories (*e.g.* quantum chromodynamics), but when considering the massive case like the $\mathbb{EWSM}$ theory, this method does not maintain gauge invariance, consequently it can not be regularized by this method.

4. Dimensional regularization

The philosophy of this method lies in the fact that divergent specific dimensional integrals would be finite for lowering (or rising in some cases) dimensions continuously. So for the 4-dimensions space-time, we assign a continuous parameter d such for example the integral in 2.1.11 would be:

$$i\Pi_{\alpha\beta}(k) = -4e^2\mu^\epsilon \int \frac{d^dp}{(2\pi)^d} \frac{p_\alpha p_\beta + p_\alpha k_\beta + p_\beta k_\alpha - g_{\alpha\beta}(p^2+pk-m^2)}{(p^2-m^2+i\varepsilon)\left((p+k)^2-m^2+i\varepsilon\right)} \tag{2.1.13}$$

Where we have redefined the electric charge[7] as:

$$e \to e\mu^{\epsilon/2} \qquad \begin{cases} \mu & : mass\ scale \\ & \epsilon = 4-d \end{cases}$$

This integral is convergent for a small enough value of d. When carrying out the calculation a pole of the form $\frac{1}{\epsilon}$ appears ($\epsilon \to 0$ for $d \to 4$) which represent the UV-divergence of the original integral. Since in this method nothing has been violated, all the physical requirements are maintained therefore making it the most suitable regularization scheme for gauge theories. In fact, the calculation of one-loop radiative corrections have been done in this thesis with a symbolic software Mathematica through a package called FeynCalc[7]. And this package utilizes the dimensional regularization scheme in its internal code. For more details on how this method works, we have treated as an example the QED vacuum polarization in appendix C

2.1.2 Renormalization schemes

In the example of QED vacuum polarization treated in details in appendix C, the full photon propagator reads as:

$$\begin{aligned} iG_{\mu\nu} &= (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\frac{1}{k^2(1+\Pi(k))} + \xi\frac{k_\mu k_\nu}{k^4} \\ &= P^T_{\mu\nu}\frac{1}{k^2(1+\Pi(k))} + \xi\frac{k_\mu k_\nu}{k^4} = iG^T_{\mu\nu} + iG^L_{\mu\nu} \end{aligned} \tag{2.1.14}$$

[7]this is due to the dimensionful of the coupling constant $[e] = \frac{4-d}{2}$ in mass scales

Where $\Pi(k)$ is given by(seeC.0.26):

$$\Pi(k) = \frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\left[\frac{2}{\epsilon} - \gamma + \ln(4\pi) - \ln\left(\frac{m^2 - x(1-x)k^2}{\mu^2}\right)\right] \tag{2.1.15}$$

Since the longitudinal part of the full propagator does not matter (is not observable), we shall consider only the transversal part, so after introducing the photon-field renormalization constantZ_3 ($A^0_\mu = \sqrt{Z_3}A_\mu$) then the renormalized photon-propagator:

$$\begin{aligned} iG^{T,R}_{\mu\nu} &= Z_3^{-1} iG^T_{\mu\nu} \\ &= iG^T_{\mu\nu}(1+\delta Z_3)^{-1} \\ &= \frac{P^T_{\mu\nu}}{k^2}\frac{1}{1+\Pi(k)}\frac{1}{1+\delta Z_3} \end{aligned} \tag{2.1.16}$$

Taking

$$\delta Z_3 = \delta Z_3^{(1)}(\alpha) + \delta Z_3^{(2)}(\alpha^2) + \cdots \tag{2.1.17}$$

and considering only the α order:

$$\frac{1}{1+\Pi(k)}\frac{1}{1+\delta Z_3} = \frac{1}{1+\Pi+\delta Z_3} \tag{2.1.18}$$

Then the renormalized photon (transversal) propagator will take the form:

$$iG^{T,R}_{\mu\nu} = P^T_{\mu\nu}\frac{1}{k^2}\frac{1}{1+\Pi(k)+\delta Z_3} \tag{2.1.19}$$

In this equation 2.1.19, the renormalized propagator should be finite (no divergence) as a result $\Pi(k)+\delta Z_3$ must be finite. Therefore the divergence in $\Pi(k)$ must be canceled by the one in δZ_3. This condition determines the constant δZ_3 up to a finite additive constant. Fixing this arbitrary constant requires an additional condition. The choice of this condition is what establishes a renormalization scheme.

1. Minimal (MS) and modified minimal ($\overline{MS}$) subtraction schemes

This subtraction scheme is specific to dimensional regularization only. The condition of this scheme is for the renormalization constant to be exactly the divergent part of the Green function, which is only the term that contains the pole $\frac{1}{\epsilon}$. this simple condition results in a simplest form of the renormalization constant. In this scheme we would have in the case of photon propagator in QED,

$$\delta Z_3^{MS} = -\frac{2\alpha}{3\pi}\frac{1}{\epsilon} \tag{2.1.20}$$

Which yields to the following form for the photon propagator:

$$iG^{T,R,MS}_{\mu\nu}(k) = P^T_{\mu\nu}\frac{1}{k^2}\left[1 + \frac{2\alpha}{\pi}\int_0^1 dx\ x(1-x)\left(-\gamma + \ln(4\pi) - \ln\left(\frac{m^2 - x(1-x)k^2}{\mu^2}\right)\right)\right]^{-1} \tag{2.1.21}$$

Due to its clean cut condition, it is frequently used in many applications of QCD and some other gauge-field theories.

As for the $(\overline{MS})$ scheme, the only difference from the (MS) is, instead of eliminating only the $\frac{1}{\epsilon}$ term, the renormalization will eliminate additional constant terms that always appear, due to dimensional regularization namely γ and $\ln 4\pi$. The renormalization constant δZ_3 then, in the $\overline{MS}$ scheme, becomes:

$$\delta Z_3^{\overline{MS}} = \frac{\alpha}{3\pi}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi)\right) \tag{2.1.22}$$

And the renormalized propagator shows:

$$iG_{\mu\nu}^{T,R,\overline{MS}}(k) = P_{\mu\nu}^T \frac{1}{k^2}\left[1 - \frac{2\alpha}{\pi}\int_0^1 dx\ x(1-x)\ln\left(\frac{m^2 - x(1-x)k^2}{\mu^2}\right)\right]^{-1} \tag{2.1.23}$$

This $\overline{MS}$ scheme offers a relatively compact expression for the propagator, while keeping the simpleness of the MS scheme.

2. On-shell (OS) and off-shell (MOM) subtraction schemes

The on-shell scheme is specified by most natural condition. The idea is to join the asymptotic state particles which are on-shell observable. So:

$$\lim_{k^2\to 0} k^2 G_{\mu\nu}^{T,R}(k) = 1\ \cdot\ P_{\mu\nu}^T \tag{2.1.24}$$

This imply that:

$$\begin{aligned}\delta Z_3^{OS} &= -\Pi(0)\\ &= -\frac{\alpha}{3\pi}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi) - \ln\left(\frac{m^2}{\mu^2}\right)\right)\end{aligned} \tag{2.1.25}$$

And the renormalized propagator would be:

$$iG_{\mu\nu}^{T,R,OS}(k) = P_{\mu\nu}^T \frac{1}{k^2}\left[1 - \frac{2\alpha}{\pi}\int_0^1 dx\ x(1-x)\ln\left(\frac{m^2 - x(1-x)k^2}{m^2}\right)\right]^{-1} \tag{2.1.26}$$

This is the conventional renormalization scheme used for QED and even for the $\mathbb{EWSM}$, this is because of its naturaless for obtaining physical quantities (*e.g.* cross section, decay width $\cdots$). If for instance, we give k^2an unphysical (off mass shell) value, $k^2 = \lambda^2$, the 2.1.24 condition would take instead the form:

$$\lim_{k^2\to \lambda^2} k^2 G_{\mu\nu}^{T,R}(k) = 1\ \cdot\ P_{\mu\nu}^T \tag{2.1.27}$$

Thus we are now applying the prescription of the off-shell renormalization scheme, which is often referred to as the momentum-space subtraction scheme (MOM). The renormal-

ization constant δZ_3 is then:

$$\begin{aligned}\delta Z_3^{MOM} &= -\Pi(\lambda) \\ &= -\frac{2\alpha}{\pi}\int_0^1 dx\ x(1-x)\left[\frac{2}{\epsilon}-\gamma+\ln(4\pi)-\ln\left(\frac{m^2-x(1-x)\lambda^2}{\mu^2}\right)\right]\end{aligned} \tag{2.1.28}$$

The renormalized propagator acquire the form:

$$iG_{\mu\nu}^{T,R,MOM}(k) = P_{\mu\nu}^T\frac{1}{k^2}\left[1-\frac{2\alpha}{\pi}\int_0^1 dx\ x(1-x)\ln\left(\frac{m^2-x(1-x)k^2}{m^2-x(1-x)\lambda^2}\right)\right]^{-1} \tag{2.1.29}$$

These different renormalization schemes produce contrasting forms of renormalized propagators. This is called the renormalization scheme dependence. Applying any one of these renormalization schemes, we get rid of all ultraviolet divergences. However, when including the radiative correction of the one-loop diagrams, one does not only encounter UV-divergences, but we confront another type of divergences, an infra-red (IR) divergences. this kind of divergences appears whenever there is a photon in a loop diagram due to the null photon mass. This kind of divergences are not eliminated by any procedure of renormalization. We shall see how to deal with it in the next chapter when considering a leptonic scattering process. In the next section, we shall exploit the $\mathbb{EWSM}$ renormalization conditions in the form of the (OS) scheme.

2.2 On-Shell renormalization conditions for $\mathbb{EWSM}$

In the following we generalize the $\mathbb{EWSM}$ further to include quarks and different generations of fermions. Then we choose the set of physical parameters to renormalize and provide their on-shell renormalization condition.

1. Taking into account quarks and multi-generations in $\mathbb{EWSM}$ Lagrangian

In first chapter we have constructed the $\mathbb{EWSM}$ Lagrangian for the first generation of leptons only, and we said that it is a straight forward generalization to include quarks and other generations. Now we will do that[1]. As has been mentioned before, we define fermions as follows:

$$R_i = (\psi_i)_R \quad , \quad R_I = (\psi_I)_R \quad , \quad L_{i=I} = \begin{pmatrix}\psi_I \\ \psi_i\end{pmatrix}_L \tag{2.2.1}$$

Fermions are taking to be in singlet representation of $SU_{I_3}(2)$ if right handed, and doublet if left handed.

Where the indices $i, I(=1,2,3)$ indicates the 3-generations of leptons and $i, I(=4,5,6)$ indicates the 3-generations of quarks[8] with ψ_i and ψ_I correspond to the up ($T^3 = 1/2$) and down ($T^3 = -1/2$) type of the fermion respectively. While suffixes R and L are as in the first chapter symbolizing right-handed and left-handed fermions component. With this extension, only the parts where there are fermions will change, the rest[9] is unchanged.

[8] the notation $i = I$ in L doublet means that there is no generation mixing between left-hand fermions

[9] $\mathcal{L}_G, \mathcal{L}_{GF}, \mathcal{L}_{FP}$ and $\mathcal{L}_h$ don't change

Thus $\mathcal{L}_F$ and $\mathcal{L}_Y$ are the only changing parts taking the following forms:

$$\mathcal{L}_F = i\sum_I \overline{L}_{I=i} \not{D}_L L_I + i \sum_{n=i,I} \overline{R}_n \not{D}_R R_n \tag{2.2.2}$$

$$\mathcal{L}_Y = -\sum_i f_i \overline{L}_{I=i} \Phi R_i - \sum_I f_I \overline{L}_{I=i} (i\tau_2 \Phi^*) R_I + h.c. \tag{2.2.3}$$

Where f_i and f_I are the Yukawa coupling constants and $h.c.$ is an abbreviation for hermician conjugate ($h.c.$ of the first two terms). We can see that up type fermions do not interact with the Higgs field as the down type do. Also one should note that the standard model does not prohibit neutrinos from having mass, it was ruled out by construction. The masses of the fermions generated by(2.2.3) are given by[1]:

$$m_{i,I} = f_{i,I}\ v/\sqrt{2} \tag{2.2.4}$$

Now, with this generalization, we present the full form of the Lagrangian once again:

$$\mathcal{L}' = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_{GF} + \mathcal{L}_{FP} + \mathcal{L}_H + \mathcal{L}_Y \tag{2.2.5}$$

$$\begin{aligned}
\mathcal{L}_G &= -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \\
\mathcal{L}_F &= i\sum_I \overline{L}_{I=i} \not{D}_L L_I + i \sum_{n=i,I} \overline{R}_n \not{D}_R R_n \\
\mathcal{L}_{GF} &= B^a \partial^\mu W^a_\mu + \frac{\alpha}{2}(B^a)^2 \\
\mathcal{L}_{FP} &= (\partial^\mu \bar{c}^a) D^{ab}_\mu c^b \\
\mathcal{L}_H &= (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi) \\
\mathcal{L}_Y &= -\sum_i f_i \overline{L}_{I=i} \Phi R_i - \sum_I f_I \overline{L}_{I=i} (i\tau_2 \Phi^*) R_I + h.c.
\end{aligned}$$

This Lagrangian embeds the dynamics and all the information of the electroweak theory. We see that it is expressed in terms of the parameters $\{\ g, g', \mu, \lambda, v, f_i$ and $f_I\ \}$. However, the set of parameters that we will choose for the renormalization program is the one accessible by laboratory. This means that EW\$M must be first incorporated to get a Lagrangian in term of masses and coupling $\{e, M_Z, M_W, m_H, m_i$ and $m_I\ \}$. We draw attention that the number of the first set of parameters is seven, while we only have six physical parameters to replace independent parameters. Although it was introduced as if it were independent. The V term (1.1.40) will produce a linear term in $H(x)$ (the Higgs field) which will generate the tadpole diagrams. We require that the tadpole term disappears which implies that the coefficient of the term linear in H vanishes, we identify this coefficient by :

$$T = v(\mu^2 - \lambda v^2) \tag{2.2.6}$$

So we have introduced the parameter v as an independent parameter, and added the condition that tadpole$= 0$ which will give us the tadpole counter term T.

The relation between these sets of parameters is given by identification, it reads[1]:

$$
\begin{aligned}
g &= e\frac{M_z}{\sqrt{M_Z^2 - M_W^2}} \\
g' &= e\frac{M_Z}{M_W} \\
v &= 2\frac{M_W\sqrt{M_Z^2 - M_W^2}}{eM_Z} \\
\lambda &= \frac{e^2 M_Z^2}{8M_W^2(M_Z^2 - M_W^2)}\left(m_H^2 - \frac{eTM_Z}{M_W\sqrt{M_Z^2 - M_W^2}}\right) \\
\mu^2 &= \frac{m_H^2}{2} \\
f_{i,I} &= \frac{\sqrt{2}m_{i,I}}{v}
\end{aligned}
\tag{2.2.7}
$$

Going now to the next step in the renormalization recipe. We introduce the renormalization constants as follows:

$$
\begin{aligned}
W_0^{\pm} &= Z_W^{1/2} W^{\pm} \quad , \quad (W_\mu^{\pm} = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}) \\
\begin{cases} Z_{0\mu} = Z_{ZZ}^{1/2} Z_\mu + Z_{ZA}^{1/2} A_\mu \\ A_{0\mu} = Z_{AZ}^{1/2} Z_\mu + Z_{AA}^{1/2} A_\mu \end{cases} &, \quad \begin{cases} Z_\mu = \frac{gW_\mu^3 - g'W_\mu^0}{\sqrt{g^2+g'^2}} \\ A_\mu = \frac{g'W_\mu^3 + gW_\mu^0}{\sqrt{g^2+g'^2}} \end{cases} \\
H_0 &= Z_H^{1/2} H \\
\psi_{0,R}^n &= (Z_R^{1/2})^{nm} \psi_R^m \\
\psi_{0,L}^n &= (Z_L^{1/2})^{nm} \psi_L^m \\
M_{W,0}^2 &= M_W^2 + \delta M_W^2 \\
M_{Z,0}^2 &= M_Z^2 + \delta M_Z^2 \\
m_{H,0}^2 &= m_H^2 + \delta m_H^2 \\
m_{n,2} &= m_n + \delta m_n \\
e_0 &= Z_e e
\end{aligned}
\tag{2.2.8}
$$

Where quantities with index 0 denote bare quantities, and those without, are the renormalized ones. Since the renormalization constants of the unphysical fields (FP ghost and Goldstons) do not affect the Green functions of physical particles, they are not presented[10]

Noting that if a renormalization constant Z_α connect the bare and the renormalized same fields, then: $Z_\alpha - 1 = \delta Z_\alpha$ as for $Z_W = 1 + \delta Z_W$ and $Z_L^{nn} = 1 + \delta Z_L^{nn}$. And if it connects the bare and renormalized different fields, then $Z_\alpha = \delta Z_\alpha$ as for $Z_{AZ} = \delta Z_{AZ}$ and $Z_L^{mn} = \delta Z_L^{mn}$, $m \neq n$. We have also $Z_e = 1 + \delta Z_e$
So the bare Lagrangian of the $\mathbb{EWSM}$ $\mathcal{L}_0$ can be split into a renormalized Lagrangian $\mathcal{L}_R$ and a counter term Lagrangian $\mathcal{L}_{CT}$,

[10]for the treatment of these fields see Aoki[1]

$$\begin{aligned}\mathcal{L}_0(e_0, M_{W,0}, M_{Z,0}, m_{H,0}, m_{n,0}, \psi_0) &= \mathcal{L}_R(e, M_W, M_Z, m_H, m_n, \psi) \\ &+ \mathcal{L}_{CT}(e, M_W, M_Z, m_H, m_n, \psi, Z_e, \delta M_Z^2, \delta M_W^2, \delta m_H^2, \delta m_n, Z)\end{aligned} \tag{2.2.9}$$

Where here ψ and Z represent all the relevant fields and their respective renormalization constants and we have ignored mixing between fermion generations. As the title of this section suggests, the scheme that we will apply to determine the relevant renormalization constants and renormalized quantities is the on-shell renormalization scheme. The on-shell conditions are presented as follows[11][1]

W boson Self-energy

This case of the W boson will be treated in detail to serve as an example on how to determine the Feynman rules for the counter terms from the Lagrangian. So we pick only the term that correspond to the propagator of the W boson which is [1]:

$$\mathcal{L}_0 = W^+_{\mu,0}[(g^{\mu\nu}\Box - \overleftarrow{\partial}^\mu \vec{\partial}^\nu) + \frac{1}{\alpha}\overleftarrow{\partial}^\mu \vec{\partial}^\nu + M^2_{\mu,0} g^{\mu\nu}]W^-_{\nu,0} \tag{2.2.10}$$

In the t'Hooft gauge ($\alpha = 1$) it reduces to:

$$\mathcal{L}_0 = W^+_{\mu,0}[g^{\mu\nu}\Box + M^2_{\mu,0} g^{\mu\nu}]W^-_{\nu,0} \tag{2.2.11}$$

This is the bare Lagrangian which correspond to the inverse propagator of W. Applying(2.2.8), we get:

$$\mathcal{L} = Z_W W^+_\mu [g^{\mu\nu}(\Box + M^2_\mu + \delta M^2_W)]W^-_\nu \tag{2.2.12}$$

Writing now the operator $\Box$ as $-k^2$ due to the correspondence change[9][41]:

$$\begin{aligned}&\vec{\partial}^\mu \to -ik^\mu \qquad ; \qquad \overleftarrow{\partial}^\mu \to ik^\mu \\ &\Rightarrow \Box = \overleftarrow{\partial}_\mu \vec{\partial}^\mu \to -k^2\end{aligned}$$

That gives:

$$\begin{aligned}\mathcal{L}_0 &= (1 + \delta Z_W) W^+_\mu [-k^2 + M^2_W + \delta M^2_W] g^{\mu\nu} W^-_\nu \\ &= W^+_\mu [-g^{\mu\nu}(k^2 - M^2_W)]W^-_\nu \\ &+ W^+_\mu [-g^{\mu\nu}\delta Z_W (k^2 - M^2_W - \delta Z^2_W) + g^{\mu\nu}\delta M^2_W]W^-_\nu\end{aligned} \tag{2.2.13}$$

[11] we have excluded the tadpole diagrams since they have no contribution in actual calculation

Which yield to:

$$= g_{\mu\nu}(k^2 - M_W^2)$$

$$+$$

$$\begin{aligned} &= -ig_{\mu\nu}[\delta Z_W(k^2 - M_W^2) - \delta M_W^2] \\ &= -i\left(g_{\mu\nu} - \tfrac{k^\mu k^\nu}{k^2}\right)[\delta Z_W(k^2 - M_W^2) - \delta M_W^2] \\ &\quad -i\tfrac{k^\mu k^\nu}{k^2}[\delta Z_W(k^2 - M_W^2) - \delta M_W^2] \end{aligned}$$

And with the self-energy diagram,

$$= \left(g_{\mu\nu} - \tfrac{k_\mu k_\nu}{k2}\right) A^W(k^2) + \tfrac{k_\mu k_\nu}{k^2} B^W(k^2)$$

The on-shell condition is defined according to:

- The pole of the propagator is at the physical mass, (for the W boson at M_W^2) this implies
$$A^W(M_W^2) = 0 \tag{2.2.14}$$
- The residue of the renormalized propagator mus be aqual to the residue free propagator which implies the second condition
$$A'^W(M_W^2) = 0 \tag{2.2.15}$$

with these two conditions we obtain :

$$\delta Z_W \qquad , \qquad \delta M_W^2$$

Same reasoning is applied to every other propagator. We give their condition as follows

A and Z bosons self-energies[1][6]

$$= \left(g_{\mu\nu} - \tfrac{k_\mu k_\nu}{k2}\right) A^Z(k^2) + \tfrac{k_\mu k_\nu}{k^2} B^Z(k^2)$$

$$+$$

$$\begin{aligned} &= -i\left(g_{\mu\nu} - \tfrac{k_\mu k_\nu}{k2}\right)[\delta Z_{ZZ}(k^2 - M_Z^2) - \delta M_Z^2] \\ &\quad -i\tfrac{k_\mu k_\nu}{k^2}[\delta Z_{ZZ}(k^2 - M_Z^2) - \delta M_Z^2] \end{aligned}$$

$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k2}\right) A^{ZA}(k^2) + \frac{k_\mu k_\nu}{k^2} B^{ZA}(k^2)$$

$+$

$$= -\frac{i}{2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k2}\right)[\delta Z_{ZA}(k^2 - M_Z^2) + \delta Z_{AZ} k^2]$$

$$-\frac{i}{2}\frac{k_\mu k_\nu}{k^2}[\delta Z_{ZA}(k^2 - M_Z^2) + \delta Z_{AZ} k^2]$$

$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k2}\right) A^{A}(k^2) + \frac{k_\mu k_\nu}{k^2} B^{A}(k^2)$$

$+$

$$= -i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k2}\right)\delta Z_{AA} k^2$$
$$-i\frac{k_\mu k_\nu}{k^2}\delta Z_{AA} k^2$$

Their corresponding renormalization are :

$$\begin{array}{ccc} A^Z(M_Z^2) = 0 & , & A'^Z(M_Z^2) = 0 \\ A^A(0) = 0 & , & A'^A(0) = 0 \\ A^{ZA}(M_Z^2) = 0 & , & A^{ZA}(0) = 0 \end{array}$$

From where we extract their relevant renormalization constants

$$\delta Z_{ZZ}\ ,\ \delta Z_{ZA}\ ,\ \delta Z_{AZ}\ ,\ \delta Z_{AA}\ ,\ \delta Z_Z$$

Charged lepton self-energy

For the fermions, we present their renormalization conditions for the specific case of charged leptons self-energies due to its importance for the application on the next chapter.

$$= \Sigma_1^l(k^2)\mathbb{1} + \Sigma_\gamma^l(k^2)\not{k} + \Sigma_{5\gamma}^l(k^2)\not{k}\gamma_5$$

$+$

$$= i[-\delta m_l \mathbb{1} + \frac{1}{2}\left(\delta Z_L^l + \delta Z_R^l\right)(\not{k} - m_l)\mathbb{1}$$
$$-\frac{1}{2}\left(\delta Z_L^l - \delta Z_R^l\right)\not{k}\gamma_5]$$

We get :

$$\delta Z_L^l\ ,\ \delta Z_R^l\ ,\ \delta m_l$$

By the following conditions :

$$\begin{aligned}
&\Sigma_1^l(m_l^2) + m_l\Sigma_\gamma^l(m_l^2) = 0\\
&\Sigma_{5\gamma}^l(m_l^2) = 0\\
&2m_l\left(\Sigma_1'^l(m_l^2) + m_l\Sigma_\gamma'^l(m_l^2)\right) + \Sigma_\gamma^l(m_l^2) = 0
\end{aligned}$$

Neutrino self-energy part (for any type)
Here we suppose the neutrinos have zero mass, so we have:

$$= \Sigma_\nu(k^2)\,\not{k}\,(1-\gamma_5)$$

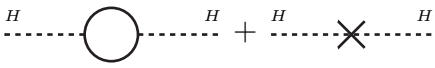
$$+$$

$$= \tfrac{1}{2}\delta Z_L^\nu\,\not{k}\,(1-\gamma_5)$$

Where the condition that determines Z_L^ν is :

$$\Sigma_\gamma(0) + \frac{1}{2}Z_L^\nu = 0 \tag{2.2.16}$$

Higgs self-energy diagram



$$= \Sigma_R^H(k^2)$$

Where,

$$= i[\delta Z_H(k^2 - m_H^2) - \delta m_H^2]$$

With the conditions:

$$\begin{aligned}
&\Sigma_R^H(m_H^2) = 0\\
&\Sigma_R'^H(m_H^2) = 0
\end{aligned}$$

That determines:

$$\delta Z_H \qquad , \qquad \delta m_H^2$$

Charge renormalization constant δZ_e
This constant can be determined by any charged particles vertex with the photon due to the universality of the on-shell electric charge[1]. We will consider the eeA vertex for δZ_e For extracting the counter term connected to the vertex aaA, since there is a mixing between Z and A. We need to pic both the eeA and eeZ terms in the bare fermionic

interaction lagrangian,

$$\mathcal{L}^{int}_{F(e),0} = \frac{g_0 g_0'}{\sqrt{g_0^2 + g'^2}} Q_e \bar{\psi}_{e,0} \gamma^\alpha \psi_{e,0} A_{\alpha,0} + \frac{\sqrt{g_0^2 + g'^2}}{2} \left\{ \bar{\psi}_{e,0} \gamma^\alpha \left(\frac{1-\gamma_5}{2} - \frac{2 Q_e g_0'^2}{g_0^2 + g'^2} \right) \psi_{e,0} \right\} Z_{\alpha,0} \tag{2.2.17}$$

Where we have:

$$Q_e = -1 \quad , \quad g_0 = e_0 \frac{M_{Z,0}}{\sqrt{M_{Z,0}^2 - M_{W,0}^2}} \quad , \quad g_0' = e_0 \frac{M_{Z,0}}{M_{W,0}}$$

$$\Rightarrow \sqrt{g_0^2 + g'^2} = e_0 \frac{M_{Z,0}^2}{M_{W,0}\sqrt{M_{Z,0}^2 - M_{W,0}^2}}$$

$$\frac{g_0'^2}{g_0^2 + g'^2} = 1 - \frac{M_{W,0}^2}{M_{Z,0}^2}$$

$$\frac{g_0 g_0'}{\sqrt{g_0^2 + g_0'^2}} = e_0$$

Then, $\mathcal{L}^{int}_{F(e),0}$ becomes:

$$\mathcal{L}^{int}_{F(e),0} = - e_0 \bar{\psi}_{e,0} \gamma^\alpha \psi_{e,0} A_{\alpha,0} + \frac{e_0 M_{Z,0}^2}{2 M_{W,0}\sqrt{M_{Z,0}^2 - M_{W,0}^2}} \left\{ \bar{\psi}_{e,0} \gamma^\alpha \left[\frac{1-\gamma_5}{2} + 2\left(1 - \frac{M_{W,0}^2}{M_{Z,0}^2}\right) \right] \psi_{e,0} \right\} Z_{\alpha,0} \tag{2.2.18}$$

Now, with the fact that ψ_e can be written in terms of left and right components:

$$\psi_{e,0} = \psi_{e_L,0} + \psi_{e_R,0}$$
$$\bar{\psi}_{e,0} = \bar{\psi}_{e_L,0} + \bar{\psi}_{e_R,0}$$

with:

$$\begin{cases} \psi_{e_L,0} = \frac{1-\gamma_5}{2} \psi_{e,0} \\ \psi_{e_R,0} = \frac{1+\gamma_5}{2} \psi_{e,0} \end{cases} \quad ; \quad \begin{cases} \bar{\psi}_{e_L,0} = \bar{\psi}_{e,0} \frac{1-\gamma_5}{2} \\ \bar{\psi}_{e_R,0} = \bar{\psi}_{e,0} \frac{1+\gamma_5}{2} \end{cases} \tag{2.2.19}$$

Substituting in 2.2.18 we get:

$$\begin{aligned} \mathcal{L}^{int}_{F(e),0} = & -e_0 (\bar{\psi}_{e_L,0} + \bar{\psi}_{e_R,0}) \gamma^\alpha (\psi_{e_L,0} + \psi_{e_R,0}) A_{\alpha,0} \\ & + \frac{e_0 M_{Z,0}^2}{2 M_{W,0}\sqrt{M_{Z,0}^2 - M_{W,0}^2}} \Big\{ (\bar{\psi}_{e_L,0} + \bar{\psi}_{e_R,0}) \gamma^\alpha \frac{1-\gamma_5}{2} (\psi_{e_L,0} + \psi_{e_R,0}) Z_{\alpha,0} \\ & + 2\left(1 - \frac{M_{W,0}^2}{M_{Z,0}^2}\right) (\bar{\psi}_{e_L,0} + \bar{\psi}_{e_R,0}) \gamma^\alpha (\psi_{e_L,0} + \psi_{e_R,0}) Z_{\alpha,0} \Big\} \end{aligned} \tag{2.2.20}$$

Omitting the vanishing combination we obtain:

$$\begin{aligned}\mathcal{L}^{int}_{F(e),0} = &-e_0\bar{\psi}_{e_L,0}\gamma^\alpha\psi_{e_L,0}A_{\alpha,0} - e_0\bar{\psi}_{e_R,0}\gamma^\alpha\psi_{e_R,0}A_{\alpha,0} \\ &+ \frac{e_0 M^2_{Z,0}}{4M_{W,0}\sqrt{M^2_{Z,0}-M^2_{W,0}}}\Big\{\bar{\psi}_{e_L,0}\gamma^\alpha(1-\gamma_5)\psi_{e_L,0}Z_{\alpha,0} \\ &+ 2\left(1-\frac{M^2_{W,0}}{M^2_{Z,0}}\right)\left(\bar{\psi}_{e_L,0}\gamma^\alpha\psi_{e_L,0}+\bar{\psi}_{e_R,0}\gamma^\alpha\psi_{e_R,0}\right)Z_{\alpha,0}\Big\}\end{aligned} \tag{2.2.21}$$

Introducing now the renormalization constants and the renormalized quantities,

$$\begin{aligned} Z_{\alpha,0} = Z^{1/2}_{ZZ}Z_\alpha + Z^{1/2}_{ZA}A_\alpha \quad &; \quad A_{\alpha,0} = Z^{1/2}_{AZ}Z_\alpha + Z^{1/2}_{AA}A_\alpha \\ \psi_{e_L,0} = Z_L^{e\frac{1}{2}}\psi_{e_L} \quad &; \quad \psi_{e_R,0} = Z_R^{e\frac{1}{2}}\psi_{e_R} \\ M^2_{Z,0} = M^2_Z + \delta M^2_Z \quad &; \quad M^2_{W,0} = M^2_W + \delta M^2_W \\ e_0 = Z_e e \quad & \end{aligned}$$

Applying these redefinition into $\mathcal{L}^{int}_{F(e),0}$ we get:

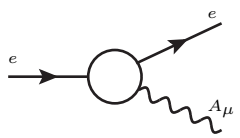
$$\begin{aligned}\mathcal{L}^{int}_{F(e),0} = &- Z_e e Z^e_L\bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L}(Z^{1/2}_{AZ}Z_\alpha + Z^{1/2}_{AA}A_\alpha) \\ &- Z_e e Z^e_R\bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R}(Z^{1/2}_{AZ}Z_\alpha + Z^{1/2}_{AA}A_\alpha) \\ &+ \frac{Z_e e(M^2_Z+\delta M^2_Z)}{4(M^2_W+\delta M^2_W)^{1/2}(M^2_Z+\delta M^2_Z-M^2_W-\delta M^2_W)^{1/2}} \\ &\times\Big\{Z^e_L\bar{\psi}_{e_L}\gamma^\alpha(1-\gamma_5)\psi_{e_L}\,(Z^{1/2}_{ZZ}Z_\alpha + Z^{1/2}_{ZA}A_\alpha) \\ &+ 2\left(1-\frac{M^2_W+\delta M^2_W}{M^2_Z+\delta M^2_Z}\right)\left(Z^e_L\bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L} + Z^e_R\bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R}\right) \\ &\times\left(Z^{1/2}_{ZZ}Z_\alpha + Z^{1/2}_{ZA}A_\alpha\right)\Big\}\end{aligned} \tag{2.2.22}$$

Since we are looking only for eeA vertex, we pick only the terms that are proportional to

A_α:

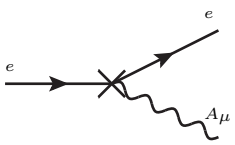
$$
\begin{aligned}
\mathcal{L}^{int}_{F(eeA),0} =& - Z_e e Z^e_L \bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L} Z^{1/2}_{AA} A_\alpha - Z_e e Z^e_R \bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R} Z^{1/2}_{AA} A_\alpha \\
&+ \frac{Z_e e (M_Z^2+\delta M_Z^2)}{4(M_W^2+\delta M_W^2)^{1/2}(M_Z^2+\delta M_Z^2-M_W^2-\delta M_W^2)^{1/2}} \{ Z^e_L\, \bar{\psi}_{e_L}\gamma^\alpha(1-\gamma_5)\psi_{e_L} Z^{1/2}_{ZA} A_\alpha \\
&+ 2\left(1-\frac{M_W^2+\delta M_W^2}{M_Z^2+\delta M_Z^2}\right)(Z^e_L\bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L} + Z^e_R\bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R}) Z^{1/2}_{ZA}\, A_\alpha\} \\
=& - (1+\delta Z_e) e (1+\delta Z^e_L)\bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L}(1+\delta Z^{1/2}_{AA}) A_\alpha \\
&- (1+\delta Z_e) e (1+\delta Z^e_R)\bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R}(1+\delta Z^{1/2}_{AA}) A_\alpha \\
&+ \frac{(1+\delta Z_e) e (M_Z^2+\delta M_Z^2)}{4(M_W^2+\delta M_W^2)^{1/2}(M_Z^2+\delta M_Z^2-M_W^2-\delta M_W^2)^{1/2}} \\
&\times \{(1+\delta Z^e_L)\, \bar{\psi}_{e_L}\gamma^\alpha(1-\gamma_5)\psi_{e_L}\delta Z^{1/2}_{ZA} A_\alpha \\
&+ 2\left(1-\frac{M_W^2+\delta M_W^2}{M_Z^2+\delta M_Z^2}\right)((1+\delta Z^e_L)\bar{\psi}_{e_L}\gamma^\alpha\psi_{e_L} + (1+\delta Z^e_R)\bar{\psi}_{e_R}\gamma^\alpha\psi_{e_R})\delta Z^{1/2}_{ZA}\, A_\alpha\}\,.
\end{aligned}
\tag{2.2.23}
$$

And with further simplification, we find:



$$= \Gamma_\mu(p'_e, p_e)$$

$$+$$



$$
\begin{aligned}
&= -e\gamma_\mu\left(1+\delta Z_e + \delta Z^{1/2}_{AA} + \tfrac{\delta Z^e_L+\delta Z^e_R}{2} - \tfrac{\delta Z^e_L-\delta Z^e_R}{2}\gamma_5\right) \\
&\quad + \tfrac{e\delta Z^{1/2}_{ZA}\gamma_\mu}{4M_W\sqrt{M_Z^2-M_W^2}}(3M_Z^2 - 4M_W^2 + M_Z^2\gamma_5) \equiv \Gamma^{CT}_\mu
\end{aligned}
$$

And the constant δZ_e is determined by :

$$\overline{u}(m_e)\Gamma^R_\mu u(m_e)|_{k^\mu=0} = 0$$

Where

$$\Gamma^R_\mu(p'_e, p_e) = \Gamma_\mu(p'_e, p_e) + \Gamma^{CT}_\mu$$

In the next section we shall provide some results.

2.3 Computation of δZ_{ZA} and δZ_{AZ}

In this section we will provide a Mathematica (FeynCalc and FeynArts) program to show how to calculate some of the renormalization constants[12] of $\mathbb{EW\$M}$. We start by a code that calculates the Z_{ZA} and Z_{AZ} renormalization constants. This correspond to the diagrams of $Z - A$ transition self-energy diagrams.
First, we start by generating the corresponding Feynman diagrams by the aid of FeynArts. But before that, we need to load FeynCalc and FeynArts in Mathematica. The codes for that would be:

```
$LoadFeynArts = True;
<< FeynCalc`

```

Now we can generate the Feynman diagrams by

```
za = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles];
zaf = InsertFields[za, {V[2]} -> {V[1]},
InsertionLevel -> {Particles}, Model -> "SM",
ExcludeParticles -> {}];
Paint[zaf, ColumnsXRows -> 5]
Paint[DiagramSelect[zaf, IRDivergentQ]]

```

The first line create the different topologies for a two external lines with one loop. And we have excluded those of which they have tadpoles because they don't contribute in calculations. The second line specify[13] which particle is going in, and which is going out. Also, we specify the model of which the particles obeys. The third line command is for displaying all the diagrams. Where in the fourth line we check whether there are diagrams that contains IR-divergence and paint them. If there aren't any, it won't do anything. The result of these commands is

Next we execute

```
SetOptions[PaVeReduce, A0ToB0 -> True];
$LimitTo4 = True
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2],
    SMP["cos_W"] -> SMP["m_W"]/SMP["m_Z"]}

```

Since FeynCalc is programmed with the Passarino-Veltman[14] scheme. The first line is then just a command to replace in the results A_0 by B_0. And the second line takes the

[12] This program (and the appendix D programs) was possible to do only after we were able to understand the work of Jorge.C.Romao[30][31]

[13] For the full table of symbols of particles see [guide FeynArts]

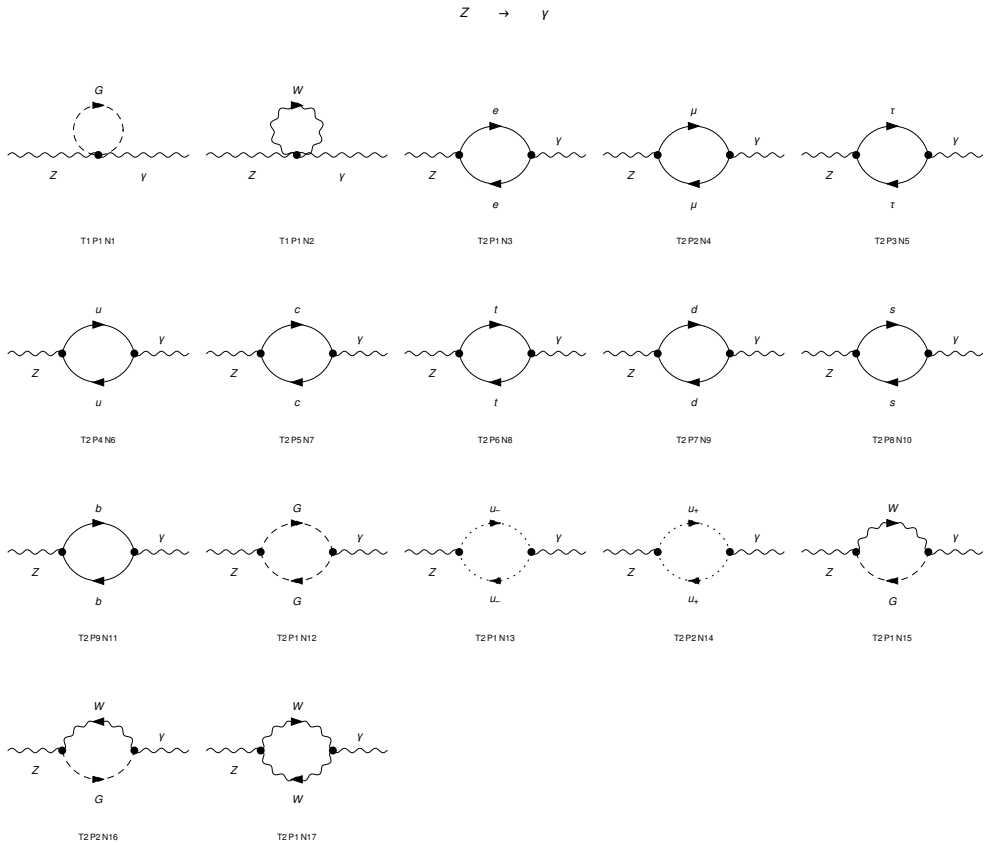


Figure 2.3: Z-A transition process

parameter D of the dimensional regularization scheme to be 4 when ever is possible. For the third line, it is just a substitution of the Weinberg angle by the weak bosons masses,

$$\cos^2(\theta_W) = \frac{M_W^2}{M_Z^2} \tag{2.3.1}$$

$$\sin^2(\theta_W) = 1 - \frac{M_W^2}{M_Z^2} \tag{2.3.2}$$

After that, we demand for FeynArts to create the Feynman amplitudes for all the diagrams of the process. Then convert these amplitudes to FeynCalc input for further treatment. This is done by the below commands[14]

```
zaamp = FCFAConvert[
CreateFeynAmp[zaf, Truncated -> True, PreFactor -> I (4 Pi)^-2],
IncomingMomenta -> {k}, OutgoingMomenta -> {k},
LoopMomenta -> {q},
LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \
\[Beta]}, SMP -> True, DropSumOver -> True,
UndoChiralSplittings -> True] /. wr;

```

The result of this command is given as a list of amplitudes. To sum all of them and

[14] the prefactor $\frac{i}{16\pi^2}$ is added because it is not included in the code definition of the loop integral

perform the loop integration we define:

```
za[n_] := zaamp[[n]] /. DiracTrace -> Tr // Simplify
zares[n_] := -I/Pi^2 OneLoop[q, za[n]] // PaVeReduce
pt[\[Mu]_, \[Nu]_] :=
MT[\[Mu], \[Nu]] - FV[k, \[Mu]] FV[k, \[Nu]]/SP[k, k]
zasum = Sum[zares[n], {n, 17}];
zas = MT[\[Alpha], \[Mu]] zasum pt[\[Alpha], \[Nu]];/(3) // Contract

```

Where the first line is to extract the n-th element of the list and perform the Mathematica trace (Tr) instead of FeynCalc's "Dirac trace. The second line calculates the loop integral by the dimensional regularization scheme. The result is given in terms of the scalar functions $B_0(k^2, m_1^2, m_1^2)$. In the third line, we defined the transversal operator $P^T_{\mu\nu} = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$ to obtain the transversal part of the self-energy operator only. With the fourth and fifth lines we get the transversal part of the Z-A transformation self-energy operator $A^T_{ZA}(m^2)$. Adding the contribution of the counter term, the renormalized operator is then:

$$A^{T,R}_{ZA}(k^2) = A^T_{ZA}(k^2) + \delta Z^{1/2}_{ZA}(M_Z^2 - k^2) - \delta Z^{1/2}_{AZ} k^2 \tag{2.3.3}$$

And with the on-shell conditions:

$$A^T_{ZA}(M_Z^2) - M_Z^2 \delta Z^{1/2}_{AZ} = 0 \tag{2.3.4}$$

$$A^T_{ZA}(0) - M_Z^2 \delta Z^{1/2}_{ZA} = 0 \tag{2.3.5}$$

We get the explicit relations for δZ_{AZ} and δZ_{ZA} as:

$$\delta Z^{1/2}_{AZ} = \frac{A^T_{ZA}(M_Z^2)}{M_Z^2} \tag{2.3.6}$$

$$\delta Z^{1/2}_{ZA} = -\frac{A^T_{ZA}(0)}{M_Z^2} \tag{2.3.7}$$

Implementing that to the program by:

```
Zaz = zas/SMP["m_Z"]^2 /. ScalarProduct[k, k] -> SMP["m_Z"]^2
aux = -zas/SMP["m_Z"]^2 /. ScalarProduct[k, k] -> k2
Zza = Limit[aux, k2 -> 0, Analytic -> True]

```

Due to the scalar functions B_0, the results can not be simplified even with the command "Simplify". Therefore, to compare with the analytically calculated results of [1] or any other reliable papers, we first need to replace in the obtained results the expressions of the scalar function and simplify. Fortunately for us, we were able to compare the result of the renormalization constant δZ_{ZA} with what Aoki found in his paper[1], our result is:

$$\text{ouroutput} = \quad -\frac{\mathrm{e}^2 m_W m_Z \sqrt{1 - \frac{m_W^2}{m_Z^2}} \mathrm{B}_0\left(0, m_W^2, m_W^2\right)}{8\pi^2\left(m_W^2 - m_Z^2\right)} \tag{2.3.8}$$

And Aoki's result was:

$$\delta Z_{ZA}^{1/2} = -\frac{e^2 M_W}{8\pi^2\sqrt{M_Z^2 - M_Z^2}}\Big(C_{UV} - \ln(M_W^2)\Big) \tag{2.3.9}$$

Where C_{UV} is the divergent part given by:

$$C_{UV} = \frac{1}{\epsilon} - \gamma + \ln(4\pi) \tag{2.3.10}$$

To show that they are identical expressions, we have to replace B_0 by its explicit expression:

$$B_0(0, m^2, m^2) = C_{UV} - \ln(M_W^2) \tag{2.3.11}$$

In the case of $m = M_Z$ and $\mu^2 = 1$[15], we get:

$$\text{ouroutput} = \quad -\frac{e^2 M_W}{8\pi^2\sqrt{M_Z^2 - M_Z^2}}\Big(C_{UV} - \ln(M_W^2)\Big) \tag{2.3.12}$$

This shows that this structure of few lines of commands is a powerful tool to facilitate the enormous complications of calculations. With a similar programs (see appendix D) like this one, we have calculated $\{\delta Z_{ZZ}, \delta Z_{AA}, \delta Z_W, \delta M_W^2, \delta M_Z^2, \delta Z_L^e, \delta Z_R^e, \delta Z_L^\nu, \delta Z_R^\nu, \delta m_e$ and $\delta Z_e\}$. But unfortunately and due to the long and complicated results, we were unable to compare with Aoki's results. And we let this verification for future work. The following diagram gives an overview of the connection between the renormalization constants, and from where we extract them.

These renormalization constants were calculated in appendix D for their importance in a practical real process of scattering particles. The treatment of this process is the main subject of the next chapter.

[15] in Aoki's paper[1], he took $\mu^2 = 1$ in his calculations.

Chapter 3

Application to $e^-\nu_\mu \to e^-\nu_\mu$ process

In this final chapter we shall consider the electron muon-neutrino scattering process ($e^-\ \nu_\mu \to e^-\ \nu_\mu$). First we calculate the tree level total cross section with the help of FeynCalc package of Mathematica, then we will include the radiative correction via another package, FormCalc[18]

3.1 Tree level total cross section

We start immediately with the FeynCalc program by loading it in Mathematica.

```
$LoadFeynArts = True;
<< FeynCalc`
```

Then we generate the relevant Feynman diagram of the process by:

```
en = CreateTopologies[0, 2 -> 2];
enf = InsertFields[en, {F[2, {1}], F[1, {2}]} -> {F[2, {1}], F[1, {2}]},
    InsertionLevel -> {Particles}, Model -> "SM", ExcludeParticles -> {}];
Paint[enf]
```

Where these commands are the same as in the previous chapter except that it creates a 2 by 2 topology with zero loop. Then insert the corresponding fields of the process. Its output is then:

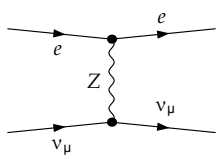


Figure 3.1: Tree level of the process $e^-\nu_\mu \to e^-\nu_\mu$

Next we define the same substitutions as in the last code with an additional one. This latter correspond to introducing the $i\varepsilon$[1] (`I \[Epsilon]` in the program) prescription of

[1]note here that the ε is just a notation, ε does not mean a small value

the propagator[2]. Also we have identified:

$$(p^2 - k^2) = t \tag{3.1.1}$$

Where t is a Mandelstam variable[29]. There are three variables in 2 by 2 scattering processes and they are defined as follows:

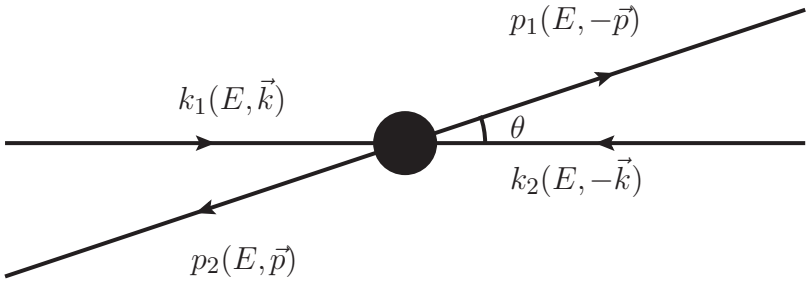


Figure 3.2: Kinematics of 2 by 2 scattering in the center of mass frame

$$s = (k_1 + k_2)^2 = (p_1 + p_2)^2 \tag{3.1.2}$$
$$t = (k_1 + p_1)^2 = (k_2 + p_2)^2 \tag{3.1.3}$$
$$u = (k_1 + p_2)^2 = (k_2 + p_1)^2 \tag{3.1.4}$$

With a relation between these variables:

$$s + t + u = \sum_{i=1}^{4} m_i^2 \tag{3.1.5}$$

They are preferred to work with, because of their frame independence (*i.e.* Lorentz invariant). So we write:

```
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2], SMP["cos_W"] ->
    SMP["m_W"]/SMP["m_Z"], PropagatorDenominator[p2-k2,SMP["m_Z"]]->1/(t -
    SMP["m_Z"]^2 + I \[Epsilon])}
```

Now we generate the Feynman amplitude of the diagram by practically the same lines of codes as in the Z-A transition program.

```
enamp = FCFAConvert[CreateFeynAmp[enf, PreFactor -> 1],IncomingMomenta -> {
    k1, k2}, OutgoingMomenta -> {p1, p2},LoopMomenta ->{q},LorentzIndexNames
    -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[Beta]}, SMP -> True,
    DropSumOver -> True, UndoChiralSplittings -> True] /. wr // Contract
```

As mentioned before, the output is a list of one element, which is the wanted amplitude. We extract it from the list by:

[2]this is a crucial detail for obtaining the correct total cross section due to the pole at M_Z^2

```
im = enamp[[1]] // Simplify
```

To calculate the cross section of this process, where in the CM[3] frame have the form [16]

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi(E_1+E_2)}\right)^2 \frac{|p_f|}{|p_i|}|\mathcal{M}|^2 \tag{3.1.6}$$

Where in the relativistic limit ($m \simeq 0$) and with Mandelstam variables, it reduces to:

$$\frac{d\sigma}{d\Omega} = \frac{(\hbar c)^2}{64\pi^2 s}|\mathcal{M}|^2 \tag{3.1.7}$$

Where $d\Omega$ is the differential solid angle ($d\Omega = d\cos\theta d\phi$) and $\hbar$ (c) being the reduced Plank's (speed of light in vacuum) constant, while $\mathcal{M}$ is the Feynman amplitude which we have obtained from the last line of code. According to 3.1.7 we need to calculate the square of the amplitude. This is done by the following lines:

```
ims = FCRenameDummyIndices[ComplexConjugate[im]]
msqrd = im ims // PropagatorDenominatorExplicit
```

Where the first line is for calculating the complex conjugate and renaming its dummy indices (Lorentz indices). And the second line calculates their product, which gives us $|\mathcal{M}|^2$. Now, to employ Mandelstam variables and average over fermions sums[4]. We execute:

```
FCClearScalarProducts[];
SetMandelstam[s, t, u, k1, k2, -p1, -p2, 0, 0, 0, 0];
res = FermionSpinSum[msqrd, ExtraFactor -> 1/4] /. {DiracTrace -> Tr} //
    Contract // PropagatorDenominatorExplicit // Simplify
res = res /. SMP["m_e"] -> 0
```

Here the first command is just for clearing any previous definitions of scalar products. Where the second command defines the Mandelstam variables (s,t,u) in terms of incoming (k_1,k_2) and outgoing momenta (p_1,p_2)[5] and sets them on-shell.

$$k_1^2 = m_e^2 \tag{3.1.8}$$
$$k_2^2 = 0 \tag{3.1.9}$$
$$p_1^2 = m_e^2 \tag{3.1.10}$$
$$p_2^2 = 0 \tag{3.1.11}$$

But since they are scattering at high energy (50 to 2500 GeV), it is a good approximation setting the electron's mass (that's of the order $0,511$ GeV) to be zero. The third line is

[3] center of mass

[4] averaging over initial and final fermions spins is a result of the ignorance on the initial and final spin states of the fermions, thus it's an unpolarized process

[5] note the "Set Mandelstam" command defines $t = (k_1+p_1)^2$ and $u = (k_1+p_2)^2$, hence the minus "-" signs for p_1 and p_2 in the command

for evaluating the trace method which is based on the relation:

$$\frac{1}{2}\sum_s \overline{u}_\alpha(p,s)A_{\alpha\beta}u_\beta(p,s) = Tr(\not{p}\, A) \tag{3.1.12}$$

And computes the traces. Then outputs the result in terms of Mandelstam variables. The fourth line is just to ensure that there are no electron mass terms. We got the following output:

$$\frac{e^4 m_Z^4\left(-4m_W^2 m_Z^2\left(3tm_e^2+s^2+2u^2\right)+4m_W^4\left(2tm_e^2+s^2+u^2\right)+m_Z^4\left(4tm_e^2+s^2+4u^2\right)\right)}{16m_W^4\left(m_W^2-m_Z^2\right)^2\left(-2tm_Z^2+m_Z^4+t^2+\varepsilon^2\right)} \tag{3.1.13}$$

We see that it is in terms of all three Mandelstam variables. But these variables are not independent. Only two of them are. In the case of zero mass, and in the CM frame, the relation of t and u in terms of s(which is the square of the total energy of collision) and the angle of diffusion θ, is given by:

$$\begin{matrix} t = -\frac{s}{2}(1-x) \\ u = -\frac{s}{2}(1+x) \end{matrix} \quad ; \quad \{x = \cos\theta \tag{3.1.14}$$

Substituting these relations in the obtained result is done by:

```
ans0 = res.{t -> -s/2 (1 - x), u -> - s/2 (1 + x)} // Simplify
```

We now have the desired expression to calculate the total cross section.

$$\sigma_{TOT}(\sqrt{s}) = \frac{2\pi(\hbar c)^2}{64\pi^2 s}\int f(s,x)dx \tag{3.1.15}$$

Where here $f(s,x)$ is the result of $|\mathcal{M}|^2$ in term of s and $x(=\cos\theta)$. The 2π factor comes from the integration aver $d\phi$. The next lines of commands are:

```
hbarc2 = (0.19732697*10^-13)^2*10^36;
ans = ans0*2 Pi (hbarc2)/(64 Pi^2 s) /. {SMP["m_Z"] -> 91, SMP["m_W"] ->
    80, SMP["e"] -> Sqrt[4 Pi/137], s -> y^2} // Simplify
```

We want for the cross section to be in Pico-barn unit. This is taken care of by the first line. The second command is just $\frac{d\sigma}{dx}$, where we have substituted the masses and the electric charge by:

$$M_Z = 91\ GeV$$
$$M_W = 80\ GeV$$
$$e^2 = \frac{4\pi}{137}$$

In addition to that, we made a change of variable $s \to y^2$ for the purpose of plotting the graph of σ_{TOT}(Figure3.3) in terms of $\sqrt{s}$. Finally, by executing the following lines:

```
resf = Integrate[ans, x, Assumptions -> {x, y} \[Element] Reals];
```

```
int1 = resf /. x -> -0.99;
int2 = resf /. x -> 0.99;
res0 = int2 - int1 /. \[Epsilon] -> 200 // Simplify;
gr = Plot[res0, {y, 50, 2000}, PlotTheme -> "Scientific", PlotRange -> Full
    ]
```

Where we have performed the $x(=\cos\theta)$ integration over the interval $[-0.99, 0.99]$ with the value of ε to be 200 GeV[6]. We obtain:

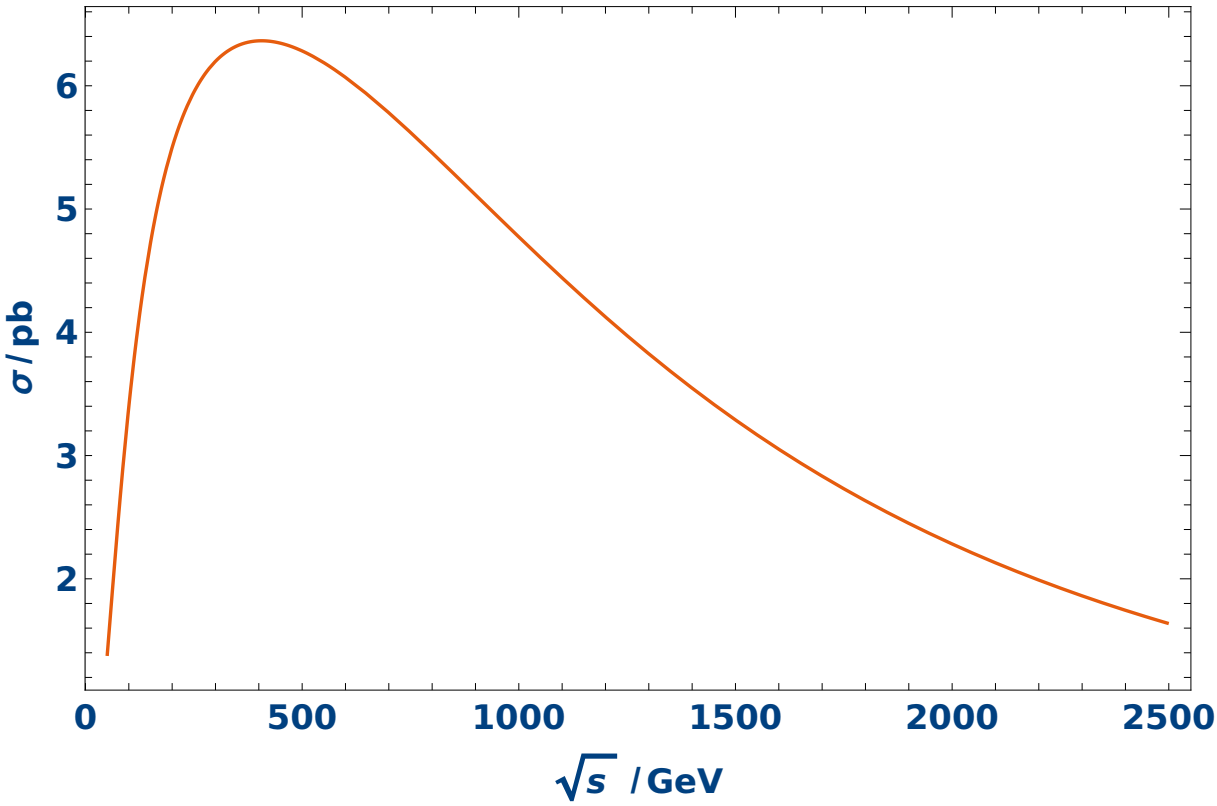


Figure 3.3: Total cross section of tree level

[6]the value of ε is irrelevant in the final result

Now that we have calculated the tree level total cross section with the help of FeynCalc(and FeynArts), we shall consider in the next section the electroweak radiative corrections of one-loop order.

3.2 Next to Leading Order (NLO) total cross section

- **Steps for NLO calculation:**

The plan to carry out, to perform Next to Leading Order (NLO) calculation for the process $e\nu_\mu \to e\nu_\mu$, can be drawn up as follows:

1. Draw all Feynman diagrams for the process $e\nu_\mu \to e\nu_\mu$ up to one loop using FeynArts.

2. Introduce through FeynArts Feynman rules of $\mathbb{EWSM}$ to create Feynman amplitude $\mathcal{M}_{tree}$ for tree diagram and $\mathcal{M}_{loop}$ for all the loops diagrams. Here we introduce a fictitious mass for the virtual photons.

3. Generate through FormCalc Feynman amplitude $\mathcal{M}_{CT}$ for self energies and vertex counter terms appearing in our process.

4. The loop contribution jointed to counter terms is then: $\delta\mathcal{M} = \mathcal{M}_{loop} + \mathcal{M}_{CT}$.

5. We can than get the total amplitude by:

$$\mathcal{M}_v = \mathcal{M}_{tree} + \delta\mathcal{M} = \mathcal{M}_{tree} + \mathcal{M}_{loop} + \mathcal{M}_{CT}$$

   which is UV divergences free.

6. We then get:

$$\frac{d\sigma_v}{d\Omega} = \frac{(\hbar c)^2}{64\pi^2 s}|\mathcal{M}_v|^2 \quad \text{and} \quad \sigma_v = \int \frac{d\sigma}{d\Omega} d\Omega \tag{3.2.1}$$

$$|\mathcal{M}_v|^2 = |\mathcal{M}_{tree} + \delta\mathcal{M}|^2 \simeq |\mathcal{M}_{tree}|^2 + 2Re(\mathcal{M}_{tree}\delta\mathcal{M}) \tag{3.2.2}$$

   this σ_v is still IR divergent.

7. To get rid of this IR divergence, we have to introduce initial and final soft photon radiations connected to the charge fermionic line of the process $e\nu_\mu \to e\nu_\mu$.

8. So we draw the tree level Feynman diagrams for the process $e\nu_\mu \to e\nu_\mu\gamma$

9. We include Feynman rules to get the amplitude $\mathcal{M}_{soft}$ for all diagrams of the previous point.

10. Then we get $\frac{d\sigma_R}{d\Omega} \sim |\mathcal{M}_{soft}|^2$ and $\sigma_R = \int_\lambda^{\Delta E} \frac{d\sigma_R}{d\Omega} d\Omega$ which is also IR divergent[7].

[7]Note that here we integrate the photon energy from the fictitious mass of photon till the calorimeter threshold

11. Finally, by adding all terms we get:

$$\sigma = \sigma_v + \sigma_R \tag{3.2.3}$$

and thanks to Kinoshita-Lee-Neuenberg theorem[27] that ensure the IR divergent terms then compensates them self. We get then a result simultaneously UV and IR divergences free.

12. Those steps are incorporated in the Mathematica code listed in appendix F, written initially by T.Hann[18] and that we adapted to the process $e\nu_\mu \to e\nu_\mu$.

13. The execution to this program for unpolarized particles leads to total cross section ploted in figure 3.6

- **The origin of IR divergences in loops**

Since massless gauge boson propagator can be written as $\frac{g^{\mu\nu}}{k^2}$, when contained inside a loop, it leads to an integral of the form (in D dimension):

$$\int \frac{d^D k}{k^2},$$

which is a special case of a general one,

$$\int \frac{d^D q}{(-q^2)^\alpha} \qquad ; \qquad \alpha > 0 \tag{3.2.4}$$

Examining this latter by first preforming a wick rotation, we get for $Q^2 = -q^2$:

$$\begin{aligned}\int \frac{d^D q}{(-q^2)^\alpha} &= i\frac{\pi^{D/2}}{\Gamma(D/2)}\int_0^{+\infty}(Q^2)^{\frac{D}{2}-\alpha-1}dQ^2 \\ &= i\frac{\pi^{D/2}}{\Gamma(D/2)}\left[\frac{(Q^2)^{\frac{D}{2}-\alpha}}{\frac{D}{2}-\alpha}\right]_0^{+\infty}\end{aligned} \tag{3.2.5}$$

A first look inspection reveals, if $\frac{D}{2}-\alpha > 0 \Rightarrow D > 2\alpha$ then 3.2.4 diverge for $Q^2 \to +\infty$. We say that we have a UV divergence. And if $\frac{D}{2}-\alpha < 0 \Rightarrow D < 2\alpha$ then 3.2.4 show another divergent behavior for $Q^2 \to 0$. We say that we have an IR divergence. To separate these two kinds of divergences manifests from the same integral, we cut this integral into two parts:

$$\begin{aligned}\int \frac{d^D q}{(-q^2)^\alpha} &= i\frac{\pi^{D/2}}{\Gamma(D/2)}\left[\int_0^{\Lambda^2}(Q^2)^{\frac{D}{2}-\alpha-1}dQ^2 + \int_{\Lambda^2}^{+\infty}(Q^2)^{\frac{D}{2}-\alpha-1}dQ^2\right] \\ &= \frac{i\pi^{D/2}}{\Gamma(D/2)}\int_0^{\Lambda^2}(Q^2)^{\frac{D}{2}-\alpha-1}dQ^2 + \frac{i\pi^{D/2}}{\Gamma(D/2)}\int_{\Lambda^2}^{+\infty}(Q^2)^{\frac{D}{2}-\alpha-1}dQ^2 \\ &= \frac{i\pi^{D_{IR}/2}}{\Gamma(D_{IR}/2)}\left[\frac{(Q^2)^{\frac{D_{IR}}{2}-\alpha}}{\frac{D_{IR}}{2}-\alpha}\right]_0^{\Lambda^2} + \frac{i\pi^{D_{UV}/2}}{\Gamma(D_{UV}/2)}\left[\frac{(Q^2)^{\frac{D_{UV}}{2}-\alpha}}{\frac{D_{UV}}{2}-\alpha}\right]_{\Lambda^2}^{+\infty}\end{aligned} \tag{3.2.6}$$

Where for the first term in the right hand side of 3.2.6 $D_{IR} > 2\alpha$ get a regularized term (without divergence), and for the second term, we must take $D_{UV} < 2\alpha$ to avoid

divergence at high energies area.
Thus we have:

$$\int \frac{d^D q}{(-q^2)^\alpha} = \frac{i\pi^{D_{IR}/2}}{\Gamma(D_{IR}/2)} \frac{(\Lambda^2)^{\frac{D_{IR}}{2}-\alpha}}{\frac{D_{IR}}{2}-\alpha} - \frac{i\pi^{D_{UV}/2}}{\Gamma(D_{UV}/2)} \frac{(\Lambda^2)^{\frac{D_{UV}}{2}-\alpha}}{\frac{D_{UV}}{2}-\alpha} \tag{3.2.7}$$

by taking these definitions:

$$\varepsilon_{UV} = \frac{D_{UV}}{2} - \alpha = \frac{D_{UV} - 2\alpha}{2} \tag{3.2.8}$$

$$\varepsilon_{IR} = \frac{D_{IR}}{2} - \alpha = \frac{D_{IR} - 2\alpha}{2} \tag{3.2.9}$$

Then 3.2.6 becomes:

$$\int \frac{d^D q}{(-q^2)^\alpha} = \frac{i\pi^{\varepsilon_{IR}+\alpha}}{\Gamma(\varepsilon_{IR}+\alpha)} \frac{(\Lambda^2)^{\varepsilon_{IR}}}{\varepsilon_{IR}} - \frac{i\pi^{\varepsilon_{UV}+\alpha}}{\Gamma(\varepsilon_{UV}+\alpha)} \frac{(\Lambda^2)^{\varepsilon_{UV}}}{\varepsilon_{UV}} \tag{3.2.10}$$

We see that for $D_{IR} \to 2\alpha$ and $D_{UV} \to 2\alpha$, we have poles corresponding to infrared and ultraviolet divergences.

- **More details on dealing with IR divergences**

Having cured the UV divergences of the process $e\nu_\mu \to e\nu_\mu$ through renormalization procedure, we are left with only IR divergences. To fix this issue[8], we need to add the contribution of the process $e\nu_\mu \to e\nu_\mu\gamma$ (real soft photon emission) to the original one. This is due to the fact that detectors have an energy threshold ΔE that must be exceeded in order to trigger the detection of the particle.
This means that photon that are produced by the scattering event and have an energy less than ΔE are not detected. This gap of ignorance must be taking into account and summed over all the photon energy from 0 to ΔE.
We have then in our one-loop calculation to deal also with the process:

$$e(p_1) + \nu_\mu(p_2) \to e(p_3) + \nu_\mu(p_4) + A(p_5) \tag{3.2.11}$$

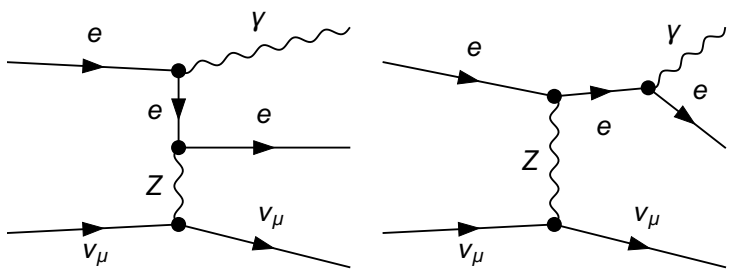


Figure 3.4: Tree level Feynman diagrams for the process $e\nu_\mu \to e\nu_\mu\gamma$

A straightforward application of Feynman rules (Appendix B) results in the following

expressions for $\mathcal{M}_1$(initial state radiation) and $\mathcal{M}_2$(final state radiation):

$$\begin{aligned}\mathcal{M}_1 =& ie^3 m_{Z,W}\bar{u}_e(p_3)\gamma_\nu \frac{(\not{p}_1 - \not{p}_5) + m_e}{(p_1-p_5)^2 - m_e^2}\gamma_\mu u_e(p_1) \\ &\times \frac{\varepsilon^\mu(p_5)}{(p_2-p_4)^2 - M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\left(\frac{1-\gamma^5}{2}\right)u_{\nu_\mu}(p2) \qquad (3.2.12)\\ \mathcal{M}_2 =& ie^3 m_{Z,W}\bar{u}_e(p_3)\gamma_\mu \frac{(\not{p}_3 + \not{p}_5) + m_e}{(p_3+p_5)^2 - m_e^2}\gamma_\nu u_e(p_1) \\ &\times \frac{\varepsilon^\mu(p_5)}{(p_2-p_4)^2 - M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\left(\frac{1-\gamma^5}{2}\right)u_{\nu_\mu}(p2) \qquad (3.2.13)\end{aligned}$$

Where,

$$m_{Z,M} = \frac{M_Z^2}{2M_W\sqrt{M_Z^2 - M_W^2}} \qquad (3.2.14)$$

Since the external lines are taken to be on mass shell particles. The denominator of the propagator in $\mathcal{M}_1$ an $\mathcal{M}_2$ is then:

$$\begin{aligned}(p_1-p_5)^2 - m_e^2 &= p_1^2 + p_5^2 - 2p_1\cdot p_5 - m_e^2 \\ &= -2p_1\cdot p_5 \qquad ; \quad (p_1^2 = m_e^2\,;\, p_5^2 = 0) \qquad (3.2.15)\\ &= -2E_5(E_1 - |\vec{p}_1|\cos(\theta_{15})) \quad ; \quad (E_5 = |\vec{p}_5|)\end{aligned}$$

and

$$\begin{aligned}(p_3+p_5)^2 - m_e^2 &= p_3^2 + p_5^2 + 2p_3\cdot p_5 - m_e^2 \\ &= 2p_3\cdot p_5 \qquad (3.2.16)\\ &= 2E_5(E_3 - |\vec{p}_3|\cos(\theta_{35}))\end{aligned}$$

Which means that for photons of energy E_5 going to 0, the amplitude diverges. This is exactly what we need to extract from this process. We are interested in calculating only the divergent part of $|\mathcal{M}_{soft}|^2 = |\mathcal{M}_1 + \mathcal{M}_2|^2$. Therefore, we will take $p_5 = 0$ wherever it is possible (when it does not diverges). Hence we get the following expressions:

$$\begin{aligned}\mathcal{M}_1 =& ie^3 m_{Z,W}\bar{u}_e(p_3)\gamma_\nu \frac{(\not{p}_1 + m_e)}{-2p_1\cdot p_5}\gamma_\mu u_e(p_1) \\ &\times \frac{\varepsilon^\mu(p_5)}{(p_2-p_4)^2 - M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\left(\frac{1-\gamma^5}{2}\right)u_{\nu_\mu}(p2) \qquad (3.2.17)\\ \mathcal{M}_2 =& ie^3 m_{Z,W}\bar{u}_e(p_3)\gamma_\mu \frac{(\not{p}_3 + m_e)}{2p_3\cdot p_5}\gamma_\nu u_e(p_1) \\ &\times \frac{\varepsilon^\mu(p_5)}{(p_2-p_4)^2 - M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\left(\frac{1-\gamma^5}{2}\right)u_{\nu_\mu}(p2) \qquad (3.2.18)\end{aligned}$$

Applying now Dirac matrix properties then making use of Dirac's equation:

$$(\not{p}_1 + m_e)\gamma_\mu = 2p_{1\mu} - \gamma_\mu(\not{p}_1 - m_e) \qquad (3.2.19)$$

$$\gamma_\mu(\not{p}_3 + m_e) = 2p_{1\mu} - (\not{p}_3 - m_e)\gamma_\mu \qquad (3.2.20)$$

$$\bar{u}(p_3)(\not{p}_3 - m_e) = 0 \qquad (3.2.21)$$

$$(\not{p}_1 - m_e)u(p_1) = 0 \qquad (3.2.22)$$

We obtain:

$$\mathcal{M}_1 = -e\frac{p_{1\mu}}{p_1 \cdot p_5}\varepsilon^\mu(p_5)\left[ie^2 m_{Z,W}\bar{u}_e(p_3)\gamma_\nu u_e(p_1)\frac{1}{(p_2-p_4)^2-M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\frac{1-\gamma^5}{2}u_{\nu_\mu}(p_2)\right] \tag{3.2.23}$$

$$\mathcal{M}_2 = e\frac{p_{3\mu}}{p_3 \cdot p_5}\varepsilon^\mu(p_5)\left[ie^2 m_{Z,W}\bar{u}_e(p_3)\gamma_\nu u_e(p_1)\frac{1}{(p_2-p_4)^2-M_Z^2}\bar{u}_{\nu_\mu}(p_4)\gamma^\nu\frac{1-\gamma^5}{2}u_{\nu_\mu}(p_2)\right] \tag{3.2.24}$$

The expression between brackets $[\cdots]$ is exactly the amplitude of the $e\nu_\mu \to e\nu_\mu$ tree level process $\mathcal{M}$, so we get:

$$\mathcal{M}_1 = -e\frac{p_{1\mu}}{p_1 \cdot p_5}\varepsilon^\mu(p_5)\mathcal{M}_{tree} \tag{3.2.25}$$

$$\mathcal{M}_2 = e\frac{p_{3\mu}}{p_3 \cdot p_5}\varepsilon^\mu(p_5)\mathcal{M}_{tree} \tag{3.2.26}$$

In this infra-red approximation $\mathcal{M}_{tree}$ is written by:

$$\begin{aligned}\mathcal{M}_R \equiv \mathcal{M}_{soft} &= \mathcal{M}_1 + \mathcal{M}_2 \\ &= e\left[\frac{p_{3\mu}}{p_3p_5} - \frac{p_{1\mu}}{p_1p_5}\right]\mathcal{M}_{tree}\end{aligned} \tag{3.2.27}$$

To calculate $|\mathcal{M}_{soft}|^2$ we will make use of the formula:

$$\sum_{polarization}\varepsilon^\mu(p_5)\varepsilon^{*\nu}(p_5) = g^{\mu\nu} \tag{3.2.28}$$

So the square of the soft photon amplitude is obtained by the following way:

$$\begin{aligned}|\mathcal{M}_{soft}|^2 &= \mathcal{M}_{soft}\mathcal{M}^*_{soft} \\ &= e^2\left[\frac{p_{3\mu}}{p_3p_5} - \frac{p_{1\mu}}{p_1p_5}\right]\left[\frac{p_{3\nu}}{p_3p_5} - \frac{p_{1\nu}}{p_1p_5}\right]\varepsilon^\mu(p_5)\varepsilon^{*\nu}(p_5)|\mathcal{M}_{tree}|^2 \\ &= e^2\left[\frac{p_3}{p_3p_5} - \frac{p_1}{p_1p_5}\right]^2|\mathcal{M}_{tree}|^2 \\ &= e^2\left[\frac{m_e^2}{(p_3p_5)^2} + \frac{m_e^2}{(p_1p_5)^2} - 2\frac{p_1p_3}{p_1p_5p_3p_5}\right]|\mathcal{M}_{tree}|^2\end{aligned} \tag{3.2.29}$$

By this formula of $|\mathcal{M}_{soft}|^2$ the cross section can be calculated. When doing so, we must integrate the real photon's four-momentum p_5 over the interval 0 to ΔE. If previously in the loop integrals, IR divergences are regulated through dimensional regularization; and over the interval λ to ΔE if we regulate through giving a fictitious mass λ to photons. The total unpolarized cross for the process $e\nu_\mu \to e\nu_\mu$ as seen by detectors is:

$$\sigma_T = \sigma_v + \sigma_{soft} \tag{3.2.30}$$

Where σ_{soft} denote the real soft photon cross section, below the Hirshold resolution of detector for seeing photons. And σ_v is the virtual cross section including tree level terms, one loop terms and counter-terms.

In summary:

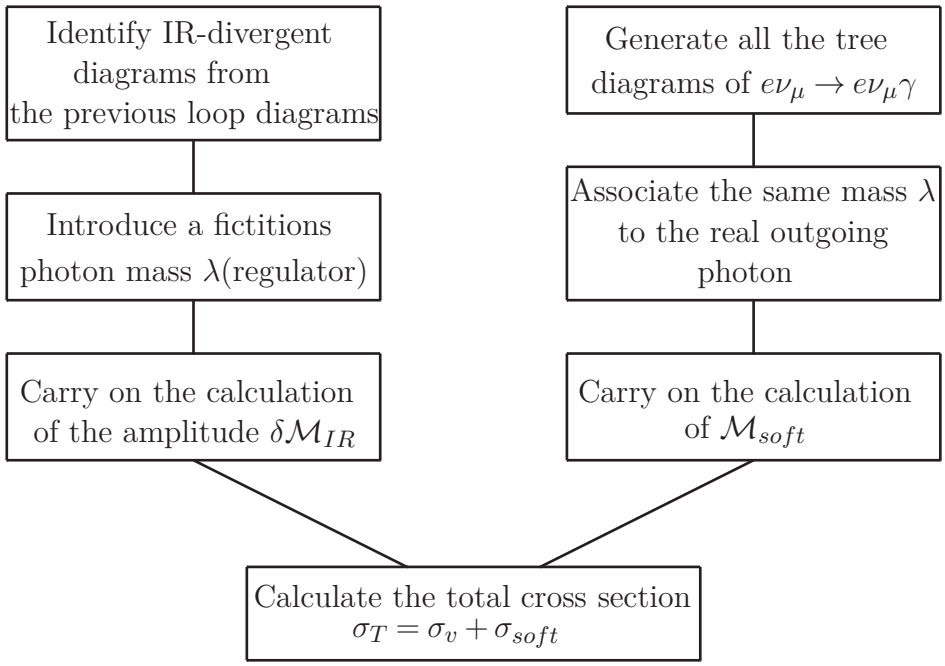


Figure 3.5: Treatment of IR-divergences

σ_T is the finite cross section. It contains no UV-divergences nor IR-divergences.

- **Numerical result and discussion:**

Even with FeynCalc this is still a complicated time consuming task to incorporate all of the above steps. Fortunately this is done by Tomas Hahn [18] in a more automatic Mathematica package called FormCalc[18]. This package deals with all of the above-mentioned problems and eliminates them automatically giving in the end a nice finite cross section of one-loop order correction. So we switch from FeynCalc (which is semi-automatic) to FormCalc (the fully automatic) knowing all the internal steps of the calculation. With the following parameter values implemented in FormCalc.

SM parameters	
α	1/137.03599907399999
G_f	$1.1663786999999999 \times 10^{-5} GeV^{-2}$
θ_W	0.22289722252391825
Gauge-boson masses	
M_W	80.385000000000005 GeV
M_Z	91.187600000000003 GeV
Fermion masses	
m_e	$5.1099892800000001 \times 10^{-4}$ GeV
m_{ν_μ}	0.10565837150000000 GeV
m_τ	1.7768200000000001 GeV
m_u	$7.3559471591835346 \times 10^{-2}$ GeV
m_c	1.2749999999999999 GeV
m_t	173.21000000000001 GeV
m_d	$7.3559471591835346 \times 10^{-2}$ GeV
m_s	$9.5000000000000001 \times 10^{-2}$ GeV
m_b	4.6600000000000001 GeV
Higgs masses	
m_H	125.00000000000000 GeV

And call for LoopTools[18] to perform numerical loop integration using Passarino-Veltmane scalar integrals[14]. For unpolarized beams, we obtain the total cross section presented in figure3.6:

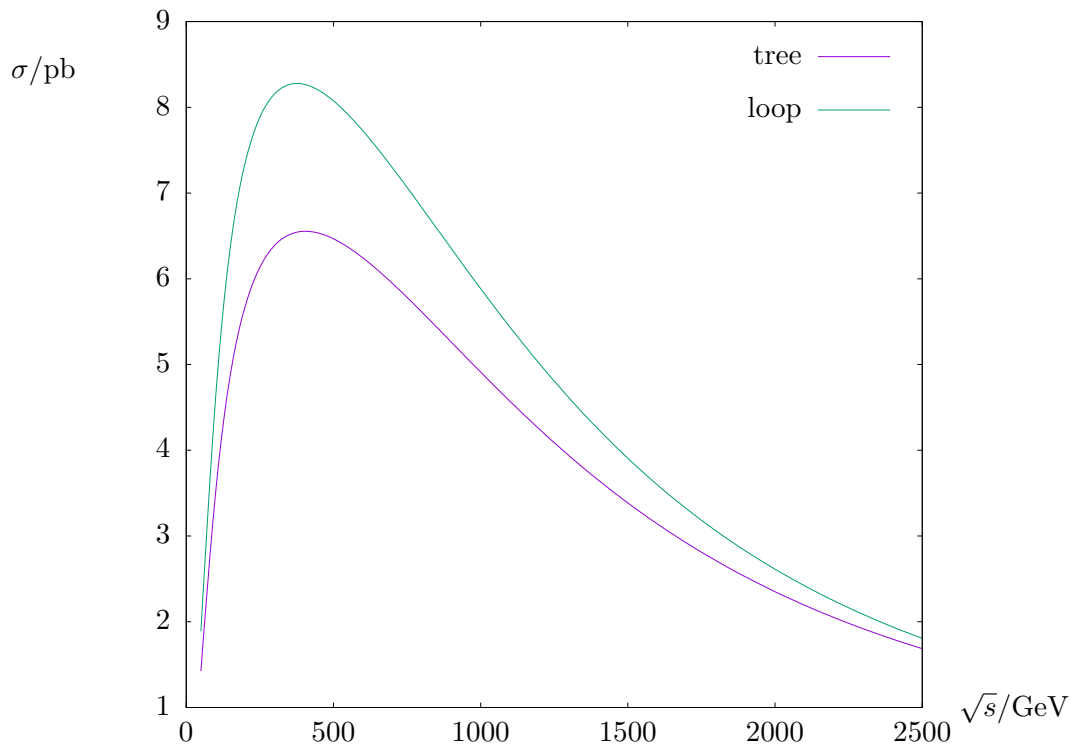


Figure 3.6: Total cross section for the unpolarized process $e\nu_\mu \to e\nu_\mu$ versus $\sqrt{s}$

- **Discussion:**

This result is what is expected to be connected with experimental data. We can see that the tree level cross section (in figure 3.6) matches exactly our previous cross section ploted in Figure 3.3. And the radiative corrections effects are shown by the one-loop level plot. We notice that the contribution of these corrections varies as energy does. Where at low energies it is practically the same as the tree level (insignificant contribution). And then it increases until it reaches a maximum effect around 400 GeV[8], afterwards the radiative corrections contributions decreases as energy increase, but most importantly it is free from any divergences.

[8]For the peak at $\sim$ 400 GeV, we initially obtained a resonance high peak at $M_Z \approx 91$ GeV. Then we introduced the width $M_Z^2 \to M_Z^2 - i\Gamma_Z M_Z$ in the Z boson propagator by replacing Γ_Z . By doing so the sharp resonance become smooth with apparently low peak at around 400 GeV

Conclusion

In this master thesis, after pointing out that sticking to the picture of isolated asymptotically free one particle entering in a zone of interaction provides us with a tool to absorb UV-divergences, hence renormalization. We have presented the on-shell renormalization method in the frame of electroweak theory to cure UV-divergences. We have provided a computational program for determining the relevant renormalization constant and calculating the tree level cross section of a concrete unpolarized process $e\nu_\mu \to e\nu_\mu$ with the help of the Mathematica packages FeynCalc and FeynArts. To implement the radiative corrections of one-loop order to the cross section formula, we used another Mathematica package FormCalc which in its internal code contains all the steps that gets rid of the plague of infinities and gives the desired finite result. By that we believe that we have achieved the goal of this thesis, which is to provide the necessary background knowledge of the quantum field theory framework to be able to compare with experimental data. Mastering thesis information, one can go further and analyses recent experimental data and try new models, ideas and hopeful contribute to develop the current understanding of the physical world.

> "Part of the art of making progress in science is recognizing which problems are ready to be solved."

Frank Wilczek

Appendix A

Some computation details

A.1 Proof of $\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = 0$

From chapter one, we have used

$$\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = 0 \tag{A.1.1}$$

To determine the Faddeev-Popov Lagrangian term $\mathcal{L}_{FP}$.
Here we provide a proof for it

$$\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = \tilde{\lambda}c^a\delta^c(\delta^b(F_i^a\phi^i)c^b) \tag{A.1.2}$$

Where here we have used 1.1.35

$$\begin{aligned}
\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) &= \delta^{BRS}[\delta^b(F_i^a\phi^i)]c^b + \delta^b(F_i^a\phi^i)\delta^{BRS}c^b \\
&= \tilde{\lambda}c^c\delta^c\delta^b(F_i^a\phi^i)c^b - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \\
&= \tilde{\lambda}\delta^c\delta^b(F_i^a\phi^i)c^c c^b - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \\
&= \frac{\tilde{\lambda}}{2}\delta^c\delta^b(F_i^a\phi^i)c^c c^b + \frac{\tilde{\lambda}}{2}\delta^b\delta^c(F_i^a\phi^i)c^b c^c - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \\
&= \frac{\tilde{\lambda}}{2}\underbrace{[\delta^c\delta^b - \delta^b\delta^c]}_{[\delta^c,\delta^b]=g^c f^{cbd}\delta^d}(F_i^a\phi^i)c^c c^b - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \\
&= \frac{\tilde{\lambda}}{2}g^c f^{cbd}\delta^d(f_i^a\phi^i)c^c c^b - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d
\end{aligned} \tag{A.1.3}$$

Using a cyclic change og indices $\{d \to b \to c \to d\}$ on the first term

$$\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = \frac{\tilde{\lambda}}{2}g^d f^{dcb}\delta^b(f_i^a\phi^i)c^d c^c - \frac{\tilde{\lambda}}{2}g^b f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \tag{A.1.4}$$

With rearranging the order of indices, we get

$$\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = \frac{\tilde{\lambda}}{2}(g^d - g^b)f^{bcd}\delta^b(f_i^a\phi^i)c^c c^d \tag{A.1.5}$$

Now thanks to 1.1.15 and $f^{0bc} = 0$, we finally find

$$\delta^{BRS}(\delta^b(F_i^a\phi^i)c^b) = 0 \tag{A.1.6}$$

A.2 An LSZ identity proof

In the last section of the first chapter, we have made use of the identity 1.2.16 to derive the LSZ formula without proof, here we give its proof as follows

$$\begin{aligned}
a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = & \int_{-\infty}^{+\infty} dt \partial_0 a_1^\dagger(t) \\
& \text{(Fundamantal theorem of calculations)} \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x \partial_0 (e^{-ikx} \overleftrightarrow{\partial_0} \phi(x)) \\
& \text{(Using 1.2.10, 1.2.17 and } \int dt \int d^3x = \int d^4x) \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{-ikx} (\partial_0^2 + \omega^2)\phi(x) \\
& \text{(evaluation of time derivation)} \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{-ikx} (\partial_0^2 + k^2 + m^2)\phi(x) \\
& \text{(using } \omega^2 = k^2 + m^2) \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{-ikx} (\partial_0^2 - \overleftarrow{\nabla}^2 + m^2)\phi(x) \\
& \text{(writing } k^2 \text{ as } -\nabla^2 \text{ acting on } e^{-ipx}) \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{-ikx} (\partial_0^2 - \overrightarrow{\nabla}^2 + m^2)\phi(x) \\
& \text{(intagrating by parts two times)} \\
= & -i \int d^3k\, f_1(\vec{x}) \int d^4x e^{ikx} (\partial^2 + m^2)\phi(x) \\
& \text{(identifuing } \partial_0^2 - \nabla^2 \text{ as } \partial^2)
\end{aligned}$$

And this completes the proof of the relation 1.2.16

A.3 A relation between $X_0 = Z_X X$ and $X_0 = X + \delta X$

In literature, renormalization of masses and couplings can be presented into two different equivalent pictures. We provide here a simple link between these two pictures.

A.3.1 Link between δm^2 and Z_m

We have: $m_0^2 = m^2 + \delta m^2$ and $m_0 = Z_m m$
So:

$$\begin{aligned} m_0^2\phi^2 &= (m^2 + \delta m^2)\phi^2 \\ m_0^2\phi^2 &= Z_m^2 m^2\phi^2 = [m^2 + (Z_m^2 - 1)m^2]\phi^2 \\ &\Rightarrow \delta m^2 = (Z_m^2 - 1)m^2 \end{aligned} \tag{A.3.1}$$

A.3.2 Link between δg and Z_g in a ϕ^n theory

We have: $g_0 = g + \delta g$ and $g_0 = Z_g g$ So:

$$g_0\phi^n = (g + \delta g)\phi^n \tag{A.3.2}$$

$$\begin{aligned} g_0\phi^n &= Z_g g\phi^n = [g + (Z_g - 1)g]\phi^n \\ &\Rightarrow \delta g = (Z_g - 1)g \end{aligned} \tag{A.3.3}$$

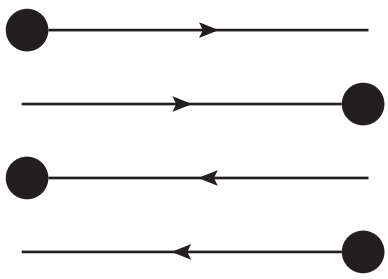
Appendix B

$\mathbb{EWSM}$ Feynman rules for use in the process $e\nu_\mu \to e\nu_\mu$ at loop level

To draw our Feynman diagrams, we should define the Feynman rules[1] which we will assume the relation between the diagrams and the mathematical formula of the amplitude. In this appendix, we will just exhibiting its. So we have in Feynman gauge ($\alpha = 1$)

B.1 Feynman rules for tree level

External lines



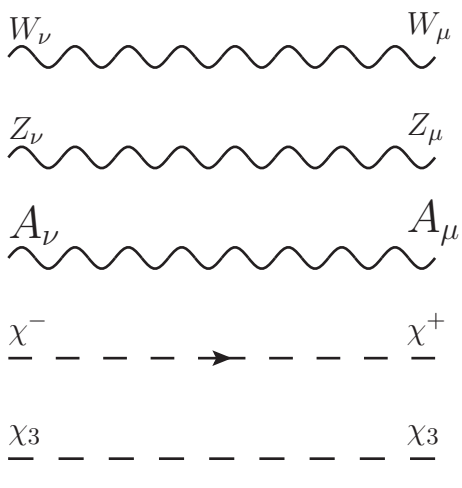
$\overline{u}(p; m_{I,i})$

$u(p; m_{I,i})$

$\overline{v}(p; m_{I,i})$

$v(p; m_{I,i})$

Propagators



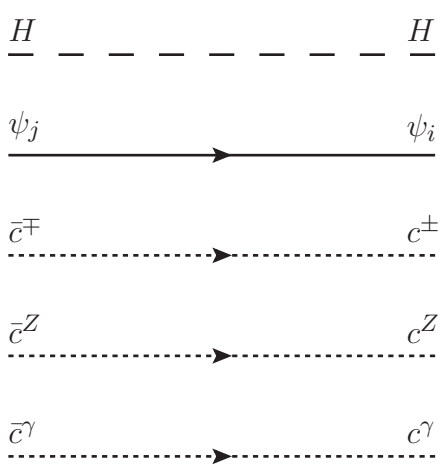
$$\frac{-ig^{\mu\nu}}{k^2 + i\varepsilon - M_W^2}$$

$$\frac{-ig^{\mu\nu}}{k^2 + i\varepsilon - M_Z^2}$$

$$\frac{-ig^{\mu\nu}}{k^2 + i\varepsilon}$$

$$\frac{-i}{M_W^2 - k^2 - i\varepsilon}$$

$$\frac{-i}{M_Z^2 - k^2 - i\varepsilon}$$



$$\frac{-i}{m_\phi^2 - k^2 - i\varepsilon}$$

$$\frac{-i}{m - k - i\epsilon} = \frac{m+k}{m^2 - k^2 - i\varepsilon}$$

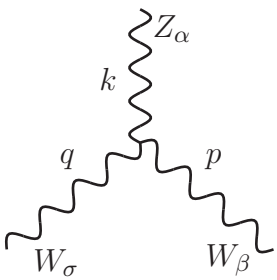
$$\frac{-i}{M_W^2 - k^2 - i\varepsilon}$$

$$\frac{-i}{M_z^2 - k^2 - i\varepsilon}$$

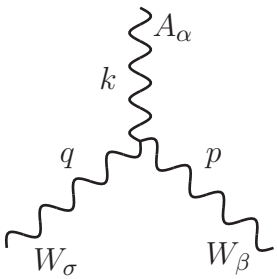
$$\frac{-i}{-k^2 - i\varepsilon}$$

Vertices in $\mathcal{L}_G$

(1) three vertices

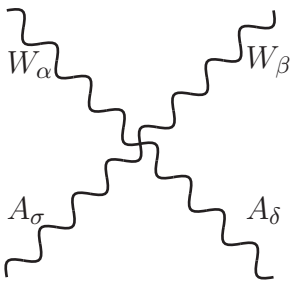


$$i\frac{eM_W}{\sqrt{M_Z^2 - M_W^2}}\{(k-p)^\sigma g^{\alpha\beta} + (p-q)^\alpha g^{\beta\sigma} + (q-k)^\beta g^{\sigma\alpha}\}$$

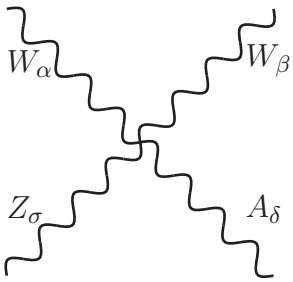


$$ie\{(k-p)^\sigma g^{\alpha\beta} + (p-q)^\alpha g^{\beta\sigma} + (q-k)^\beta g^{\sigma\alpha}\}$$

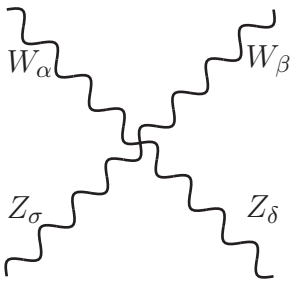
(2) four vertices



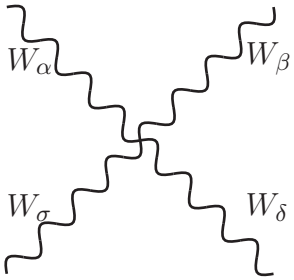
$$ie^2(g^{\alpha\sigma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\sigma} - 2g^{\alpha\beta}g^{\sigma\delta})$$



$$i\frac{e^2 M_W}{\sqrt{M_Z^2 - M_W^2}}(g^{\alpha\sigma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\sigma} - 2g^{\alpha\beta}g^{\sigma\delta})$$

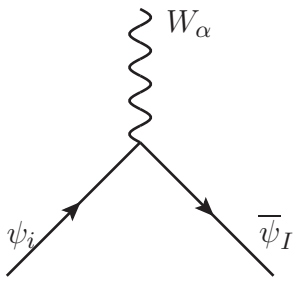


$$i\frac{e^2 M_W^2}{M_Z^2 - M_W^2}(g^{\alpha\sigma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\sigma} - 2g^{\alpha\beta}g^{\sigma\delta})$$

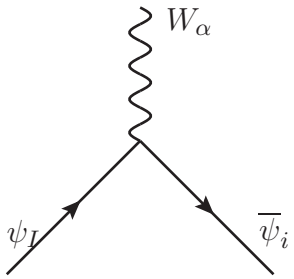


$$i\frac{e^2 M_W^2}{M_Z^2 - M_W^2}(g^{\alpha\sigma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\sigma} - 2g^{\alpha\beta}g^{\sigma\delta})$$

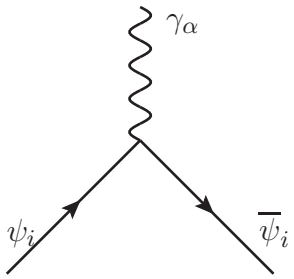
Vertices in $\mathcal{L}_F$



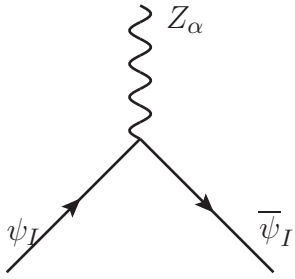
$$i\frac{eM_Z}{2\sqrt{2(M_Z^2 - M_W^2)}}\gamma_\alpha(1-\gamma_5)$$



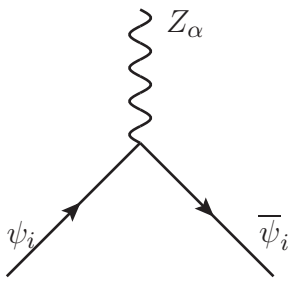
$$i\frac{eM_Z}{2\sqrt{2(M_Z^2 - M_W^2)}}\gamma_\alpha(1-\gamma_5)$$



$$ieQ_i\gamma_\alpha$$



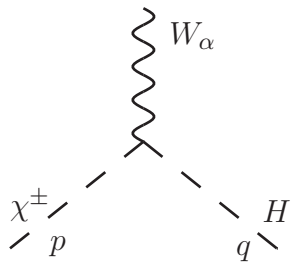
$$i\frac{eM_Z}{2M_W\sqrt{M_Z^2 - M_W^2}}\gamma_\alpha(\tfrac{1}{2} - 2Q_I\tfrac{M_Z^2 - M_W^2}{M_Z^2} - \tfrac{1}{2}\gamma_5)$$



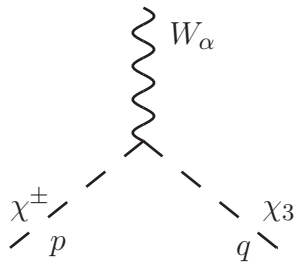
$$i\frac{eM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_\alpha(-\tfrac{1}{2}-2Q_I\tfrac{M_Z^2-M_W^2}{M_Z^2}+\tfrac{1}{2}\gamma_5)$$

Vertices in $\mathcal{L}_H$

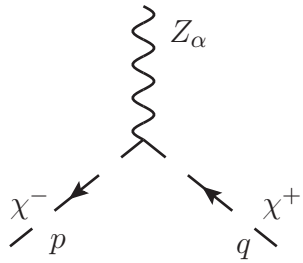
(1) three vertices



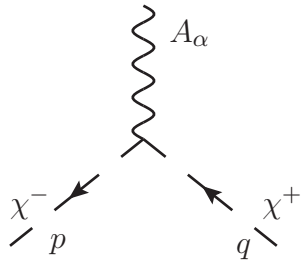
$$i\frac{ieM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



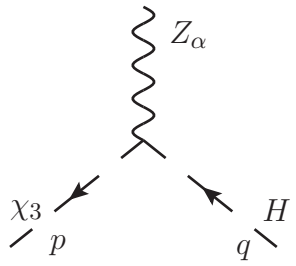
$$i\frac{\mp eM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



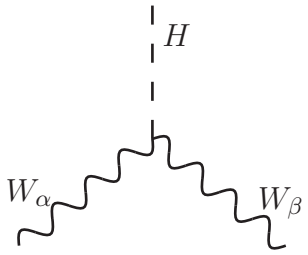
$$i\frac{e(M_Z^2-2M_W^2)}{2M_W\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



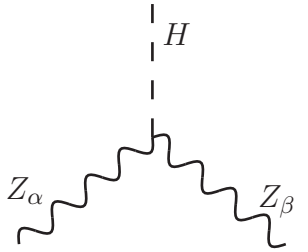
$$-ie(p-q)_\alpha$$



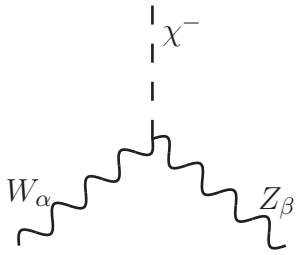
$$i\frac{ieM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



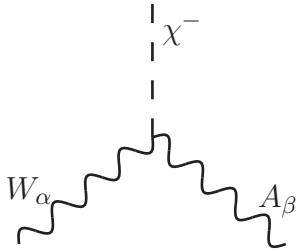
$$i\frac{eM_W M_Z}{\sqrt{M_Z^2 - M_W^2}} g_{\alpha\beta}$$



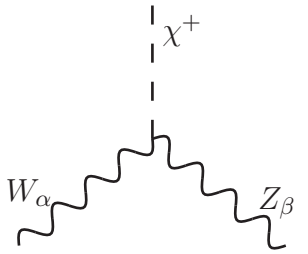
$$i\frac{eM_Z^3}{M_W\sqrt{M_Z^2 - M_W^2}} g_{\alpha\beta}$$



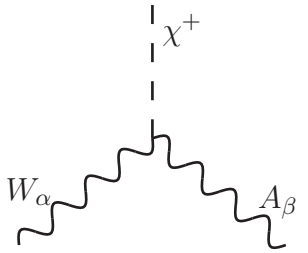
$$ie\sqrt{M_Z^2 - M_W^2}\, g_{\alpha\beta}$$



$$-ieM_W g_{\alpha\beta}$$

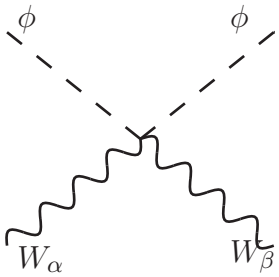


$$-ie\sqrt{M_Z^2 - M_W^2}\, g_{\alpha\beta}$$

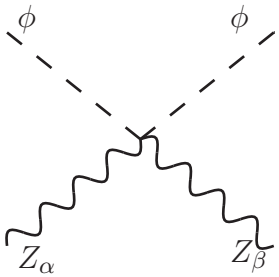


$$ieM_W g_{\alpha\beta}$$

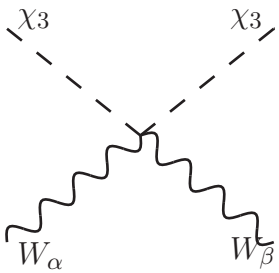
(2) four vertices



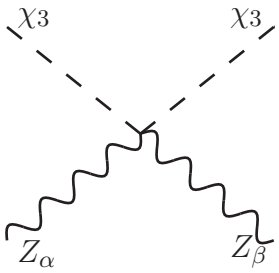
$$i\frac{e^2 M_Z^2}{2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$



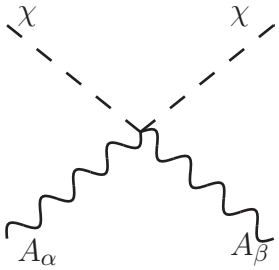
$$i\frac{e^2 M_Z^4}{2M_W^2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$



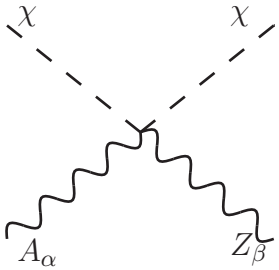
$$i\frac{e^2 M_Z^2}{2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$



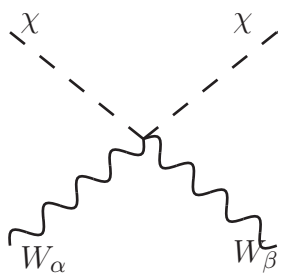
$$i\frac{e^2 M_Z^4}{2M_W^2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$



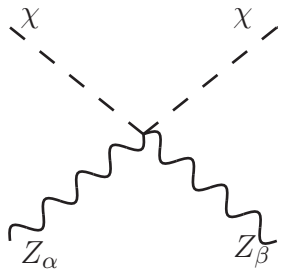
$$2ie^2 g^{\alpha\beta}$$



$$i\frac{e^2(2M_W^2 - M_Z^2)}{M_W\sqrt{M_Z^2 - M_W^2}} g^{\alpha\beta}$$

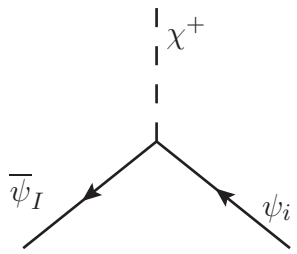


$$i\frac{e^2 M_Z^2}{2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$

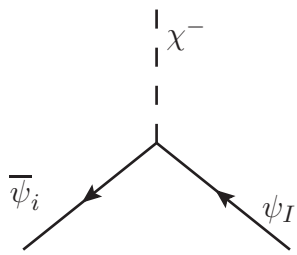


$$i\frac{e^2(2M_W^2 - M_Z^2)^2}{2M_W^2(M_Z^2 - M_W^2)} g^{\alpha\beta}$$

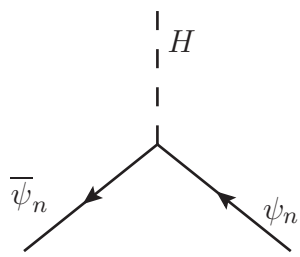
Vertices in $\mathcal{L}_M$



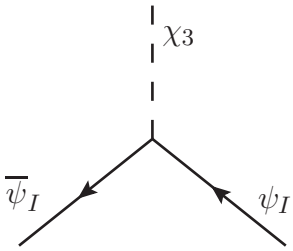
$$\frac{-ieM_Z}{2M_W\sqrt{2(M_Z^2 - M_W^2)}} U_{Ii}^{+}\{(m_i - m_I) + (m_i + m_I)\gamma_5\}$$



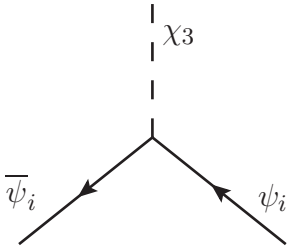
$$\frac{-ieM_Z}{2M_W\sqrt{2(M_Z^2 - M_W^2)}} U_{Ii}\{(m_I - m_i) + (m_I + m_i)\gamma_5\}$$



$$\frac{-iem_n M_Z}{2M_W\sqrt{2(M_Z^2 - M_W^2)}}$$

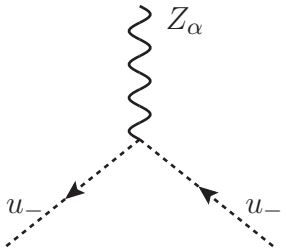


$$\frac{-iem_I M_Z}{2M_W\sqrt{2(M_Z^2-M_W^2)}}$$

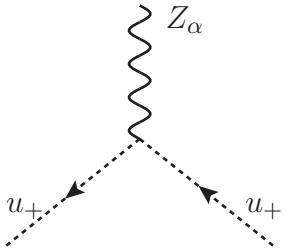


$$\frac{iem_i M_Z}{2M_W\sqrt{2(M_Z^2-M_W^2)}}$$

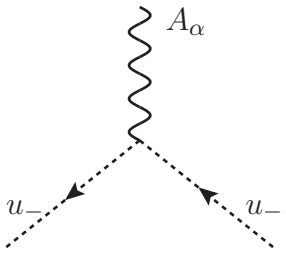
Vertices in $\mathcal{L}_{FP}$



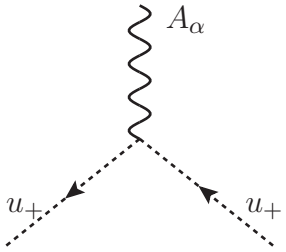
$$\frac{ieM_W}{\sqrt{M_Z^2-M_W^2}}\mathcal{P}_\alpha$$



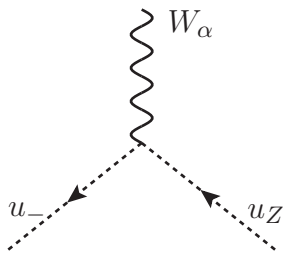
$$\frac{-ieM_W}{\sqrt{M_Z^2-M_W^2}}\mathcal{P}_\alpha$$



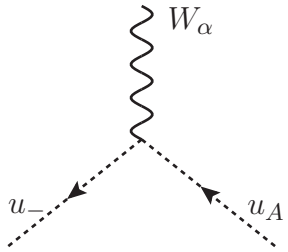
$$-ie\mathcal{P}_\alpha$$



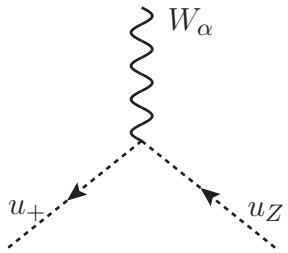
$$ie\mathcal{P}_\alpha$$



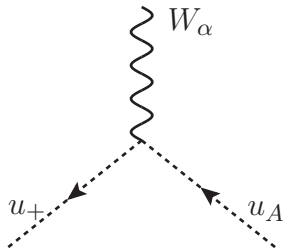
$$\frac{-ieM_W}{\sqrt{M_Z^2 - M_W^2}}\mathcal{P}_\alpha$$



$$ie\mathcal{P}_\alpha$$



$$\frac{ieM_W}{\sqrt{M_Z^2 - M_W^2}}\mathcal{P}_\alpha$$



$$-ie\mathcal{P}_\alpha$$

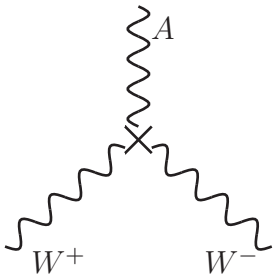
B.2 Some of Feynman rules for counter terms

now, we present the Feynman rules for the counter terms, but because of the heaviness of the task, we will just present one counter-term for the case of a propagator and one for a vertex. For more details see Aoki[1]

Counter term of the Higgs propagator

$$-\delta m_H^2 Z_H + (k^2 - m_H^2) Z_H$$
$$+3TY\frac{eM_Z}{2m_W\sqrt{m_Z^2 - m_W^2}}G_3 H^{-1} Z_H$$

Counter term of WWA vertex



$$YZ_W Z_{AA}^{1/2} + YG_1 Z_W Z_{ZA}^{1/2}$$

We have the following definition:

$$G_1 \equiv \frac{G_W}{H} \qquad ; \qquad G_3 \equiv \frac{G_Z}{G_W}$$

And

$$H = \sqrt{1 + \frac{\delta M_Z^2 - \delta M_W^2}{M_Z^2 - M_W^2}}$$

$$G_W = \sqrt{1 + \frac{\delta M_W^2}{M_W^2}}$$

$$G_Z = \sqrt{1 + \frac{\delta M_Z^2}{M_Z^2}}$$

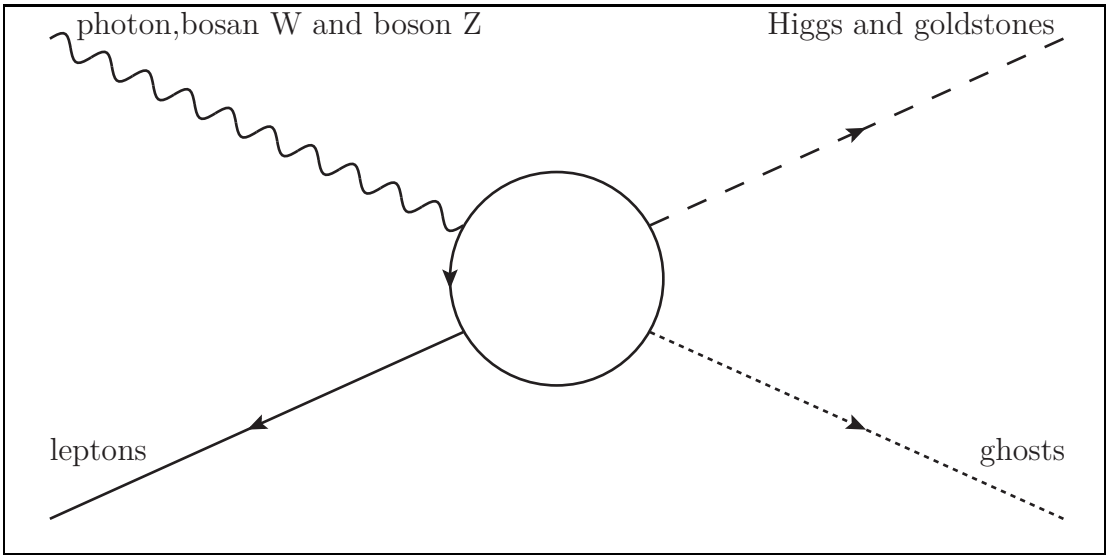
B.3 How to draw a Feynman diagram with LaTeX

To draw a Feynman diagram we need first the package `\usepackage{axodraw4j}` which define the environment and the command which give as the possibility to draw our diagrams.
To illustrate this, we give the following example.

```
\documentclass[12pt,a4paper,final]{article}

\usepackage{axodraw4j}
\usepackage{pstricks}
\usepackage{color}

\usepackage[utf8]{inputenc}
\usepackage[T1]{fontenc}

\usepackage[english]{babel}


\begin{center}
    \fcolorbox{black}{white}{
        \begin{picture}(400,200)
        \SetWidth{1.0}
        \SetColor{Black}
        \Arc[arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset
=0.2](200,100)(40,0,0)
```

```
            \Line[arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset=0.2](165,80)(10,10)
            \Photon(165,120)(10,190){4}{10}
            \Line[dash,dashsize=8,arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset=0.2](235,120)(390,190)
            \Line[dash,dashsize=2,arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset=0.2](235,80)(390,10)
            \Text(20,190)[lb]{\Black{photon,bosan W and boson Z}}
            \Text(10,30)[lb]{\Black{leptons}}
            \Text(280,190)[lb]{\Black{Higgs and goldstones}}
            \Text(360,30)[lb]{\Black{ghosts}}
            \end{picture}
            }
    \end{center}


```

And this code give us the following diagram



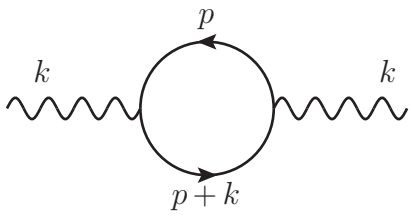
Note: before we compile, we must know certain point

- To compile our script with the package "axodraw4j" the default compiler should be "*latex* → *dvips* → *ps2pdf*".
- With this scheme of compilation, the command `\includegraphics{../../}` can't execute any image's extension, the only extension that we can use is ".ps" and ".eps"

Appendix C

Computation details for QED vacuum polarization

This is a relatively detailed example[1] of on-shell renormalization scheme with the dimensional regularization method on the QED vacuum polarization (photon self-energy). the corresponding of the first order radiative correction is



$$G^{(1)}_{\mu\nu}(k) = G^{(0)}_{\mu\mu'}(k) i\Pi_{\mu'\nu'}(k) G^{(0)}_{\nu'\nu}(k) \tag{C.0.1}$$

Where

$$\begin{aligned} i\Pi_{\mu\nu}(k) &= -(ie)^2 \int \frac{d^4p}{(2\pi)^4} Tr\left(\gamma_\mu \frac{i}{\not{p} - m + i\varepsilon} \gamma_\nu \frac{i}{\not{p} + \not{k} - m + i\varepsilon}\right) \\ &= -e^2 \int \frac{d^4p}{(2\pi)^4} \frac{Tr[\gamma_\mu(\not{p} + m)\gamma_\nu(\not{p} + \not{k} + m)]}{(p^2 - m^2 + i\varepsilon)\left((p+k)^2 - m^2 + i\varepsilon\right)} \\ &= -4e^2 \int \frac{d^4p}{(2\pi)^4} \frac{p_\mu p_\nu + p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(p^2 + pk - m^2)}{(p^2 - m^2 + i\varepsilon)\left((p+k)^2 - m^2 + i\varepsilon\right)} \end{aligned} \tag{C.0.2}$$

As we have seen in the first section of chapter 2, this integral diverges so we must regularize it first to apply the on-shell renormalization procedure on it, using dimensional regularization, we define

$$\epsilon = 4 - d$$
$$e \to e\mu^{\frac{4-d}{2}} = e\mu^{\frac{\epsilon}{2}}$$

[1] This example was strongly relied on the work of Jorge.C.Romao[30]

Then the integral would be

$$\begin{aligned} i\Pi_{\mu\nu}(k,\epsilon) &= -4e^2\mu^\epsilon \int \frac{d^dp}{(2\pi)^4} \frac{p_\mu p_\nu + p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(p^2 + pk - m^2)}{(p^2 - m^2 + i\varepsilon)\left((p+k)^2 - m^2 + i\varepsilon\right)} \\ &= -4e^2\mu^\epsilon \int \frac{d^dp}{(2\pi)^4} \frac{N_{\mu\nu}(p,k)}{(p^2 - m^2 + i\varepsilon)\left((p+k)^2 - m^2 + i\varepsilon\right)} \end{aligned} \tag{C.0.3}$$

We now need to rewrite the denominator as a single term by using the Feynman parameterization

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[ax + b(1-x)]^2} \tag{C.0.4}$$

With

$$\begin{aligned} a &= (p+k)^2 - m^2 \\ b &= p^2 - m^2 \end{aligned}$$

We obtain

$$\begin{aligned} i\Pi_{\alpha\beta}(k,\epsilon) &= -4e^2\mu^\epsilon \int_0^1 dx \int \frac{d^dp}{(2\pi)^4} \frac{N_{\mu\nu}(p,k)}{[x(p+k)^2 - xm^2 + (1-x)(p^2 - m^2) + i\varepsilon]^2} \\ &= -4e^2\mu^\epsilon \int_0^1 dx \int \frac{d^dp}{(2\pi)^4} \frac{N_{\mu\nu}(p,k)}{[p^2 + k \cdot px + xk^2 - m^2 + i\varepsilon]^2} \\ &= -4e^2\mu^\epsilon \int_0^1 dx \int \frac{d^dp}{(2\pi)^4} \frac{N_{\mu\nu}(p,k)}{[(p+kx)^2 + k^2x(1-x) - m^2 + i\varepsilon]^2} \end{aligned} \tag{C.0.5}$$

Now that the integral is convergent for small enough d, we can perform a variable change

$$p \to p - kx \tag{C.0.6}$$

we then get

$$i\Pi_{\mu\nu}(k,\epsilon) = -4e^2\mu^\epsilon \int_0^1 dx \int \frac{d^dp}{(2\pi)^d} \frac{N_{\nu\mu}(p-kx,k)}{[p^2 - C + i\epsilon]^2} \tag{C.0.7}$$

$$C = m^2 - k^2x(1-x)$$

Because of the following relations[2]

$$\int \frac{d^dp}{(2\pi)^d} \frac{p^\mu}{[p^2 - C + i\epsilon]^2} = 0 \tag{C.0.8}$$

$$\int \frac{d^dp}{(2\pi)^d} \frac{p^\mu p^\nu}{[p^2 - C + i\epsilon]^2} = \frac{1}{d} g^{\mu\nu} \int \frac{d^dp}{(2\pi)^d} \frac{p^2}{[p^2 - C + i\epsilon]^2} \tag{C.0.9}$$

We only have to calculate integrals of the form

$$\begin{aligned} I_{r,m} &= \int \frac{d^dp}{(2\pi)^d} \frac{(p^2)^r}{[p^2 - C + i\epsilon]^m} \\ &= \int \frac{d^dp - 1}{(2\pi)^d} \int dp^0 \frac{(p^2)^r}{[p^2 - C + i\epsilon]^m} \end{aligned} \tag{C.0.10}$$

[2]for their proofs see[32] appendix

To evaluate this integral, we first perform a Wick's rotation that is defined by

$$p^0 \to ip_E^0 \qquad ; \qquad \int_{-\infty}^{+\infty} dp^0 \to i\int_{-\infty}^{+\infty} dp_E^0 \tag{C.0.11}$$

And then writes

$$\int d^d p_E = \int d\bar{p}\bar{p}^{d-1} d\Omega_{d-1} \tag{C.0.12}$$

Where $\bar{p} = \sqrt{(p_E^0)^2 + (\vec{p})^2}$ and $d\Omega_{d-1}$ is the solid angle of generalized spherical coordinates with

$$\int d\Omega_{d-1} = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \tag{C.0.13}$$

And with performing the $\bar{p}$ integral we finally get

$$I_{r,m} = iC^{r-m+\frac{d}{2}} \frac{(-1)^{r-m}}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(r+\frac{d}{2})}{\Gamma(\frac{d}{2})} \frac{\Gamma(m-r-\frac{d}{2})}{\Gamma(m)} \tag{C.0.14}$$

With the condition $m - r - \frac{d}{2} \neq 0, -1, -2, \cdots$. By the substitution of $d = 4 - \epsilon$ in C.0.14 we get

$$I_{r,m} = i\frac{(-1)^{r-m}}{(4\pi)^2} \left(\frac{4\pi}{C}\right)^{\frac{\epsilon}{2}} C^{2+r-m} \frac{\Gamma(2+r-\frac{\epsilon}{2})}{\Gamma(2-\frac{\epsilon}{2})} \frac{\Gamma(m-r-2+\frac{\epsilon}{2})}{\Gamma(m)} \tag{C.0.15}$$

Now, we can go back and calculate $\Pi_{\mu\nu}$. First, from the previous change of variable C.0.6, $N_{\mu\nu}$ becomes

$$N_{\mu\nu}(p - kx, k) = 2p_\mu p_\nu + 2x^2 k_\mu k_\nu - g_{\mu\nu}(p^2 + x^2k^2 - xk^2 - m^2) \tag{C.0.16}$$

And therefore we get

$$\begin{aligned} \mathcal{N}_{\mu\nu} &\equiv \mu^\epsilon \int \frac{d^d p}{(2\pi)^d} \frac{N_{\mu\nu}(p-kx,k)}{[p^2 - C + i\epsilon]^2} \\ &= (\frac{2}{d} - 1) g_{\mu\nu}\mu^\epsilon I_{1,2} + [-2x(1-x)k_\mu k_\nu + x(1-x)k^2 g_{\mu\nu} m^2]\mu^\epsilon I_{0,2} \end{aligned} \tag{C.0.17}$$

using now the result C.0.15, we can write

$$\begin{aligned} \mu^\epsilon I_{0,2} &= \frac{i}{16\pi^2} \left(\frac{2\pi\mu^2}{C}\right)^{\frac{\epsilon}{2}} \frac{\Gamma(\frac{\epsilon}{2})}{\Gamma(2)} \\ &= \frac{i}{16\pi^2} \left(\Delta_\epsilon - \ln\left(\frac{C}{\mu^2}\right)\right) + \mathcal{O}(\epsilon) \end{aligned} \tag{C.0.18}$$

With the help of the expansion of Γ function

$$\Gamma(\frac{\epsilon}{2}) = \frac{2}{\epsilon} - \gamma + \mathcal{O}(\epsilon) \tag{C.0.19}$$

And defined

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma + \ln(4\pi) \qquad ; \qquad \gamma\text{: Euler constant} \tag{C.0.20}$$

Similarly

$$\mu^{\epsilon} I_{1,2} = \frac{i}{16\pi^2} C \left(1 + 2\Delta_\epsilon - 2\ln\left(\frac{C}{\mu^2}\right)\right) + \mathcal{O}(\epsilon) \tag{C.0.21}$$

Substituting back into C.0.21 with expanding the quantity $(\frac{2}{d} - 1)$ up to $\mathcal{O}(\epsilon)$

$$\frac{2}{d} - 1 = \frac{2}{4-\epsilon} - 1 = -\frac{1}{2} + \frac{1}{8}\epsilon + \mathcal{O}(\epsilon^2) \tag{C.0.22}$$

We get

$$\begin{aligned}\mathcal{N}_{\mu\nu} =& g_{\mu\nu}\left[-\frac{1}{2} + \frac{1}{8}\epsilon + \mathcal{O}(\epsilon^2)\right]\left[\frac{i}{16\pi^2} C\left(1 + 2\Delta_\epsilon - 2\ln\left(\frac{C}{\mu^2}\right)\right) + \mathcal{O}(\epsilon)\right] \\ &+ \left[-2x(1-x)k_\mu k_\nu + x(1-x)k^2 g_{\mu\nu} + g_{\mu\nu} m^2\right]\left[\frac{i}{16\pi^2}\left(\Delta_\epsilon - \ln\frac{C}{\mu^2}\right) + \mathcal{O}(\epsilon)\right]\end{aligned} \tag{C.0.23}$$

After eliminating higher terms and simplifying, we finally get

$$\mathcal{N}_{\mu\nu} = \frac{i}{16\pi^2}\left(\Delta_\epsilon - \ln\left(\frac{C}{\mu^2}\right)\right)\left(g_{\mu\nu}k^2 - k_\mu k_\nu\right) 2x(1-x) \tag{C.0.24}$$

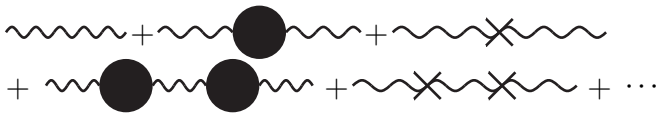
Now using C.0.3, we obtain

$$\Pi_{\mu\nu} = -(g_{\mu\nu}k^2 - k_\mu k_\nu)\Pi(k^2, \epsilon) \tag{C.0.25}$$

Where

$$\Pi(k^2, \epsilon) \equiv \frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\left[\Delta_\epsilon - \ln\left(\frac{m^2 - x(1-x)k^2}{\mu^2}\right)\right] \tag{C.0.26}$$

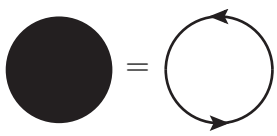
Going back to the definition of the full photon propagator which is the 2-point function 2.1.4, by rearranging its terms, the propagator can be represented as

$G_{\mu\nu} =$ 

Where

$\equiv i\Pi_{\mu\nu}(k) =$ sum of all one-particle irreducible diagrams to all orders. (C.0.27)

And in lowest order (one-loop order) of C.0.27 we have



Which is that was just been calculated. Continuing now by rewriting the free propagator

as

$$\begin{aligned} iG^0_{\mu\nu} &= P^T_{\mu\nu}\frac{1}{k^2} + \xi\frac{k_\mu k_\nu}{k^2} \\ &\equiv iG^{0T}_{\mu\nu} + iG^{0L}_{\mu\nu} \end{aligned} \tag{C.0.28}$$

where the transversal projector $P^T_{\mu\nu}$ was introduced

$$P^T_{\mu\nu} = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \tag{C.0.29}$$

Which satisfies

$$\begin{cases} k^\mu P^T_{\mu\nu} = 0 \\ P^T_{\mu\rho}P^T_{\rho\nu} = P^T_{\mu\nu} \end{cases} \tag{C.0.30}$$

the full photon propagator also can be written as

$$G_{\mu\nu} = G^T_{\mu\nu} + G^L_{\mu\nu} \tag{C.0.31}$$

where

$$G^T_{\mu\nu} = P^T_{\mu\nu}G_{\mu\nu} \tag{C.0.32}$$

From the result of $\Pi_{\mu\nu}$ in first order, the vacuum polarization tensor is transversal

$$i\Pi_{\mu\nu}(k) = ik^2 P^T_{\mu\nu}\Pi(k) \tag{C.0.33}$$

In fact, this is valid to all order of perturbation theory, which means that the longitudinal part is not renormalized

$$G^L_{\mu\nu} = G^{0L}_{\mu\nu} \tag{C.0.34}$$

For the transversal part we obtain

$$\begin{aligned} iG^T_{\mu\nu} &= P^T_{\mu\nu}\frac{1}{k^2} + P^T_{\mu\mu'}\frac{1}{k^2}(-i)k^2 P^T_{\mu'\nu'}\Pi(k)(-i)P^T_{\nu'\nu}\frac{1}{k^2}\cdots \\ &= P^T_{\mu\nu}\frac{1}{k^2}[1 - \Pi(k) + \Pi^2(k^2) + \cdots] \end{aligned} \tag{C.0.35}$$

Where after summing the geometric series, we get

$$iG^T_{\mu\nu} = P^T_{\mu\nu}\frac{1}{k^2[1+\Pi(k)]} \tag{C.0.36}$$

To renormalize this propagator properly we must also add the contribution of the counter terms Lagrangian

$$\Delta\mathcal{L} = -\frac{1}{4}\delta Z_3 F_{\mu\nu}F^{\mu\nu} \tag{C.0.37}$$

Which yields to the following Feynman rule

$$-i\delta Z k^2 P^T_{\mu\nu} \tag{C.0.38}$$

We have then

$$i\Pi_{\mu\nu} = i\Pi^{loop}_{\mu\nu} - i\delta Z_3 k^2 \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \tag{C.0.39}$$

$$= -i(\Pi(k) + \delta Z) P^T_{\mu\nu} \tag{C.0.40}$$

Thus, we should only make the substitution

$$\Pi(k) \to \Pi(k) + \delta Z_3 \tag{C.0.41}$$

In C.0.36, we obtain

$$iG^T_{\mu\nu} = P^T_{\mu\nu} \frac{1}{k^2[1 + \Pi(k) + \delta Z_3]} \tag{C.0.42}$$

Now to fix the renormalization constant δZ_3 with the on-shell renormalization scheme, C.0.42 must satisfy

$$\lim_{k \to 0} k^2 iG^{RT}_{\mu\nu} = 1 \cdot P^T_{\mu\nu} \tag{C.0.43}$$

This imply that

$$\Pi(k) + \delta Z_3 = 0 \tag{C.0.44}$$

From which we determine

$$\begin{aligned} \delta Z_3 &= -\Pi(0) \\ &= -\frac{2\alpha}{\pi} \int_0^1 dx\ x(1-x) \left[\Delta_\epsilon - \ln\left(\frac{m^2}{\mu^2} \right) \right] \\ &= -\frac{2\alpha}{3\pi} \left[\Delta_\epsilon - \ln\left(\frac{m^2}{\mu^2} \right) \right] \end{aligned} \tag{C.0.45}$$

The renormalization photon propagator is then written as

$$iG_{\mu\nu}(k) = \frac{P^T_{\mu\nu}}{k^2[1 + \Pi(k,\epsilon) - \Pi(0,\epsilon)]} + iG^L_{\mu\nu} \tag{C.0.46}$$

Appendix D

Computation with FeynCalc of renormalization constant connected to the process $e\nu_\mu \to e\nu_\mu$

D.1 Photon self-energy

From this program, we obtain, from the last line of code, the result of

$$Zaa \equiv \delta Z_{AA}^{1/2} \tag{D.1.1}$$

```
$LoadFeynArts = True;
<< FeynCalc`

aa = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles];
aaf = InsertFields[aa, {V[1]} -> {V[1]},InsertionLevel -> {Particles},
    Model -> "SM",ExcludeParticles -> {}];
Paint[aaf]
Paint[DiagramSelect[aaf, IRDivergentQ]]

SetOptions[PaVeReduce, A0ToB0 -> True];
$LimitTo4 = True

wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2],SMP["cos_W"] ->
    SMP["m_W"]/SMP["m_Z"]}

aaamp = FCFAConvert[CreateFeynAmp[aaf, Truncated -> True, PreFactor -> (4
    Pi)^-2], IncomingMomenta -> {k}, OutgoingMomenta -> {k},LoopMomenta -> {
    q}, LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[
    Beta]}, SMP -> True, DropSumOver -> True, UndoChiralSplittings -> True]
    /. wr;

aa[n_] := aaamp[[n]] /. DiracTrace -> Tr // Simplify
aares[n_] := Simplify[-I/Pi^2 OneLoop[q, aa[n]] // PaVeReduce]
pt[\[Mu]_, \[Nu]_] := MT[\[Mu], \[Nu]] - FV[k, \[Mu]] FV[k, \[Nu]]/SP[k, k]
aasum = Sum[aares[n], {n, 17}];
aas = MT[\[Alpha], \[Mu]] aasum pt[\[Alpha], \[Nu]]/(3) // Contract

Za0 = 1/2 D[aas /. ScalarProduct[k, k] -> k2, k2]^2 // Simplify
```

```

Zaa = Limit[Za0, k2 -> 0, Analytic -> True]

```

D.2 Electron self-energy

This program calculates, from the last three lines of code

$$Zre \equiv \delta Z_L^e \tag{D.2.1}$$

$$Zle \equiv \delta Z_R^e \tag{D.2.2}$$

$$dme \equiv \delta m_e \tag{D.2.3}$$

```
$LoadFeynArts = True;
<< FeynCalc`
es = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles];
esf = InsertFields[es, {F[2, {1}]} -> {F[2, {1}]},
InsertionLevel -> {Particles}, Model -> "SM",ExcludeParticles -> {S}];
Paint[esf]
Paint[DiagramSelect[esf, IRDivergentQ]]
SetOptions[PaVeReduce, A0ToB0 -> True];
$LimitTo4 = True;
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2],SMP["cos_W"] ->
    SMP["m_W"]/SMP["m_Z"],PropagatorDenominator[q - k, 0] ->
    PropagatorDenominator[q - k, \[Lambda]]};
esamp = FCFAConvert[CreateFeynAmp[esf, Truncated -> True, PreFactor -> (4
    Pi)^-2],IncomingMomenta -> {k}, OutgoingMomenta -> {k},LoopMomenta -> {q
    },LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[
    Beta]}, SMP -> True, DropSumOver -> True,UndoChiralSplittings -> True,
    List -> False] /. wr;
fesamp = Simplify[esamp /. DiracTrace -> Tr]
res := -I/Pi^2 OneLoop[q, fesamp]
res1 = PaVeReduce[res, PaVeAutoReduce -> True] // Simplify
res2 = DiracSimplify[res1, DiracSubstitute67 -> True] // Simplify
A1 := Coefficient[Coefficient[res2, DiracSlash[k], 0], DiracSlash[k].
    DiracMatrix[5], 0]
Ak := Coefficient[res2, DiracSlash[k], 1]
Ak5 := Coefficient[res2, DiracSlash[k].DiracMatrix[5], 1]
dA1 := D[A1 /. ScalarProduct[k, k] -> k2, k2 ] /. k2 -> SMP["m_e"]^2
dAk := D[Ak /. ScalarProduct[k, k] -> k2, k2] /. k2 -> SMP["m_e"]^2
Zre = -(Ak5 + Ak + 2 SMP["m_e"] dA1 + 2 SMP["m_e"]^2 dAk) /.ScalarProduct[k
    , k] -> SMP["m_e"]^2 // Simplify
Zle = (Ak5 - Ak - 2 SMP["m_e"] dA1 - 2 SMP["m_e"]^2 dAk) /.ScalarProduct[k,
    k] -> SMP["m_e"]^2 // Simplify
dme = (A1 + SMP["m_e"] Ak - SMP["m_e"]/2 (Zle + Zre)) /.ScalarProduct[k, k]
    -> SMP["m_e"]^2 // Simplify

```

D.3 Neutrino self-energy

We obtain from the neutrino self-energy the following renormalization constant

$$Zln \equiv \delta Z_L^{\nu} \tag{D.3.1}$$

```
$LoadFeynArts = True;
<< FeynCalc`
ns = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles, CTOrder ->
    0];
nsf = InsertFields[ns, {F[1, {2}]} -> {F[1, {2}]}, InsertionLevel -> {
    Particles}, Model -> "SM",ExcludeParticles -> {S}];
Paint[nsf]
Paint[DiagramSelect[nsf, IRDivergentQ]]
SetOptions[PaVeReduce, A0ToB0 -> True]
$LimitTo4 = True
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2], SMP["cos_W"] ->
     SMP["m_W"]/SMP["m_Z"]}
nsamp = FCFAConvert[CreateFeynAmp[nsf, Truncated -> True, PreFactor -> (4
    Pi)^-2], IncomingMomenta -> {k}, OutgoingMomenta -> {k},LoopMomenta -> {
    q},LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[
    Beta]}, SMP -> True, DropSumOver -> True] /. wr
ns[n_] := nsamp[[n]] /. DiracTrace -> Tr // Simplify
nsres[n_] := -I/Pi^2 OneLoop[q, ns[n]] // PaVeReduce
s = (nsres[1] + nsres[2]) // Simplify
k7 = DiracGamma[Momentum[k]].DiracGamma[7]
sr0 = s/k7 /. ScalarProduct[k, k] -> k2
Zln = -Limit[sr0, k2 -> 0, Analytic -> True] // Simplify

```

D.4 Z-boson self-energy

The following program gives the result of

$$dmz2 \equiv \delta M_Z^2 \tag{D.4.1}$$

$$sqrZzz \equiv \delta Z_{ZZ}^{1/2} \tag{D.4.2}$$

```
$LoadFeynArts = True;
<< FeynCalc`
zz = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles];
zzf = InsertFields[zz, {V[2]} -> {V[2]}, InsertionLevel -> {Particles},
    Model -> "SM",ExcludeParticles -> {}];
Paint[zzf]
Paint[DiagramSelect[zzf, IRDivergentQ]]
SetOptions[PaVeReduce, A0ToB0 -> True];
$LimitTo4 = True
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2], SMP["cos_W"] ->
     SMP["m_W"]/SMP["m_Z"]}
```

```
zzamp = FCFAConvert[CreateFeynAmp[zzf, Truncated -> True, PreFactor -> (4 Pi)^-2], IncomingMomenta -> {k}, OutgoingMomenta -> {k}, LoopMomenta -> {q}, LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[Beta]}, SMP -> True, DropSumOver -> True, List -> True] /. wr;
zz[n_] := zzamp[[n]] /. DiracTrace -> Tr // Simplify
zzres[n_] := -I/Pi^2 OneLoop[q, zz[n]] // PaVeReduce
pt[\[Mu]_, \[Nu]_] := MT[\[Mu], \[Nu]] - FV[k, \[Mu]] FV[k, \[Nu]]/SP[k, k]
zzsum = Sum[zzres[n], {n, 24}];
zzs = MT[\[Alpha], \[Nu]] zzsum pt[\[Mu], \[Alpha]]/(3) // Contract
dmz2 = -zzs /. ScalarProduct[k, k] -> SMP["m_Z"]^2
sqrZzz = 1/2 D[zzs /. ScalarProduct[k, k] -> k2, k2] /. k2 -> SMP["m_Z"]^2 // Simplify

```

D.5 W-boson self energy

The last program provides the following constants

$$dmw2 \equiv \delta M_W^2 \tag{D.5.1}$$

$$Zw \equiv \delta Z_W \tag{D.5.2}$$

```
$LoadFeynArts = True;
<< FeynCalc`
ww = CreateTopologies[1, 1 -> 1, ExcludeTopologies -> Tadpoles];
wwf = InsertFields[ww, {V[3]} -> {V[3]}, InsertionLevel -> {Particles}, Model -> "SM", ExcludeParticles -> {}];
Paint[wwf]
Paint[DiagramSelect[wwf, IRDivergentQ], ColumnsXRows -> 4]
SetOptions[PaVeReduce, A0ToB0 -> True];
$LimitTo4 = True
wr = {SMP["sin_W"] -> Sqrt[ 1 - SMP["m_W"]^2/SMP["m_Z"]^2], SMP["cos_W"] -> SMP["m_W"]/SMP["m_Z"], PropagatorDenominator[q, 0] -> PropagatorDenominator[q, \[Lambda]]}
wwamp = FCFAConvert[CreateFeynAmp[wwf, Truncated -> True, PreFactor -> (4 Pi)^-2], IncomingMomenta -> {k}, OutgoingMomenta -> {k}, LoopMomenta -> {q}, LorentzIndexNames -> {\[Mu], \[Nu], \[Rho], \[Sigma], \[Alpha], \\[Beta]}, SMP -> True, DropSumOver -> True] /. wr;
ww[n_] := wwamp[[n]] /. DiracTrace -> Tr // Simplify
wwres[n_] := Simplify[-I/Pi^2 OneLoop[q, ww[n]] // PaVeReduce]
pt[\[Mu]_, \[Nu]_] := MT[\[Mu], \[Nu]] - FV[k, \[Mu]] FV[k, \[Nu]]/SP[k, k]
wwsum = Sum[wwres[n], {n, 23}];
wws = MT[\[Alpha], \[Nu]] wwsum pt[\[Mu], \[Alpha]]/(3) // Contract
dmw2 = -wws /. ScalarProduct[k, k] -> SMP["m_W"]^2
Zw = D[wws /. ScalarProduct[k, k] -> k2, k2] /. k2 -> SMP["m_W"]^2

```

D.6 Charge renormalization constant

The $Z_e(= 1 + \delta Z_e)$ renormalization constant of the electric charge can be calculated directly from the following relation[42, 39]

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \left(\frac{m_Z^2}{M_W^2} - 1\right)^{1/2} \delta Z_{ZA} \tag{D.6.1}$$

where δZ_{AA} is given in appendix D.1
and δZ_{ZA} in section 2.3

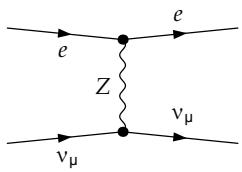
Appendix E

One-loop Feynman diagrams of $e\nu \to e\nu$ process

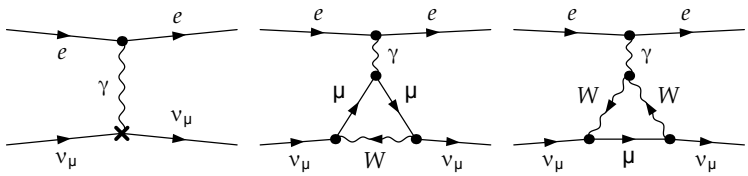
Here we show in Feynman gauge Feynman diagrams of $e\nu_\mu \to e\nu_\mu$ process generated by FeynArts package

---

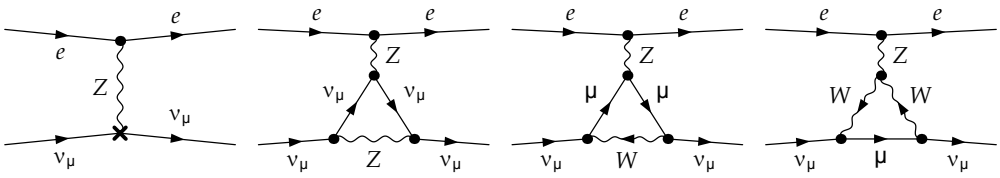
Tree level diagram



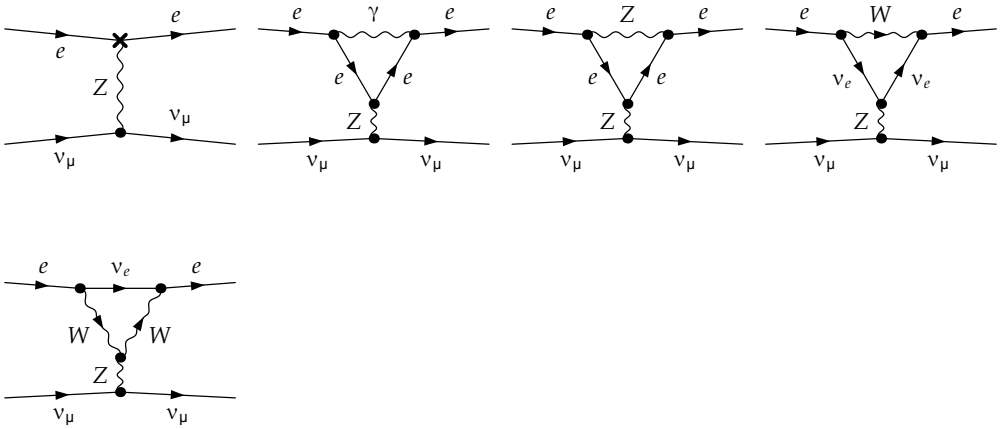
Diagrams for counter-term and one loop corrections to the vertex $\gamma\nu\nu$



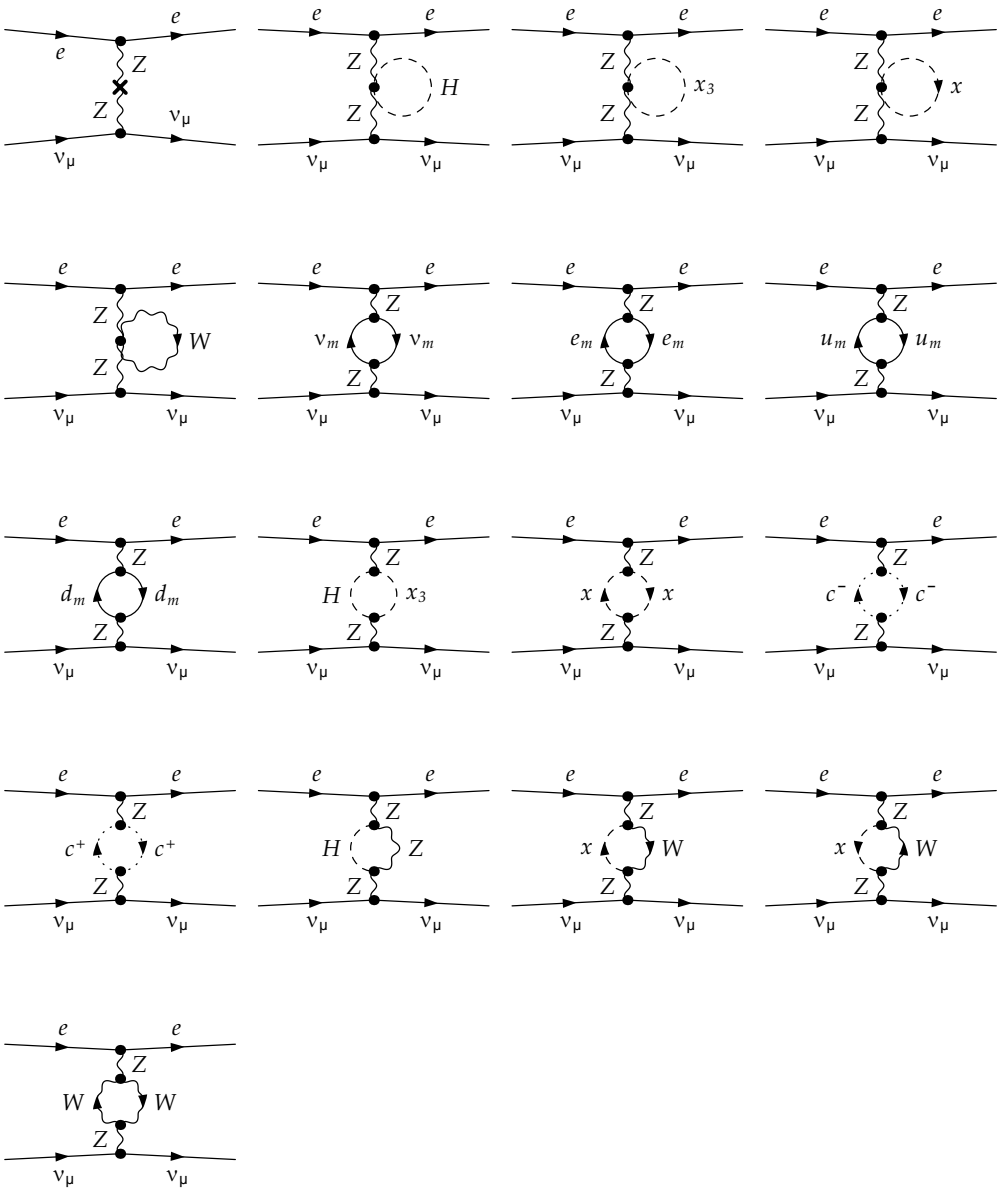
Diagrams for counter-term and one loop corrections to the vertex $Z\nu\nu$



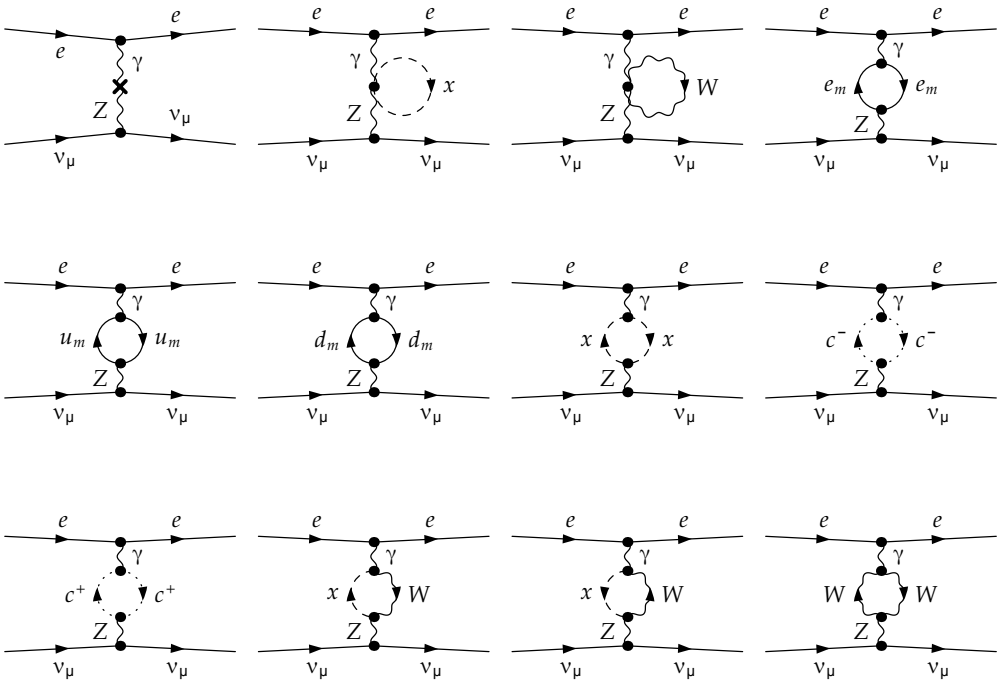
Diagrams for counter-term and one loop corrections to the vertex Zee



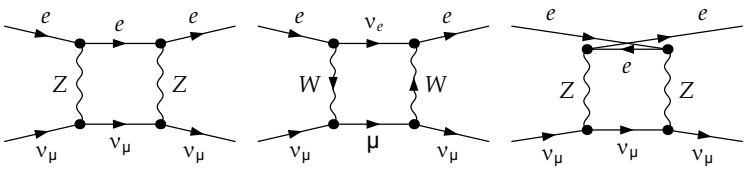
Diagrams for counter-term and one loop corrections to the self energy ZZ



Diagrams for counter-term and one loop corrections to the self energy $Z\gamma$



Box diagrams



Appendix F

FormCalc code to obtain figure 3.6

The following code is T.Hann's code that we adapted to the process $e\nu_\mu \to e\nu_\mu$

```
Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

Neglect[ME] = Neglect[ME2] = 0

process = {F[2, {1}], F[1, {2}]} -> {F[2, {1}], F[1, {2}]}

name = "/../../Bureau/Formcalc/
eMeM-SM-test"

SetOptions[InsertFields, Model -> "SM", Restrictions -> NoLightFHCoupling]

SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

Print["Born"]

tops = CreateTopologies[0, 2 -> 2];
ins = InsertFields[tops, process];
DoPaint[ins, "born"];
born = CalcFeynAmp[CreateFeynAmp[ins]]

Print["Counter terms"]

tops = CreateCTTopologies[1, 2 -> 2,
ExcludeTopologies -> {TadpoleCTs, WFCorrectionCTs}];
ins = InsertFields[tops, process];
DoPaint[ins, "counter"];
counter = CreateFeynAmp[ins]

Print["Self energies"]

tops = CreateTopologies[1, 2 -> 2, SelfEnergiesOnly];
ins = InsertFields[tops, process];
ins = DiagramSelect[ins, FreeQ[#, Field[5|6] -> S]&];
```

```
DoPaint[ins, "self"];
self = CalcFeynAmp[
CreateFeynAmp[ins],
Select[counter, DiagramType[#] == 2 &]]

Print["Vertices"]

tops = CreateTopologies[1, 2 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
ins = DiagramSelect[ins, FreeQ[#, Field[5] -> S]&];
DoPaint[ins, "vert"];
vert = CalcFeynAmp[
CreateFeynAmp[ins],
Select[counter, DiagramType[#] == 1 &]]

Print["Boxes"]

tops = CreateTopologies[1, 2 -> 2, BoxesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "box"];
box = CalcFeynAmp[
CreateFeynAmp[ins],
Select[counter, DiagramType[#] == 0 &]]

amps = {born, self, vert, box}

{born, self, vert, box} = Abbreviate[amps, 6,
Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[born, {self, vert, box}, abbr, subexpr, dir]

WriteRenConst[amps, dir]

Print["time used: ", SessionTime[] - time1]
```

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