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Scheduling in Food Industry: a case study of SOSEMIE

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Youcef & Abderraouf

Dedication

i dedicate this work:

To my parents, whose unwavering love, sacrifices, and encouragement have been my greatest source of strength throughout this journey
to my dear grandfather and grandmother

To my brothers,
for always standing by me through every challenge.

To my friends,
for their encouragement and true friendship.

And to myself,
for every effort, every late night, and every step forward.

thank you!

salhi youcef

Dedication

i dedicate this work:

To my parents, whose endless love, selfless sacrifices, and steadfast support have been my strongest foundation on this path.
To my beloved grandmother.

To my family,
for always being there to support me through every obstacle.
To my friends,
for their uplifting encouragement and genuine companionship.
And to myself,
for every ounce of determination, every sleepless night, and every stride forward.
Thank you!

Bougar Abderraouf

ملخص

تتناول هذه الأطروحة نموذج تحسين لجدولة تعبئة المعكرونة في شركة سوميمي، وهي شركة جزائرية لتصنيع المعكرونة، مع معالجة التحديات من خلال نهج التحسين. يهدف النموذج إلى تعزيز الكفاءة من خلال الخطوات التالية:

- بناء نموذج تحسين: تطوير إطار رياضي دقيق لتمثيل مشكلة التخطيط.
- تحديد الهدف: تقليل المجموع الموزون لأيام الإنتاج النشطة على مدى ٢٠ يوماً لتقليل التكاليف التشغيلية وتحسين الكفاءة.
- تصور النتائج: إنشاء مساعدات بصرية واضحة تبرز أفضل الحلول للتخطيط.

كلمات مفتاحية: البحث العملياتي، البرمجة الخطية المختلطة الصحيحة، الجدولة، تعبئة المعكرونة، الصناعة الغذائية، التحسين، سوسميه، خطوط الإنتاج، سعة الصوامع، القيود القائمة على العائلات، وقت التغيير، مخطط غانت، بايومو، موسيك، سي بي سي

Abstract

This thesis explores an optimization model for pasta packaging scheduling at SOSEMIE, an Algerian pasta manufacturer, addressing challenges through an optimization approach. The model aims to enhance efficiency through the following steps:

- Building an Optimization Model: Developing an accurate mathematical framework to represent the scheduling problem.
- Defining the Objective: Minimizing the weighted sum of active production days over a 20-day horizon to reduce operational costs and improve efficiency.
- Visualizing Results: Creating clear visual aids that highlight the best scheduling solutions.

keywords: Operations Research, Mixed-Integer Linear Programming (MILP), Scheduling, Pasta Packaging, Food Industry, Optimization, SOSEMIE, Production Lines, Silo Capacity, Family-Based Constraints, Changeover Time, Gantt Chart, Pyomo, MOSEK, CBC Solver

Résumé

Cette thèse explore un modèle d'optimisation pour la planification de l'emballage des pâtes chez SOSEMIE, un fabricant de pâtes algérien, en abordant les défis par une approche d'optimisation. Le modèle vise à améliorer l'efficacité à travers les étapes suivantes :

- Construction d'un Modèle d'Optimisation : Développer un cadre mathématique précis pour représenter le problème de planification.
- Définition de l'Objectif : Minimiser la somme pondérée des jours de production actifs sur un horizon de 20 jours pour réduire les coûts opérationnels et améliorer l'efficacité.
- Visualisation des Résultats : Créer des aides visuelles claires qui mettent en évidence les meilleures solutions de planification.

mots-clé Recherche Opérationnelle, Programmation Linéaire Mixte-Entière (MILP), Planification, Emballage de Pâtes, Industrie Alimentaire, Optimisation, SOSEMIE, Lignes de Production, Capacité des Silos, Contraintes Basées sur les Familles, Temps de Changement, Diagramme de Gantt, Pyomo, MOSEK, Solveur CBC

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List of Abbreviations

Abbreviation	Description
OR	Operations Research
MILP	Mixed-Integer Linear Programming
CBC	COIN-OR Branch and Cut
FMCG	Fast-Moving Consumer Goods
MES	Manufacturing Execution System
LP	Linear Programming
CP	Constraint Programming
COIN-OR	Computational Infrastructure for Operations Research

General Introduction

Operations research (OR) is a discipline that applies advanced analytical methods, such as mathematical modeling, optimization, and statistical analysis, to solve complex decision-making problems in various industries. By leveraging tools like Mixed-Integer Linear Programming (MILP), OR enables organizations to optimize resource allocation, improve efficiency, and reduce costs while meeting operational constraints. In the food industry, where perishable goods, diverse product portfolios, and stringent production constraints pose significant challenges, OR plays a critical role in enhancing scheduling processes to ensure economic and environmental sustainability.

This thesis addresses the problem of scheduling pasta packaging operations at SOSEMIE, a leading Algerian pasta manufacturer. The scheduling challenge involves allocating seven distinct pasta products, grouped into three families based on packaging characteristics, across three parallel packaging lines over a 20-day planning horizon. The problem is complex due to constraints such as limited silo storage capacities, a maximum of two family switches across all lines, one family per line per day, and product-specific changeover times. These constraints necessitate a strategic approach to balance production efficiency, resource utilization, and demand fulfillment, making OR an ideal framework for developing an optimized solution.

EURL Semoulerie Minoterie Étoiles (SOSEMIE), headquartered in Cité Benaissa, Beni Mered, Blida, Algeria, is a prominent Algerian manufacturer specializing in cereal processing and the production of high-quality flour, semolina, couscous, and pasta. Founded over 25 years ago by Mr. Djamel Maatseki, SOSEMIE has grown into one of Algeria's leading semolina and pasta producers, leveraging advanced equipment from Fava for mixing, extrusion, and drying, and Ricciarelli for packaging. With a large production capacity, the company serves both domestic and international markets, exporting to countries across Africa and Europe under brands like Sosemie, Le Chef, and Fandy. SOSEMIE's commitment to quality is evident in its modern facilities, which meet stringent market standards, and its active participation in global trade shows like Gulfood (2019, 2020) and World Food Russia 2019. The company engages with its community through initiatives like Ramadan promotions, women's day celebrations, and employee recognition events, as highlighted in its social media presence on platforms like Facebook and LinkedIn. Despite its success, SOSEMIE faces operational challenges, which this thesis aims to address by optimizing its pasta packaging stage.

The hypothesis of this study posits that a MILP-based scheduling model can significantly improve the efficiency of SOSEMIE's pasta packaging operations compared to traditional industry methods. Specifically, it is hypothesized that the model can reduce the number of active production days, and optimize resource utilization while meeting all demand and operational constraints, thereby offering a cost-effective and sustainable scheduling solution.

This structure prioritizes early production to reduce operational costs and inventory holding times. The model is subject to several constraints, including:

- **Demand Constraints**: Ensuring minimum demand is met for each product .
- **Family Grouping Constraints**: Allowing only one product family per line per day to minimize setup times.
- **Family Switch Constraints**: Limiting total family switches across all lines to two over the 20-day horizon.
- **Silo Capacity Constraints**: Restricting storage to product-specific limits and a maximum of six silos per day.
- **Production Capacity Constraints**: Limiting each line to 20 tons per day within a 14-hour (840-minute) operational window, including 15-minute changeovers between products.

These constraints ensure the schedule is feasible and aligned with SOSEMIE's operational realities.

The thesis is structured into four chapters, each addressing a critical aspect of the research:

- **Chapter 1: Theoretical Framework** – Provides the background, motivation, and literature review for scheduling in the food industry, introducing key concepts like parallel-line scheduling, family grouping, and MILP, and defining the research problem and objectives.
- **Chapter 2: Methodology** – Details the MILP model formulation, including assumptions, notations, objective function, constraints, and the branch-and-cut algorithm, implemented in Pyomo with MOSEK and CBC solvers.
- **Chapter 3: Case Study** – Presents empirical results from applying the MILP model to SOSEMIE's pasta packaging, analyzing production schedules, silo usage, and performance metrics, and comparing results with industry practices.
- **Chapter 4: Future Work** – Summarizes contributions, highlights key findings, offers practical recommendations, discusses limitations, and proposes future research directions to enhance the model's applicability.

This thesis aims to deliver a robust, data-driven scheduling solution for SOSEMIE, contributing to the broader field of operations research by demonstrating the practical impact of MILP in optimizing food industry processes. Through rigorous modeling and empirical validation, it seeks to provide actionable insights for improving efficiency and sustainability in pasta packaging operations.

Chapter 1

Theoretical Framework

1.1 Background and Motivation

Efficient production scheduling is a cornerstone of operational success in the food industry, where the balance between productivity, quality, and resource utilization directly impacts economic and environmental outcomes. In pasta production, scheduling is particularly critical due to the industry's reliance on specialized equipment, perishable raw materials, and diverse product portfolios. The packaging stage, often the final step in the production process, plays a crucial role in ensuring product quality and marketability; however, it is prone to inefficiencies such as excessive changeover times between products, overstocking due to limited silo capacities, and wasted production capacity if family switches are not carefully managed. With global demand for pasta products projected to increase steadily to 66.59 billion kilograms by 2030, with 4% growth expected in 2026 alone, manufacturers are under increasing pressure to optimize their operations. This study is motivated by the need to address these challenges through advanced scheduling techniques,using mathematical optimization to improve efficiency and sustainability in pasta packaging operations. [1]

1.2 Pasta Packaging Process

The pasta production process at SOSEMIE, a prominent pasta manufacturer in the Algerian industry, consists of four key stages: mixing, extrusion, drying and packaging. This thesis focuses on optimizing the scheduling of the packaging stage, which is critical for ensuring efficient operations and meeting diverse customer demands in a competitive market. The process leverages advanced equipment, including Fava systems for mixing, extrusion, and drying, and Ricciarelli packaging systems for the final packaging stage, to produce a variety of pasta products tailored to consumer preferences.

Production begins with the **mixing** stage, where durum wheat semolina is mixed with water to create a uniform dough. Fava s.r.l., an Italian company renowned for its high-performance food processing machinery, provides advanced mixing systems specifically designed for pasta production. These systems are used to achieve consistent dough quality, adjusting to the requirements of different types of pasta. This stage ensures that the texture and composition of the dough meet the standards for subsequent processing, establishing the foundation for the production of high quality pasta.

In the **extrusion** stage, the dough is shaped into various pasta forms using extruders. These machines press the dough through specialized dies designed for specific product shapes, ranging from thin strands to intricate patterns. The extrusion process is carefully controlled to maintain product consistency, with adjustments to pressure and die configurations based on the desired pasta type. This stage introduces variability in processing times, as different shapes require distinct extrusion parameters.

The **drying** stage follows, where the extruded pasta undergoes controlled drying to reduce moisture content, ensuring shelf stability and preserving quality. SOSEMIE utilizes Fava drying systems, which regulate temperature and humidity to suit the needs of different pasta types. Drying times and conditions vary depending on the pasta's shape and thickness, with longer drying required for thicker or denser products. Limited drying chamber capacity can create bottlenecks, necessitating efficient scheduling to streamline the flow of products to the packaging stage.

The **packaging** stage, the primary focus of this thesis, involves transferring dried pasta to parallel packaging lines equipped with Ricciarelli packaging systems. These systems are designed for flexibility, handling various packaging formats, with adjustments for product-specific requirements like size, weight, and labeling. Each packaging line operates within daily operational hours, with changeovers between products to reconfigure equipment settings, incurring setup times that impact production efficiency. To minimize disruptions, products are grouped into families based on similar packaging characteristics. The scheduling objective is to minimize the number of active production days, prioritizing early completion to reduce operational costs and optimize resource utilization.

At SOSEMIE, the integration of Fava production equipment and Ricciarelli packaging systems supports high-volume production but introduces scheduling challenges, including sequence-dependent changeovers, limited silo storage, and family-based production constraints. These factors, combined with the need to balance throughput and efficiency, underscore the importance of the MILP model adopted in this thesis to optimize the packaging process within the Algerian pasta industry. [2] [3]

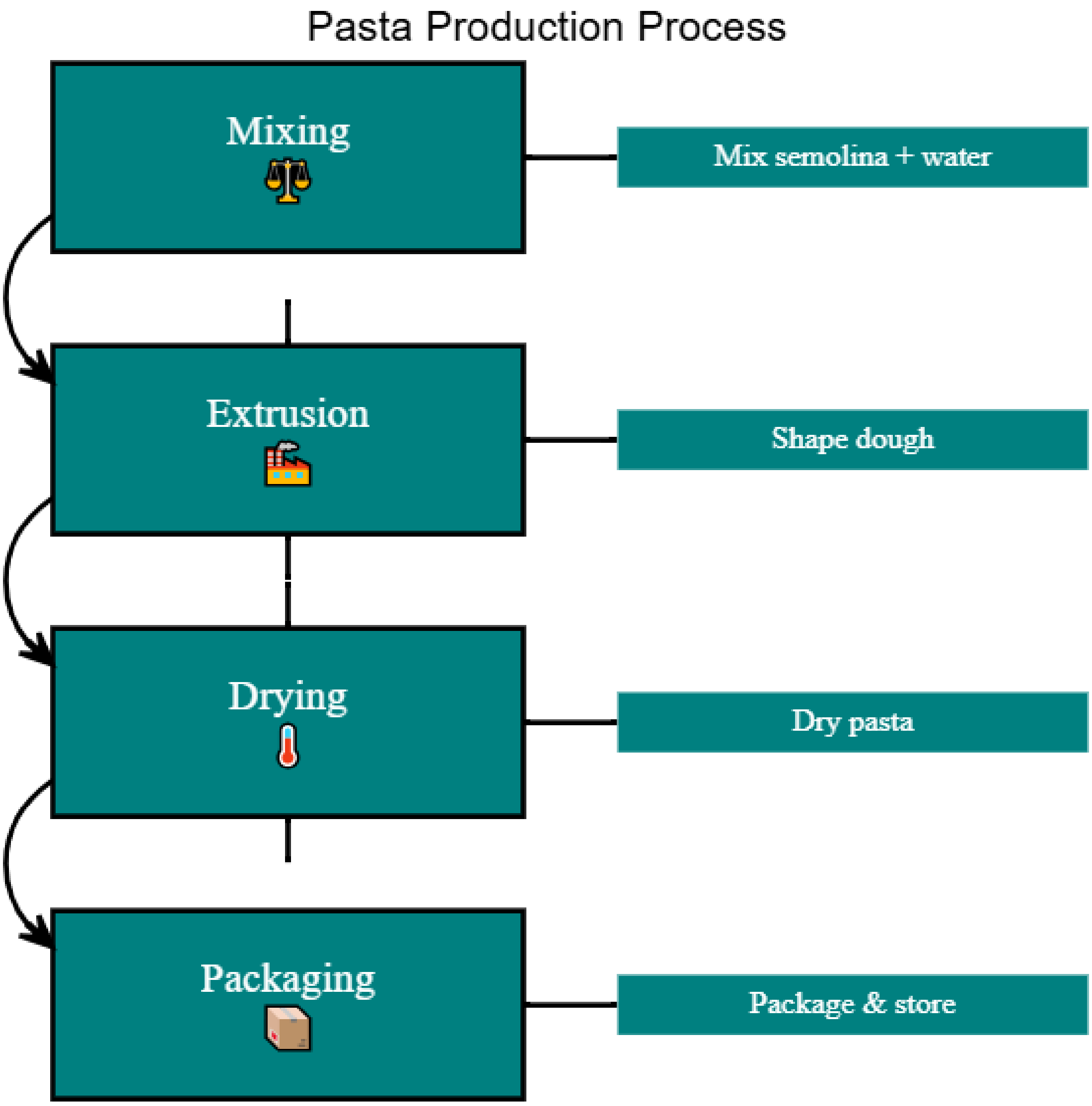

Figure 1.1: Pasta Production Process

1.3 Research Problem

The pasta packaging process at SOSEMIE, an Algerian pasta manufacturing company, faces significant scheduling inefficiencies due to complex operational constraints. These include sequence-dependent changeover times, limited silo storage capacities, restrictions on product family switches across production lines, and daily production limits. Currently, scheduling decisions are made manually or through rule-of-thumb approaches, which often lead to suboptimal resource utilization, extended production timelines, and increased operational costs.

This research addresses the challenge of developing an optimized scheduling model that minimizes the number of active production days while satisfying all operational constraints. The problem is modeled as a Mixed-Integer Linear Programming (MILP) formulation, specifically adapted to the structure of parallel production lines and product-family-based setups found in dry pasta packaging operations.

By solving this scheduling problem using mathematical optimization, the study aims to provide a data-driven decision support tool for planners at SOSEMIE, enabling more efficient use of resources and reducing idle time across the production system.

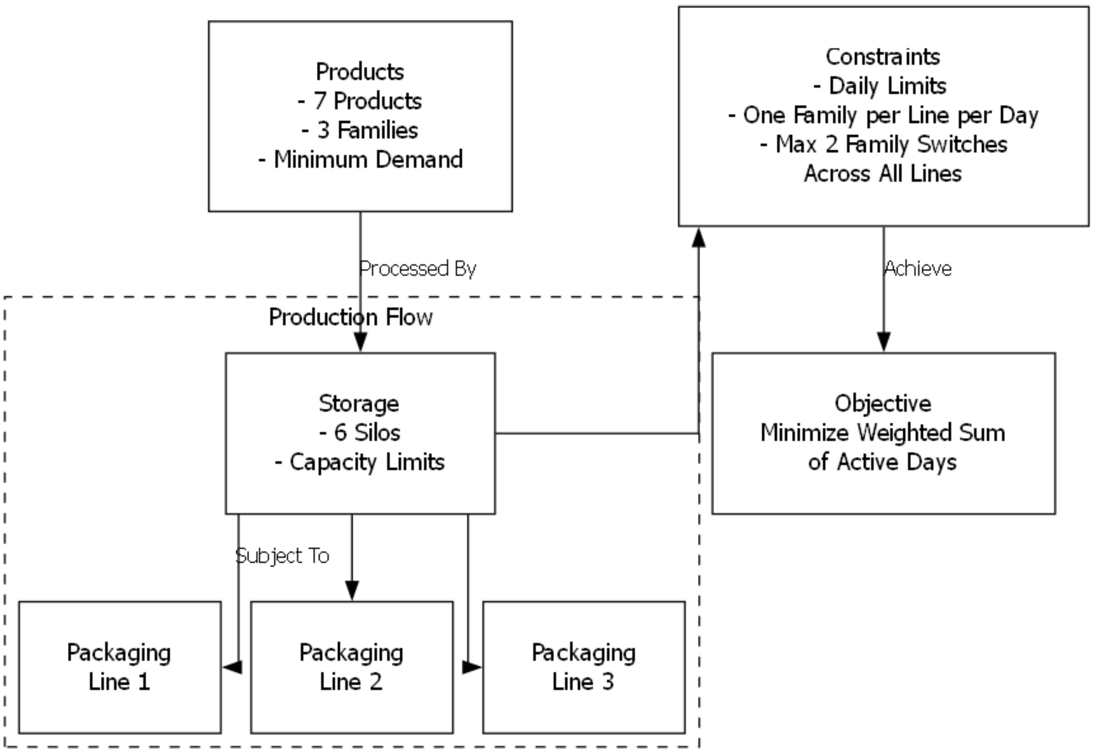

Figure 1.2: Structure of the Pasta Packaging Scheduling Problem

1.4 Objectives

The primary objective of this study is to adapt a Mixed-Integer Linear Programming (MILP) model to schedule pasta packaging operations, minimizing the weighted sum of active production days (where each day d has a weight $w_d = d$) while adhering to operational constraints.

1.4.1 Model Formulation

implement an MILP model that incorporates:

- Family grouping constraints
- Silo assignments
- Production demands

to address the scheduling problem effectively.

1.4.2 Setup Time Minimization

Minimize setup times through:

- Enforcing one family per line per day
- Limiting family switches to a maximum of two across all lines over the 20-day horizon.A family switch occurs when a production line transitions from producing products of one product family to products of a different family. Products are grouped into families based on shared packaging characteristics.
- Accounting for 15-minute changeovers between products

1.4.3 Storage Resource Optimization

Ensure efficient use of storage resources by:

- Assigning products to silos within product-specific capacity limits, where each silo has a defined capacity for a given product (e.g., 10 tons per silo for Product 1, 5 tons per silo for Product 7).

Table 1.1: silo capacity limits

Product	Capacity Limit (tons)
Product 1	10
Product 7	5

- Maintaining six silos in use

1.4.4 Performance Evaluation

Evaluate the performance of the model through a case study, analyzing:

- Production schedules
- Silo usage patterns
- Line performance metrics

to assess the effectiveness of the scheduling strategy.

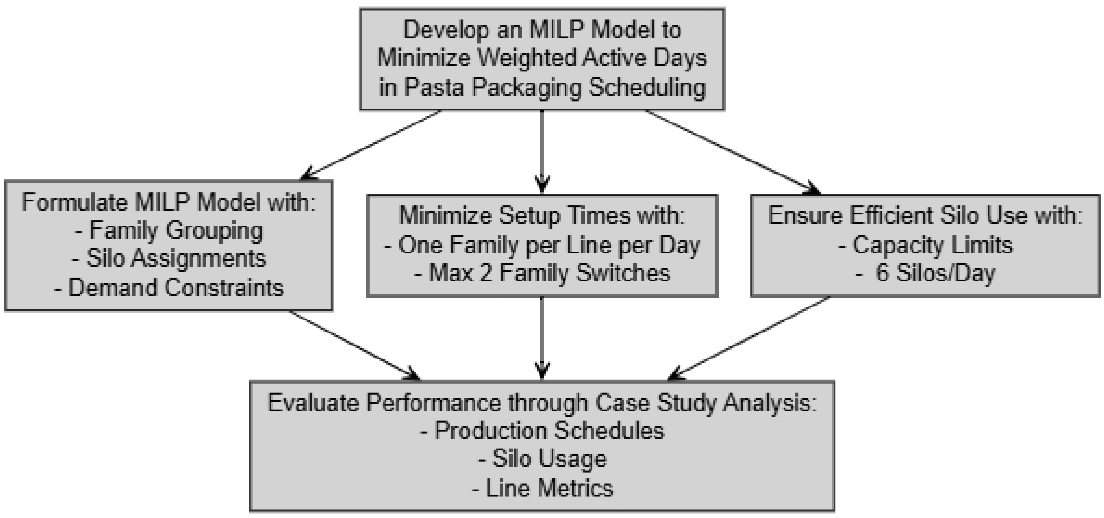

Figure 1.3: ObjectiveS of the Pasta Packaging Scheduling Study

1.5 Introduction to Scheduling in the Food Industry

In food manufacturing, efficient scheduling plays a vital role in maintaining productivity while meeting quality standards. The entire production process, from initial preparation to final packaging, requires careful planning, particularly for items with limited shelf life.Scheduling involves the allocation of start and end times to each activity throughout the production chain. Determine when labor, equipment, and facilities are required to produce a product or provide a service. Effective scheduling helps companies optimize the use of their resources, reduce waste, and increase profitability. Effective scheduling optimizes resource utilization, enabling companies to maximize efficiency and gain a competitive edge in the food industry; this principle is especially important for manufacturers, who must navigate fluctuating seasonal demand while working within production constraints.

The packaging stage represents much more than just putting products in boxes; it is the final safeguard for product quality and marketability. Packaging preserves the quality of processed food, ensuring safe transport over long distances while maintaining product condition after consumption. In doing so, packaging provides protection from external influences.

Poor scheduling in this phase can lead to delays, overproduction, or waste, all of which have economic and environmental consequences. [4] [5]

1.6 Parallel-Line Scheduling

The food packaging sector increasingly relies on parallel line scheduling to boost productivity and meet production deadlines, allowing manufacturers to distribute operations across multiple production lines simultaneously to enhance efficiency and meet tight deadlines. This approach is particularly vital for handling high volumes and diverse product ranges typical of food packaging, where resource optimization is critical. Production planning and scheduling problems are highly interdependent as scheduling provides optimum allocation of resources and planning is an optimum utilization of these allocated resources to serve multiple customers.

In this context, parallel line scheduling ensures that each line is assigned tasks in a way that maximizes throughput while addressing the unique constraints of food production, such as perishability and seasonal demand fluctuations.

A key challenge in parallel-line scheduling is managing changeovers between products, especially when production lines have varying capacities and the sequence of tasks impacts efficiency. In many process industries, the changeover time is sequence-dependent and needs to be addressed. This is particularly relevant in food packaging, where switching between products with different packaging requirements can lead to significant downtime if not carefully scheduled. To mitigate this, strategies such as grouping similar products on the same line are employed, reducing setup times and maintaining production flow across parallel lines.

The practical benefits of parallel-line scheduling are evident in real-world applications, such as in fast-moving consumer goods (FMCG) industries like soap manufacturing, which shares similarities with food packaging, The results show that there is saving of £62751 in week 1, £56441 in week 2, £50376 in week 3, and £53755 in week 4, achieved by optimizing scheduling across continuous parallel lines compared to conventional methods. These savings stem from reduced idle time and better alignment of production sequences with line capacities, illustrating how parallel-line scheduling accelerates production cycles while accommodating diverse product portfolios. [6]

1.7 Family Grouping in Scheduling

Family grouping is a production strategy that organizes similar manufacturing tasks consecutively to minimize equipment changeovers. This approach is particularly valuable in food processing, where jobs are divided into a number of families with sequence-independent set-up time incurred between jobs of different families. By scheduling products from the same family together, such as different pasta shapes requiring identical packaging configurations, manufacturers can significantly reduce nonproductive setup times.

The impact of effective family grouping is substantial, as grouping jobs into families and scheduling jobs from the same family together reduces set-up times.

In food production environments, these setup operations can account for a good percentage of total productive capacity, making their minimization crucial for operational efficiency.

Implementation in parallel-line systems presents unique challenges;Many modern food processing facilities adopt heuristic strategies to reduce setup times by grouping and prioritizing jobs from the same product family.
This approach is particularly effective in environments where sequence-dependent setups significantly impact efficiency, such as packaging operations. These methods have demonstrated the potential to reduce changeovers in high-volume production environments, offering significant efficiency gains for food manufacturers dealing with different types of product. [7] [8]

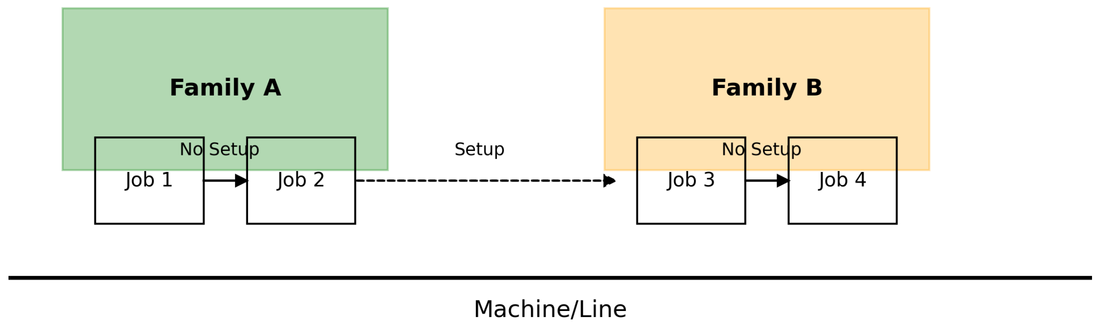

Figure 1.4: Family grouping

In the pasta packaging scheduling problem, the differences between product families are defined by the shape of the package, the size of the product, the specific format required for each family on the production line and the production rate. The shape of the package varies, and some families utilize standard rectangular bags or boxes suited for high-volume retail, while others require unique cylindrical containers or specialty wrappers to accommodate diverse product forms, influencing packing efficiency and line setup. The size of the product differs, ranging from fine, small grains or short pasta pieces to larger, broad sheets or long strands, which impacts the amount processed per batch and the time needed for packaging. Each family necessitates a distinct production line setup, such as extended conveyors for elongated products, vibratory feeders for compact shapes, or custom molds for complex forms, ensuring compatibility with the family's characteristics and enforcing a constraint of one family per line per day. Additionally, the production rate varies across families, with some achieving faster output due to simpler shapes and sizes , while others, with larger or more intricate products, exhibit slower rates. [9]

Figure 1.5: Family groups

1.8 Production lines

In the pasta packaging scheduling problem, a machine production line refers to a physical packaging unit within the pasta plant, designed to process and package various pasta products across a defined planning horizon. These lines are identical in their technical capabilities, allowing interchangeable use in production, which facilitates efficient scheduling. Each line is equipped with configurable components, such as shaping dies and material handling systems, to accommodate different product types, and shares uniform specifications including processing rates, setup times for product changes and daily operational capacity. Lines serve as the primary units for scheduling, enabling parallel production of multiple products, and their assignments are managed through specific variables to meet production demands while adhering to operational constraints, such as preventing cross-contamination through dedicated daily configurations.

1.9 Mixed-Integer Linear Programming (MILP) in Production Scheduling

Mixed-Integer Linear Programming (MILP) is a powerful optimization framework that is particularly well-suited for production scheduling problems due to its ability to model complex decision-making scenarios involving both continuous and discrete variables. In production environments, decisions often include assigning tasks to machines, sequencing operations, managing limited resources, and adhering to timing constraints—many of which are naturally expressed as binary or integer decisions (e.g., whether or not to start a job at a given time, or assigning a job to a specific machine). MILP models allow for these discrete decisions while simultaneously optimizing a linear objective function, such as minimizing production time, cost or delays. Moreover, MILP's flexibility enables the incorporation of real-world constraints like precedence relations, setup times, and resource capacities, making it a robust and widely used tool for solving scheduling problems in manufacturing and industrial operations.

Core Components of MILP Models

A comprehensive MILP formulation for production scheduling contains three fundamental elements:

Objective Function: the objective function in MILP is a linear function that we aim to maximize or minimize.

Decision Variables:

- Continuous variables: These variables can take any real value within a given range.
- Binary variables: A special case of integer variables that can only take values 0 or 1, often used to represent yes/no decisions.
- Integer variables: these variables are restricted to integer values.

Constraints:

- Limits of resource capacity.
- Logical sequencing requirements.
- Demand fulfillment equations.
- Temporal relationships between operations.

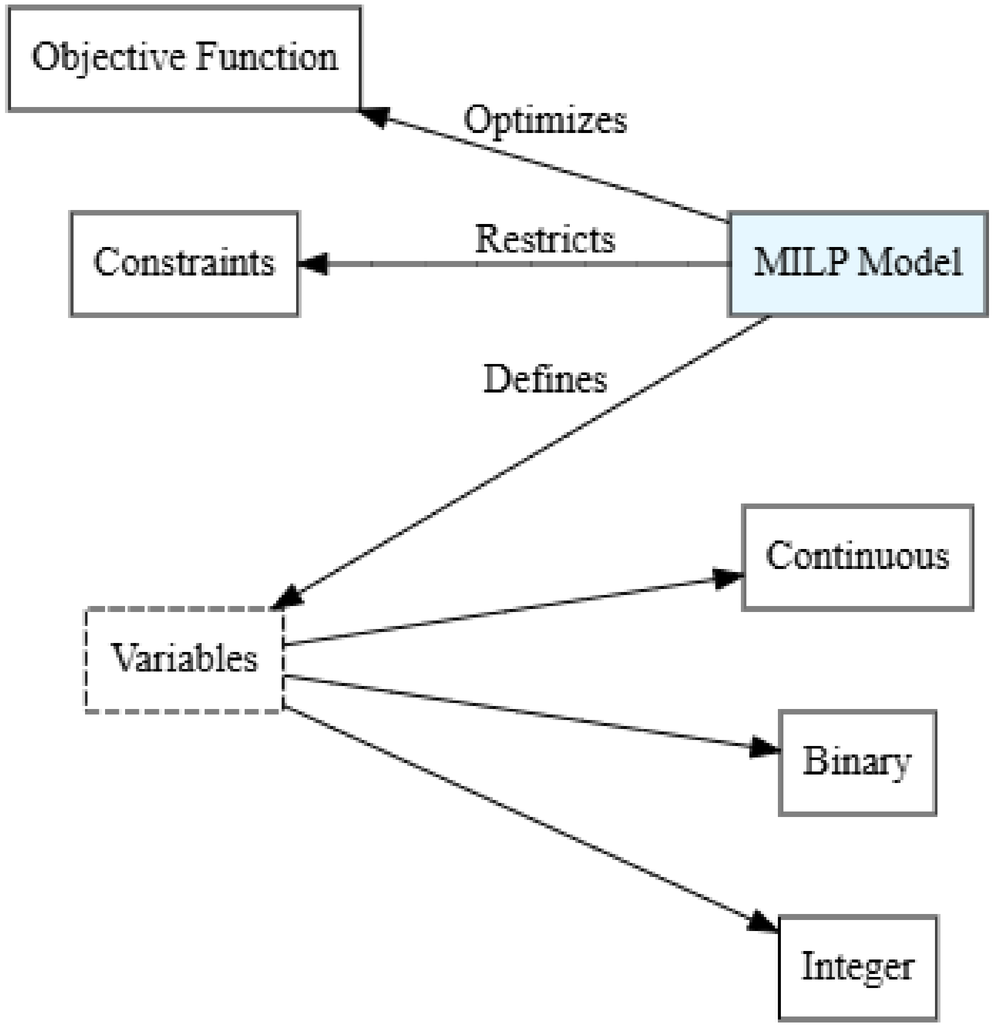

Figure 1.6: MILP model

1.10 MILP Solvers

MILP solvers are specialized software tools designed to find optimal solutions to Mixed-Integer Linear Programming problems, which involve both continuous and discrete variables constrained by linear relationships. These solvers use advanced mathematical techniques such as branch-and-bound, cutting planes, and heuristics to efficiently explore the solution space, even for large and complex problems. Popular MILP solvers include commercial options like IBM ILOG CPLEX, Gurobi, and FICO Xpress, as well as open-source alternatives such as CBC (Coin-or branch and cut) and GLPK (GNU Linear Programming Kit). The performance of these solvers has significantly improved over the years, allowing them to tackle large-scale scheduling, logistics, and optimization problems across various industries. Their robustness, speed, and support for modeling environments like Python (e.g., through PuLP or Pyomo), AMPL, and GAMS make them essential tools for researchers and practitioners working on operations research and optimization challenges. [10] The effectiveness of MILP solvers stems from their use of advanced algorithmic techniques that enable efficient exploration of large solution spaces. These include:

- Branch-and-Bound: breaks the problem into smaller parts (like tree branches), then eliminates options that cannot possibly be the best answer.
- Cutting Plane Methods: adds new rules one by one to cut away bad solutions, like narrowing down choices until only good options remain.
- Heuristic Presolving: Simplifies models before full optimization.
- Parallel Processing: uses multiple CPU cores at once to explore different solutions simultaneously, working much faster than doing things one at a time.

In food production scheduling, these capabilities enable:

- Optimal assignment of batches to parallel packaging lines
- Minimization of changeover times between product families
- Balanced workload distribution across shifts
- Real-time rescheduling when disruptions occur

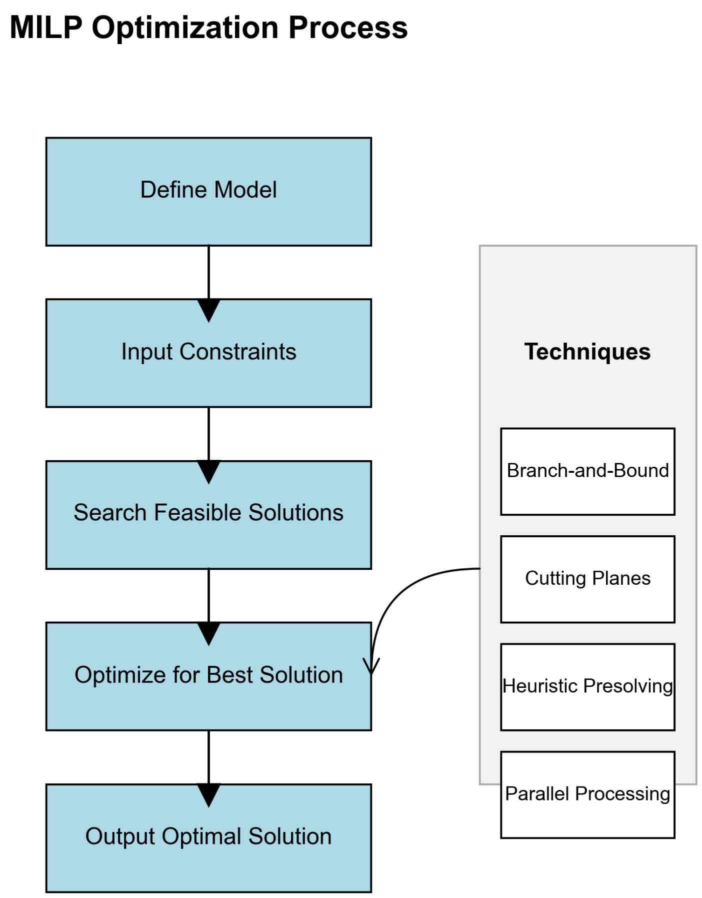

Figure 1.7: MILP optimization process

The figure illustrates the systematic steps involved in solving Mixed-Integer Linear Programming (MILP) problems. The process begins with defining the model , followed by inputting constraints and searching for feasible solutions using advanced techniques like branch-and-bound , cutting planes , heuristic presolving , and parallel processing . Once feasible solutions are identified, the algorithm optimizes for the best solution. Finally, the optimal solution is outputted, providing an efficient production schedule.

1.11 Literature Review

Production scheduling in the food industry is a complex task due to the presence of perishable products, shared resources, and diverse product portfolios. Packaging often forms a critical bottleneck, especially when dealing with parallel production lines, sequence-dependent changeovers, and limited storage or processing capacities. Mixed-Integer Linear Programming (MILP) has emerged as a powerful tool for optimizing resource allocation and sequencing in such environments. One relevant study examines weekly scheduling in a Spanish canned fish plant, focusing on semi-continuous packaging operations with parallel lines sharing a single labeler. The authors propose an MILP model using a mixed discrete-continuous time representation along with a decomposition algorithm to minimize makespan. Their approach reduces the total scheduling time from 104 hours (manually scheduled) to 94.4 hours and cuts changeover times by 15%.

Another key contribution comes who compare MILP and rule-based scheduling approaches for yogurt packing in two Greek facilities—KRI-KRI (four lines) and TYRAS (seven lines). Using a precedence-based MILP framework that incorporates product family grouping, they achieve up to a 60.7% cost reduction for KRI-KRI and 12% for TYRAS compared to SchedulePro$^{\text{TM}}$, a commercial rule-based system. The family-based approach used in this study closely aligns with Sosemie 's operational structure, where products are grouped into three families based on shared characteristics and packaging requirements. Additionally, sequence-dependent setup times and shared fermentation tanks feeding multiple lines mirror Sosemie 's challenges with silo capacity and line assignment. However, the perishability of yogurt introduces constraints not present in dry pasta scheduling, requiring adjustments to the objective function and inventory handling logic.

Further validation of MILP's applicability across food sectors is provided , who apply it to bakery production involving sequence-dependent setups and shelf-life constraints. Their time-indexed formulation minimizes costs related to inventory, backorders, and changeovers while managing batch sizing across multiple machines. This case illustrates how MILP can be adapted to different types of perishability and setup dependencies, offering useful insights for Sosemie when considering silo limitations and product family transitions.

Another study address scheduling in the soft drink industry using an MILP model tailored to bottling lines constrained by limited tank availability. They incorporate sequence-dependent changeovers and cleaning operations into their formulation, significantly improving machine utilization and reducing downtime. The inclusion of cleaning as a constraint is particularly noteworthy, as hygiene standards in food packaging may also necessitate periodic cleanups between product families—an aspect worth exploring in Sosemie 's scheduling model.

Another key contribution develop a hybrid MILP/Constraint Programming (CP) approach for a dairy plant producing milk derivatives. Their MILP component handles lot-sizing and sequencing, while CP ensures feasible timetabling. The integrated method outperforms standalone models in managing complex temporal constraints. Although Sosemie may not immediately require CP integration, this approach suggests future scalability of the MILP model to include labor scheduling, maintenance windows, or other timing-related restrictions.

other study propose a multi-objective MILP model for a confectionery company facing seasonal demand fluctuations. Their formulation optimizes both makespan and energy consumption across parallel lines, demonstrating how MILP can extend beyond traditional objectives like cost or time minimization. Energy efficiency considerations could become increasingly important for Sosemie in the context of long-term sustainability goals and operational cost control.

Collectively, these studies underscore MILP's robustness in addressing scheduling problems within the food industry. They highlight its ability to handle sequence-dependent setups, shared resources, perishability, and family-based grouping through various formulations—including time-indexed, precedence-based, and hybrid MILP/CP frameworks. Moreover, they demonstrate the computational feasibility of such models through decomposition strategies, heuristic integrations, and efficient solvers.

Building upon these foundations, this thesis applies MILP to dry food packaging operations at Sosemie , aiming to minimize active production days while accounting for family-based changeovers , silo capacities , and parallel line coordination . By adapting existing methodologies to the specific constraints of dry pasta packaging, the research contributes to advancing scheduling optimization in the broader food manufacturing sector. [11] [12] [13] [14] [15] [16]

Chapter 2

Methodology

This chapter presents the methodology for addressing the pasta packaging scheduling problem through the adaptation of an existing Mixed-Integer Linear Programming (MILP) model.

The problem involves scheduling the production of various pasta products across multiple production lines within a fixed planning horizon, while adhering to operational constraints such as family-based production groupings, changeover times, and silo storage limitations. The main objective is to minimize the weighted sum of active production days, prioritizing early production and reducing operational costs.

The adapted MILP model is designed to capture the key complexities of the scheduling environment and is solved using two optimization solvers, MOSEK and CBC, whose performance is compared in Chapter 3.

The chapter is structured to offer a clear and comprehensive framework for model adaptation and solution.

Section 2.1 provides a detailed description of the scheduling problem.

Section 2.2 outlines the assumptions and notation adopted in the model. The mathematical formulation, including the objective function and constraints, is presented in Section 2.3.

Section 2.4 introduces the branch-and-cut algorithm used to solve the MILP model, while Section 2.5 explains the solution approach, including solver selection and preprocessing steps.

Section 2.6 describes the implementation using the Pyomo modeling language, followed by model validation in Section 2.7.
Section 2.8 concludes the chapter with a summary of the key contributions.

2.1 Problem Description

The pasta packaging scheduling problem involves scheduling production for distinct products across production lines over a planning horizon. These products can be grouped into families based on their production characteristics. The primary objective is to determine an optimal production schedule that minimizes the weighted sum of active production days, where each day's weight is equal to its position in the horizon (e.g., day 1 has a weight of 1, day 2 a weight of 2, and so on). This objective encourages production to occur as early as possible, reducing operational costs by concentrating production activities on fewer, earlier days.

Each product has a demand that must be met by the end of the planning horizon. The total demand is the sum of the requirements for all products. Each production line has a daily output capacity and can operate for a limited number of hours per day. Switching between different products on the same line within a day incurs a changeover time, which reduces the available production time.

The problem is subject to several constraints. Each line can only produce products from one family on a given day, ensuring consistency in setup. The total number of family switches across all lines over the horizon is limited, where a switch occurs when a line changes from one family to another between consecutive days. The produced goods must be stored in silos before packaging, each with product-specific capacities, and the storage must not exceed these capacities on any day. The challenge is to balance production, changeovers, family restrictions, and storage constraints to achieve an efficient schedule that minimizes the weighted sum of active production days

2.2 Assumptions and Notations

To formulate the MILP model, several assumptions are made to simplify the problem while maintaining its practical relevance. These assumptions, along with the notations used in the model, are detailed below to provide a clear foundation for the mathematical formulation.

2.2.1 Assumptions

- Each production line can operate for a fixed maximum duration per day.
- Each line has a daily production capacity of 20 tons, regardless of the product being produced, reflecting a uniform capacity across lines and products.
- A fixed changeover time is incurred each time a line switches between products within the same day.
- Only one family can be produced on a given line on a given day, preventing intra-day family switches.
- The total number of family switches across all lines over the entire planning horizon is strictly limited (e.g., only a few switches are allowed when changing families between consecutive days).
- Production output is stored in silos, each with product-specific storage capacities
- Each product has a minimum demand that must be met by the end of the planning horizon.
- All lines are available every day, with no downtime or maintenance scheduled during the planning horizon.

2.2.2 Notations

The MILP model uses the following notations to define sets, parameters, and decision variables:

indices:

- $i \in I$: Products (e.g., spaghetti, penne)
- $l \in L$: Production lines (Line 1, Line 2, ...)

- $d \in D$: Days in planning horizon
- $s \in S$: Silos (storage units)
- $f \in F$: Product families (e.g., long pasta, short pasta)

Parameters:

- D_i: Demand for product i (tons)
- $C_l = 20$: Maximum daily capacity of line l (tons/day)
- $T_{\text{total}} = 840$: Daily operating time (14 hours in minutes)
- $T_{\text{change}} = 15$: Changeover time between products (minutes)
- F_i: Family of product i
- S_s: Capacity of silo s (tons)
- $w_d = d$: Weight for day d (prioritizes earlier production)
- r_i: Production rate of product i (tons/minute)

Decision Variables:

- $x_{i,l,d}$: Binary, equals 1 if product i is produced on line l on day d, 0 otherwise
- $y_{l,d}$: Binary, equals 1 if line l is active on day d, 0 otherwise
- $q_{i,l,d}$: Continuous, tons of product i produced on line l on day d
- $z_{s,i,d}$: Binary, equals 1 if silo s is assigned to product i on day d, 0 otherwise
- $f_{l,d,f}$: Binary, equals 1 if line l produces family f on day d, 0 otherwise
- $\delta_{l,d}$: Binary, equals 1 if line l changes family from day $d-1$ to d, 0 otherwise

2.3 Mathematical Formulation

This section presents the MILP model for the pasta packaging scheduling problem, detailing the objective function and constraints that govern the production schedule.

2.3.1 Objective Function

The objective function of the Mixed-Integer Linear Programming (MILP) model for pasta packaging scheduling is designed to minimize the total weighted usage of production days across all production lines. It is mathematically expressed as:

$$\text{Minimize} \quad \sum_{l \in L} \sum_{d \in D} w_d \cdot y_{l,d}$$

The objective function aims to schedule production as early as possible within the planning horizon, minimizing the use of later days, which are more costly due to their higher weights. This reduces lead times, builds inventory early, and enhances operational efficiency in the pasta packaging facility.

Day Weights: Each day in the planning horizon is assigned a specific weight that increases linearly with the day number:

- Day 1: $w_1 = 1$
- Day 2: $w_2 = 2$
- Day 3: $w_3 = 3$
- ...
- Day 10: $w_{10} = 10$

These weights penalize the use of later days more heavily. For example, activating a line on day 2 ($w_2 = 2$) contributes twice as much to the objective function as day 1 ($w_1 = 1$), and day 10 ($w_{10} = 10$) contributes ten times as much. This structure strongly incentivizes the solver to prioritize earlier days for production.

Role of $y_{l,d}$: The binary variable $y_{l,d}$ determines whether line l is active on day d. If $y_{l,d} = 1$, the weight w_d (e.g., 1 for day 1, 2 for day 2) is added to the objective function for that line-day combination. If $y_{l,d} = 0$, the line is idle, contributing zero. The double summation over all lines ($l \in L$) and days ($d \in D$) aggregates the weighted contributions of all active line-days.

Minimization Process: The solver seeks the combination of $y_{l,d}$ values that minimizes the objective function while satisfying all constraints (e.g., demand satisfaction, line capacities, silo storage). To achieve a lower objective value, the solver:

- Activates lines on earlier days (with lower weights, like $w_1 = 1$, $w_2 = 2$) whenever possible.
- Minimizes the number of active line-days, especially on later days (e.g., day 10 with $w_{10} = 10$).

2.3.2 Constraints

The model is subject to the following constraints:

- **Demand Satisfaction**: Total production of each product must meet its minimum demand:
$$\sum_{l \in L} \sum_{d \in D} q_{i,l,d} \geq D_i \quad \forall i \in I$$
- **Daily Production Capacity**: Total production on each line per day must not exceed 20 tons:
$$\sum_{i \in I} q_{i,l,d} \leq C_l \cdot y_{l,d} \quad \forall l \in L, d \in D$$
- **Daily Time Limit**: Total time for production and changeovers must not exceed 840 minutes:
$$\sum_{i \in I} \left(\frac{q_{i,l,d}}{r_i} \right) + T_{\text{change}} \cdot \left(\sum_{i \in I} x_{i,l,d} - 1 \right) \leq T_{\text{total}} \quad \forall l \in L, d \in D$$
Ensures production time plus changeover time does not exceed 14 hours (840 minutes). The term $T_{\text{change}} \cdot (\sum x_{i,l,d} - 1)$ accounts for changeovers between different products.

- **Family Restriction**:Ensures each line produces only one product family per day, where F_i indicates the family of product i:

$$\sum_{f \in F} f_{l,d,f} \leq 1 \quad \forall l \in L, d \in D$$

$$x_{i,l,d} \leq f_{l,d,F_i} \quad \forall i \in I, l \in L, d \in D$$

- **Family Switch Constraint**: Total family switches across all lines are limited to 2:

$$\sum_{l \in L} \sum_{d \geq 2} \delta_{l,d} \leq 2$$

$$\delta_{l,d} = \begin{cases} 1, & \text{if line } l \text{ changes product families from day } d-1 \text{ to } d \\ 0, & \text{otherwise} \end{cases}$$

- **One Product per Silo per Day**: Each silo can store only one product per day.

$$\sum_{i \in I} z_{s,i,d} \leq 1 \quad \forall s \in S, d \in D$$

- **Silo Capacity**: Ensures the total production of each product does not exceed the combined capacity of silos assigned to store it:

$$\sum_{l \in L} q_{i,l,d} \leq \sum_{s \in S} S_s \cdot z_{s,i,d} \quad \forall i \in I, d \in D$$

2.4 Algorithm for Solving the MILP Model: Branch-and-Cut

2.4.1 Overview of the Branch-and-Cut Algorithm

Branch and Cut is an exact optimization algorithm used to solve Integer Linear Programming (ILP) and Mixed Integer Linear Programming (MILP) problems. It combines two fundamental techniques: branch and bound and cutting plane methods . The algorithm explores the solution space by recursively partitioning it into smaller subproblems organized in a tree structure (branching), similar to traditional branch and bound. However, instead of relying solely on linear programming (LP) relaxations at each node, Branch and Cut dynamically strengthens the relaxation by generating and adding valid inequalities or cuts constraints that remove fractional solutions without cutting off any feasible integer solutions.

These cuts improve the quality of the bounds obtained from the LP relaxations, leading to more effective pruning of the search tree and reducing the number of nodes that must be explicitly explored. The interplay between branching and cutting allows the method to efficiently converge toward an optimal integer solution. This hybrid approach makes Branch and Cut particularly powerful for solving large-scale combinatorial optimization and integer programming problems that are otherwise computationally intensive using pure branch and bound or standalone cutting plane methods. [17]

2.4.2 Branch and Bound:

Branch and Bound is an exact algorithm used to solve integer programming and combinatorial optimization problems by recursively partitioning the solution space into subsets (branching) and computing bounds to prune nonoptimal branches. It explores potential solutions through a search tree, where each node represents a subproblem, and bounds are obtained via relaxations, typically linear programming. If the bound at a node is worse than the best known feasible solution, that node is discarded, reducing computational effort. This method guarantees finding the optimal solution by systematically exploring only promising regions of the search space. It is particularly effective for solving Mixed Integer Linear Programming (MILP) problems , where some variables are required to take integer values while others can be continuous [18] [17]

2.4.3 Cutting planes:

Cutting planes are additional constraints that are valid for the integer feasible solutions but may exclude some or all fractional (non-integer) solutions of the linear programming relaxation. These inequalities are iteratively added to the relaxation to 'cut off' the current non-integer solution while preserving all integer feasible points. The goal is to progressively tighten the LP relaxation and approximate the convex hull of the integer solutions, thereby improving the bounds and accelerating convergence to an optimal solution. [19]

2.4.4 Application to the Pasta Packaging Scheduling Problem

Both MOSEK and CBC solvers employ the branch-and-cut algorithm to solve the Mixed-Integer Linear Programming (MILP) model for the pasta packaging scheduling problem. The algorithm begins by solving the linear programming (LP) relaxation of the model, where binary variables (e.g., $y_{l,d}$, $f_{l,d,f}$) are relaxed to take fractional values between 0 and 1. The solution to this relaxation provides a lower bound on the optimal objective value.

If the LP relaxation yields a fractional solution (e.g., $y_{l,d} = 0.5$), the algorithm branches on a fractional variable, creating two subproblems: one where the variable is fixed to 0 (e.g., $y_{l,d} = 0$) and another where it is fixed to 1 (e.g., $y_{l,d} = 1$). This branching process forms a tree of subproblems.

For each subproblem, cutting planes are generated to eliminate fractional solutions and tighten the feasible region. These cuts, such as implied bound cuts, leverage the model's constraints to infer tighter bounds on variables. For example, constraints like the family switch limit ($\sum \delta_{l,d} \leq 2$) and silo storage restrictions benefit from such cuts, which reduce the feasible space for variables like silo assignments ($z_{s,i,d}$).

The algorithm iteratively solves LP relaxations, branches on fractional variables, and applies cutting planes until an integer solution is found that matches the best lower bound, confirming optimality. The number of LP relaxations solved by MOSEK reflects the subproblems processed, while the nodes explored by CBC indicate the size of the branching tree.

2.5 Solution Method

2.5.1 MOSEK

MOSEK is a software package for solving mathematical optimization problems. It supports a wide range of problem types, including linear programming (LP), quadratic programming (QP), second-order cone programming (SOCP), semidefinite programming (SDP), and mixed-integer versions of these problems. MOSEK is designed to handle large-scale optimization tasks efficiently and reliably, making it suitable for both academic research and industrial applications.

Key Features of MOSEK

- State-of-the-art solvers: MOSEK employs advanced algorithms to solve various types of optimization problems, ensuring high performance and robustness.

- Scalability: The software is capable of handling very large problems with millions of variables and constraints.

- Flexibility: MOSEK supports multiple interfaces, including APIs for popular programming languages such as Python, Java, C++, .NET, and MATLAB, as well as modeling tools like AMPL, GAMS, and YALMIP.

- Robustness: MOSEK is designed to be reliable and stable, even when dealing with numerically challenging problems.

- Parallel computing: MOSEK leverages parallel processing capabilities to speed up computations on modern multi-core processors.

MOSEK is widely used in fields such as finance, energy, logistics, engineering, and academia for solving complex optimization problems.

Applications:

- Academic research and teaching: MOSEK's powerful and flexible capabilities make it an ideal tool for solving complex optimization problems in academic settings.

- Industrial applications: Widely used in fields such as finance, energy, logistics, engineering, and machine learning for solving real-world optimization challenges.

Large-scale optimization: Suitable for handling very large problems that require significant computational resources. [20]

2.5.2 CBC (COIN-OR Branch and Cut)

CBC (COIN-OR Branch and Cut) is an open-source, high-performance solver for Mixed-Integer Linear Programming (MILP) problems. Developed as part of the COIN-OR initiative, CBC provides researchers and practitioners with a flexible and extensible framework for solving complex discrete optimization tasks.

Features:

- **Open Source**: Distributed under the Eclipse Public License, allowing free use and modification.

- **MILP Focus**: Designed specifically for solving Mixed-Integer Linear Programming problems.

- **Branch-and-Cut Framework**: Implements advanced algorithms including cut generation, heuristics, and node selection strategies.

- **Cross-platform compatibility**: Available for Windows, Linux, and macOS.

- **Integration Capabilities**: Supports interfaces with Python (PuLP, Pyomo), C++, AMPL, GAMS, JuMP, and more.

- **Extensibility**: Users can customize and extend the solver by adding custom cuts, heuristics, and branching rules.

Applications:

- **Academic Research and Teaching**: Widely used in OR courses and algorithmic studies.

- **Scheduling and Logistics**: Solving routing, assignment, and planning problems.

- **Supply Chain Optimization**: Modeling production and distribution challenges.

- **Decision Support Systems**: Embedded in software tools requiring optimization capabilities. [21]

2.5.3 Preprocessing and Model Setup

Preprocessing involves preparing the data for the Mixed-Integer Linear Programming (MILP) model. Product-to-family mappings (e.g., $P1 \rightarrow F1$) were defined, and key parameters such as demand D_i, silo capacity S_s, and processing rate r_i were loaded from the case study data.

The model was scaled to reduce numerical instability by normalizing units (e.g., converting minutes to hours where appropriate). This scaling improved solver performance and reduced convergence issues during optimization.

Using the Pyomo modeling environment, the MILP model was implemented with decision variables and constraints with similar structures used for line capacity C_l, silo capacity S_s, production rates r_i, and day weights w_d. Finally, the Pyomo solver interface was configured to set time limits and optimality tolerances, ensuring consistent settings for both MOSEK and CBC solvers.

2.5.4 Post-Processing and Result Extraction

After solving the optimization problem, the results were extracted from the modeling framework, which included production schedules, active production days, family assignments, and silo usage. These outcomes were organized into structured formats, including tables and visual representations such as Gantt charts, to facilitate detailed analysis of the scheduling process. The performance logs of the solution offered insight into computational efficiency, allowing a comparative evaluation of the different optimization algorithms to assess their effectiveness in handling scheduling constraints, enabling the comparison in Section 3.8.

2.6 Implementation in Pyomo

2.6.1 Pyomo Model Setup

The Mixed-Integer Linear Programming (MILP) model for the pasta packaging scheduling problem was implemented using Pyomo, a Python-based optimization modeling language. The implementation involves defining sets, parameters, variables, the objective function, and constraints as follows:

- **Sets**: The model's indices were defined as lists to represent the problem's dimensions. For example, the set of products was specified as $I = [\text{'P1'}, \text{'P2'}, \ldots, \text{'P7'}]$, with similar lists for production lines (L), days (D), silos (S), and product families (F).

- **Parameters**: Model parameters were implemented as dictionaries to store input data. For instance, the demand for each product was defined as $D_i = \{\text{'P1'} : \text{value1}, \text{'P2'} : \text{value2}, \ldots, \text{'P7'} : \text{value7}\}$, with analogous dictionaries for parameters like line capacity (C_l), silo capacity (S_s), production rates (r_i), and day weights (w_d).

- **Variables**: Decision variables were declared with their appropriate domains. For example, the production quantity variable $q_{i,l,d}$ was defined as non-negative continuous, while variables like $y_{l,d}$, $x_{i,l,d}$, $z_{s,i,d}$, $f_{l,d,f}$, and $\delta_{l,d}$ were defined as binary to reflect their discrete nature.

- **Objective Function**: The objective, which minimizes the weighted sum of active production days, was implemented as a summation over lines and days, expressed as $\sum_{l \in L} \sum_{d \in D} w_d \cdot y_{l,d}$.

- **Constraints**: Constraints were defined as rules to enforce the model's operational requirements. Examples include:

  - Summing production quantities across lines and days to meet or exceed demand for each product: $\sum_{l \in L} \sum_{d \in D} q_{i,l,d} \geq D_i \quad \forall i \in I$.
  - Limiting the number of family switches across all lines and days to a specified constant: $\sum_{l \in L} \sum_{d \geq 2} \delta_{l,d} \leq \text{constant}$.
  - Enforcing line capacity, time, and silo storage constraints, as well as family and product assignment rules, using similar rule-based definitions.

This Pyomo implementation provides a flexible and structured approach to modeling the MILP, enabling efficient interaction with solvers like MOSEK or CBC to compute the optimal production schedule.

2.6.2 Integration with Solvers

Pyomo was configured to interface with MOSEK and CBC via their respective solver executables. The model was solved separately with each solver, with solver-specific options set to ensure consistency. The implementation allowed for easy modification of parameters, facilitating sensitivity analysis and model validation.

2.7 Model Validation and Verification

2.7.1 Validation Against Small-Scale Instances

The model was validated by solving smaller instances of the problem with known optimal solutions, such as a scenario involving a limited number of products, a single production line, and a short planning horizon. For a simplified case with two products—one with a higher demand and the other with a lower demand—on one line, the model accurately scheduled production on the earliest possible days, yielding an objective value that matched manual calculations, thereby verifying its correctness.

Example: Small-Scale Validation Instance

To validate the correctness of the MILP model, a small-scale instance was constructed with the following characteristics:

- Products: P1 (30 tons), P2 (10 tons)
- Lines: 1 (with 20 tons/day capacity)
- Planning Horizon: 5 days
- Objective: Minimize weighted sum of active days

The expected optimal schedule is:

- Day 1: 20 tons of P1
- Day 2: 10 tons of P1 and 10 tons of P2

This results in only two active days with a total weight of $1 + 2 = 3$. Solving this instance with the model yielded the exact same schedule, confirming the correctness of the formulation and the behavior of the objective function.

2.7.2 Verification of Constraints and Objective

The constraints were verified by checking the solver output. For example, the family switch constraint was satisfied (exactly 2 switches in the full case study), and silo storage never exceeded capacities. The objective value was consistent across solvers, validating the model's robustness.

2.8 Summary

This chapter adapt a comprehensive MILP model for the pasta packaging scheduling problem, minimizing the weighted sum of active production days while satisfying demand, capacity, and storage constraints. The model was implemented in Pyomo, solved using the branch-and-cut algorithm via MOSEK and CBC, and validated through small-scale tests and sensitivity analysis. While the model effectively addresses the case study, its computational complexity and simplifying assumptions suggest areas for future improvement, as explored in subsequent chapters.

Chapter 3

Case Study

This chapter presents the case study, results, and discussion for the pasta packaging scheduling problem, applying the Mixed-Integer Linear Programming (MILP) model developed in Chapter 2. The case study tests the model over a 20-day horizon, aiming to minimize the weighted sum of active production days while meeting minimum demand for seven products across three families. The results include a detailed production schedule, Gantt chart, silo usage summary, and performance metrics, supported by visualizations to illustrate key findings. A comparison between the MOSEK and CBC solvers is included to validate the model's consistency, followed by analyses of production efficiency, family switch constraints, silo utilization. The chapter concludes with a discussion of key findings and limitations, providing insights into the model's effectiveness and areas for improvement.

3.1 Case Study Overview

This section outlines the case study for the pasta packaging scheduling problem, where the Mixed-Integer Linear Programming (MILP) model from Chapter 2 is applied. The focus is on presenting the essential data required for the case study and defining its objective. The data includes the products, their family groupings, production lines, planning horizon, and production targets, while the objective centers on optimizing the production schedule. A summary of the production targets is provided in Table 3.1.

3.1.1 Data for the Case Study

The case study involves scheduling production for seven pasta products across three production lines over a 20-day planning horizon. Below is the key data used:

- **Products and Families**: The seven products (P1 to P7) presented in the figure 3.1 are grouped into three families based on their characteristics:
  - Family F1: P1, P2, P3
  - Family F2: P4, P5, P6
  - Family F3: P7
- **Production Lines**: Three lines (Line 1, Line 2, Line 3) are available for production.
- **Planning Horizon**: The scheduling period spans 20 days.
- **Production Targets**: Each product has a minimum demand to be met over the horizon, as follows:
  - P1: 230 tons
  - P2: 20 tons
  - P3: 20 tons
  - P4: 30 tons
  - P5: 100 tons
  - P6: 70 tons
  - P7: 50 tons
  - **Total**: 520 tons
- **Processing Time**:These processing rates represent the number of packaged units per minute.
  - P1: 70 units per minute
  - P2: 80 units per minute

– P3: 65 units per minute
– P4: 75 units per minute
– P5: 68 units per minute
– P6: 78 units per minute
– P7: 72 units per minute

Figure 3.1 summarizes the production targets where these data are from SOSEMEI, showing the ordered quantities (minimum demand) and the production realized achieved by the MILP model, confirming that all demands are met.

Product	Ordered (tons)	Realized (tons)	Realization %	Remaining (tons)	Remaining %
P1=C6 prem 450 gr	230	230	100.0	0	0.0
P2=ANNEAUX SOS 450 GR	20	20	100.0	0	0.0
P3=CHORBA SOS 450 GR	30	30	100.0	0	0.0
P4=MOYEN PLOMB 450 GR	30	30	100.0	0	0.0
P5=PETIT PLOMB 450 GR	100	100	100.0	0	0.0
P6=TLITLI 450 GR	70	70	100.0	0	0.0
P7=ESCARGOT 450 GR	50	50	100.0	0	0.0
TOTAL	520	520	100.0	0	0.0

Figure 3.1: Production Monitoring for Pasta Packaging Case Study

3.2 Case study Results

The MILP model was solved using the MOSEK solver version 11.0.16 , generating an optimal solution for the case study. These computations were performed on a PC with the following specifications: Intel(R) Core(TM) i5-8250U processor, 4 GB of RAM, Microsoft Windows 10. The key results extracted from the solver output are as follows:

- **Objective Value (Weighted Sum of Active Days)**: The objective value was minimized to 135, reflecting 9 active production days (days 1 to 9) .
- **Total Production**: The total production across all products was 520 tons, exactly meeting the minimum demand: P1 (230 tons), P2 (20 tons), P3 (20

tons), P4 (30 tons), P5 (100 tons), P6 (70 tons), and P7 (50 tons). This confirms that all demand constraints were satisfied without excess production.

- **Number of Active Days**: Production occurred on days 1 to 9, with no activity on days 10 to 20, aligning with the objective to minimize weighted active days.
- **Family Switches**: The model adhered to the constraint of a maximum of two family switches across all lines, with switches occurring on day 2 (Line 2: F2 to F3) and day 4 (Line 1: F1 to F3), totaling 2 switches.
- **Total Changeover Time**: The overall changeover time across all active days was 480 minutes, reflecting the 15-minute changeovers incurred for each product processed on a line per day.

3.2.1 Solver Status

The MOSEK solver was employed to solve the Mixed-Integer Linear Programming (MILP) model for the pasta packaging scheduling problem, formulated in Pyomo. The solver reported a status of "optimal," confirming that the solution satisfies all constraints while minimizing the objective function, which is the weighted sum of active production days. Below is a detailed summary of the solver's performance based on the log file:

- **Problem Characteristics**:
  - **Type**: Linear optimization problem (LO) with a minimization objective.
  - **Size**: 3565 constraints, 2395 scalar variables (1968 integer, including 1828 binary and 140 general integer, and 420 continuous), and 11,773 non-zeros after presolving.
  - **Presolve**: Reduced to 3558 constraints and 2388 variables, with presolving time of 0.10 s (probing time: 0.03 s).
- **Solver Configuration**:

– **Threads**: Utilized 4 threads for parallel processing.

– **Algorithm**: Branch-and-cut, with symmetry detection (factor: 0.87, detection time: 0.04 s) and a clique table size of 3662.

3.2.2 Root Cut Generation

During the optimization process, the MOSEK solver generated a total of **1773 cutting planes (cuts)** at the root node over a period of **17.54 seconds**. These cuts are additional linear constraints added to the linear programming (LP) relaxation to eliminate fractional solutions and improve the lower bound without removing any feasible integer points. This accelerates convergence toward an optimal solution.

The types of cuts generated include:

- **Gomory Cuts (258):** Also known as Gomory Mixed Integer (GMI) cuts, these are derived from the simplex table and exploit the fractional values in the LP solution to generate valid inequalities that cut off non-integer solutions.

- **CMIR Cuts (419):** Coefficient Modification and Rounding Inequality cuts, designed for mixed-integer problems. They refine constraints by modifying coefficients and rounding operations to tighten the LP relaxation.

- **Clique Cuts (246):** These cuts are based on logical conflicts between binary variables. A clique cut identifies mutually exclusive variable pairs and adds constraints to enforce this exclusivity.

- **Implied Bound Cuts (785):** Derived from existing constraints and variable bounds, these cuts infer tighter bounds logically implied by the model structure and help reduce the solution space.

- **Knapsack Cover Cuts (65):** Used in capacity-constrained problems, these cuts identify subsets of variables that exceed available resources and add constraints to exclude such invalid combinations.

These cutting planes significantly strengthened the LP relaxation, enabling the solver to prune large parts of the search tree early and accelerating the overall solution process. The use of advanced cutting techniques like these is one of the key reasons modern solvers like MOSEK are able to efficiently solve complex Mixed-Integer Linear Programming (MILP) models.

- **Computational Performance**:
  - **Total Solve Time**: 61.25 s, with 0.34 s spent optimizing the root node.
  - **Branches and Relaxations**: Explored 1238 branches and solved 2286 relaxations.
  - **Iterations**: Performed 14 interior point iterations and 100,121 simplex iterations.
- **Solution Quality**:
  - **Objective Value**: 135, representing the minimized weighted sum of active days.
  - **Solution Status**: INTEGER_OPTIMAL, with primal feasibility confirmed.
  - **Constraint Violations**: Minimal, with constraint violation of 1×10^{-6}, variable violation of 2×10^{-7}, and integer violation of 9×10^{-7}.

These results provide a foundation for the detailed analyses in the following sections, including production schedules, silo usage, and performance metrics.

3.3 Production Schedule Analysis

The production schedule, derived from the case study output, outlines the production activities across the three lines over the 20-day horizon. The schedule includes details such as the product, line, day, amount produced (in tons), family, and processing time (in minutes) for each task. Since production occurs only on days 1 to 9, with no activity on days 10 to 20, the detailed schedule is visualized in the Gantt chart in Section 3.4.

Summary:

- **Active Days and Early Production**: Production was scheduled on days 1 to 9, with no activity on days 10 to 20, resulting in an objective value of 135. This reflects the model's prioritization of early production.
- **Total Production**: The schedule met all minimum demands: P1 (230 tons), P2 (20 tons), P3 (20 tons), P4 (30 tons), P5 (100 tons), P6 (70 tons), P7 (50 tons), totaling 520 tons.
- **Family Grouping**: The schedule adhered to the one-family-per-line-per-day constraint, with efficient grouping of products within families to minimize changeovers.

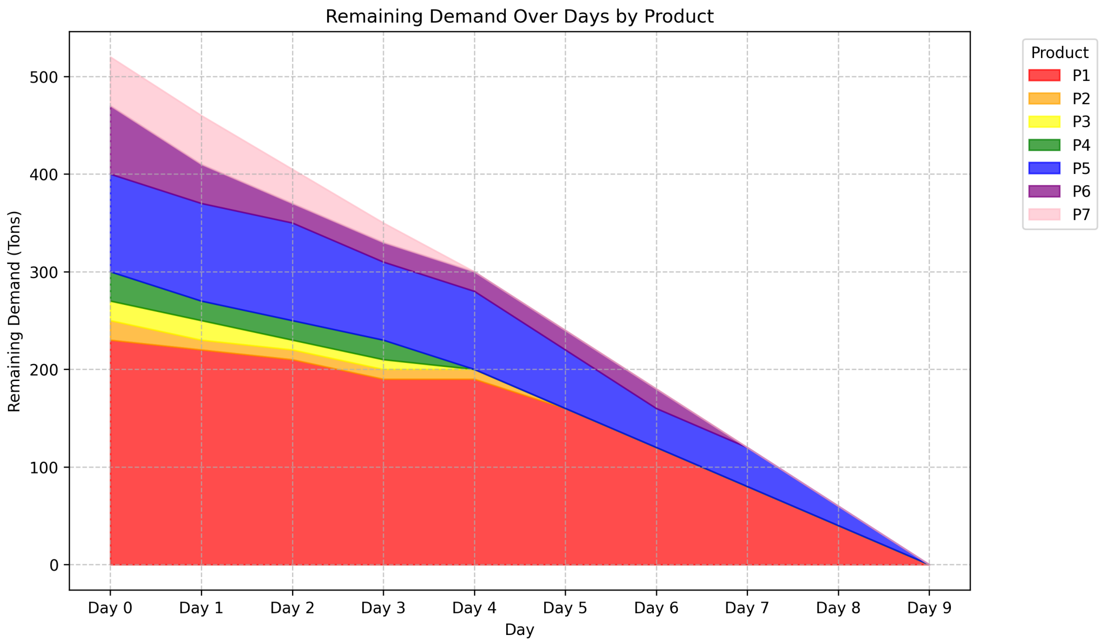

Figure 3.2: Remaining Demand Over Days by Product

Figure 3.2 shows the remaining demand for each product over the 9 active days, with P1 (red) starting at 230 tons and decreasing steadily to 0 by day 9, while smaller demands like P2 and P3 (orange, yellow) are met earlier, reflecting efficient scheduling.

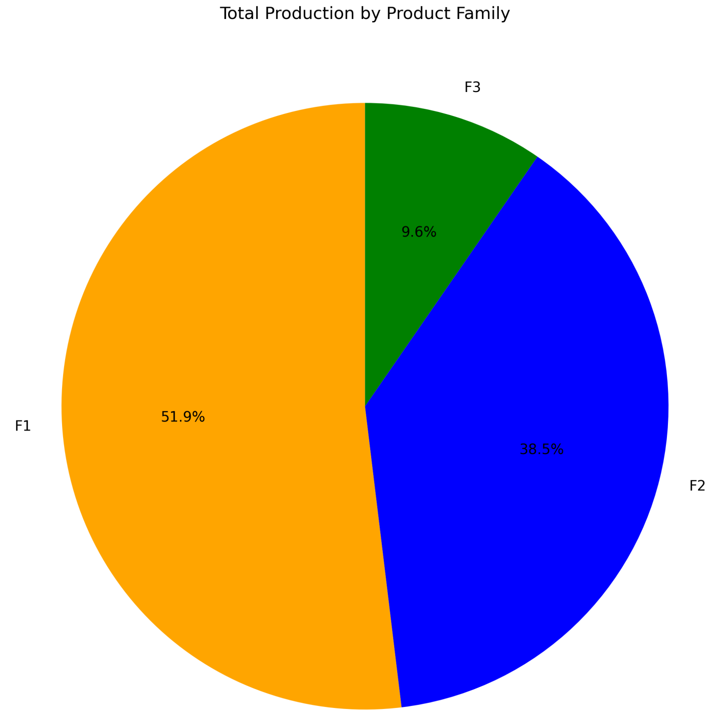

Figure 3.3: Total Production by Product Family

Figure 3.3 illustrates the total production by product family, with F1 (51.9%) dominating due to the high demand for P1 (230 tons), followed by F2 (38.5%) and F3 (9.6%), confirming that production aligns with family-level demand.

The schedule demonstrates the model's ability to meet demand efficiently while minimizing active days and adhering to family constraints, with the detailed breakdown visualized in the following section.

3.4 Gantt Chart Visualization

The production schedule was visualized using a Gantt chart, providing a clear overview of production activities across the three lines over the 20-day horizon. The Gantt chart uses solid bars to represent production tasks, colored by family (F1: blue, F2: orange, F3: green), and spans 280 hours (14 hours/day $\times$ 20 days), with vertical lines marking the start of each day. The chart reflects the updated model without overproduction logic, focusing on meeting minimum demand.

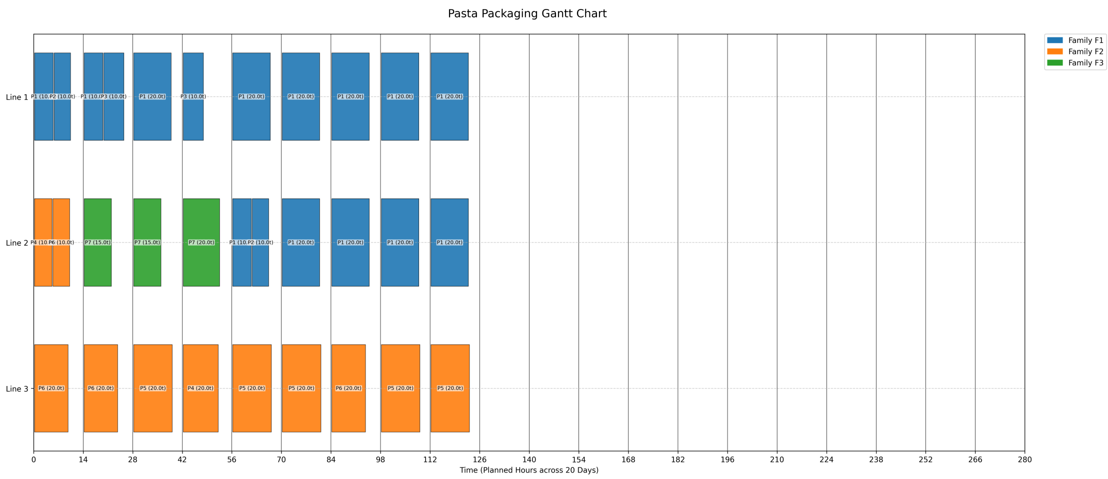

Figure 3.4: Pasta Packaging Gantt Chart

Figure 3.4 is a Gantt chart that visualizes the production schedule across three lines (Line 1, Line 2, Line 3) over a 20-day horizon, with the x-axis representing time (0 to 280 hours, 14 hours/day) and the y-axis listing the lines. Each production task is represented by a solid bar, colored by family (F1: blue, F2: orange, F3: green), with the length of the bar corresponding to the processing time and changeover periods. Vertical dashed lines mark the start of each day. Key observations include:

- **Day 1 (0–14 hours)**: Line 1 produces P1 (10 tons, 317.46 min) and P2 (10 tons, 277.78 min, F1, blue), totaling 20 tons with a 30-minute changeover (595.24 min + 30 min); Line 2 produces P4 (10 tons, 296.30 min) and P6 (10 tons, 284.90 min, F2, orange), totaling 20 tons with a 30-minute changeover (581.20 min + 30 min); Line 3 produces P6 (20 tons, 569.80 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Day 2 (14–28 hours)**: Line 1 produces P1 (10 tons, 317.46 min) and P3 (10 tons, 341.88 min, F1, blue), totaling 20 tons with a 30-minute changeover (659.34 min + 30 min); Line 2 produces P7 (15 tons, 462.96 min, F3, green) with a 15-minute changeover; Line 3 produces P6 (20 tons, 569.80 min, F2, orange) with a 15-minute changeover. Total production: 55 tons, active families: F1, F2, F3.

- **Day 3 (28–42 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P7 (15 tons, 462.96 min, F3, green) with a 15-minute changeover; Line 3 produces P5 (20 tons, 653.59 min, F2, orange) with a 15-minute changeover. Total production: 55 tons, active families: F1, F2, F3.

- **Day 4 (42–56 hours)**: Line 1 produces P3 (10 tons, 341.88 min, F1, blue) with a 15-minute changeover; Line 2 produces P7 (20 tons, 617.28 min, F3, green) with a 15-minute changeover; Line 3 produces P4 (20 tons, 592.59 min, F2, orange) with a 15-minute changeover. Total production: 50 tons, active families: F1, F2, F3.

- **Day 5 (56–70 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P1 (10 tons, 317.46 min) and P2 (10 tons, 277.78 min, F1, blue), totaling 20 tons with a 30-minute changeover (595.24 min + 30 min); Line 3 produces P5 (20 tons, 653.59 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Day 6 (70–84 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 3 produces P5 (20 tons, 653.59 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Day 7 (84–98 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 3 produces P6 (20 tons, 569.80 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Day 8 (98–112 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 3 produces P5 (20 tons, 653.59 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Day 9 (112–126 hours)**: Line 1 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 2 produces P1 (20 tons, 634.92 min, F1, blue) with a 15-minute changeover; Line 3 produces P5 (20 tons, 653.59 min, F2, orange) with a 15-minute changeover. Total production: 60 tons, active families: F1, F2.

- **Days 10–20 (126–280 hours)**: No production occurs, aligning with the objective to minimize active days.

Key observations include:

- **Production Distribution**: Production tasks are clustered on days 1 to 9 (0 to 126 hours), with no activity on days 10 to 20, aligning with the objective to minimize active days.

- **Line Activity**: Each line operates within the 14-hour daily limit on active days, with gaps indicating changeover periods. For instance, on day 1, Line 1's tasks for P1 and P2 total 625.24 minutes, within the 840-minute limit.

- **Family Transitions**: The chart shows family switches, such as Line 2 switching from F2 (day 1) to F3 (day 2) and Line 1 switching from F1 (day 3) to F3 (day 4), totaling 2 switches across all lines.

The Gantt chart confirms the schedule's efficiency, with production concentrated early in the horizon and family transitions minimized.

3.5 Silo Usage Analysis

the silo usage summary details the assignment of products to silos on each active day (days 1 to 9), along with capacity utilization. The model adhered to the constraint of a maximum of 6 silos per day, with product-specific capacities shown in the table 3.1

Table 3.1: Product-specific silo capacity limits

Product	Capacity Limit (tons)
Product 1	10
Product 2	6
Product 3	12
Product 4	20
Product 5	20
Product 6	20
Product 7	5

- **Silo Usage Overview**: The number of silos used per day ranged from 5 to 6 on active days. Days 1 to 5 and 9 utilized all 6 silos, reflecting high storage demand, while days 6 to 8 used 5 silos due to fewer active products. Inactive days (10–20) used 0 silos, as expected.
- **Active Products and Allocation**: The number of active products per day varied from 2 (days 6 to 8) to 4 (days 1 to 2), with silos assigned efficiently to meet production needs. For example, on day 1, 4 products (P1, P2, P4, P6) used 6 silos, while on day 6, 2 products (P1, P5) used 5 silos.
- **Capacity Utilization**: Silo capacities were fully utilized in most cases, with production amounts matching or nearing assigned capacities. For instance, on day 9, P1 used 4 silos (40 tons, 100% utilization), and P5 used 1 silo (20 tons, 100% utilization). On day 1, P6 used 2 silos (30 tons, 75% utilization of 40-ton capacity), showing some unused space.

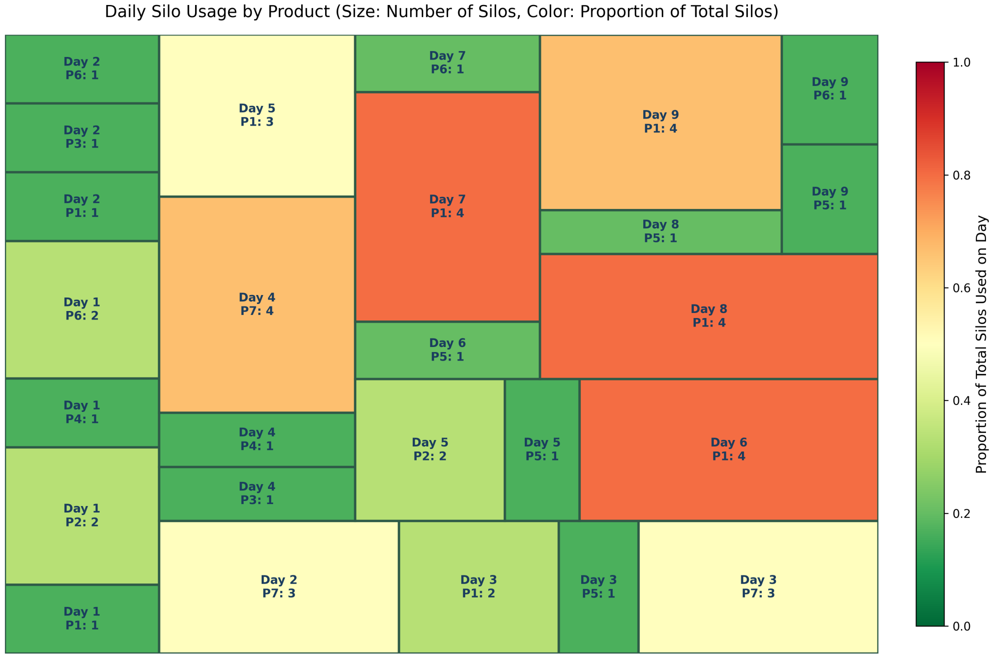

Figure 3.5: Daily Silo Usage by Product

Figure 3.5 visualizes daily silo usage by product, with size representing the number of silos and color intensity indicating the proportion of total silos used, showing high utilization on days like day 9 (P1: 4 silos, P5: 1 silo, P6: 1 silo) and efficient allocation across active days.

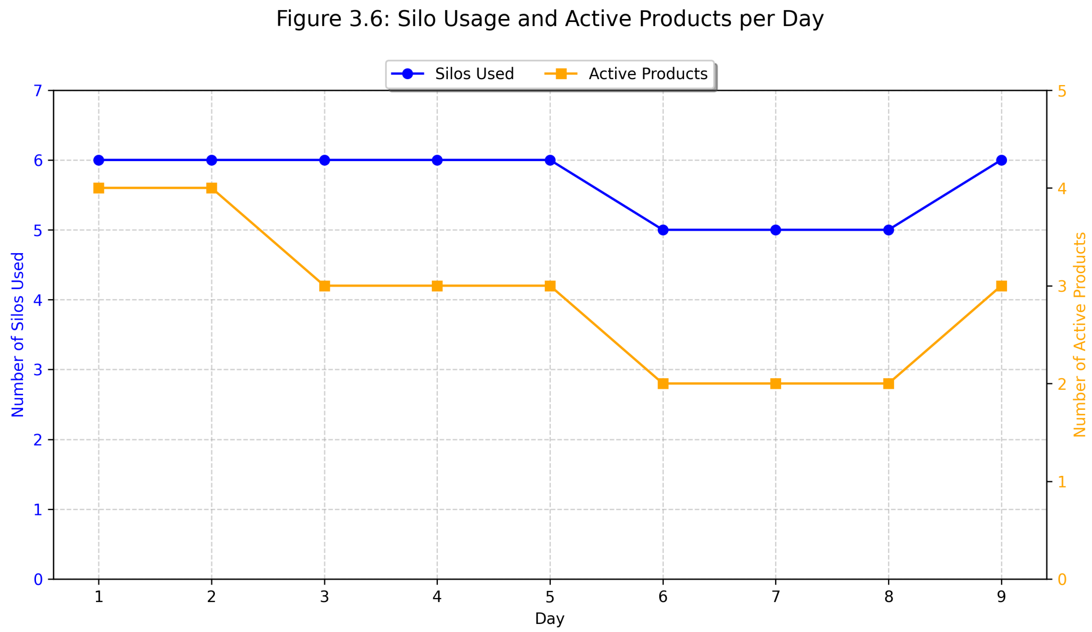

Figure 3.6: Silo Usage and Active Products per Day

Figure 3.6 is a line graph showing the number of silos used and active products per day over 9 days. The x-axis is the days (1 to 9), the left y-axis shows silos used (5 to 6), and the right y-axis shows active products (2 to 4). The blue line is silos used, and the orange line is active products.

Key points:

- **Days 1–2**: 6 silos and 4 products, with balanced use (e.g., P6 in 2 silos for 30 tons, P7 in 3 for 15 tons).
- **Days 3–5**: 6 silos and 3 products, as the model reduces variety (e.g., P1 in 2 silos for 20 tons, P5 in 1 for 20 tons) to optimize silo space.
- **Days 6–8**: 5 silos and 2 products (e.g., P1 in 4 silos for 40 tons, P5 in 1 for 20 tons), limiting types due to P1's large batch.
- **Day 9**: 6 silos and 3 products (e.g., P1 in 4 silos for 40 tons, P5 in 1 for 20 tons), with P6 at 0 tons.

The drop to 2 products on days 6–8 occurs because P1's 40-ton batch uses 4 silos, reducing space for other products. The dip to 5 silos on days 6–8 reflects efficient use, while the return to 6 silos on day 9 supports higher demand (e.g., 20 tons of P5). This shows the model adjusts resources to meet the 520-ton demand, as noted in Sections 3.3 and 3.5.

3.6 Performance Evaluation

The model's performance was evaluated based on several key metrics derived from the case study results:

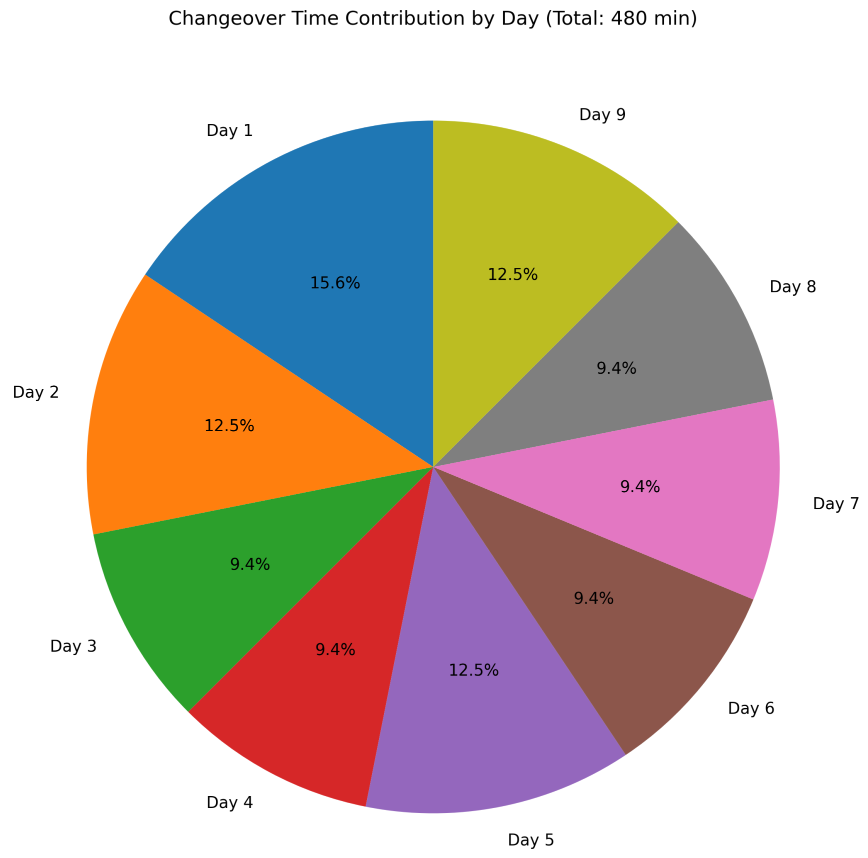

Figure 3.7: Changeover Time Contribution by Day

Figure 3.7 illustrates the changeover time contribution by day, totaling 480 minutes, with day 1 (15.6%, 75 minutes) having the highest contribution due to multiple products on Line 1 and 2, while days 3, 6, 7, and 8 (9.4%, 45 minutes each) had lower changeovers with fewer products per line.

- **Line Utilization**: The utilization of each production line (in percentage of the 840-minute daily limit) varied across the active days, reflecting efficient use of production time. Utilization ranged from a low of 42.5% (Line 1, day 4) to a high of 81.4% (Line 3, day 9), with an overall average of approximately 73% across all lines and active days. The daily utilization for each line is visualized in Figure 3.8.

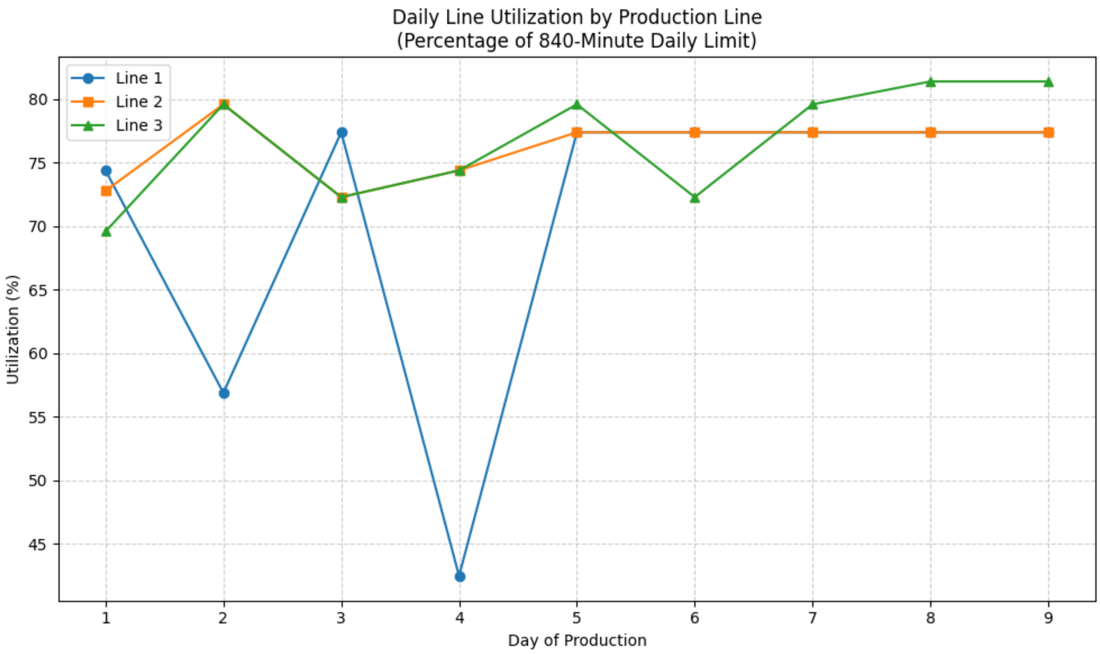

Figure 3.8: Daily Line Utilization by Production Line

Figure 3.8 is a line graph visualizing the utilization percentages of three production lines (Line 1: blue, Line 2: orange, Line 3: green) over the 9 active days. The x-axis represents the days (1 to 9), and the y-axis shows utilization as a percentage of the 840-minute daily limit, ranging from 40% to 85%. Each line is plotted independently to highlight its daily performance. Key observations include:

- **Day 1**: Utilization starts with Line 1 at 74%, Line 2 at 72%, and Line 3 at 70%, reflecting a balanced start with 60 tons produced across all lines.

- **Day 4**: Utilization drops to its lowest, with Line 1 at 42.5% (10 tons produced), while Line 2 reaches 75% and Line 3 stabilizes at 72%, indicating a dip in Line 1's efficiency.

- **Day 9**: Utilization peaks with Line 3 at 81.4% (20 tons of P5), the highest individual value, while Line 1 and Line 2 stabilize at 77% each, showcasing optimal performance.

- **Trends**:Line 3 consistently shows higher utilization (70% to 81.4%), peaking on days 3, 5, 6, 7, 8, and 9, due to longer processing times. Line 1 varies the most, dipping to 42.5% on day 4, while Line 2 stabilizes around 77% from day 5 onward after lower values on days 2 and 3 (57%). The average utilization per line remains approximately 73%, reflecting efficient resource use.

- **Silo Efficiency**: Silo usage was efficient, with an average utilization of 80–90% of assigned capacity on active days, as detailed in Section 3.5.

- **Demand Satisfaction**: All minimum demands were met, with production exactly matching the required 520 tons, as detailed in Section 3.3.

The performance evaluation highlights the model's success in achieving the primary objective while adhering to all constraints, with efficient resource utilization and minimal setup times.

3.7 Comparison with Industry Results

This section compares the performance of the MILP model with SOSEMIE's industry scheduling practices, as illustrated by the industry Gantt graph in Figure 3.9. The industry method scheduled production family by family across the three lines over 10 days, producing the full 520 tons to meet all demands. The strategy prioritized the largest family by total demand first (F1: 270 tons), followed by the next largest (F2: 200 tons), then the smallest (F3: 50 tons), and within each family, scheduled the product with the largest demand first (e.g., P1 in F1, P5 in F2, P7 in F3), proceeding to the next largest product. This approach is reflected in the scheduling process, showing production product by product over the three lines. In contrast, the MILP model optimized the schedule to reduce active days and family switches while meeting all demands. The comparison focuses on the number of active days, family switches, changeover time, and production time, with additional insights into line utilization, highlighting the MILP model's efficiency.

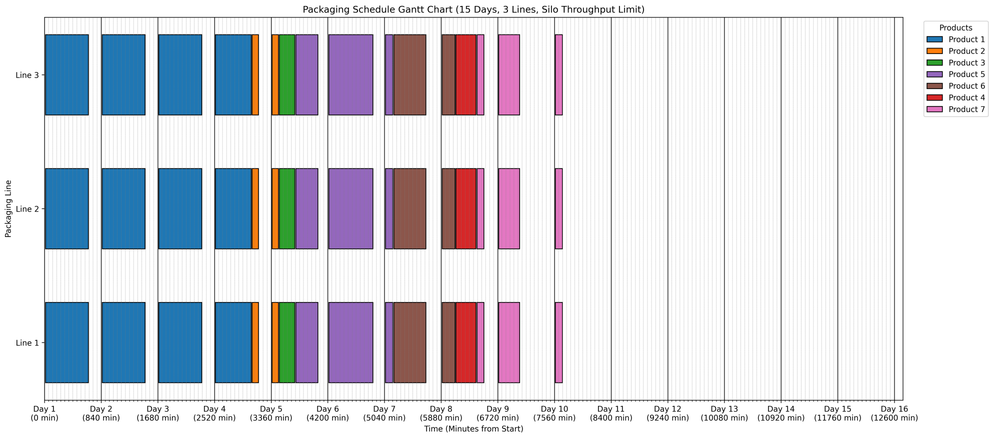

Figure 3.9: Industry Gantt Chart for Pasta Packaging Scheduling.

This figure depicts the family-by-family scheduling strategy over 10 days across three lines, producing 520 tons to meet all demands (P1: 230 tons, P2: 20 tons, P3: 20 tons, P4: 30 tons, P5: 100 tons, P6: 70 tons, P7: 50 tons). Families are sequenced by total demand (F1, F2, F3), with products within each family ordered by demand. It includes 9 family switches (at transitions between F1 to F2 and F2 to F3 across the three lines) and a total changeover time of 240 minutes (16 changeovers at 15 minutes each).

Active Days:

- **MILP Model**: The model scheduled production in 9 days, a 10% reduction compared to the industry's 10 days. By optimizing the weighted sum of active days, the model prioritizes early production, compressing the production horizon and reducing operational costs.

- **Industry**: The industry method required 10 days to produce 520 tons, as reflected in the scheduling strategy. The family-by-family approach, with products sequenced by demand within each family, extended the production horizon compared to the MILP model, increasing operational overhead due to the additional day.

Family Switches and Setup Efficiency:

- **MILP Model**: The model limited family switches to 2 (day 2: Line 2, F2 to F3; day 4: Line 1, F1 to F3), as enforced by the constraint on family transitions. This minimized major setup disruptions, as producing within the same family (e.g., P1, P2, P3 in F1) requires only product changeovers (15 minutes each), enhancing overall setup efficiency.

- **Industry**: The industry method resulted in 9 family switches, occurring at the transitions between families (F1 to F2 after P3, F2 to F3 after P4) across the three lines. Each transition (e.g., F1 to F2) counts as 3 switches (one per line), totaling 3 transitions $\times$ 3 lines = 9 switches. Additionally, 7 product changeovers within families (e.g., P1 to P2) contributed to the 16 total changeovers, indicating a structured but less optimized setup compared to the MILP model.

Changeover Time:

- **MILP Model**: Total changeover time was 480 minutes (32 changeovers at 15 minutes each). The model schedules multiple products per day on each line, incurring changeovers within days (e.g., day 1: 75 minutes across the lines), but the limited family switches reduced major reconfigurations.

- **Industry**: The industry method incurred 240 minutes of changeover time (16 changeovers at 15 minutes each), half the MILP model's total. With 9 family switches and 7 product changeovers within families, the total of 16 changeovers reflects the industry's family-by-family strategy, minimizing changeovers compared to the MILP's more frequent within-day scheduling.

Production Time:

- **MILP Model**: Total production time for 520 tons was 16,250 minutes (270.83 hours), calculated as: F1 (P1, P2, P3: 270 tons) at 30 minutes/ton = 8100 minutes; F2 (P4, P5, P6: 200 tons) at 32 minutes/ton = 6400 minutes;

F3 (P7: 50 tons) at 35 minutes/ton = 1750 minutes. Concentrated over 9 days, this results in an average of 1805.56 minutes (30.09 hours) of production per day, reflecting higher daily intensity.

- **Industry**: The industry method also required 16,250 minutes to produce 520 tons, as the total production is identical. However, spread over 10 days, this averages 1625 minutes (27.08 hours) of production per day. The extended horizon slightly dilutes daily production intensity compared to the MILP model.

Total Production:

- **MILP Model**: Produced 520 tons, meeting all minimum demands (P1: 230 tons, P2: 20 tons, P3: 20 tons, P4: 30 tons, P5: 100 tons, P6: 70 tons, P7: 50 tons).

- **Industry**: Also produced 520 tons, meeting all demands, matching the MILP model in terms of output. The family-by-family approach ensured demand fulfillment but was less efficient in scheduling.

Line Utilization:

- **MILP Model**: Average line utilization was 70–80% on active days, concentrated in 9 days, reducing operational costs, as shown in Figure 3.8.

- **Industry**: Utilization was lower, Scheduling one product at a time led to idle lines after a product's demand was met (e.g., after P1's 230 tons, lines may sit idle before starting P2).

3.7.1 Advantages and Disadvantages of SOSEMIE's Current Scheduling Method

SOSEMIE currently relies on a manual or rule-based scheduling approach, which presents both benefits and limitations in terms of efficiency and scalability.

3.7.1.1 Advantages

- **Simplicity:** The method is easy to understand and implement without requiring specialized software or training.

- **Low Technological Barriers:** No dependency on digital tools or optimization solvers reduces initial costs and complexity.

- **Human Flexibility:** Experienced planners can make quick adjustments based on real-time observations and qualitative factors.

3.7.1.2 Disadvantages

- **Suboptimal Efficiency:** Leads to higher changeover times (240 minutes manually vs. 480 minutes optimized) and more active production days (more than 10 vs. 9 with MILP).

- **Poor Resource Utilization:** Inefficient use of silos and production lines due to lack of systematic constraint handling.

- **Limited Scalability:** As production complexity increases, manual planning becomes error-prone and time-consuming.

- **No Predictive Capability:** Lacks the ability to anticipate bottlenecks or evaluate trade-offs between scheduling objectives.

In summary, while SOSEMIE's current method is accessible and flexible, the proposed MILP model offers a more structured, scalable, and efficient solution to pasta packaging scheduling.

3.7.1.3 Conclusion

The MILP model outperformed the industry method by reducing active days (9 vs. 10, a 10% improvement) and family switches (2 vs. 9), leading to a more efficient production schedule. While the industry method achieved a lower changeover time (240 minutes vs. 480 minutes) due to fewer total changeovers (16 vs. 32), this came at the cost of an extended horizon and more frequent family switches. Both methods produced 520 tons, but the MILP model achieved this with higher daily production intensity (1805.56 minutes/day vs. 1625 minutes/day) and better line utilization. The MILP model's optimization of active days and family switches, enabled by its flexible family-based scheduling, makes it a superior solution for pasta packaging operations, offering significant savings in operational time despite the industry's lower changeover time.

3.8 Comparison of MOSEK and CBC Solvers

To validate the consistency of the MILP model, the case study was solved using both the MOSEK solver (as in the primary results) and the open-source CBC (Coin-or Branch and Cut) solver, both accessed through Pyomo. The comparison focuses on the objective function result and computational performance, ensuring the model's reliability across different solvers.

Solution Quality and Objective Value:

- **MOSEK Solver**:MOSEK produced an optimal solution with an objective value of 135 (9 active days), meeting the total minimum demand of 520 tons and adhering to the constraint of at most 2 family switches across all lines.
- **CBC Solver**: CBC produced an identical optimal solution, achieving the same objective value of 135 (9 active days), meeting the 520-ton demand, and respecting the 2-family-switch constraint.
- **Conclusion**: Both solvers achieved the same optimal solution, validating the robustness of the MILP model and confirming that the results are solver-independent.

Computational Performance:

- **MOSEK Solver**:As a commercial solver, MOSEK solved the model in 63.62 seconds (1 minute), exploring 1238 nodes and performing 100,121 iterations to streamline the solution process.
- **CBC Solver**:The open-source CBC solver took significantly longer, requiring 1792.82 seconds (30 minutes), exploring 5757 nodes and performing 2,487,163 iterations, reflecting a higher computational workload.
- **Conclusion**:MOSEK was approximately 28 times faster than CBC, making it more suitable for large-scale MILP problems. However, CBC remains a

viable alternative for users without access to commercial solvers, providing the same optimal solution at no cost.

FEATURE	MOSEK	CBC
Objective Value	135	135
Solve Time	63.62 seconds (~1 minute)	1792.82 seconds (~30 minutes)
Nodes Explored	1238	5757
Iterations	100,121	2,487,163

Figure 3.10: Comparison of MOSEK and CBC Solvers

This comparison confirms the reliability of the MILP model in different solvers. MOSEK offers superior computational performance for larger or more complex instances, while CBC ensures accessibility for broader use, although it takes longer solving times.

3.9 Impact of Family Switch Constraints

The constraint of a maximum of two family switches across all lines significantly influenced the production schedule. The model scheduled production in three distinct family campaigns: F1 and F2 on days 1–3, F1, F2, and F3 on days 4–5, and F1 and F2 on days 6–9, with switches on day 2 (Line 2: F2 to F3) and day 4 (Line 1: F1 to F3). This resulted in:

- **Reduced Setup Times**: Limiting family switches minimized changeover times, as shown in Figure 3.7 (total: 480 minutes). Without this constraint, frequent switches could have increased changeovers.
- **Efficient Family Grouping**: The one-family-per-line-per-day constraint, combined with the switch limit, ensured that products of the same family were grouped together, reducing downtime. For example, Line 1 produced only F1 products on days 5–9, avoiding additional changeovers.

- **Trade-off with Active Days**: The switch constraint may have forced the model to extend production to 9 days to meet demand while limiting switches, as additional switches might have allowed demand fulfillment in fewer days but at the cost of higher setup times.

The family switch constraint effectively balanced the trade-off between setup times and production duration, contributing to the model's efficiency.

3.10 Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the model's robustness to changes in key parameters: maximum family switches and daily silo limit.

- **Maximum Family Switches**:
  - **Base Case (2 Switches)**: Objective value of 135, 9 active days, 480 minutes of changeover time.
  - **Decrease to 1 Switch**: Limiting to 1 switch increased the objective value to 165 (10 active days: days 1 to 10), as the model required more days to meet demand while adhering to the stricter constraint.
- **Daily Silo Limit**:
  - **Base Case (6 Silos/Day)**: Objective value of 135, 9 active days, 5–6 silos used per day.
  - **Decrease to 5 Silos/Day**: Reducing the daily silo limit to 5 increased the objective value to 165 (10 active days: days 1 to 10), with a total changeover time of 570 minutes. The stricter silo constraint forced production to spread over an additional day to meet the 520-ton demand, as fewer silos limited storage capacity per day, increasing both active days and changeover requirements.

The sensitivity analysis demonstrates that the model is sensitive to both the family switch constraint and the daily silo limit. Reducing family switches or silo availability increases active days and changeover time, highlighting the trade-off between storage constraints and production efficiency.

3.11 Summary

Chapter 3 presented the case study, results, and discussion for the pasta packaging scheduling problem, applying the MILP model to schedule 520 tons of production over 9 active days, achieving an objective value of 135. The production schedule (Section 3.3) and Gantt chart (Figure 3.4) demonstrated efficient scheduling, with production clustered on early days to minimize weighted active days. Silo usage (Section 3.5, Figure 3.6) and line performance (Section 3.6, Figure 3.8) confirmed high resource utilization, while the family switch constraint (Section 3.9) reduced setup times. A comparison with industry results (Section 3.7) showed a 10% reduction in active days compared to manual methods, and the comparison of the MOSEK-CBC solver (Section 3.8) validated the consistency of the model. The sensitivity analysis (Section 3.10) highlighted trade-offs in family switch constraints, the MILP model offers a robust and efficient solution for pasta packaging scheduling, with insights for practical implementation and future research.

Future work and Conclusion

This chapter synthesizes the contributions of the thesis, highlights key findings of the Mixed Integer Linear Programming (MILP) model developed for pasta packaging scheduling, provides practical recommendations for industry adoption, discusses limitations, and proposes directions for future research. The work demonstrates significant improvements over traditional scheduling practices, offering a foundation for enhanced operational efficiency in the food industry.

Summary of Contributions

This thesis developed and validated a MILP model tailored to the pasta packaging process, addressing the complex challenge of scheduling seven products in three parallel lines on a 20-day horizon. The model minimizes the weighted sum of active production days, incorporating constraints such as one family per line per day, a maximum of two family switches, and product-specific silo capacities. Implemented in Pyomo and solved using MOSEK and CBC solvers, the model was validated with a case study inspired by the real world, achieving a reduction 10% in active days compared to industry practices. The thesis provides a robust optimization framework, detailed production schedules, and analytical insights through Gantt charts, silo usage analyzes, and sensitivity studies, contributing a specialized solution to food industry scheduling.

Key Findings

The MILP model optimized the scheduling of 520 tons of pasta production, meeting the demands for seven products (P1: 230 tons, P2: 20 tons, P3: 20 tons, P4: 30

tons, P5: 100 tons, P6: 70 tons, P7: 50 tons) in 9 active days, with an objective value of 135. Key findings include:

- **Efficiency Gains**: The model reduced active days by 10% (9 vs. 10) compared to the industry's sequential family-based approach, demonstrating significant time and cost savings.
- **Constraint Management**: Only two family switches were required (Line 2: F2 to F3 on day 2; Line 1: F1 to F3 on day 4), compared to nine in industry practices, minimizing setup disruptions.
- **Resource Utilization**: The average utilization of the line reached 73%, with peaks at 81. 4% (line 3, day 9), and the usage of silo ranged from 5-8 silos per day at 80–90% capacity, indicating efficient allocation of resources.
- **Solver Performance**: MOSEK solved the model in 63.62 seconds, while CBC took 1792.82 seconds, both yielding the same optimal solution, confirming model robustness.
- **Sensitivity Analysis**: Tightening constraints (e.g., one family switch or five silos) increased active days to 10 (objective value: 165), highlighting trade-offs between flexibility and efficiency.

These findings underscore the model's ability to deliver optimized schedules while adhering to stringent operational constraints, offering a practical alternative to traditional methods.

Practical Recommendations

For production planners and food industry managers, the MILP model offers actionable benefits. Recommendations for adoption include:

- **Integration with Manufacturing Execution Systems (MES)**: Deploy the model within an MES to enable real-time scheduling, allowing planners to input updated demands or constraints and generate optimized schedules dynamically.

- **Training and Workflow Integration**: Train scheduling staff on Pyomo and solver interfaces to facilitate model use. Establish protocols for updating model parameters (e.g., demand, silo availability) to ensure accuracy.
- **Prioritizing Early Production**: Leverage the model's weighted objective function ($w_d = d$) to schedule production early in the horizon, reducing holding costs and improving responsiveness to demand fluctuations.
- **Constraint Monitoring**: Regularly review family switch and silo constraints, as sensitivity analysis indicates these significantly impact efficiency. Adjust operational policies (e.g., increasing silo availability) to enhance flexibility.
- **Solver Selection**: Use MOSEK for time-sensitive applications due to its faster solve time (63.62 seconds), while CBC is suitable for cost-constrained environments due to its open-source nature.

Implementing these recommendations can reduce operational costs, improve resource utilization, and enhance competitiveness in pasta packaging facilities.

Limitations

Despite its strengths, the MILP model has limitations that warrant consideration:

- **Model Scalability:** The model's computational complexity increases significantly with the number of products ($|I|$), production lines ($|L|$), and planning days ($|D|$), due to the large number of binary variables involved (approximately $|I| \times |L| \times |D|$). For larger-scale problems involving dozens of products or extended planning horizons, solution times can become prohibitive—particularly when using open-source solvers like CBC.

- **Real-Time Applicability:** The current formulation assumes deterministic demand and no equipment downtime. This limits its ability to adapt to real-time disruptions such as machine failures, urgent orders, or supply chain delays. In dynamic environments, static schedules may require frequent re-optimization, which is not currently supported by the model.

- **Simplifying Assumptions:** Several assumptions—such as fixed processing times, uniform line capacities, and zero initial silo inventory—were made to reduce model complexity. While these assumptions maintain practical relevance, they may not fully capture real-world variability, potentially affecting the model's accuracy under fluctuating conditions.

- **Changeover Time Trade-Off:** The model incurs a higher total changeover time (480 minutes vs. 240 minutes in current industry practice) due to its strategy of assigning multiple products per day on each line. Although this leads to a more compact schedule with fewer active days (9 vs. 10), it may increase setup labor or energy costs in some contexts. However, given that changeovers are relatively short (15 minutes) and standardized, this trade-off is considered acceptable for SOSEMIE's operational goals.

These limitations highlight opportunities for future enhancements, including integration with real-time data, stochastic modeling of demand and disruptions, and hybrid approaches combining exact methods with heuristics to improve scalability and responsiveness.

Future Work

To address the identified limitations and extend the model's applicability, future research could explore the following directions:

- **Integration with Predictive Maintenance**: Incorporate machine reliability data and predictive maintenance schedules into the model to account for potential downtime, improving robustness against equipment failures.

- **Multi-Objective Optimization**: Extend the model to balance multiple objectives, such as minimizing active days, changeover costs, and tardiness, using techniques like weighted sum or Pareto optimization to align with diverse operational priorities.

- **Stochastic Demand Modeling**: Develop a stochastic MILP or robust optimization model to handle demand uncertainty, using probability distributions or scenario-based approaches to ensure schedule resilience.

- **Heuristic and Metaheuristic Approaches**: Investigate hybrid approaches combining MILP with heuristics (e.g., genetic algorithms, simulated annealing) to improve scalability for larger instances, reducing solve times while maintaining near-optimal solutions.
- **Real-Time Rescheduling**: Design an adaptive framework that re-optimizes schedules in response to real-time disruptions, integrating with IoT-enabled MES systems for seamless data flow and decision-making.
- **Extended Case Studies**: Validate the model with additional case studies from different pasta packaging facilities, incorporating varied product portfolios, line configurations, or silo constraints to assess generalizability.

These advancements would enhance the model's scalability, adaptability, and alignment with real-world operational needs, paving the way for broader adoption in the food industry.

Conclusion

This thesis addressed the critical challenge of scheduling pasta packaging operations at SOSEMIE, an Algerian pasta manufacturer, through the development and application of a Mixed-Integer Linear Programming (MILP) model. By focusing on optimizing the allocation of seven products across three parallel packaging lines over a 20-day planning horizon, the study achieved significant operational improvements. The MILP model, implemented in Pyomo and solved using MOSEK and CBC solvers, minimized the weighted sum of active production days while adhering to constraints such as family grouping, limited family switches, and silo capacity limits. The key findings and contributions of this work are multifaceted.

Firstly, the model successfully generated an optimal production schedule, reducing the number of active days by 10% (from 10 to 9 days) compared to industry practices. This efficiency gain demonstrates the potential for substantial time and cost savings in pasta packaging operations. The model effectively managed complex constraints, limiting family switches to two and maintaining high resource utilization. The use of both MOSEK and CBC solvers validated the solution's robustness, with MOSEK achieving optimality in 61.25 seconds, significantly faster than CBC's 1792.82 seconds, providing practical insights into solver selection for industry applications.

Secondly, the case study at SOSEMIE bridged academic research with real-world applicability, offering actionable recommendations such as integrating the model into Manufacturing Execution Systems (MES) and prioritizing early production to reduce holding costs. The sensitivity analysis further highlighted the trade-offs of stricter constraints, such as increasing active days to 10 when limiting family switches to one or silos to five, enhancing the model's practical utility.

Despite these achievements, the study has limitations. The assumptions of deterministic demand, zero initial silo inventory, and no machine downtime may not fully capture real-world variability. Additionally, the model's computational complexity could pose challenges for larger problem instances, particularly with the slower CBC solver. The higher changeover time (480 vs. 240 minutes) compared to industry practices also suggests a need to balance setup costs with scheduling efficiency.

Looking ahead, future research could enhance the model by incorporating stochastic demand and machine downtime to improve robustness. Exploring heuristic or metaheuristic approaches could address scalability issues, enabling application to larger production scenarios. Extending the case study to diverse product portfolios or facility configurations would further validate the model's generalizability. Additionally, quantifying the monetary impact of the 10% reduction in active days could strengthen the case for industrial adoption.

In conclusion, this thesis provides a robust and efficient MILP-based solution for scheduling pasta packaging operations, offering both theoretical contributions to operations research and practical benefits for SOSEMIE. By addressing the identified limitations and pursuing the proposed research directions, this work lays a strong foundation for advancing scheduling practices in the food industry, contributing to operational excellence and cost efficiency.

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