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Intelligent Control of the Lateral Motion of Boeing 747

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Dedication

To the memory of my beloved father, who was a source of inspiration, and inherent guidance.

To my beloved mother, my flower of life, and whose love, sacrifices, encouragement, support are invaluable.

To my dear sisters and brother for always being there for me, supporting me, helping me and encouraging me during my studies, and to their families.

To my dear grandparents "Mima and Jadi", and all my family.

To my best thesis partner Mohammed Alim.

To all my friends and colleagues of the class: Avionics
To anyone who encouraged or helped me during my studies.

Zouheir Belkhatir

Dedication

To my dear and beloved parents, for their sacrifices, love, tenderness, support and prayers along my studies.

To my dear brother Bachir and my sister Zehor for their constant encouragements, help and support.

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To all my friends and colleagues of the class: Avionics
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Abstract:

The rapid advancement of the aircraft design from the first successful airplane, which is due to Wright's brothers, with very limited capabilities to today's high-performance commercial, military, and general aviation aircraft required the development of many technologies, including flight controls. Today's aircraft design rely heavily on automatic control systems to monitor and control many of the aircraft's subsystems. The development of automatic control systems have played a key role in the growth of civil and military aviation, and have aided the flight crew in flight management, navigation, and increasing the stability features of the airplane. This thesis investigates the control design problem for the lateral motion of the Boeing 747 aircraft using model-based control strategies. In fact, mathematical modeling is a powerful tool that is extensively used to describe real-life problems illuminating different disciplines of science and engineering including the field of aerospace engineering. Therefore, to achieve the investigated control objective, the important step of deriving the mathematical model of the aircraft's dynamics is given. Then, a background review of two different types of control techniques is provided. The first control method, which is Proportional-Integral-Derivative (PID) controller, belongs to the classical control theory field while the second one, which is fuzzy logic controller, belongs to modern control theory. The performance of both synthesized control strategies is tested on the dynamics of Boeing 747 plane. Furthermore, qualitative comparative study between the aforementioned control techniques is presented.

Résumé:

L'avancement rapide de la conception des avions du premier avion réussi, qui est dû aux frères 'Wright', avec des capacités très limitées à celles des avions hautes performances d'aujourd'hui. avions commerciaux, militaires et d'aviation générale ont nécessité le développement de nombreux technologies, y compris les commandes de vol. Aujourd'hui, la conception des avions repose largement sur les systèmes de contrôle pour surveiller et contrôler de nombreux sous-systèmes de l'avion. Le développement des systèmes de contrôle automatique ont joué un rôle clé dans la croissance de la société civile et l'aviation militaire et ont aidé l'équipage de conduite dans la gestion des vols, la navigation augmente les caractéristiques de stabilité de l'avion. Cette thèse examine le problème de conception de contrôle pour le mouvement latéral du Boeing 747 utilisant des stratégies de contrôle basées sur un modèle. En fait, la modélisation mathématique est un outil puissant qui est largement utilisé pour décrire des problèmes de la vie réelle éclairant différentes disciplines des sciences et du génie, y compris le domaine du génie aérospatial. Par conséquent, pour atteindre l'objectif de contrôle recherché, l'importante étape qui consiste à dériver le modèle mathématique de la dynamique de l'avion est donné. Ensuite, une revue de fond sur deux types différents de techniques de contrôle est fournie. La première méthode de contrôle, qui est un contrôleur PID (Proportional-Integral-Derivative), appartient à la théorie du contrôle classique tandis que la seconde, qui est un contrôleur de logique floue, appartient au contrôle moderne . La performance des deux stratégies de contrôle synthétisées est testée sur la dynamique de Boeing 747 . En outre, une étude comparative qualitative entre les deux techniques de contrôle susmentionnées est présenté.

ملخص :

التقدم السريع لتصميم الطائرات لأول طائرة ناجحة ، واالتي تعود إلى الأخوان رايت ، مع قدرات محدودة للغاية مقارنة مع الطائرات عالية التجهيز في وقتنا الراهن. الطائرات التجارية والعسكرية والطيران العام يتطلب تطوير مختلف التقنيات ، بما في ذلك ضوابط الطيران. يعتمد تصميم الطائرات اليوم بشكل كبير على أنظمة التحكم للمراقبة والتحكم في العديد من النظم الفرعية للطائرة. لعبت أنظمة التحكم الآلي دورا رئيسيا في نمو المجتمع المدني و الطيران العسكري ومساعدة طاقم الرحلة في إدارة الطيران والملاحة مما زاد من خصائص استقرار الطائرة. تتناول هذه الرسالة مشكلة تصميم التحكم للحركة الجانبية لبوينج ٧٤٧ باستخدام استراتيجيات التحكم القائمة على النموذج. النمذجة الرياضية أداة قوية تستخدم على نطاق واسع لوصف قضايا الحياة الحقيقية التي تتضمن مختلف تخصصات العلوم والهندسة ، بما في ذلك مجال هندسة الطيران. وذلك لبلوغ الهدف المطلوب الا وهو السيطرة و التحكم و ذلك بالاعتماد على ميزة هامة وهي الاشتقاق. يتم إعطاء النموذج الرياضي لديناميكيات الطائرة. ثم مراجعة الخلفية على نوعين مختلفين من تقنيات التحكم المقدمة. الطريقة الأولى للسيطرة وهي وحدة تحكم (متناسب مشتق لا يتجزأ) والذي ينتمي إلى نظرية التحكم الكلاسيكية، بينما الثاني وهو وحدة تحكم منطق غامض، ينتمي إلى انظمة التحكم الحديثة. يتم اختبار أداء وحدتي التحكم المركبتين على الديناميكيات بوينغ ٧٤٧. بالإضافة إلى مقارنة نوعية بين تقنيات التحكم مذكورة أعلاه.

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Notations and Acronyms

A	State matrix
B	Input matrix.
C	The output matrix
D	The feedforward matrix
n	Number of states
m	Number of inputs
p	Number of outputs
$\dot{x}(t)$	Derivative of the state with respect to time
$x(t)$	State of the system
$u(t)$	The control input
$y(t)$	The output of the system
s	The Laplace transform parameter
$H(s)$	Transfer function of the system
$N_r(s)$	The numerator of the RMFD
$D_r(s)$	The denominator of the RMFD
$N_L(s)$	The numerator of the LMFD
$D_L(s)$	The denominator of the LMFD
w_c	Reachability matrix of index
T_c	Similarity transformation for the controller canonical form
$x_c(t)$	State of the system under similarity transformation
A_c	The system matrix in controller canonical form
B_c	The control matrix in controller canonical form
C_c	The output matrix in controller canonical form
b	Wing span, $[ft]$
S	wing area, $[m^2]$
β	Angle of sideslip,$[rad]$
α	Angle of Attack, $[rad]$
P_a	Dynamic Pressure,[P]
C_z	Lift Coeffcient
C_N	Yawing moment coeffcient, N/P_aSb
C_L	Rolling moment coeffcient, L/P_aSb
C_Y	Lateral Force coeffcient, $LateralForce/P_aS$

g	Gravity,$[m/s^2]$
$I_{xx}, I_{yy}, I_{zz}, I_{xz}$	Product of inertias according to axes
I_x, I_z	Moment of inertias according to axes
F_X	Drag force,[N]
F_z	Lift force,[N]
F_Y	Side force,[N]
m	Aircraft mass, $[Kg]$
δ_a	aileron control de ection, $[rad]$
δ_r	rudder control de ection, $[rad]$
δ_e	elevator control de ection, $[rad]$
L	Rolling Moment,$[N.m]$
N	Yawing moment, $[N.m]$
ϕ	angle of roll,$[rad]$
ψ	angle of yaw, $[rad]$
θ	angle of Pitch,$[rad]$
p	Roll rate
q	Pitch rate
r	Yaw rate
N_r	Lateral-Directional stability parameter,$[s^{-1}]$
N_p	Lateral-Directional stability parameter,$[s^{-1}]$
$N_{\delta a}$	Lateral-Directional stabilit parameter,$[s^{-2}]$
$N_{\delta r}$	Lateral-Directional stability parameter,$[s^{-2}]$
L_β	Lateral-Directional stability parameter,$[s^{-2}]$
L_p	Lateral-Directional stability parameter,$[s^{-1}]$
L_r	Lateral-Directional stability parameter,$[s^{-1}]$
$L_{\delta a}$	Lateral-Directional stability parameter,$[s^{-2}]$
$L_{\delta r}$	Lateral-Directional stability parameter,$[s^{-2}]$
$C_{N\delta r}$	directional or rudder control power
C_{Nr}	Yaw damping derivative
$C_{N\beta}$	directional static stability
C_{Lp}	Roll damping derivative
C_{Lr}	Cross derivative
$C_{L\beta}$	lateral static stability
$C_{L\delta a}$	lateral or aileron control power

General Introduction

Since the first successful airplane, more than a century ago, invented by Wright's brothers it is still a hot topic to enhance in airplanes hardware and software especially in the era of digitization and data science. In particular, improving the stability and control characteristics of an airplane, which are referred to as the vehicle's handling or flying qualities, remains of interest in the engineering field.

Flight control is, in fact, an intriguing and challenging subject that needs the efforts and skills of different engineering disciplines to be aligned and combined for successful system design. The first step towards assessing the airplane's handling or flying qualities is to develop a deep understanding of its dynamic characteristics. Moreover, some of the main difficulties to approach towards the flight control problem are associated to (i) complexity of the mathematical model of an aircraft, (ii) presence of different types of constraints, e.g., the ones related to high tolerances of flight characteristics and flight safety constraints, ...etc.

Motivated by the need to develop more efficient stabilizing flight control systems, aircraft manufacturers are constantly improving in the flight control systems ($FCSs$). In this work, we are interested in investigating a particular control problem related to the stabilization and tracking of the aircraft's path, more specifically the Boeing 747 aircraft, in its lateral motion. To approach towards the solution of such a control problem, we design and apply a modern control strategy, the so-called fuzzy logic controller, in comparison to the well-known classical Proportional-integral-derivative (PID) controller.

The structure of this project is briefly outlined in the following:

- Chapter 1 introduces the basics and background concepts of the aerodynamics, aircraft modeling, and different control strategies.

- Chapter 2 is concerned with the derivation of the mathematical dynamical model of B747 airplane in lateral motion.

- Chapter 3 presents general notions about multi-input multi-output PID controller and its application to an academic example.
- Chapter 4 introduces the theory of fuzzy logic controller.
- Chapter 5 presents the application of PID and fuzzy logic controllers to Boeing747 aircraft and also highlights the comparative study between both synthesized control strategies.

We finish this work with a general conclusion along with some potential perspectives.

Chapter 1

Background and Preliminaries

1.1 Introduction

The primary focus of this chapter is to introduce some background notions and concepts that allow a better understanding of the problem of controlling aircraft surfaces in general, and an application to Boeing 747 in particular which are explained in next chapters of this thesis. First, we provide definitions of the main mechanical parts and features of an aircraft. Second, models of both aerodynamic forces and torques are introduced. Then, stability notions, both static and dynamic, are briefly explained. Finally, a non-exhaustive literature review on the well-known linear and non-linear controllers is provided.

1.2 Flight Control Systems

In addition to the control surfaces which are used for steering, every aircraft contains motion sensors which provide measures of changes in motion variables which occur as the aircraft responds to the pilot's commands or as it encounters some disturbance. The signals from these sensors can be used to provide the pilot with a visual display, or they can be used as feedback signals for the automatic flight control system (AFCS). Thus, the general structure of an AFCS can be represented as the block schematic of (Fig 1.1) The purpose of the controller is to compare the commanded motion with the measured motion and, if any discrepancy exists, to generate, in accordance with the required control law, the command signals to the actuator to produce the control surface deflections which will result in the correct control force or moment being applied. This, in turn, causes the aircraft to respond appropriately so that the measured motion and commanded motion are finally in correspondence [1].

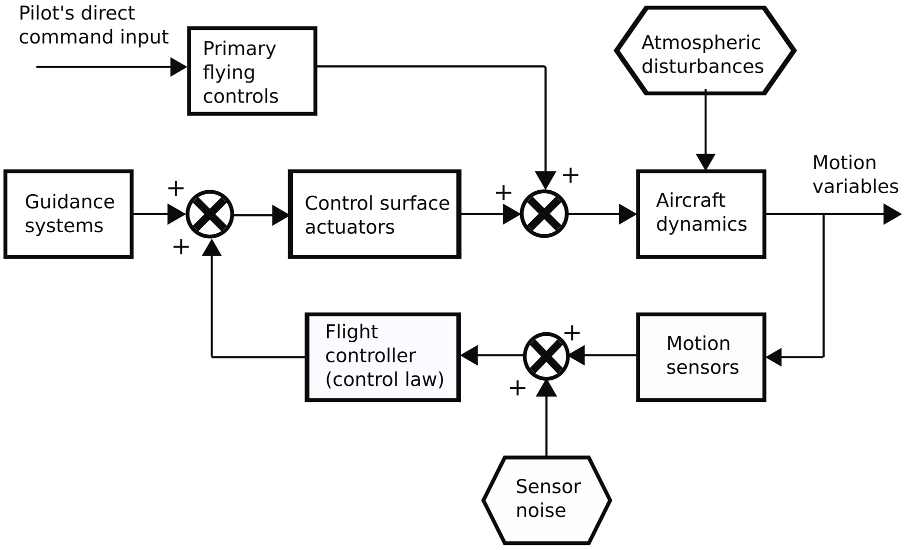


Figure 1.1: General structure of an AFCS.

1.3 Mechanical Preliminaries

The study of flight dynamics, as is the case in any field of physical sciences and technologies, requires a common acceptance of several basic definitions. These definitions which, e.g., may include unambiguous nomenclature help in the understanding of the relevant physical properties, the related mechanics and the appropriate mathematics of the investigated flight dynamics. For this purpose, the most important notions that will be used throughout the rest of this thesis are defined in this section.

- **Mass** The quantitative measure of the inertia of a body is a physical quantity called the mass of that body [2]. A constant mass will be assumed, that is : $\frac{dm}{dt} \cong 0$
- **Rigid Body** A rigid body is an idealized system of particles. Furthermore, it will be assumed that the body does not undergo any change in size or shape. Consequently, the rigid body can be treated as a particle whose mass is that of the body and is concentrated at the center of mass [2].
- **Center of Mass** For the objects that have a uniform mass per unit volume, the center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry. The origin of the body axes is usually the mass center (cm) [2].

- **Center of Gravity** The forces due to gravity are always present in an aircraft (or missile) and act at the center of gravity (cg). Since the centers of mass and gravity in an aircraft practically coincide, there is no external moment produced by gravity about the cg [2]. The gravitational force acting upon an aircraft is commonly expressed in terms of the Earth axes,

  Remark 1.1. *In assuming a rigid body, the aero-elastic effects are not included in the equations. With this assumption, the forces acting between individual elements of mass are eliminated. Furthermore, it allows the airframe motion to be described completely by a translation of the center of gravity and by a rotation about this point.*

- **Angle of Attack** (α) The angle between the relative wind and the chord line [2].

- **Aerodynamic Center** The point on the chord of an airfoil about which the moment coefficient is practically constant for all angles of attack [2].

- **Center of Pressure** The point on the chord of an airfoil through which all of the aerodynamic forces act. The center of pressure (cp) in general will not be located at the center of gravity of the airfoil; thus a moment will be produced [2].

- **Dynamic Pressure** The aerodynamic pressure appears frequently in the derivation of aerodynamic formulas. Dynamic pressure, denoted by the symbol P_a , is given by the expression $P_a = \frac{1}{2}\rho V^2$ where is the air density and V is the free-stream velocity [2].

- **Relative Wind** Refers to the motion of air relative to an airfoil and is equal and opposite to the forward velocity of the air vehicle [2].

- **Resultant Aerodynamic Force** The vector summation of all of the aerodynamic forces acting on the airfoil. Its point of application is at the center of pressure [2].

1.4 Model of the Aerodynamic Forces

It is conventional in aerodynamics to resolve the sum of the normal (or pressure) forces and the tangential (or viscous shear) forces that act on the surface due to the fluid motion around a vehicle into three components along axes parallel and perpendicular to the free-stream direction. These forces are lift, drag, and side force.

- **Lift force** Is the component of the resultant aerodynamic force that is perpendicular (i.e., upward) to the relative wind (direction of flight) or to the undisturbed free stream velocity. The aerodynamic lift is produced primarily by the pressure forces acting on the vehicle surface [3]. The lift force is calculated by multiplying the lift coefficient by the wing surface S and the dynamic pressure P_a as follows:

$$F_z = P_a S C_z(\alpha)^2 \tag{1.1}$$

The dimensionless lift coefficient is modeled as a linear function of the angle of attack (α) as

$$C_z(\alpha) = C_{z1} + \alpha C_{z\alpha} \tag{1.2}$$

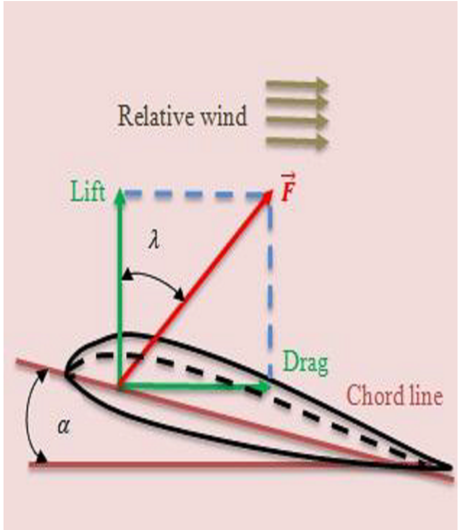


Figure 1.2: Aerodynamic Forces on an airfoil.

- **Drag force**

Is the component of the resultant aerodynamic force that is parallel to the relative wind. In other words, it is net aerodynamic force acting in the same direction as the undisturbed free-stream velocity. The aerodynamic drag is produced by the pressure forces and by skin friction forces that act on the surface [3]. The drag force is measured along the velocity vector, but in the opposite direction. Drag is obtained for an angle of attack α different from zero. The drag force is obtained by multiplying the drag coefficient by the dynamic pressure and the wing surface resulting in $F_X = P_a S C_X(\alpha, \beta)$. The dimensionless drag coefficient is approximated by a quadratic function in and (is the sideslip angle) according to

$$C_X(\alpha, \beta) = C_{X1} + C_{X\alpha}\alpha + C_{X\alpha 2}\alpha^2 + C_{X\beta 2}\beta^2 \tag{1.3}$$

- **Side force**

The lateral force (i.e. Side force) is the component of force in a direction perpendicular to both the lift and the drag and is measured in the horizontal plane [3]. The lateral force acting on the aircraft is mainly due to the fuselage, which is considered to be an inefficient wing with zero offset due to the symmetry of the airplane in the (xb, zb) plane, yielding $C_Y(\beta) = C_{Y1}\beta$ and $F_Y = P_a S C_Y(\beta)$.

1.5 Model of the Aerodynamic Torques

In a similar manner to the aerodynamic forces of the previous section, the moments (torques) on the flight body can be divided into moments created by the aerodynamic load distribution and the thrust force not acting through the center of gravity. Specifically, the moment due to the resultant force acting at a distance from the origin may be divided into three components, referring to the flight's body reference axes. The three moment components are the pitching moment, the rolling moment, and the yawing moment. In order to change the attitude of the aircraft, torques are applied to the airframe. They are generated by control surfaces such as ailerons, elevators, and rudders.$\delta_a, \delta_e, \delta_r$. These moments will now be defined more closely [3].

1.5.1 Pitching Moment

Is the torque about the plane's lateral axis (i.e., the Y_b-axis). The pitching moment is the result of the lift and the drag forces acting on the vehicle. A positive moment is in the nose-up direction. The generation of the pitch torque M expressed in the aircraft body-fixed frame (b) is modeled by a linear function of the elevator deflection δ_e, of the angle of attack α, and of the dimensionless pitch rate q, $M = cP_aC_M(\delta_e, \alpha, q)$. With $C_M(\delta_e, \alpha, q) = C_{M1} + C_{Me}\delta_e + C_{Mq}q + C_{M\alpha}\alpha$ and is the mean aerodynamic chord [3].

1.5.2 Rolling Moment

This torque is about the longitudinal axis of the plane (i.e., the X_b-axis). A rolling moment is often created by a differential lift, generated by some type of aileron [3]. A positive rolling moment causes the right or starboard wingtip to move downward. The generation of the roll torque is modeled by a linear function of the aileron deflection δ_a, the sideslip angle β, the angular rates p and r.

$$L = P_abSC_L(\delta_a, \beta, r, p). \tag{1.4}$$

With : $C_L(\delta_a, \beta, r, p) = C_{L\delta a}\delta_a + C_{L\beta}\beta + C_{Lp}p + C_{Lr}r$

1.5.3 Yawing Moment

The moment about the vertical axis of the plane (i.e., the Z_b-axis) is the yawing moment [3]. A positive yawing moment tends to rotate the nose to the right. The generation of the yaw torque (moment) N is modeled by a linear function of the rudder deflection δ_r, of the sideslip angle β, and of the dimensionless yaw rate r as follows:

$$N = P_a bSC_N(\delta_r, \beta, r) \tag{1.5}$$

With : $C_N(\delta_r, \beta, r) = C_{N\delta r}\delta_r + C_{Nr}r + C_{N\beta}\beta$

1.6 Engine (Model of the Power-Forces)

Probably the most significant variation in longitudinal static stability arises from the effects of power. Direct effects result from the point of application and line of action of the thrust forces with respect to the cg [3].

1.6.1 Engine Rate

The dynamics for the engine speed μ are modeled by a first-order linear system with the time constant τ_n and the engine speed reference signal μ_c as follows [3]:

$$\dot{\mu} = \frac{1}{\tau_n}(\mu_c - \mu) \tag{1.6}$$

1.6.2 Thrust Force

The thrust force is generated by the propeller and can be expressed with dimensionless coefficients. The dimensionless thrust coefficient [5] is $C_{FT}(J) = C_{FT0} + C_{FT1}J + C_{FT2}J^2$ with the ratio $J = V_T/D\pi\mu$, where the diameter of the propeller is D, the engine speed is μ, and the airspeed is V_T. The thrust force is computed as follows:

$$F_T = \rho\mu^2 D^4 C_{FT}(J) \tag{1.7}$$

1.7 Static Stability

Static stability [6] is generally defined as the initial tendency of an airplane, following a perturbation from a steady-state flight condition, to develop aerodynamic forces or moments that are in a direction to return the aircraft to the steady-state flight condition. This somewhat complex definition can be simply illustrated with an example.

1.7.1 Longitudinal Static Stability

static stability refers to the initial tendency of an airplane, following a disturbance from steady-state flight, to develop aerodynamic forces and moments that are in a direction to return the aircraft to the steady-state flight condition. For purposes of this text, longitudinal static stability will primarily refer to aircraft pitching moment characteristics and will be analyzed for the stick fixed condition. the sign of the stability derivative, C_{ma} is key in determining the static longitudinal stability of an aircraft. The requirement to trim the aircraft at usable angles of attack is also discussed with the longitudinal stability requirement because both are generally necessary to achieve acceptable flight characteristics [6].

1.7.2 Directional Stability

Directional, or weathercock, stability is concerned with the static stability of the airplane about the Z axis. Just as in the case of longitudinal static stability, it is desirable that the airplane should tend to return to an equilibrium condition when subjected to some form of yawing disturbance [7].

1.8 Dynamic stability

dynamic stability is concerned with the time history of the motion of the vehicle after it is disturbed from its equilibrium poin [7].

Aircraft dynamic stability [6] focuses on the time history of aircraft motion after the aircraft is disturbed from an equilibrium or trim condition. This motion may be first order (exponential response) or second order (oscillatory response), and will have either positive dynamic stability (aircraft returns to the trim condition as time goes to infinity), neutral dynamic stability (aircraft neither returns to trim nor diverges further from the disturbed condition), or dynamic instability (aircraft diverges from the trim condition and the disturbed condition as time goes to infinity). The study of dynamic stability is important to understanding aircraft handling qualities and the design features that make an airplane fly well or not as well while performing specific mission tasks. The differential equations that define the aircraft equations of motion (EOM) form the starting point for the study of dynamic stability.

1.9 Controller Theory

Controllers are designed for turning an unstable system to a stable one , if a system is stable controller can improve the performance of system [8]. Controllers are very important in our life. For example, if there is an unstable aircraft system, the responses of the aircraft is not good and unrelated. Also if an aircraft gives unrelated responses, it effects the ight security and maybe it can end with death. But when we apply a controller to aircraft system and make it stable, aircraft responsesvery well and provides a safe.
There are many controller's types. In what follow, we describe some of the well-known linear and nonlinear controller that exist in the literature.

1.9.1 Linear controls

In linear systems theory, control synthesis is based on a linear approximation of the dynamic vehicle model associated with a single control input [9]. Generally, it is assumed that the translation and orientation speeds are low, which makes it possible to neglect undesirable aerodynamic phenomena. The linear approach facilitates the study of the stability of each loop in the sense that there are specific indicators such as the gain margin and the phase margin. These margins determine the allowable amount of gain and phase that can be injected by the control while maintaining the stability of the loop. Several linear control architectures are presented later.

- **The command by PID**

  The PID control strategy is undoubtedly the most intuitive and most effortless approach to embed on a processor. It allows us to quickly understand the substantial role of each of the terms of the order, which helps adjusting the gains accordingly.

- **Pole placement control**

  Pole placement control is a method of determining a matrix of gains that place the eigenvalues of the closed-loop system at predefined positions. The purpose of this approach is to ensure proper behavior of the looped system closed. Indeed, the location of its eigenvalues is closely linked to temporal behavior and frequency of the system, particularly in terms of stability and performance. For that, it is necessary that eigenvalues have a real, strictly negative part. However, this genuine part is not too negative otherwise the system will admit a large bandwidth, which will induce a noise amplification [10].

- **Quadratic linear control**

  The quadratic linear command denoted LQR command where LQ , is a synthesis method which makes it possible to determine the optimal control of a system that minimizes (or maximizes) a performance criterion . This performance criterion is quadratic in the state of the system and its order. The design of such a command consists of skilfully choosing the weighting matrices involved in the approach to obtain the desired behavior of the closed-loop system. Once the weighting matrices have been selected, the optimal gains are obtained by solving an algebraic equation of "Riccati." The advantage of the linear quadratic control is that it intrinsically possesses excellent robustness properties [11].

- **Predictive control**

  The predictive control is based on the dynamic model of the system to anticipate the behavior of the process over a given time interval. It is possible to generate, over this time interval, the control sequence that optimizes this behavior prediction concerning the setpoint. As for the LQR command, the command sequence is determined using a quadratic programming algorithm that minimizes the cost while taking into account the different constraints. This attractive method in theory, however, suffers from severe practical limitations, especially in terms of the influence of computing time, which is essential compared to the sampling period causing instability [12].

- **The H^{∞} control**

  The H^{∞} approach is undoubtedly the command structure that has been most applied for the management of autonomous air vehicles. Indeed, its performances are superior to those obtained with the LQG command because it integrates elements of robustness directly in its synthesis. In general, the H^{∞} command consists of modeling the loop transfer of such so that it presents a good compromise between performance and robustness [13]. This modeling is performed by filters that are added in the control loop, around the transfer function of the system.

1.9.2 Non-linear controls

In recent years, more and more research is moving towards orders based on a non-linear representation of the dynamics of aerial vehicles. These approaches offer some theoretical contribution, but their application remains limited because of the complexity of models and control algorithms [14]. The most common nonlinear control architectures used to control aircraft are presented below

- **Nonlinear fuzzy logic control**

  The idea of fuzzy logic control is to get closer to a certain extent to the flexibility of human reasoning in controlling a system It introduces the notion of graduation when switching the control signal of a signal structure to another. Fuzzy logic control is particularly suitable for controlling complex nonlinear systems [15] .

- **The input-output linearization control**

  The first architectures of nonlinear controls are mostly based on the concept of input-output linearization. The principle of this approach is to transform the non-linear dynamics of the system into an equivalent linear momentum using a change of variables and an appropriate choice of control inputs. It is, therefore, essential that the model of the system is well known, which is difficult to guarantee in practice [16].

- **Hierarchical control strategy**

  The principle of hierarchical control consists in separating the command in translation from the command in rotation and joining thus the classic cascading architecture of control algorithms. It is important to note that each control law can be developed separately. Indeed, a first control law determines the vertical thrust and the orientation necessary for the stabilization of the translation dynamics [17].

- **The command by sliding mode**

  This is a very effective control strategy in the face of sensitivity to parametric uncertainties. This method consists in modifying the dynamics of a non-linear system by applying a high-frequency switching signal forcing it to join and subsequently remaining on a surface. This so-called sliding surface has previously been chosen according to the control objectives. It varies according to the current position in the state space. The fact that the control is discontinuous is an essential element because it allows stabilizing systems that would not be with continuous signals. This is why this command was implemented on helicopters [18].

1.10 Conclusion

In this chapter, the basic notions needed to understand the topic of controlling the dynamics of an aircraft were briefly reviewed. Definitions of the aircraft's features and models of aerodynamic forces and moments were provided. Moreover, background material on the stability notions and some of the well-known controllers that exist in the literature were presented.

Chapter 2

Mathematical Modeling of the Boeing 747 Aircraft

2.1 Introduction

The focus of our study is to control the dynamics of Boeing 747 aircraft using mathematical-based control strategies. Thus, modeling the dynamics of the aircraft is essential to solving the considered control problem. In this regard, this chapter introduces the mathematical model of the different parts of the aircraft. First, we discuss the general equations of aircraft's motion using Newton's laws. Then, we consider a particular case of small perturbations around a state of equilibrium, which makes it possible to linearize the latter dynamics. After that, we analyze the expressions of aerodynamics and propulsive forces. Finally, we show that for symmetrically shaped aircraft, low amplitude motions around the equilibrium state break down into longitudinal and lateral motions.

2.2 Motion of a Rigid Body on a Fixed Point

We have in polar coordinates: $\vec{h} = h\vec{i_r}$, with $\vec{i_r} = \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$. Notice that $\vec{i_r}$ is time varying, don't change magnitude, but its direction will change with time when the body is rotating about Z axis. As it is known in classical mechanics a moving vector $\vec{h}$ has a speed equal to the derivative of its position [3].

$$\vec{v_h} = \frac{d\vec{h}}{dt} = \frac{dh}{dt}\vec{i_r} + h\frac{d\vec{i_r}}{dt} \tag{2.1}$$

$$\frac{d\vec{i_r}}{dt} = \frac{d\theta}{dt}\left(-\sin(\theta)\vec{i} + \cos(\theta)\vec{j}\right) = \frac{d\theta}{dt}\vec{i_\theta} \tag{2.2}$$

The scalar quantities h, z_r are constant with respect to time, means that $r(\vec{t}) = h\vec{i_r} + z_r\vec{k}$ and the velocity is given by:

$$v(\vec{t}) = \frac{dr(\vec{t})}{dt} = \frac{dh}{dt}\vec{i_r} + h\frac{d\vec{i_r}}{dt} + z_r\frac{d\vec{k}}{dt} \tag{2.3}$$

With

$$\frac{dh}{dt} = \frac{dz_r}{dt} = 0 \tag{2.4}$$

and

$$\frac{d\vec{k}}{dt} = \vec{0}$$

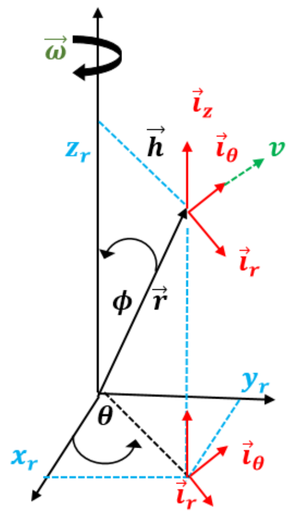


Figure 2.1: The vector $\vec{r}$ in the Cartesian coordinate.

$$v(\vec{t}) = \frac{dr(\vec{t})}{dt} = \left(h\frac{d\theta}{dt}\right)\vec{i_\theta} = \left(h\frac{d\theta}{dt}\right)\left(-\sin(\theta)\vec{i} + \cos(\theta)\vec{j}\right) \tag{2.5}$$

From the projection of the vector $\vec{r}$ in the Cartesian coordinate we obtain the following components: $x_r = h\cos(\theta)$, $y_r = h\sin(\theta)$, $z_r = r\cos(\phi)$ and $h = r\sin(\phi)$
then we can write $x_r = r\sin(\phi)\cos(\theta)$, $y_r = r\sin(\phi)\sin(\theta)$, $z_r = r\cos(\phi)$ Using the following cross-products: $\vec{i}\times\vec{j} = \vec{k}$, $\vec{k}\times\vec{i} = \vec{j}$, $\vec{k}\times\vec{k} = \vec{0}$ and x_r, y_r, z_r in the velocity equation we get

$$v(\vec{t}) = \left(h\frac{d\theta}{dt}\right)\left(\frac{y_r}{h}\vec{k}\times\vec{j} + \frac{x_r}{h}\vec{k}\times\vec{i} + \frac{z_r}{h}\vec{k}\times\vec{k}\right) \tag{2.6}$$

hence we deduce that

$$v(\vec{t}) = \left(\frac{d\theta}{dt}\vec{k}\right)\times r(\vec{t}) \tag{2.7}$$

$$v(\vec{t}) = \vec{\omega}\times r(\vec{t}) \tag{2.8}$$

Remark 2.1. *The velocity vector for any point M of a body is equal to the vector product of the angular velocity of that body and the radius vector of the point.* $\vec{v(t)} = \vec{\omega} \times \vec{r(t)}$ *In order to calculate the derivative of some rotating unit vectors* $\vec{i}, \vec{j}, \vec{k}$ *we assume that is the radius of vector* $r_A = \vec{i}$ *of a point A on the axis x at unit distance from the origin. Then* $\frac{d\vec{i}}{dt} = \frac{d\vec{r_A}}{dt} = \vec{v_A}$. *But according to what we have found in purely rotational motion (i.e. around fixed point),* $\vec{v_A} = \vec{\omega} \times \vec{r_A} = \vec{\omega} \times \vec{i}$, *where* $\vec{\omega}$ *is the angular velocity of the rotation about axis OZ. Similar relationships are obtained for the derivatives of* $\vec{j}$ *and* $\vec{k}$, *and finally we obtain:*

$$\frac{d\vec{i}}{dt} = \vec{\omega} \times \vec{i}, \qquad \frac{d\vec{j}}{dt} = \vec{\omega} \times \vec{j}, \quad \textit{and} \quad \frac{d\vec{k}}{dt} = \vec{\omega} \times \vec{k}$$

Those last equations are known as the Poisson equations.

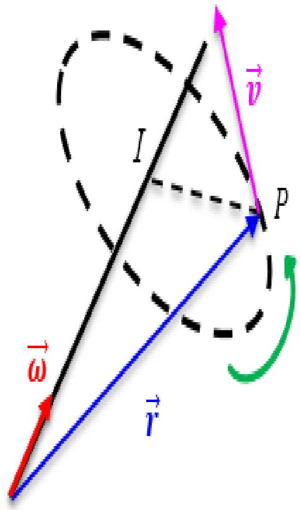


Figure 2.2: angular speed.

2.3 The General Motion of a Free Rigid Body

Let us now examine the most general motion of a rigid body free to move in any direction with respect to a reference system. The general motion of a rigid body during an instant of time may be considered in two steps [19].

- Translation of an arbitrary base-point in the body to its final position.
- Rotation of the body about this base-point so that the body consider with its final position.

$$\vec{r_M} = \vec{r_A} + \vec{r_{M/A}} \quad \textit{and} \quad \vec{v_M} = \vec{v_A} + \vec{v_{M/A}}$$

Notice that the relative position vector $\vec{r_{M/A}}$ don 't change magnitude, but the direction will be changed. Then the motion will be considered firstly as translation of the point A as observed from $(Oxyz)_I$ and a rotation about this point as observed from $(Oxyz)_B$ Hence, the speed is $\vec{v_M} = \vec{v_A} + \vec{\omega} \times \vec{r_{M/A}}$ or more generally we can write

$$\vec{v} = \frac{d\vec{r}}{dt} + \vec{\omega} \times \vec{r} \tag{2.9}$$

- **More Explanation** description Consider a fixed system $(Oxyz)_I$ assumed to be the inertial frame and another rotating system $(Oxyz)_B$ assumed to be the body's fixed frame, which rotate with respect to the first one by an angular velocity $\vec{\omega}$. Let $\vec{i}$, $\vec{j}$ and $\vec{k}$ be a unit vectors along the axes of the rotating system. Let $\vec{A}$ be an arbitrary vector with components A_x, A_y , and A_z along the rotating axes. Then:

$$\left(\frac{d\vec{A}}{dt}\right)_I = \left(\frac{dA_x}{dt}\vec{i} + \frac{dA_y}{dt}\vec{j} + \frac{dA_z}{dt}\vec{k}\right) + A_x\frac{d\vec{i}}{dt} + A_y\frac{d\vec{j}}{dt} + A_z\frac{d\vec{k}}{dt} \tag{2.10}$$

$$\left(\frac{d\vec{A}}{dt}\right)_I = \left(\frac{d\vec{A}}{dt}\right)_B + A_x\frac{d\vec{i}}{dt} + A_y\frac{d\vec{j}}{dt} + A_z\frac{d\vec{k}}{dt} \tag{2.11}$$

$$\left(\frac{d\vec{A}}{dt}\right)_I = \left(\frac{d\vec{A}}{dt}\right)_B + \vec{\omega} \times \left(A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\right) \tag{2.12}$$

$$\left(\frac{d\vec{A}}{dt}\right)_I = \left(\frac{d\vec{A}}{dt}\right)_B + \vec{\omega} \times \vec{A} \tag{2.13}$$

Since a body's motion represents a sum of elementary displacements, we finally conclude that the most general motion of a free rigid body is composed of a translation of the body, in which all its points move with a velocity $\vec{v_A}$ in the same way as an arbitrary pole A, and a series of infinitesimal rotations with an angular velocity $\vec{\omega}$ about the instantaneous axes of rotation through the pole A.

2.4 Aircraft Equations of Motion

The equation of motion of our aircraft model was assumed that the aircraft is a rigid-body, i.e. the distance between any two points in the aircraft does not change in flight and the inertial frame of reference does not itself accelerate: in other words; the Earth is taken to be fixed in space. In dealing with AFCS design, the tropocentric coordinate system; one whose origin is regarded as being fixed at the center of the Earth is generally used as a basic inertial reference frame. The aircraft itself is also assigned an axis system, which is the body fixed axis system whose origin is the aircraft's center of gravity. Atmospheric flight mechanics is the study of the motion of a vehicle inside planetary atmosphere. With respect to an inertial frame of reference, let $\vec{r}$ be the vector defining the position of the center of mass of the flying object and $\vec{v}$ is its velocity vector. They are related by the kinematic relation $\vec{v} = \frac{d\vec{r}}{dt}$. In this respect, we follow the motion of the center of mass under the application of various forces [2]. For many problems in airplane dynamics, an axis fixed to the earth can be used as an inertial frame (reference frame). Newton's second law can be applied:

Force equation:

$$\sum \vec{F} = \frac{d\vec{L}}{dt} \tag{2.14}$$

With : $\vec{L} = m\vec{v}$

Moment equation:

$$\sum \vec{M} = \frac{d\vec{H}}{dt} \tag{2.15}$$

With : $\vec{H} = m(\vec{r} \times \vec{v})$

In order to develop the force equation let we define δm be an element of mass of the airplane, $\vec{v}$ be the velocity of the element mass relative to an absolute or inertial frame. $\delta\vec{F}$ be the resulting force acting on the elemental mass then, $\delta\vec{F} = \delta m \dfrac{d\vec{v}}{dt}$ and $\vec{F} = \sum \delta\vec{F}$. The velocity of the differential mass δm is: $\vec{v} = \vec{v_c} + \dfrac{d\vec{r}}{dt}$. With v_c is the velocity of the center of mass of the airplane and $\dfrac{d\vec{r}}{dt}$ is the velocity of the element relative to center of mass [2].

$$\vec{F} = \sum \delta\vec{F} = \frac{d\sum\left(\vec{v_c} + \dfrac{d\vec{r}}{dt}\right)\delta m}{dt} \tag{2.16}$$

$$\vec{F} = m\frac{d\vec{v_c}}{dt} + \frac{d\sum \dfrac{d\vec{r}}{dt}\delta m}{dt} \tag{2.17}$$

$$\vec{F} = m\frac{d\vec{v_c}}{dt} + \frac{d^2\sum \vec{r}\delta m}{dt^2} \tag{2.18}$$

Because $\vec{r}$ is measured from the center of mass then $\sum \vec{r}\delta m = 0$ and therefore we get, $\vec{F} = m\dfrac{d\vec{v_c}}{dt}$ This last equation relates the external forces on the airplane to the motion of the center of mass. In similar manner, we can develop the moment equation referred to a moving center of mass.

$$\delta\vec{M} = \frac{d\delta\vec{H}}{dt} = \frac{d(\vec{r} \times \vec{v})\delta m}{dt} \tag{2.19}$$

With : $\vec{M} = \sum \delta\vec{M}$

The velocity of the mass element can be expressed in terms of the velocity of the center of the mass and relative velocity.

$$\vec{v} = \vec{v_c} + \frac{d\vec{r}}{dt} = \vec{v_c} + \vec{\omega} \times \vec{r} \tag{2.20}$$

Where $\vec{\omega}$ is the angular velocity of the vehicle and $\vec{r}$ is the position of the mass element measured from the center of mass. The total momentum can be written as

$$\vec{H} = \sum \delta\vec{H} = \sum \delta m \vec{r} \times \vec{v_c} + \sum [\vec{r} \times (\vec{\omega} \times \vec{r})]\delta m \tag{2.21}$$

We know that $\sum \vec{r}\delta m = 0$ then $\vec{H} = \sum [\vec{r} \times (\vec{\omega} \times \vec{r})]\delta m$

Let:

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \quad and \quad \vec{H} = \vec{H_x}\vec{i} + \vec{H_y}\vec{j} + \vec{H_z}\vec{k}$$

$$\vec{H} = \begin{cases} H_x = p\sum(y^2+z^2)\delta m - q\sum xy\delta m - r\sum xz\delta m \\ H_y = q\sum(x^2+z^2)\delta m - p\sum xy\delta m - r\sum yz\delta m \\ H_z = r\sum(x^2+y^2)\delta m - p\sum xz\delta m - q\sum yz\delta m \end{cases} \tag{2.22}$$

$$\vec{H} = (pI_x - qI_{yx} - rI_{xz})\vec{i} + (qI_y - pI_{yx} - rI_{yz})\vec{j} + (rI_z - qI_{yz} - pI_{xz})\vec{k} \tag{2.23}$$

The sum is replaced by triple integral because $\delta m = \rho dxdydz = \rho dv$ with ρ is the density of mass and dv is an infinitesimal volume, $\sum \delta m = \iiint \rho dxdydz$.

$$\begin{aligned} I_x &= \iiint (y^2+z^2)\delta m & I_y &= \iiint (x^2+z^2)\delta m \\ I_z &= \iiint (x^2+y^2)\delta m & I_{xy} &= \iiint xy\delta m \\ I_{xz} &= \iiint xz\delta m & I_{yz} &= \iiint yz\delta m \end{aligned}$$

and

$$I = \begin{pmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{pmatrix}$$

If the reference frame is not rotating, then as the airplane rotates the moments and products of the inertia will vary with time. To avoid this difficulty we will fix the axis system to the aircraft (body axis system). Now we must determine the derivatives of the vectors $\vec{v}$ and $\vec{H}$ referred to the rotating body frame of reference. It has been shown that the derivative of an arbitrary vector $\vec{X}$ referred to a rotating body frame having an angular velocity $\vec{\omega}$ can be represented by the following vector identity

$$\left(\frac{d\vec{X}}{dt}\right)_I = \left(\frac{d\vec{X}}{dt}\right)_B + \left(\vec{\omega}\times\vec{X}\right),$$

where the subscript I and B refer to the inertial and body fixed frames respectively. Applying this identity to the equation derived before yields:
Forces:

$$\vec{F} = \left(m\frac{d\vec{v_c}}{dt}\right)_B + m\left(\vec{\omega}\times\vec{v_c}\right)_B = \left(m\frac{d\vec{v_c}}{dt}\right)_I \tag{2.24}$$

Moments:

$$\vec{M} = \left(\frac{d\vec{H}}{dt}\right)_B + \left(\vec{\omega}\times\vec{H}\right)_B = \left(\frac{d\vec{H}}{dt}\right)_I \tag{2.25}$$

Let:

$$\vec{v_c} = (u,v,w)^T, \vec{\omega} = (p,q,r)^T, \vec{H} = (H_x,H_y,H_z)^T$$
$$\vec{F} = (F_x,F_y,F_z)^T, \vec{M} = (M_x,M_y,M_z)^T$$

The scalar components of forces and moments are:

$$\vec{F} = \begin{cases} F_x = m(\dot{u} + q\omega - rv) \\ F_y = m(\dot{v} + ru - p\omega) \\ F_z = m(\dot{\omega} + pv - qu) \end{cases} \qquad \vec{M} = \begin{cases} M_x = \dot{H}_x + qH_z - rH_y \\ M_y = \dot{H}_y + rH_x - pH_z \\ M_z = \dot{H}_z + pH_y - qH_x \end{cases} \tag{2.26}$$

Remark 2.2. *The components of $\vec{F}$ and $\vec{M}$ are composed of aerodynamic, gravitational and propulsive contributions.*

Remark 2.3. *By proper positioning of the body axis system, one can make the products of inertia $I_{xy} = I_{yz} = 0$, to do this we assume that the xy plane is a plane of symmetry of the airplane.*
Hence, $I^B = \begin{pmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{pmatrix}$ *is the inertia matrix.*

For rotational motion, the angular momentum is defined by $(\vec{H})^B = I^B.\vec{\omega}$

$$(\vec{H})^B = \begin{pmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} pI_x - rI_{xz} \\ qI_y \\ rI_z - pI_{xz} \end{pmatrix} \tag{2.27}$$

$$\left(\frac{d\vec{H}}{dt}\right)_B = \begin{pmatrix} \dot{p}I_x - \dot{r}I_{xz} \\ \dot{q}I_y \\ \dot{r}I_z - \dot{p}I_{xz} \end{pmatrix} \tag{2.28}$$

$$(\vec{\omega} \times \vec{H})_B = \begin{pmatrix} qr(I_{zz} - I_{yy}) - pqI_{xz} \\ pr(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz} \\ pq(I_{yy} - I_{xx}) + qrI_{xz} \end{pmatrix} \tag{2.29}$$

Thus the moment equations simplify to the following:

$$\vec{M} = \begin{cases} M_x = \dot{p}I_x - \dot{r}I_{xz} + qr(I_{zz} - I_{yy}) - pqI_{xz} \\ M_y = \dot{q}I_y + pr(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz} \\ M_z = \dot{r}I_z - \dot{p}I_{xz} + pq(I_{yy} - I_{xx}) + qrI_{xz} \end{cases} \tag{2.30}$$

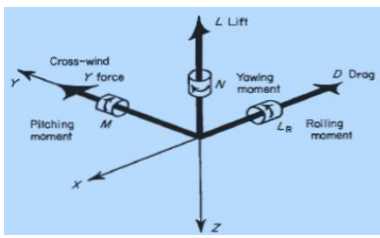
The traditional approach, after Bryan (1911), is to assume that the disturbing forces and moments are due to aerodynamic effects, gravitational effects, movement of aerodynamic controls, power effects and the effects of atmospheric disturbances [20].
The total Force : $\vec{F} = \vec{F_a} + \vec{F_g} + \vec{F_c} + \vec{F_p} + \vec{F_d}$
The total Torque : $\vec{M} = \vec{M_a} + \vec{M_g} + \vec{M_c} + \vec{M_p} + \vec{M_d}$
Such that:

a stands for aerodynamic effects g stands for gravitational effects
c stands for control effects p stands for power effects
d stands for atmospheric effects

The nonlinear equations of motion of airplane are:

$$\begin{cases} F_x = m(\dot{u} + q\omega - rv) \\ F_y = m(\dot{v} + ru - p\omega) \\ F_z = m(\dot{\omega} + pv - qu) \\ M_x = \dot{p}I_x - \dot{r}I_{xz} + qr(I_{zz} - I_{yy}) - pqI_{xz} \\ M_y = \dot{q}I_y + pr(I_{xx} - Izz) + (p^2 - r^2)I_{xz} \\ M_z = \dot{r}I_z - \dot{p}I_{xz} + pq(I_{yy} - I_{xx}) + qrI_{xz} \end{cases} \tag{2.31}$$

It is clear that those equations are nonlinear, so we need to linearize them [20]. We assume that the aircraft is flying in an equilibrium condition and we would like to linearize these equations around this nominal flight conditions. Before commencing the main task of developing linear mathematical models of the aircraft it, is first necessary to put in place an appropriate and secure foundation on which to build the models. The foundation comprises a mathematical framework in which the equations of motion can be developed in



an orderly and consistent way. Here in this work only the most basic commonly used axes system are involved, there are three basic reference frames named:

- Body ref frame (Fixed to the body).
- Aerodynamic frame (Fixed to the airplane but Ox axis is oriented parallel to $\vec{v_c}$).
- Inertial frame (Fixed to the Earth)

The rotation of one Cartesian coordinate system with respect to another can always be described by three rotations, and the angles of rotation are called Euler angles [4]. For general applications in $3D$, we need to perform three rotations in the space to relate the system (XYZ) to (xyz).

- We rotate by ψ (Yaw) about the z axis we obtain (x',y',z').

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{2.32}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = T_3(\psi) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{2.33}$$

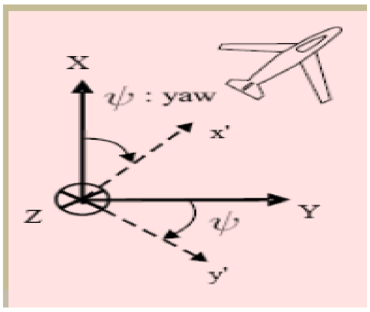


Figure 2.3: Rotate by ψ Yaw

- We rotate by θ (Pitch) about the y' axis we obtain (x'',y'',z'').

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \tag{2.34}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = T_2(\theta) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \tag{2.35}$$

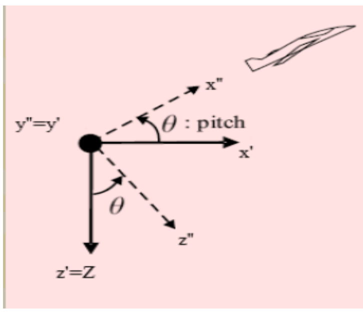


Figure 2.4: Rotate by θ Pitch

- We rotate by ϕ (Roll) about the x'' axis we obtain (x,y,z).

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \tag{2.36}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = T_1(\phi) \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \tag{2.37}$$

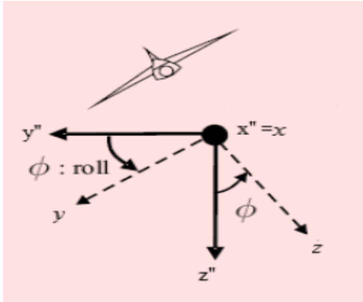


Figure 2.5: Rotate by ϕ Roll

By repeated substitution, the last equations may be combined to give the required transformation relationship:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = T_1(\phi)T_2(\theta)T_3(\psi) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R(\theta, \phi, \psi) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{2.38}$$

- **Properties** The rotation matrix $R(\theta, \phi, \psi)$ has the following properties:

  1- $RR^T = R^T R = I$

  2- $\det(R) = 1$

  3- Each column (and each row) of R is a unit vector

  4- Each columns (and each rows) of R are mutually orthogonal

The attitude transformation matrix (also called direction cosine matrix) is necessary to transform vectors and point coordinates from the aircraft's body fixed frame (xyz) to the navigation frame (XYZ) and vice versa.

We want to get the angular velocity in (xyz) axes which is defined as follows:

$$\begin{cases} \vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} & \text{in the inertial frame} \\ \vec{\omega} = \left(\dfrac{d\psi}{dt}\right)\vec{e}_\psi + \left(\dfrac{d\theta}{dt}\right)\vec{e}_\theta + \left(\dfrac{d\phi}{dt}\right)\vec{e}_\phi & \text{in term of Euler angles} \end{cases} \tag{2.39}$$

$$\text{Where:} \qquad \vec{e}_\psi = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta)\sin(\phi) \\ \cos(\theta)\cos(\phi) \end{pmatrix}, \quad \vec{e}_\theta = \begin{pmatrix} 0 \\ \cos(\phi) \\ -\sin(\phi) \end{pmatrix}, \quad \vec{e}_\phi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{\omega} = T_1(\phi)T_2(\theta)\begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} + T_1(\phi)\begin{pmatrix} 1 \\ \dot{\theta} \\ 0 \end{pmatrix} + \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} \tag{2.40}$$

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -\sin(\theta) & 0 & 1 \\ \cos(\theta)\sin(\phi) & \cos(\phi) & 0 \\ \cos(\theta)\cos(\phi) & -\sin(\phi) & 0 \end{pmatrix}\begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} \tag{2.41}$$

Remark 2.4. *For a small perturbation on the Euler angles we get*

$$\begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \Delta\dot{\phi} \\ \Delta\dot{\theta} \\ \Delta\dot{\psi} \end{pmatrix}$$

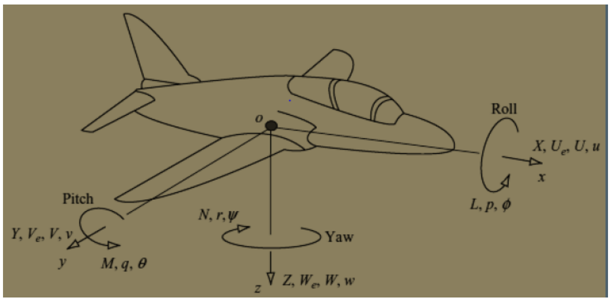


Figure 2.6: Small perturbation on the Euler angles

2.5 Small Disturbances Theory

The equations developed in the previous section can be linearized using small disturbance theory [21]. In applying the small-disturbance theory we assume that all motion of the airplane consists of small deviation about steady state flight condition. Obviously this theory cannot be applied to problems in which large amplitude motions are to be expected. All variables in the equations of motion are replaced by reference value plus a perturbation or disturbances. $\delta = \delta_0 + \Delta\delta$

$\vec{v_c}$: $u = u_0 + \Delta u$ $\quad v = v_0 + \Delta v$ $\quad w = w_0 + \Delta w$ Linear speed

$\vec{\omega}$: $p = p_0 + \Delta p$ $\quad q = q_0 + \Delta q$ $\quad r = r_0 + \Delta r$ Angular speed

$\vec{F}$: $X_0 + \Delta X$ $\quad Y = Y_0 + \Delta Y$ $\quad Z = Z_0 + \Delta Z$ Forces

$\vec{M}$: $M_0 + \Delta M$ $\quad N = N_0 + \Delta N$ $\quad L = L_0 + \Delta L$ Torques

For convenience, the reference flight condition is assumed to be symmetric and propulsive forces are assumed to remain constant (i.e. equilibrium condition) . This implies that

$$v_0 = w_0 = p_0 = q_0 = r_0 = \phi_0 = \psi_0 = 0.$$

The perturbed force equations are:

$$\begin{cases} X_0 + \Delta X = m(\dot{u_0} + \Delta\dot{u} + (q_0 + \Delta q)(\omega_0 + \Delta\omega) - (r_0 + \Delta r)(v_0 + \Delta v)) \\ Y_0 + \Delta Y = m(\dot{v_0} + \Delta\dot{v} + (r_0 + \Delta r)(u_0 + \Delta u) - (p_0 + \Delta p)(\omega_0 + \Delta\omega)) \\ Z_0 + \Delta Z = m(\dot{\omega}_0 + \Delta\dot{\omega} + (p_0 + \Delta p)(v_0 + \Delta v) - (q_0 + \Delta q)(u_0 + \Delta u)) \end{cases} \tag{2.42}$$

Neglecting the equalized and very inconsiderable terms in the force equations we get:

$$\begin{cases} \Delta X = m(\Delta\dot{u} + \Delta q\Delta\omega - \Delta r\Delta v) = m\Delta\dot{u} \\ \Delta Y = m(\Delta\dot{v} + \Delta r u_0 + \Delta r\Delta u - \Delta p\Delta\omega) = m\Delta\dot{v} + m\Delta r u_0 \\ \Delta Z = m(\Delta\dot{\omega} + \Delta p\Delta v - \Delta q\Delta u - \Delta q u_0) = m\Delta\dot{\omega} - m\Delta q u_0 \end{cases} \tag{2.43}$$

Following the same procedure we can obtain the small change of the rotation equation.

Remark 2.5. *Whenever the aircraft is disturbed from equilibrium the force and moment balance is upset and the resulting transient motion is quantified in terms of the perturbation variables. At equilibrium we have:*

$$\vec{\omega} = \begin{pmatrix} p_0 \\ q_0 \\ r_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix} = \begin{pmatrix} u_0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{\omega}_{new} = \begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix}, \quad \vec{v}_{new} = \begin{pmatrix} \Delta u + u_0 \\ \Delta v \\ \Delta w \end{pmatrix} = \begin{pmatrix} u_0 \\ 0 \\ 0 \end{pmatrix}$$

$$\theta_{new} = \theta_0 + \Delta\theta \quad \phi_{new} = \Delta\phi \quad \psi_{new} = \Delta\psi$$

$$Forces \begin{cases} \Delta X = m\Delta\dot{u} \\ \Delta Y = m\Delta\dot{v} + m\Delta r u_0 \\ \Delta Z = m\Delta\dot{\omega} - m\Delta q u_0 \end{cases} \qquad Moments \begin{cases} \Delta L = \Delta\dot{p}I_x - \Delta\dot{r}I_xz \\ \Delta M = \Delta\dot{q}I_y \\ \Delta N = \Delta\dot{r}I_z - \Delta\dot{p}I_{xz} \end{cases} \tag{2.44}$$

The traditional approach, after Bryan (1911), is to assume that the disturbing forces and moments are due to aerodynamic effects, gravitational effects, movement of aerodynamic controls, power effects and the effects of atmospheric disturbances. They may be written to include these contributions as follows:

$$\begin{cases} \Delta X = X_a + X_g + X_c + X_p \\ \Delta Y = Y_a + Y_g + Y_c + Y_p \\ \Delta Z = Z_a + Z_g + Z_c + Z_p \end{cases} \tag{2.45}$$

$$\begin{cases} \Delta L = L_a + L_g + L_c + L_p \\ \Delta M = M_a + M_g + M_c + M_p \\ \Delta N = N_a + N_g + N_c + N_p \end{cases} \tag{2.46}$$

Remark 2.6. *For seeking simplicity in the next part we drop the symbol* Δ *, and we assume that the new perturbation variables are* $u, v, w, p, q, r, \phi, \psi$ *and* θ *.*

2.6 Motion For Small Perturbations

To complete derivation and development of the linearized equation of motion, it only remains to substitute the appropriate expressions for aerodynamic, gravitational, control and thrust terms into the equation of motion [22]:

$$\begin{cases} X_a + X_g + X_c + X_p = m\dot{u} \\ Y_a + Y_g + Y_c + Y_p = m\dot{v} + mru_0 \\ Z_a + Z_g + Z_c + Z_p = m\dot{w} - mqu_0 \\ L_a + L_g + L_c + L_p = \dot{p}I_x - \dot{r}I_{xz} \\ M_a + M_g + M_c + M_p = \dot{q}I_y \\ N_a + N_g + N_c + N_p = \dot{r}I_z - \dot{p}I_{xz} \end{cases} \tag{2.47}$$

The following figure (Fig. 2.7) present the dynamic model of aircraft:

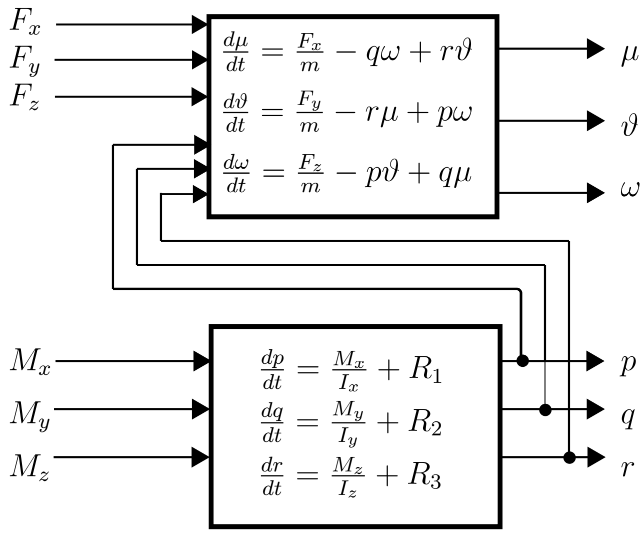


Figure 2.7: The nonlinear compact model of the B747 .

With:

$$R_1 = \dot{r}\frac{I_{xz}}{I_x} - \frac{qr}{I_x}(I_{zz} - I_{yy}) - \frac{pq}{I_x}I_{xz}$$

$$R_2 = \frac{pr}{I_y}(I_{zz} - I_{xx}) + (r^2 - p^2)\frac{Ixz}{I_y}$$

$$R_1 = \dot{p}\frac{I_{xz}}{I_z} + \frac{pq}{I_z}(I_{zz} - I_{yy}) - \frac{qr}{I_z}I_{xz}$$

Remark 2.7. *We know that there exist three aerodynamic control term which are essential (or primary). The elevator δ_e is used to control the pitch angle, the aileron δ_a is used to control the roll angle, and the rudder δ_r is used to control the yaw angle. However, it is generally, known that by acting on one of this control elements, motion about one axis produces motion about other axes, which is known as coupling. The equations of motion comprise a set of six simultaneous linear differential equations written in the traditional manner with the forcing, or input, terms on the right hand side. As written, and subject to the assumptions made in their derivation, the equations of motion are perfectly general and describe motion in which longitudinal and lateral dynamics may be fully coupled. However, for the vast majority of aeroplanes when small perturbation transient motion only is considered, as is the case here, longitudinal lateral coupling is usually negligible. Consequently it is convenient to simplify the equations by assuming that longitudinal and lateral motion is in fact fully decoupled.*

- **Longitudinal motion** defines all the parameters that change about the longitudinal axis which goes from the aircraft's tail to its nose and pass through the center of gravity.

- **Lateral motion** defines the parameters that change about the lateral axis which pass through the center of gravity along the airplane wings.

2.7 Longitudinal motion

In this type of motion, the aerodynamic coupling derivatives and control derivatives are all negligible small and may be taken zero [22].

aerodynamic derivatives: $\dfrac{\partial X}{\partial v} = \dfrac{\partial X}{\partial p} = \dfrac{\partial X}{\partial r} = \dfrac{\partial Z}{\partial v} = \dfrac{\partial Z}{\partial p} = \dfrac{\partial Z}{\partial r} = \dfrac{\partial M}{\partial v} = \dfrac{\partial M}{\partial p} = \dfrac{\partial M}{\partial r} = 0$

control derivatives: $\dfrac{\partial X}{\partial \delta_a} = \dfrac{\partial X}{\partial \delta_r} = \dfrac{\partial Z}{\partial \delta_a} = \dfrac{\partial Z}{\partial \delta_r} = \dfrac{\partial M}{\partial \delta_a} = \dfrac{\partial M}{\partial \delta_r} = 0$

Also at steady state $X_{a0} = mg\sin(\theta_0)$ and $Z_{a0} = -mg\cos(\theta_0)$, the equations of the longitudinal motion are therefore obtained as:

$$m\dot{u} - \left[\frac{\partial X}{\partial u}u + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial \dot{w}}\dot{w}\right] + mg\theta\cos(\theta_0) = \frac{\partial X}{\partial \delta_e}\delta_e + \frac{\partial X}{\partial \tau}\tau \tag{2.48}$$

$$m\dot{w} - mqu_0 - \left[\frac{\partial Z}{\partial u}u + \frac{\partial Z}{\partial w}w + \frac{\partial Z}{\partial q}q + \frac{\partial Z}{\partial \dot{w}}\dot{w}\right] + mg\theta\sin(\theta_0) = \frac{\partial Z}{\partial \delta_e}\delta_e + \frac{\partial Z}{\partial \tau}\tau \tag{2.49}$$

$$\dot{q}I_y - \left[\frac{\partial M}{\partial u}u + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial \dot{w}}\dot{w}\right] = \frac{\partial M}{\partial \delta_e}\delta_e + \frac{\partial M}{\partial \tau}\tau \tag{2.50}$$

Since the longitudinal motion of the aeroplane is described by four state variables u, w, q and θ four differential equations are required. Thus the additional equation is the auxiliary equation relating pitch rate to attitude rate, which for small perturbations is $\dot{\theta} = q$ .The state vector of this four simultaneous DE is $X(t) = (u, w, q, \theta)^T$ and the input control is $u(t) = (\delta_e, \tau)^T$. The state space representation of the longitudinal motion is given by the next equation $M\dot{X}(t) = AX(t) + Bu(t)$ where:

$$M = \begin{pmatrix} m & -\frac{\partial X}{\partial \dot{w}} & 0 & 0 \\ 0 & (m - \frac{\partial Z}{\partial \dot{w}}) & 0 & 0 \\ 0 & -\frac{\partial M}{\partial \dot{w}} & I_y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} \frac{\partial X}{\partial \delta_e} & \frac{\partial X}{\partial \tau} \\ \frac{\partial Z}{\partial \delta_e} & \frac{\partial Z}{\partial \tau} \\ \frac{\partial M}{\partial \delta_e} & \frac{\partial M}{\partial \tau} \\ 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{\partial X}{\partial u} & \frac{\partial X}{\partial w} & \frac{\partial X}{\partial q} & -mg\cos(\theta_0) \\ \frac{\partial Z}{\partial u} & \frac{\partial Z}{\partial w} & (mu_0 + \frac{\partial Z}{\partial q}) & -mg\sin(\theta_0) \\ \frac{\partial M}{\partial u} & \frac{\partial M}{\partial w} & \frac{\partial M}{\partial q} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

2.8 Lateral motion

as before, the aerodynamic coupling derivatives and control derivatives are all negligible small and may be taken zero [22].

aerodynamic derivatives: $\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial w} = \frac{\partial Y}{\partial \dot{w}} = \frac{\partial Y}{\partial q} = \frac{\partial L}{\partial u} = \frac{\partial L}{\partial w} = \frac{\partial L}{\partial \dot{w}} = \frac{\partial L}{\partial q} = \frac{\partial N}{\partial u} = \frac{\partial N}{\partial w} = \frac{\partial N}{\partial \dot{w}} = \frac{\partial N}{\partial q} = 0$

control derivatives: $$\frac{\partial Y}{\partial \delta_e} = \frac{\partial Y}{\partial \tau} = \frac{\partial L}{\partial \delta_e} = \frac{\partial L}{\partial \tau} = \frac{\partial N}{\partial \delta_e} = \frac{\partial N}{\partial \tau} = 0$$

$$m\dot{v} - mg\psi \sin(\theta_0) = \left[\frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial p}p + \left(\frac{\partial Y}{\partial r} - mu_0\right) r\right] + mg\phi\cos(\theta_0) + \frac{\partial Y}{\partial \delta_a}\delta_a + \frac{\partial Y}{\partial \delta_r}\delta_r \tag{2.51}$$

$$\dot{p}I_x - \dot{r}Ixz = \left[\frac{\partial L}{\partial v}v + \frac{\partial L}{\partial p}p + \frac{\partial L}{\partial r}r\right] + \frac{\partial L}{\partial \delta_a}\delta_a + \frac{\partial L}{\partial \delta_r}\delta_r \tag{2.52}$$

$$\dot{r}I_z - \dot{p}Ixz = \left[\frac{\partial N}{\partial v}v + \frac{\partial N}{\partial p}p + \frac{\partial N}{\partial r}r\right] + \frac{\partial N}{\partial \delta_a}\delta_a + \frac{\partial N}{\partial \delta_r}\delta_r \tag{2.53}$$

Since the lateral motion is described by five stat variables v, p, r, ϕ and ψ we need five DE. Thus we add the auxiliary equation, which is for small perturbation $\dot{\phi} = p$, hence the state equation will be $(X(t) = (v, p, r, \phi, \psi)^T$ and $u(t) = (\delta_a, \delta_r)^T)$

$$M = \begin{pmatrix} m & 0 & 0 & 0 & 0 \\ 0 & I_x & -I_{xz} & 0 & 0 \\ 0 & -I_{xz} & I_z & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} \frac{\partial Y}{\partial \delta_a} & \frac{\partial Y}{\partial \delta_r} \\ \frac{\partial L}{\partial \delta_a} & \frac{\partial L}{\partial \delta_r} \\ \frac{\partial N}{\partial \delta_a} & \frac{\partial N}{\partial \delta_r} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{\partial Y}{\partial v} & \frac{\partial Y}{\partial p} & \left(\frac{\partial Y}{\partial r} - mu_0\right) & mg\cos(\theta_0) & mg\sin(\theta_0) \\ \frac{\partial L}{\partial v} & \frac{\partial L}{\partial p} & \frac{\partial L}{\partial r} & 0 & 0 \\ \frac{\partial N}{\partial v} & \frac{\partial N}{\partial p} & \frac{\partial N}{\partial r} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The figure below shows the lateral model of aircraft:

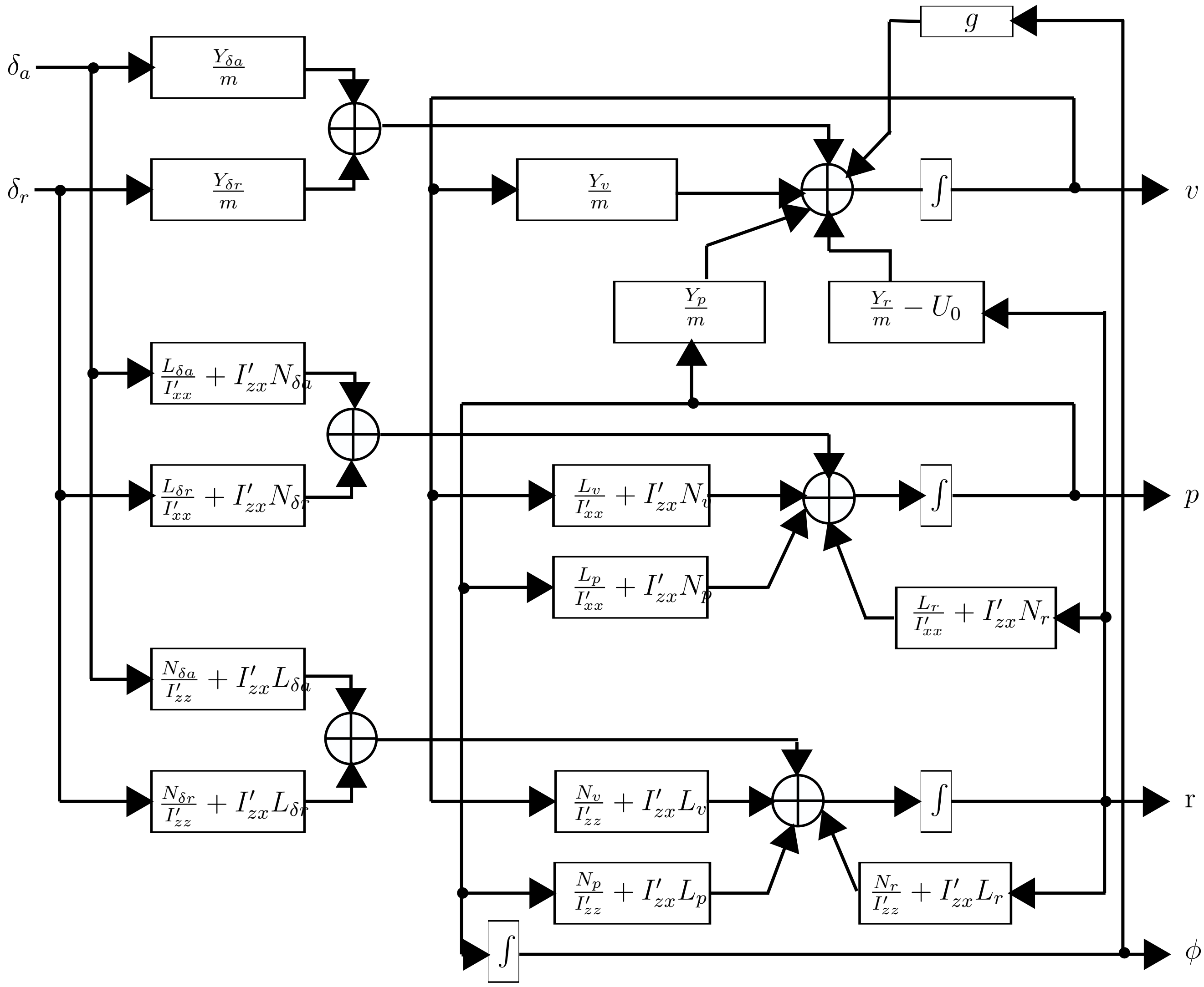


Figure 2.8: Depth expanded model of the B747.

2.9 Conclusion

This chapter presented the derivation steps of the nonlinear mathematical model of the dynamics of an aircraft. For small perturbations around the equilibrium state, the later nonlinear model was approximated by its linearized version that will be used for control purposes in Chapter 5.

Chapter 3

Multi-Input Multi-Output PID Controller

3.1 Introduction

In industrial control, proportional integral and derivative (PID) controllers still have an undisputed lead. In spite of system theory evolution, the most controllers in use are still PID because of the advantages and numerous benefits they offer, to cite few of them, e.g., their ease of implementation, simplicity, wide availability and success rate in providing satisfactory performance especially for systems that don't have complex nonlinear dynamics.

Over the past decades, an enormous amount of effort has been expended in designing this type of controllers for both Single-Input Single-Output (SISO) and Multi-Input Multi-Output (MIMO) systems. To prepare for the application of MIMO PID controller to Boeing 747 aircraft, in this chapter, we introduce the theory of the considered controller design approach, and we also test its performance on a numerical academic example.

3.2 Proportional Derivative Integral Control

Model-based PID control synthesis is a typical low-order controller design problem. The three control blocks in the PID control have different actions in the process. A proportional controller (K_p) has the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error. An integral control (K_I) has the effect of eliminating

the steady-state error, but it may make the transient response worse. A derivative control (K_D) has the effect of increasing the stability of the system, reducing the overshoot and improving the transient response [24].
The expression for the output of the PID controller in terms of the error and the corresponding transfer function are given as:

$$u(t) = K_p \left(e(t) + \tau_D \frac{de(t)}{dt} + \frac{1}{\tau_I} \int_0^t e(\tau) d\tau \right) \tag{3.1}$$

This figure Fig 3.1 show the traditional *PID* structure applied to an aircraft model.

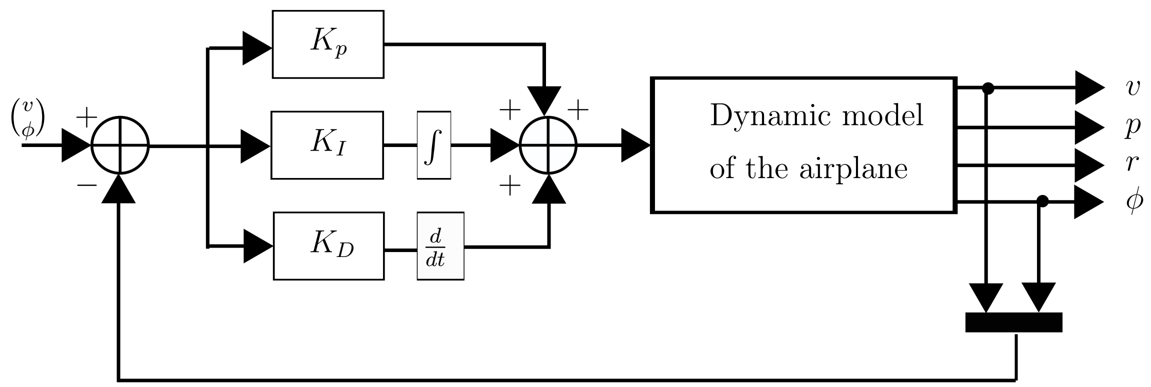


Figure 3.1: Traditional PID structure .

3.3 Definition of a matrix polynomials

Given the set of $m \times m$ complex matrices $A_i, i = 0, 1, 2...l$, the following matrix of the complex number S is called a polynomial matrix of degree m and index l ([25]).

$$\Delta(S) = A_0 S^l + A_1 S^{l-1} + ... + A_l \tag{3.2}$$

Consider the system described by the following dynamic equation:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{3.3}$$

Assuming that the system can be transformed to a block controller form, this means:

- The number $l = n/m$ is an integer.
- The controllability matrix defined by:
  $w_c = \begin{pmatrix} B & AB & A^2B & . & . & . & A^{l-1}B \end{pmatrix}$ has full rank(i.e) it has a rank n.
  Then we use the following transformation matrix:
  $$T_c = \begin{pmatrix} T_{c1} \\ T_{c1}A \\ T_{c1}A^2 \\ . \\ . \\ T_{c1}A^{l-1} \end{pmatrix}, \quad \text{Where:} \quad T_{c1} = \begin{pmatrix} 0_m & 0_m & . & . & I_m \end{pmatrix} w_c^{-1}.$$
  The new system becomes:
  $$\begin{cases} \dot{x_c} = A_c x_c + B_c u \\ y = C_c x_c \end{cases} \tag{3.4}$$

$$\text{Where: } A_c = \begin{pmatrix} 0_m & I_m & ... & 0_m \\ 0_m & ... & ... & 0_m \\ ... & ... & ... & ... \\ 0_m & 0_m & 0_m & I_m \\ -A_l & -A_{l-1} & ... & -A_1 \end{pmatrix}, \quad b_c = \begin{pmatrix} 0_m \\ 0_m \\ ... \\ ... \\ I_m \end{pmatrix}, \quad C_c = \begin{pmatrix} C_l & C_{l-1} & ... & ... & C_1 \end{pmatrix}.$$

3.4 Transfer function

The transfer function of this open-loop system is given by:

$$TF = N_R(s).D_R^{-1}(s) \tag{3.5}$$

Where:

$$N_R(S) = C_1 S^{l-1} + C_2 S^{l-2} + ... + C_l$$
$$D_R(S) = I_m S^l + A_1 S^{l-1} + ... + A_l$$

This transfer function is called the Right Matrix Fraction Description (RMFD), we need to use it in the block controller form [27]. It should be noted that the behavior of the system depends on the characteristic matrix polynomial $D(s)$.

3.5 Concept of solvents (block roots)

A root for a polynomial matrix is not well defined. If it is defined as a complex number it may not exist at all. Then we may consider a root as a matrix called block root.

3.5.1 Right solvent:

Given the matrix polynomial of degree and index l defined by:

$$D(S) = I_m S^l + D_1 S^{l-1} + ... + D_l \tag{3.6}$$

A right solvent [23], denoted by R , is a $m \times m$ matrix satisfying:

$$R^l + D_1 R^{l-1} + ... + D_{l-1} R + D_l = 0_m \tag{3.7}$$

3.5.2 Left solvent:

A left solvent of the matrix polynomial $D(s)$ defined above [23], denoted by L, is a $m \times m$ matrix satisfying:

$$L^l + L^{l-1} D_1 + ... + L D_{l-1} + D_l = 0_m \tag{3.8}$$

A right solvent, if exist, is considered as a right block root. A left solvent, if exist, is considered as a left block root.

3.5.3 Latent root and latent vector:

- A complex number λ satisfying $\det(D(\lambda)) = 0$ is called a latent root of $D(s)$.
- Any vector x_i associated with the latent root λ_i satisfying $D(\lambda_i)x_i = 0_m$ is a right latent vector of $D(s)$.

Theorem 3.1 ([26]). *If we assign a matrix R to be a right solvent of D(s) , or a matrix L to be a left solvent of D(s), it is equivalent to assigning the eigenvalues and eigenvectors of R or L, as latent roots and right, or left, latent vectors of D(s).*

Consider the block controller form described by the following dynamic equation:

$$\begin{cases} \dot{x}_c = A_c x_c + B_c u \\ y = C_c x_c \end{cases} \tag{3.9}$$

The characteristic matrix polynomial of this system is:

$$D(S) = I_m S^l + A_1 S^{l-1} + ... + A_l \tag{3.10}$$

Theorem 3.2 ([26]). *Any eigenvalue of a right solvent or a left solvent of $D(s)$ is an eigenvalue of A_c.*

Theorem 3.3 ([26]). *Assigning eigenvalues to a solvent of $D(s)$ is equivalent to assigning these eigenvalues to the matrix A_c.*

3.5.4 Complete set of solvents:

Theorem 3.4 ([27]). *Given a characteristic matrix polynomial defined by $D(\lambda)$, the set of $m \times m$ matrices $(R_1, R_2, ..., R_l)$ is said to be a complete set of right solvents if the following conditions are verified:*

1) $\sigma(R_i) \cap \sigma(R_j) = \emptyset$ *for* $i \neq j$ *and* $i = 1, 2, ...l$ *and* $j = 1, 2, ...l$

2) $\cup_{i=1}^{l} \big(\sigma(R_i)\big) = \sigma(A_c)$

3) $det\big(V(R_1, R_2, ..., R_l)\big) \neq 0$
   Where:

- $\sigma(R_i)$ *is the spectrum of the right solvent* R_i *, which is the set of its eigenvalues.*
- $\sigma(A_c)$ *is the spectrum of the matrix* A_c.
- $V\big(R_1, R_2, ..., R_l\big)$ *is the block Vandermonde matrix of the set of right solvents defined by the following:*

$$V\big(R_1, R_2, ..., R_l\big) = \begin{pmatrix} I_m & I_m & ... & I_m \\ R_1 & R_2 & ... & R_l \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ R_1^{l-1} & R_2^{l-1} & ... & R_l^{l-1} \end{pmatrix} \tag{3.11}$$

Remark 3.1. *We can define a set of left solvents in the same way as in the previous theorem.*

3.6 Constructing a matrix polynomial from a complete set of solvents

We want to construct the matrix polynomial defined by $D(\lambda)$ from a set of solvents or a set of desired poles which will determine the behaviour of the system that we want. Suppose we have a desired complete set of solvents [23]. The problem is to find the desired polynomial matrix or the characteristic equation of the block controller form defined by:

$$D(\lambda) = I_m\lambda^l + D_{d1}\lambda^{l-1} + ... + D_{dl}. \tag{3.12}$$

We want to find the coefficients D_i for $i = 1, 2, ..., l$

- **Constructing from a complete set of right solvents:**

  Consider a complete set of right solvents $\{R_1, R_2, ..., R_l\}$, for the matrix polynomial $D(\lambda)$ If R_i is a right solvent of $D(\lambda)$ so:

  $$R_i^l + D_{d1}R_i^{l-1} + ... + D_{d(l-1)}R_i + D_{dl} = 0_m \Longleftrightarrow D_{d1}R_i^{l-1} + ... + D_{d(l-1)}R_i + D_{dl} = -R_i^l$$

  Replacing i from 1 to l we get the following:

  $$\begin{pmatrix} D_l & D_{(l-1)} & ... & D_2 & D_1 \end{pmatrix} = \begin{pmatrix} R_1^l & R_2^l & ... & R_l^l \end{pmatrix} \begin{pmatrix} I_m & I_m & ... & I_m \\ R_1 & R_2 & ... & R_l \\ R_1^2 & R_2^2 & ... & R_l^2 \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ R_1^{l-1} & R_2^{l-1} & ... & R_l^{l-1} \end{pmatrix}^{-1} \tag{3.13}$$

- **Constructing from a complete set of left solvents**

  Consider a complete set of left solvents $\{L_1, L_2, ..., L_l\}$ for the matrix polynomial $D(\lambda)$. If L_i is a left solvent of $D(\lambda)$, then:

  $$L_i^l + L_i^{l-1}D_{d1} + ... + L_iD_{d(l-1)} + D_{dl} = 0_m \Longleftrightarrow L_i^{l-1}D_{d1} + ... + L_iD_{d(l-1)} + D_{dl} = -L_i^l$$

  Replacing i from 1 to l we get what follows:

  $$\begin{pmatrix} D_l \\ D_{(l-1)} \\ ... \\ D_2 \\ D_1 \end{pmatrix} = - \begin{pmatrix} I_m & L_1 & L_1^2 & ... & L_1^{l-1} \\ I_m & L_2 & L_2^2 & ... & L_2^{l-1} \\ ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... \\ I_m & L_l & L_l^2 & ... & L_l^{l-1} \end{pmatrix} \begin{pmatrix} L_1^l \\ L_2^l \\ ... \\ ... \\ L_l^l \end{pmatrix} \tag{3.14}$$

3.7 MIMO PID Control and Diophantine Equation

Objectives: the main idea of this section here is the dynamic compensator design which relates inputs to outputs when the states are not measurable, the proposed (see [27]) compensator is of special type called three actions or MIMO PID controller based on the solution of Diophantine equation to relocate some desired Block roots of matrix polynomial achieving needed control performances.

Given a system described by right or left MFD as

$$H(\lambda) = N_R(\lambda)D_R^{-1}(\lambda) = D_L^{-1}(\lambda)N_L(\lambda) \tag{3.15}$$

The matrix transfer function of the controller is [23]:

$$C(\lambda) = \left(K\lambda\right)^{-1}\left(K_I + K_P\lambda + K_D\lambda^2\right) = D_c^{-1}(\lambda)N_c(\lambda) \tag{3.16}$$

The control input signal is given by:

$$u(\lambda) = \left(K\lambda\right)^{-1}\left(K_I + K_P\lambda + K_D\lambda^2\right)e(\lambda) \tag{3.17}$$

Where: $e(\lambda) = r(\lambda) - y(\lambda)$ is the error between the input and the output. The closed loop transfer matrix is obtained as:

$$H_{closed}(\lambda) = N_R(\lambda)\left[D_c(\lambda)D_R(\lambda) + N_c(\lambda)N_R(\lambda)\right]^{-1}N_c(\lambda) \tag{3.18}$$

$$H_{closed}(\lambda) = N_R(\lambda)D_f^{-1}(\lambda)N_c(\lambda) \tag{3.19}$$

The matrix equation $D_f(\lambda)$is called Diophantine equation where:

$$D_f(\lambda) = D_c(\lambda)D_R(\lambda) + N_c(\lambda)N_R(\lambda) \tag{3.20}$$

Expanding this last equation we get:

$$\begin{pmatrix} D_l^T & N_{l-1}^T & 0 & 0 \\ D_{l-1}^T & N_{l-2}^T & N_{l-1}^T & 0 \\ D_{l-2}^T & N_{l-3}^T & N_{l-2}^T & N_{l-1}^T \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ D_1^T & N_0^T & N_1^T & N_2^T \\ D_0^T & 0 & N_0^T & N_1^T \\ 0 & 0 & 0 & N_0^T \end{pmatrix}\begin{pmatrix} K^T \\ \\ K_D^T \\ \\ K_P^T \\ \\ K_I^T \end{pmatrix} = \begin{pmatrix} D_{f(l+1)}^T \\ D_{fl}^T \\ \\ \\ \\ D_{f1}^T \\ D_{f0}^T \end{pmatrix} \tag{3.21}$$

Remark 3.2. *The existence of MIMO PID controller using this procedure depends on the solvability of the last rectangular matrix equation.*

Example 3.1 ([30])**.** *A two input-two output system in state space form given by :*

$$A = \begin{bmatrix} -0.0558 & 0 & -235.9 & 9.81 \\ -0.0127 & -0.4351 & 0.4143 & 0 \\ 0.0036 & -0.0061 & -0.1458 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0729 & 0.0000 \\ -4.75 & 0.00775 \\ 0.153 & 0.143 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \end{bmatrix}$$

Find its corresponding matrix transfer function $H(\lambda)$, then design a MIMO PID controller which will achieve the desired set of latent structure with tracking conditions. The desired set of solvent (Latent structure) is given below:

$$R_1 = \begin{bmatrix} -3.9914 & -0.1381 \\ 0.0627 & -5.0086 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -2.9657 & 00.4224 \\ -0.3279 & -7.0343 \end{bmatrix}, \quad R_3 = \begin{bmatrix} -7.9647 & -0.3234 \\ 0.1129 & -9.0353 \end{bmatrix}$$

The numerator $N_R(\lambda)$ and denominator $D_R(\lambda)$ of the proper rational matrix transfer function $H(\lambda)$ are conducted according to the next MATLAB statement.

```
theta=-A^2*B*D_2, Omega=[B A*B], D_\lambda=linsolve(Omega,theta)
D_0 = D_lambda(1:2,:), D_1 = D_lambda(3:4,:),  D_2 = eye(2,2),
M= [D_1 ; D_2; .. D_2 ; O], N_lambda=Omega_c*M,
 N_0=C*N_lambda(:,1:2), N_1=C*N_lambda(:,3:4).
```

Now we obtain the matrix transfer function of dynamic system given in its RMFD form:

$$H(\lambda) = N_R(\lambda) D_R^{-1}(\lambda) = \left(\sum_{i=0}^{2} N_i \lambda^i\right)\left(\sum_{i=0}^{2} D_i \lambda^i\right)^{-1} \tag{3.22}$$

Where:

$$D_0 = \begin{bmatrix} -0.0178 & -9.5414 \\ 0.0020 & 0.8558 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.4463 & -1.7529 \\ 0.0140 & 0.1904 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \end{bmatrix},$$

$$N_0 = \begin{bmatrix} 0.0024 & 11.4855 \\ -1.4330 & 1.1460 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0 & 0.1719 \\ 0 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \end{bmatrix}.$$

Let us define now the following matrices:

$$V_r = \begin{pmatrix} I_2 & I_2 & I_2 \\ R_1 & R_2 & R_3 \\ R_1^2 & R_2^2 & R_3^2 \end{pmatrix}, \quad M = \begin{pmatrix} D_2 & D_1 & D_0 & O_2 \\ N_1 & N_0 & O_2 & O_2 \\ O_2 & N_1 & N_0 & O_2 \\ O_2 & O_2 & N_1 & N_0 \end{pmatrix}.$$

Construction of the desired matrix polynomial coefficients form those Block spectral data

$$\left[D_{f2}, D_{f1}, D_{f0}\right] = -\left[R_1^3, R_2^3, R_3^3\right] V_R^{-1}, \text{ and } D_{f3} = I_2. \tag{3.23}$$

Here in this example the matrix M is full rank square matrix then:

$$\left[K, K_D, K_P, K_I\right] = \left[D_{f3}, D_{f2}, D_{f1}, D_{f0}\right] M^{-1} \tag{3.24}$$

$$\left[K, K_D, K_P, K_I\right] = \left(I_2 \quad -\left[R_1^3, R_2^3, R_3^3\right] \begin{pmatrix} I_2 & I_2 & I_2 \\ R_1 & R_2 & R_3 \\ R_1^2 & R_2^2 & R_3^2 \end{pmatrix} \right) \begin{pmatrix} D_2 & D_1 & D_0 & O_2 \\ N_1 & N_0 & O_2 & O_2 \\ O_2 & N_1 & N_0 & O_2 \\ O_2 & O_2 & N_1 & N_0 \end{pmatrix}^{-1} \tag{3.25}$$

Finally the PID parameters are:

$$K = \begin{bmatrix} 1.0000 & -1.5055 \\ 0 & -4.0852 \end{bmatrix}, \quad K_D = \begin{bmatrix} 8.7589 & -67.9097 \\ 29.5860 & -21.8488 \end{bmatrix},$$

$$K_p = \begin{bmatrix} 6.4415 & -47.8866 \\ 13.5706 & -8.5042 \end{bmatrix}, \quad K_I = \begin{bmatrix} 1.1158 & -10.4948 \\ 1.8964 & -0.7180 \end{bmatrix}.$$

The corresponding MIMO PID compensator can be designed using the following MATLAB statements:

```
clear all
clc
A=[-0.0558 0  -235.9 9.81; -0.0127 -0.4351 0.4143 0; 0.0036 -0.0061 -0.1458 0;0 1 0 0];
B=[0 1.7188;-0.1433 0.1146;0 -0.4859;0 0];
C=[0.1 0 0 0;0 0 0 10];
D=[0 0;0 0]; I=eye(2,2); Z=zeros(2,2); D2=eye(2,2);
R1=[-3.9914 -0.1381;0.0627 -5.0086];
R2=[-2.9657 0.4224;-0.3279 -7.0343];
R3=[-7.9647 -0.3234;0.1129 -9.0353];
theta=-A^2*B*D2;
Omegac=[B A*B] ; DM=linsolve(Omegac,theta);
D0=DM(1:2,:)
D1=DM(3:4,:)
D2=eye(2,2);
M=[D1 D2;D2 Z]; NM=Omegac*M;
N0=C*NM(:,1:2)
N1=C*NM(:,3:4)
N2=Z;
VR=[I I I;R1 R2 R3;R1^2 R2^2 R3^2]; Mm=[D2 D1 D0 Z;N1 N0 Z Z;Z N1 N0 Z;Z Z N1 N0];
Df=-[R1^3 R2^3 R3^3]*inv(VR); PID=[I Df]*inv(Mm);
K=PID(:,1:2)
KD=PID(:,3:4)
KP=PID(:,5:6)
KI=PID(:,7:8)
s=tf('s');
Nc=KI+KP*s+KD*s^2; Dc=K*s; Ns=N2*s^2+N1*s+N0; Ds=D2*s^2+D1*s+D0;
H= Ns*inv(Ds);
PID=(inv(K)/s)*(KI + KP *s + KD *s^2);
Tforward = H*PID;
Hcl = feedback(Tforward,I,-1); Hcl = minreal(Hcl);
Ts= 0.1; N = 40; t = [0 : Ts : N]'; n = length(t);
u = 10*[1.5*(1.5 -1.5*exp(-0.1*t)).* sin(0.3*t), 3.0*(1 -exp(-0.2*t)).*
```

```
    cos(-0.2*t + 30)] ;
u1=u(:,1); u2=u(:,2);
y = lsim(Hcl, u, t); y1 = y(1 : n, 1); y2 = y(1 : n, 2);
e=u-y;
plot(t, y1,'--black','linewidth', 1.5),grid on ,hold on
plot(t, u1,'black','linewidth', 1.5)
figure
plot(t, y2,'--red','linewidth', 1.5),grid on, hold on
plot(t, u2,'red','linewidth', 1.5)
figure
plot(t, e,'--red','linewidth', 1.5) ,grid
```

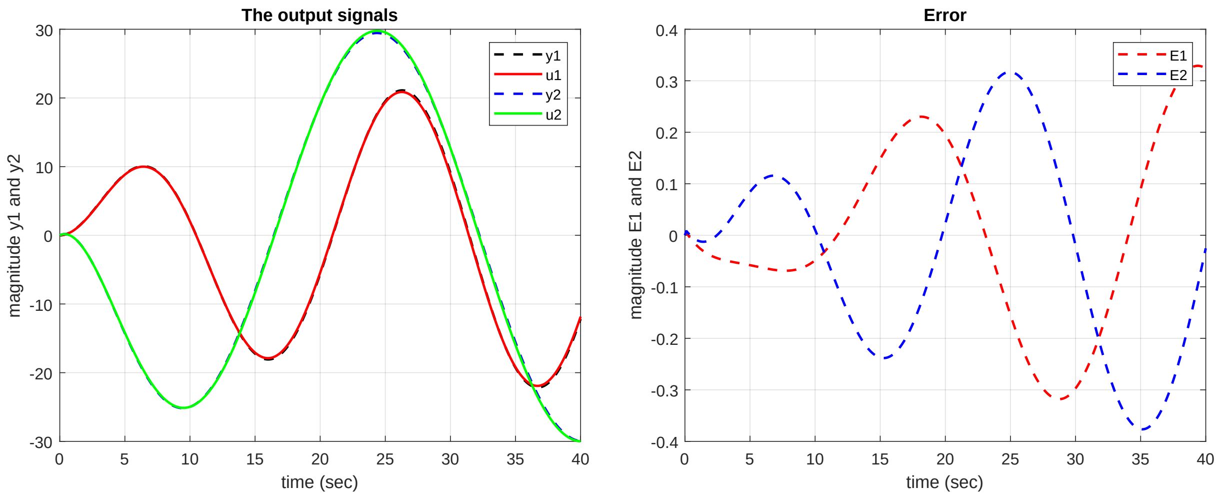


Figure 3.2: The trajectory tracking control of MIMO PID controller

The following case study illustrates best tracking, regulation and robustness with no oscillation and the ability of the proposed MIMO PID controller to robustly maintaining best dynamic performance and matching some desired latent structures or in other word Block pole placement preserving the output feedback compensator behavior. From the results obtained, in the above figures we see that the plant outputs coincides with its reference, no excess is recorded in both transient and permanent regimes which are well shown by the error signals. Also another discussion point can be considered and taken as an advantage which is the small controller gains that leads to smaller control signals, and thus to less energy consumption. Finally the global stability is guaranteed because all desired Block roots are stable matrices having specific latent roots latent vectors which implies large design degree of freedom and/or much more flexibility in syntheses.

3.8 Conclusion

In this chapter, we investigated the design problem of MIMO PID controller. We provided the theory employed for such a design, and then tested its performance on academic numerical example. This controller will be applied to Boeing 747 aircraft in Chapter 5.

Chapter 4

Fuzzy Logic Controller

4.1 Introduction

In the last few years, artificial intelligent techniques have been used to convert human experience into a form understandable by computers. There exists a mismatch between humans and machines, human's reason is based on uncertainty and summarizes in a fuzzy way, while machines and computers that run by them are based on binary reasoning.

The fuzzy logic is a way to make machines more intelligent, enabling them to reason in a fuzzy manner like a human, and allows decision making with estimated values under incomplete or uncertain information. Fuzzy logic combined with control systems, which can enhance the capabilities of industrial automation. Fuzzy control techniques rely on the human capability to understand the behaviour of the system. In this chapter, we first present the main definitions and theory of fuzzy logic control. Then, we describe and explain the Matlab fuzzy logic toolbox that will be used to implement the controller for Boeing 747 aircraft in the next chapter.

4.2 Definition of Fuzzy Logic

Fuzzy logic is a logic operations method based on many-valued logic rather than binary logic (two-valued logic). Two-valued logic often considers 0 to be false and 1 to be true. However, fuzzy logic deals with truth values between 0 and 1, and these values are considered as intensity (degrees) of truth. Fuzzy logic may be applied to many fields, including control systems, neural networks and artificial intelligence (AI) [28].

4.3 Fuzzy set theory

4.3.1 Definition of fuzzy sets

Fuzzy set theory is an extension to the classical set theory. As with classical sets, fuzzy sets are defined over an universe of discourse. For a given universe of discourse U, a fuzzy set is determined by a membership function which maps elements of U on to a membership range which is usually in the range $[0, 1]$.
Let U be a collection of objects denoted by $\{u\}$ where 'u' represents the generic element of U. A fuzzy set A in the universe of discourse U is characterised by a membership function $\mu_A(u)$ which maps each element of U to a real number in the interval $[0, 1]$, namely $(\mu_A : U \to [0, 1])$. The membership function represents the grade of membership of u in A.
The fuzzy set A can thus be represented as :

$$A = \{(u, \mu_A(u)/u \in U\}$$

A fuzzy set can be considered to be a generalisation of an ordinary set, such that in an ordinary set, an element will have a membership function $\mu_A = 0$ or 1. In the classical set theory, an element either belongs to or does not belong to a set but, the elements belonging to a fuzzy set show a gradual transition from membership to non-membership. Thus, fuzzy sets allow an element in the set to have a degree of membership of any real value between zero and one which is called the membership value. This value determines to what degree an element belongs to a set [28].

4.3.2 Membership Functions

Each linguistic value is characterised by a membership function. The membership functions commonly used are:(see [31])

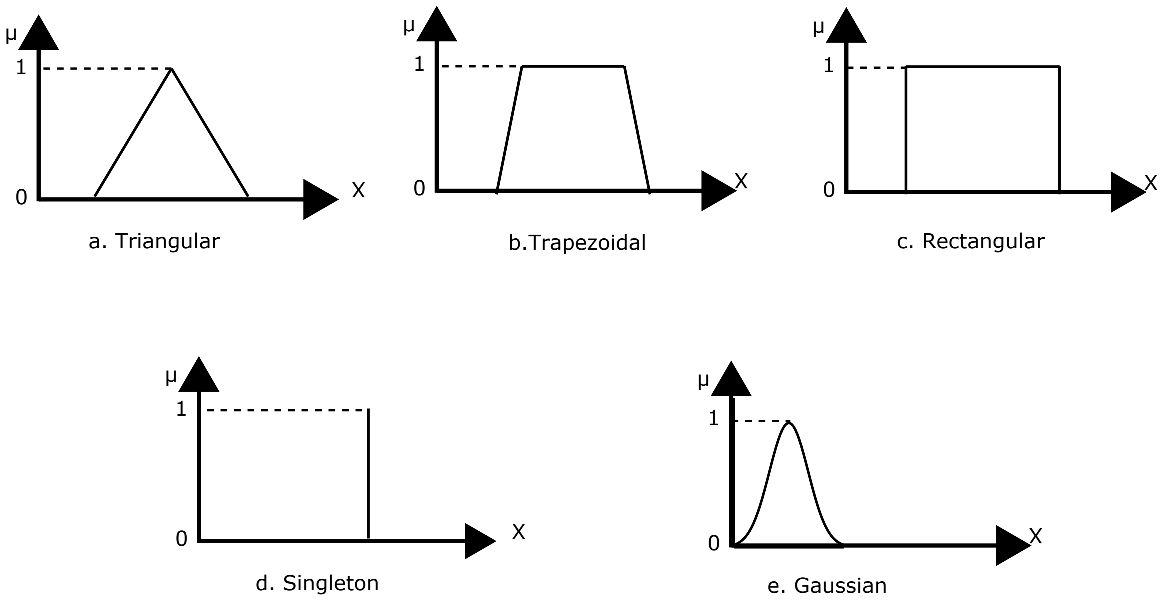


Figure 4.1: Commonly used shapes of membership functions.

4.3.3 Operations on Fuzzy Sets

All the normal set operations can be defined on fuzzy sets. Let A and B be two fuzzy sets in X with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively. The traditional set theory operations of union, intersection and complement of classical subsets of X can be extended for fuzzy sets via their membership functions [35].

- **Union** $(A \cup B)$: $\mu_{A \cup B}(x) = max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,
- **Intersection** $(A \cap B)$: $\mu_{A \cap B}(x) = min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,
- **Complement :** The compliment of A (noted $comp(A)$) $\mu_{comp(A)} = 1 - \mu_A(x)$ for all $x \in X$,
- **Empty set:** A is empty if and only if $\mu_A(x) = 0$ for All $x \in X$.

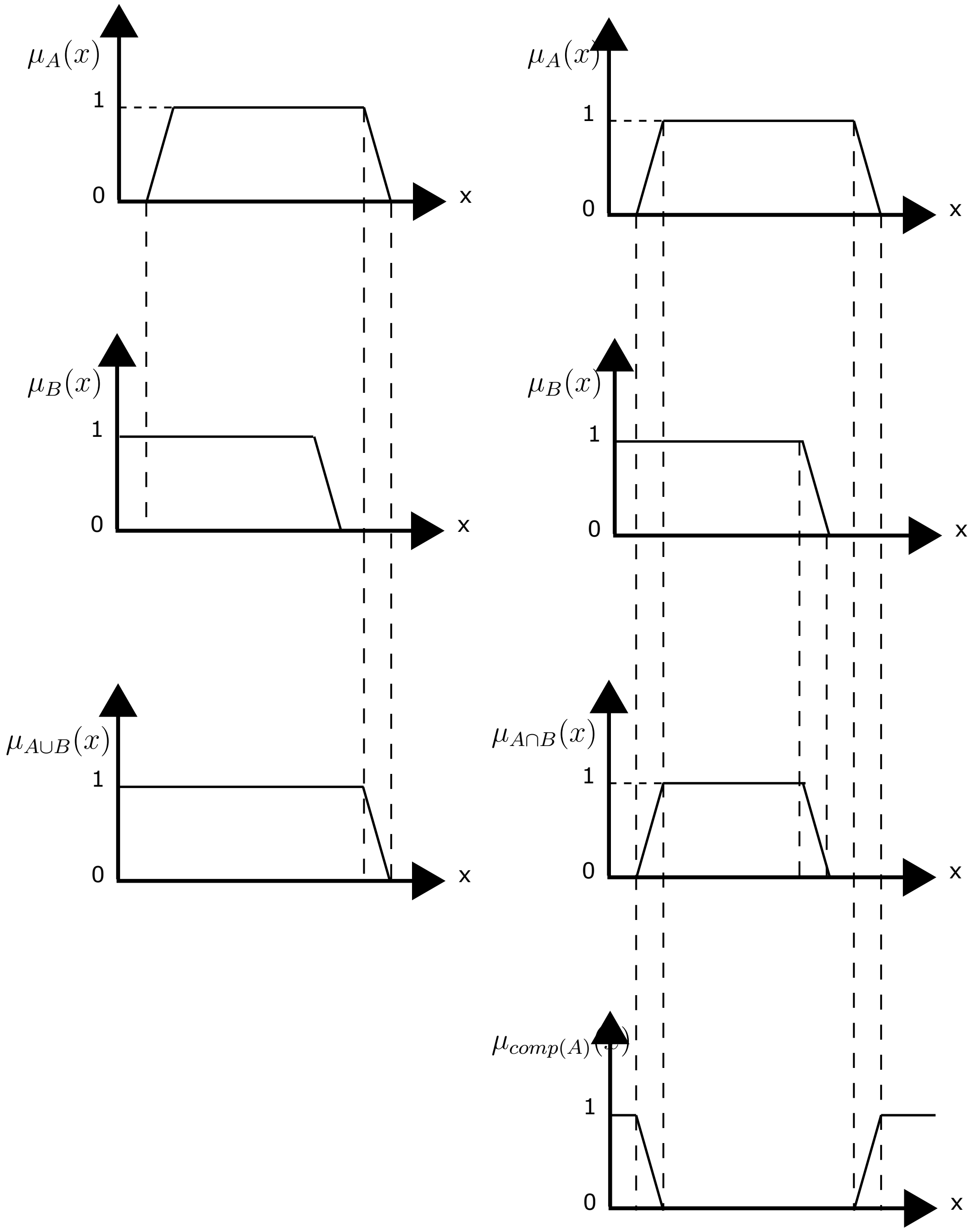


Figure 4.2: Fuzzy operations.

4.4 Fuzzy Rules

Fuzzy controllers are built from {if...then...} rules: If x is A then y is B Here A and B are fuzzy sets defined on universes X and Y respectively carrying labels such as small, medium, large. This is an implication, where the left-hand side " x is A " is the an-

tecedent, and the right-hand side " y is B " is the consequent (see [29]).
The following list gives some examples of such rules in everyday conversation:

1) If it is dark, then drive slowly.

2) If the room is cold, then increase the heat.

4.5 Reasoning on Fuzzy Logic

Approximation reasoning is a form of fuzzy logic,which include a set of inference rules whose premises are fuzzy prepositions.

4.5.1 Linguistic Variables

The use of fuzzy sets provides a basis for a systematic way for the manipulation of vague and imprecise concepts. In particular, we can employ fuzzy sets to represent linguistic variables. A linguistic variable can be regarded either as a variable whose value is a fuzzy number or as a variable whose values are defined in linguistic terms (see [33]).
A linguistic variable is characterized by a quintuple

$$L = \{x, T(x), X, G, M\}$$

Where:

- x: The name of variable.
- $T(x)$: The set of linguistic terms of x.
- X: The universe of discourse.
- G: The rules that generate the terms in $T(x)$.
- M: The semantic rule which associates each linguistic value whith its meaning by means of fuzzy set.

Example 4.1. *Let x be a linguistic variable labelled $\ll$ Age $\gg$, take $X = [0, 100]$*
T(Age)={young, very young, not very young, old, more or less old}.

4.6 Structure of Fuzzy Controller

The block diagram in (Fig 4.3) shows the different components of a fuzzy controller [34]:

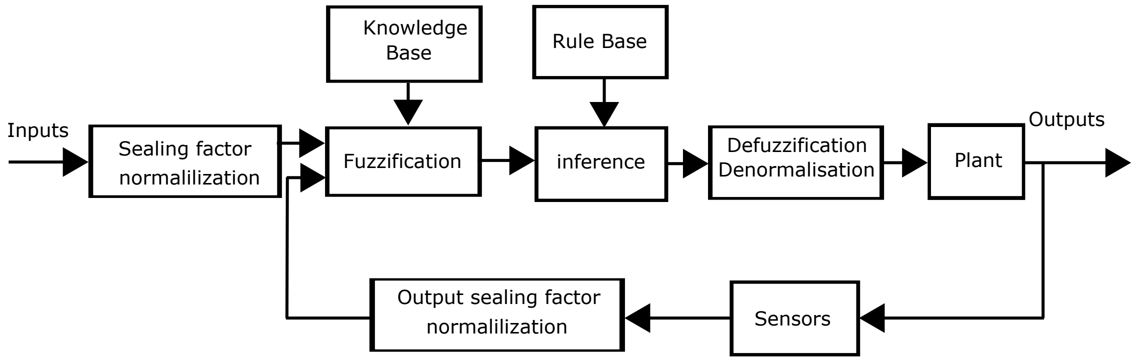


Figure 4.3: Simple Fuzzy Logic Controller

The block diagram is composed of four principle modules:

- knowledge base
- Fuzzification
- Inference engine
- Defuzzification

4.6.1 Knowledge base

The knowledge base of a fuzzy logic controller comprises two components, namely, a database and a fuzzy control rule base. The database defines the fuzzy sets for the system variables with the membership functions defined over the universe of discourse for each variable. The rule base contains the fuzzy control rules intended to achieve the control objectives [35].

4.6.2 Inference engine

In this part of the regulator, the values of the input and output linguistic variables are linked by several rules which must take into account the static and dynamic behavior of the system to be adjusted as well as the adjustment goals envisaged. In particular, the control circuit must be stable. and well cushioned. The strategy of adjustment depends essentially on the adopted inferences. It is not possible to specify specific rules, experience plays an important role here. To express the inferences exist several possibilities to know by linguistic and symbolic description, as well as tables and matrices of inference . In this course we use this last description [36].

4.6.3 Fuzzification

Fuzzification is the process of mapping from observed inputs to fuzzy sets in the universe of discourse [35]. In fuzzy control applications, the observed data is usually crisp and hence fuzzification is necessary to map the crisp inputs to the corresponding fuzzy values for the input variables. The mapped data are further converted into linguistic terms as labels for the fuzzy sets defined for the system input variables.

4.6.4 Defuzzification

The defuzzification module is in a sense the reverse of the fuzzification module: it converts all the fuzzy terms created by the rule base of the controller to crisp terms (numerical values) and then sends them to the physical system (plant, process), so as to execute the control of the system [28]. The defuzzification module performs the following functions:

- It creates a crisp, overall control signal U by combining all possible control outputs from the rule base into a weighted average formula.
- Just like the first step of the fuzzification module, this step of the defuzzification module transforms the overall control output U obtained in the previous step, to the corresponding physical values (position, voltage, degree, etc.) that the system (plant, process) can accept. This converts the fuzzy logic controller's numerical output to a physical means that can actually drive the given plant (process) to produce the expected outputs.

There are seven methods used for defuzzifying the fuzzy output functions they are [29]:

1) Max-membership principle

2) Centroid method

3) Weighted average method

4) Mean-max membership

5) Centre of sums

6) Centre of largest area

7) First of maxima or last of maxima

4.7 Fuzzy Logic in Matlab

Fuzzy logic in Matlab can be used very easily due to the existing new Fuzzy Logic Toolbox [29]. This provides a complete set of functions to design and implement various fuzzy logic processes. The major fuzzy logic operations includes fuzzification, defuzzification, and the fuzzy inference. These all are performed by means of various functions and even can be implemented using the Graphical User Interface (GUI). Many of the applications can be simulated using the *fuzzy logic controller* Simulink block present in MATLAB-SIMULINK toolbox. The features are:

- It provides tools to create and edit fuzzy inference system (FIS).
- Allows integrating fuzzy systems into simulation with Simulink.
- It is possible to create stand-alone C programs that call on fuzzy systems built with Matlab.

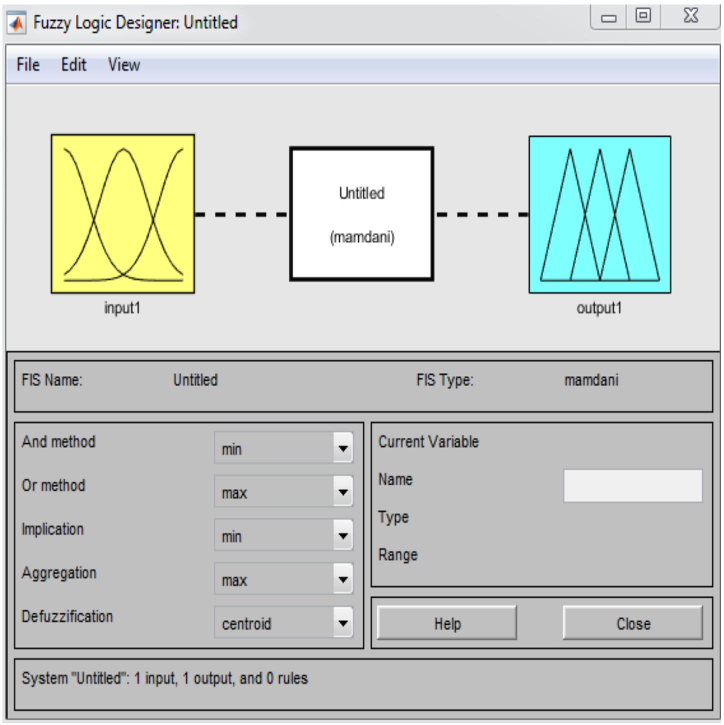


Figure 4.4: Fuzzy logic Toolbox

The Toolbox Fig. 4.4 provides three categories of tools:

- Command line functions
- Graphical or interactive tools
- Simulink blocks.

4.7.1 Command line functions

addmf	Add membership function to FIS.
addrule	Add rule to FIS.
addvar	Add variable to FIS.
defuzz	Defuzzify membership function.
evalfis	Perform fuzzy inference calculation.
evalmf	Generic membership function evaluation.
gensurf	Generate FIS output surface.
getfis	Get fuzzy system properties.
mf2mf	Translate parameters between functions.
mfstrtch	Stretch membership function.
newfis	Create new FIS. parsrule-parse fuzzy rules.
plotfis	Display FIS input-output diagram.
plotmf	Display all membership functions for one variable.
readfis	Load FIS from disk.
rmmf	Remove membership function from FIS.
rmvar	Remove variable from FIS.
setfis	Set fuzzy system properties.
showfis	Display annotated FIS.
showrule	Display FIS rules.
writefis	Save FIS to disk.

4.7.2 Graphical User Interface Editors (GUI tools)

fuzzy	Basic FIS editor.
mfedit	Membership function editor.
ruleedit	Rule editor and parser.
ruleview	Rule viewer and fuzzy inference diagram.
surfview	Output surface viewer.

4.7.3 Simulink Blocks

- Once fuzzy system is created using GUI tools or some other method, it can be directly embedded into Simulink using the fuzzy logic controller block as shown in Fig.4.5.

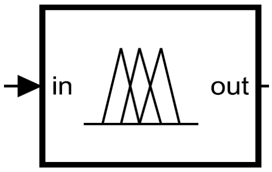


Figure 4.5: Fuzzy logic controller Simulink block.

- It is important to make sure that the FIS matrix corresponding to the fuzzy system is both in the Matlab workspace and referred to by name in the dialog box associated with this fuzzy logic controller.

4.8 Conclusion

In this chapter, we provided a comprehensive introduction to a control strategy that belongs to modern control theory, the so-called fuzzy logic control. We exposed briefly the main definitions and concepts of its theory, and we also presented its corresponding Matlab toolbox.

Chapter 5

Application of PID and Fuzzy Logic Controllers to Boeing 747 Aircraft

5.1 Introduction

This chapter is concerned with the application of the two control strategies introduced in earlier chapters, namely*PID* and *Fuzzy* logic, to control the lateral motion of Boeing 747 plane. First, simulation results assessing the performance of each controller are provided. Then, a qualitative comparative study of the designed controllers is discussed. Moreover, the MATLAB-SIMULINK block diagrams used in simulation are given along with the values of control gains.

5.2 Decoupled SISO *PID* Controller

In the first application we are going to present a decoupled classical controller named *PID* regulator, this type of command is studies via some hybridization of artificial intelligence with mathematical methods, where; in such case the controller parameters are found by genetic algorithms.

$$PID(s) = K_p + \frac{K_I}{s} + K_D s$$

The controller gain K_I, K_p, K_D are determined by genetic algorithms in order to decouple the systems.

The next figure Fig. 5.1 shows the Simulink model for two inputs and two outputs with the *PID* controller, this simulation apply to the lateral model of the B747 aircraft.

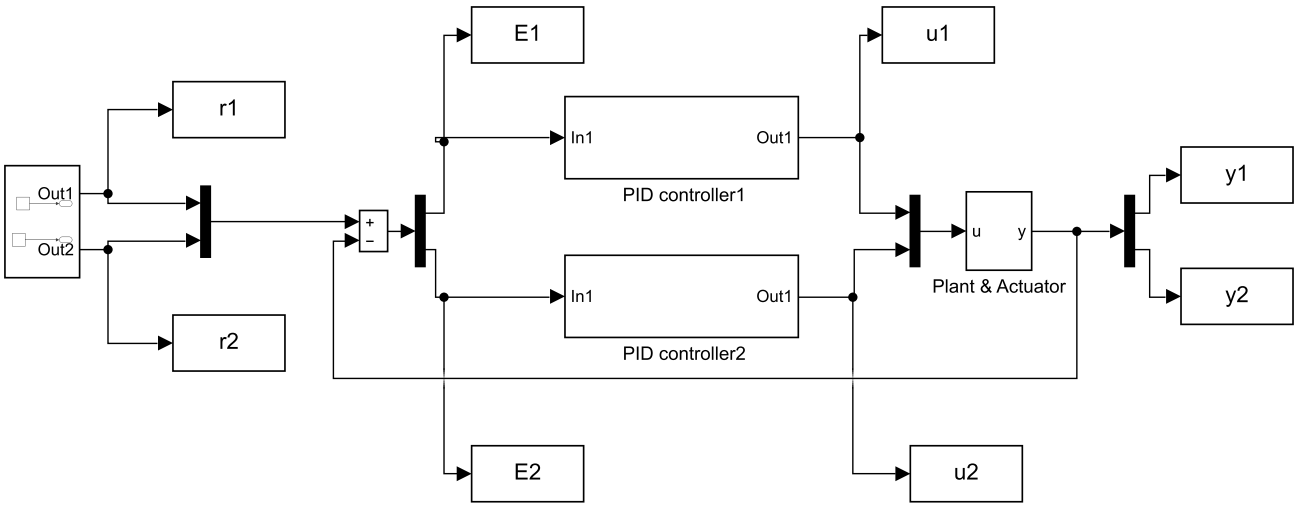


Figure 5.1: Decoupled aircraft model via *PID* controller.

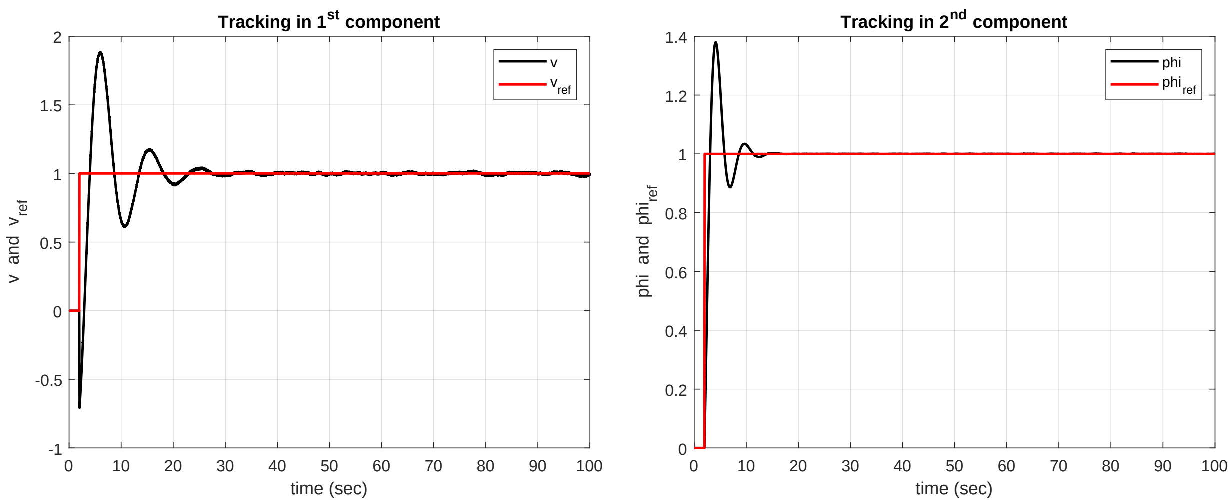


Figure 5.2: The step response of *PID*-plant system control.

The previous figure Fig. 5.2 shows a good tracking to the reference command in both

transient and steady state phases but with 89.32% overshoots and response time of about 26.41 seconds to reach the set point, this is all for the first output.
the second output shows a good track of the transient path of the reference control but with 37.94% overshoots and response time of about 10 seconds to reach the set point.

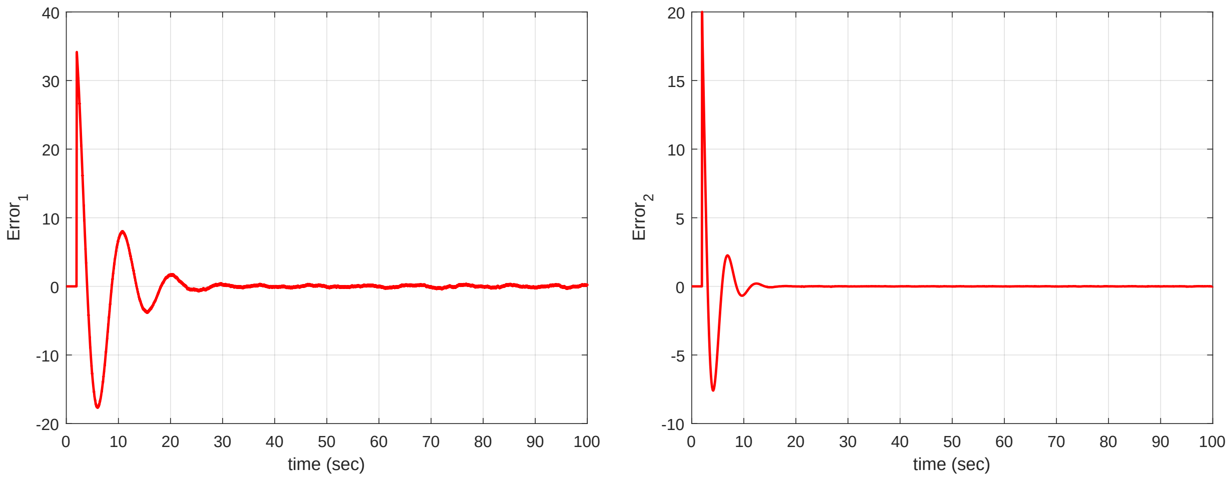


Figure 5.3: Error signals with *PID* control.

The amplitude of the error converges rapidly to zero (see Fig.5.3), for example the first error E_1, is less then 2 after 20 seconds, and the second error vanishes rapidly after 15 seconds.

The following table (see Table. 5.1) presents the performance characteristics of the simulation shown in Fig. 5.1 obtaind by MATLAB.

Variable	Value of output 1	value of output 2
RiseTime	0.8532	0.8577
SettlingTime	26.4111	10.5211
SettlingMin	0.6089	0.8864
SettlingMax	1.8851	1.3799
Overshoot	89.8455	37.9407
Undershoot	71.2812	0
Peak	1.8851	1.3799
PeakTime	6.0230	4.1170

Table 5.1: Controller simulation performance characteristics.

Variable	Value
K_{p1}	40.96347
K_{I1}	55.93934
K_{D1}	119.7381
K_{p2}	100.0662
K_{I2}	84.75314
K_{D2}	114.0171

Table 5.2: The gain of the controller *PID*.

The *PID* controller gains for this simulation are presented in Table 5.2.

5.3 Enhanced *PID* Controller

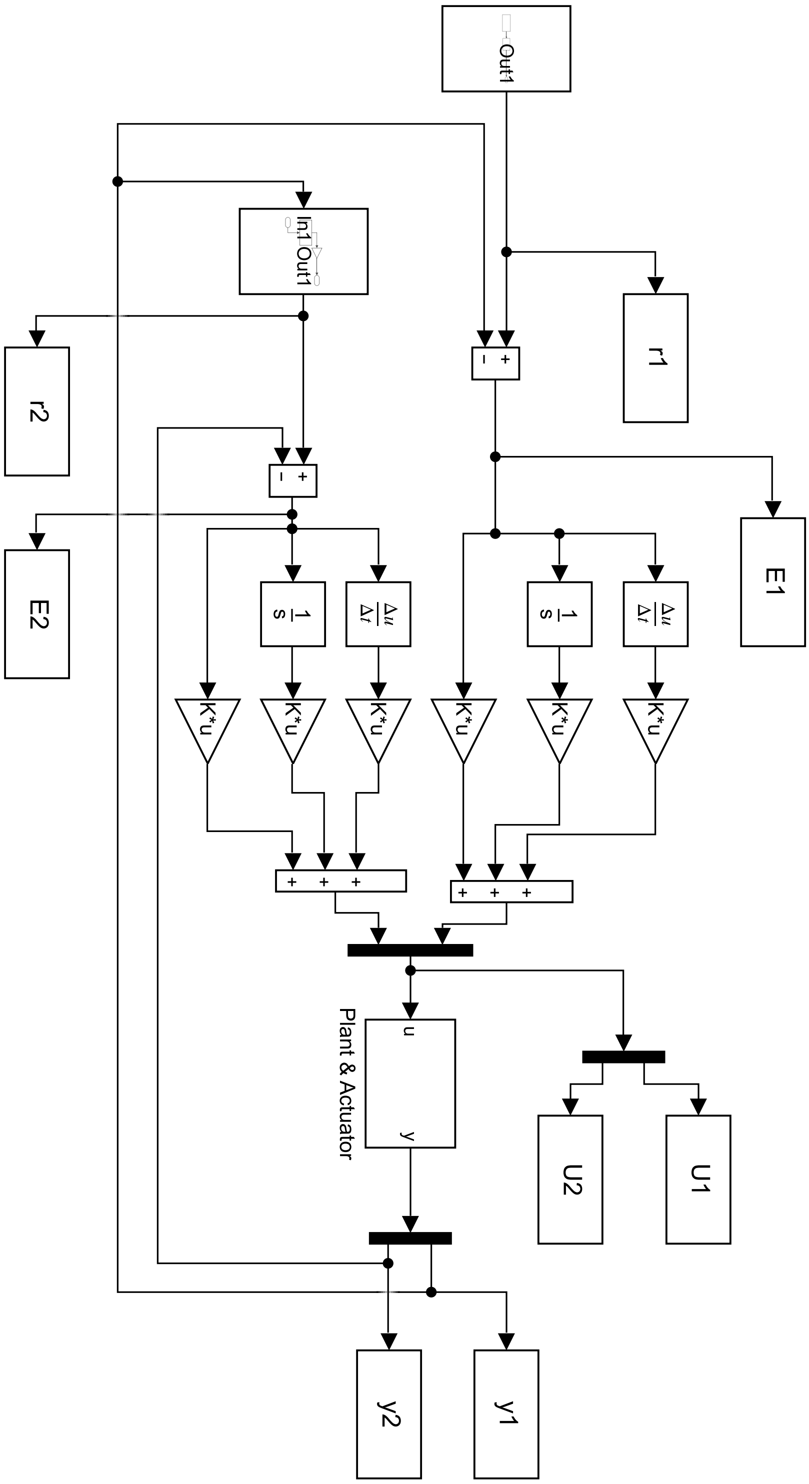


Figure 5.4: System enhanced with *PID* controller on Simulink.

There are a lot of methods and control strategies to convert multi variable plant into a decoupled subsystems with no interaction in between , in this part we provide the reader a new method to do this goal.in such manner we proposed on enhancement of the well known classical method, by cascading a new smoothing filter between the first output and the reference.

The previous figure Fig. 5.4 shows the unified input model with one varying reference,and two output with PID controller.The lateral model of an aircraft (B747) simulated with this structure.

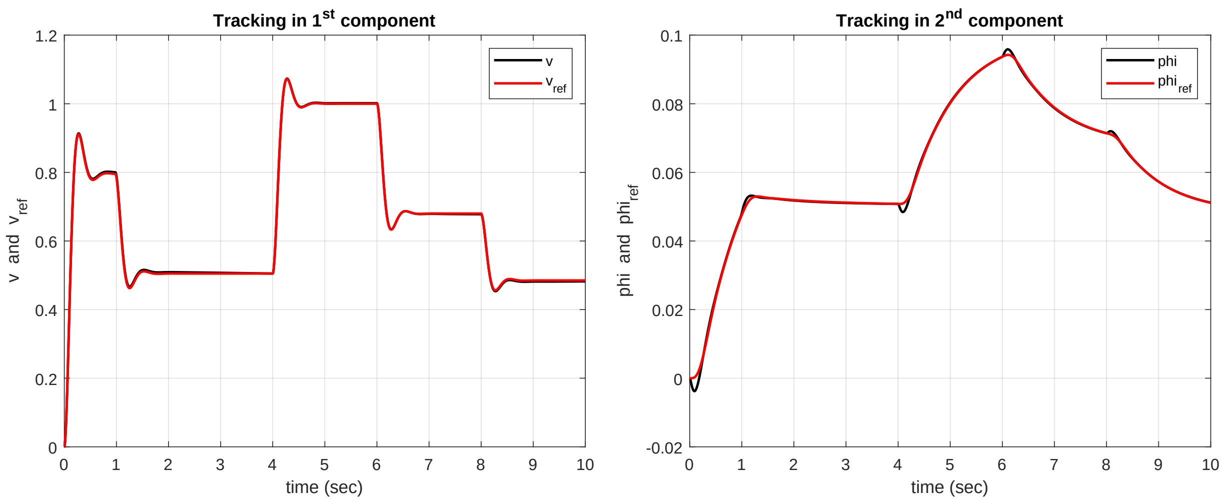


Figure 5.5: Tracking trajectory by PID control.

The previous figure Fig. 5.5 shows a good tracking to the reference control in both transient and steady state phases, but with some overshoots and short response time to reach the set point for the first output.
The second output shows a good track of the transient path of the reference control. but we have a spike in times (0.1, 4, 6 and 8 second) although the controller correct's.

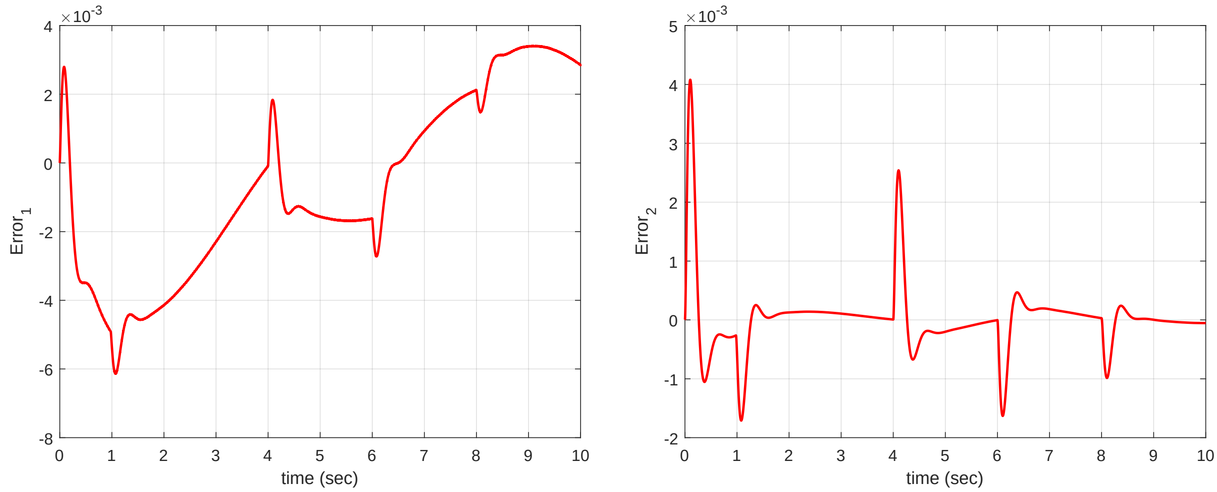


Figure 5.6: Error signals with *PID* control

The previous figure Fig. 5.6 shows that the error $(E1)$ is between two maximal values 0.0037 and -0.005 which are variate arbitrarily and the error $(E2)$ is between 0.0041 and -0.0018.

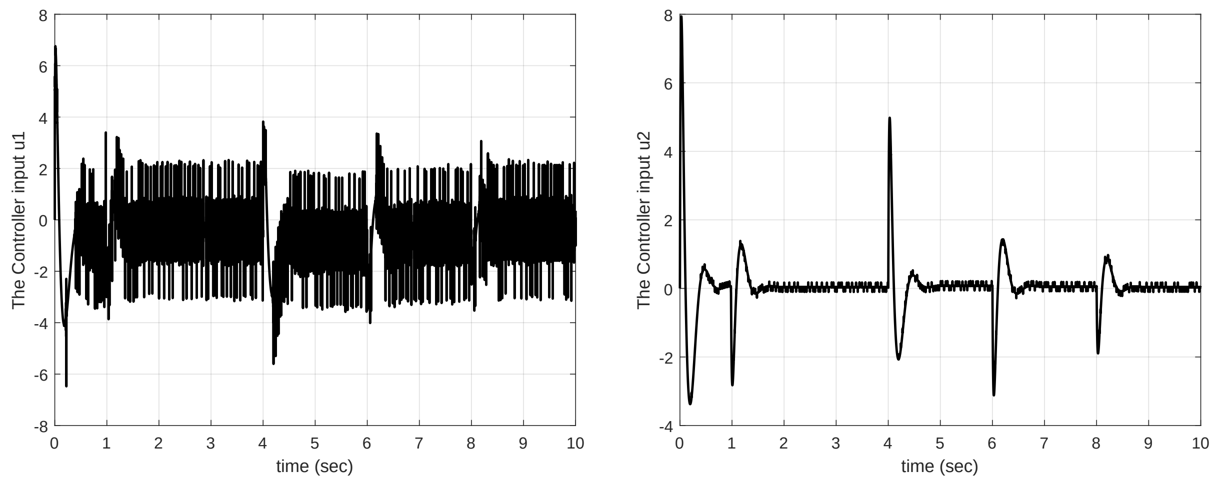


Figure 5.7: Rudder and aileron response with *PID* control.

The previous figure Fig. 5.7 shows the deflection of rudder and aileron that do not go over the limit ± 6, so it is accepted, elsewhere we have undesired chattering for *PID* controllers (for U_1, see Fig. 5.7), but in (for U_2, see Fig. 5.7) we have small chattering. To enhance the results we proposed a new schema named $Fuzzy$-PID control.

5.4 Schematic Diagram of Fuzzy Logic Control

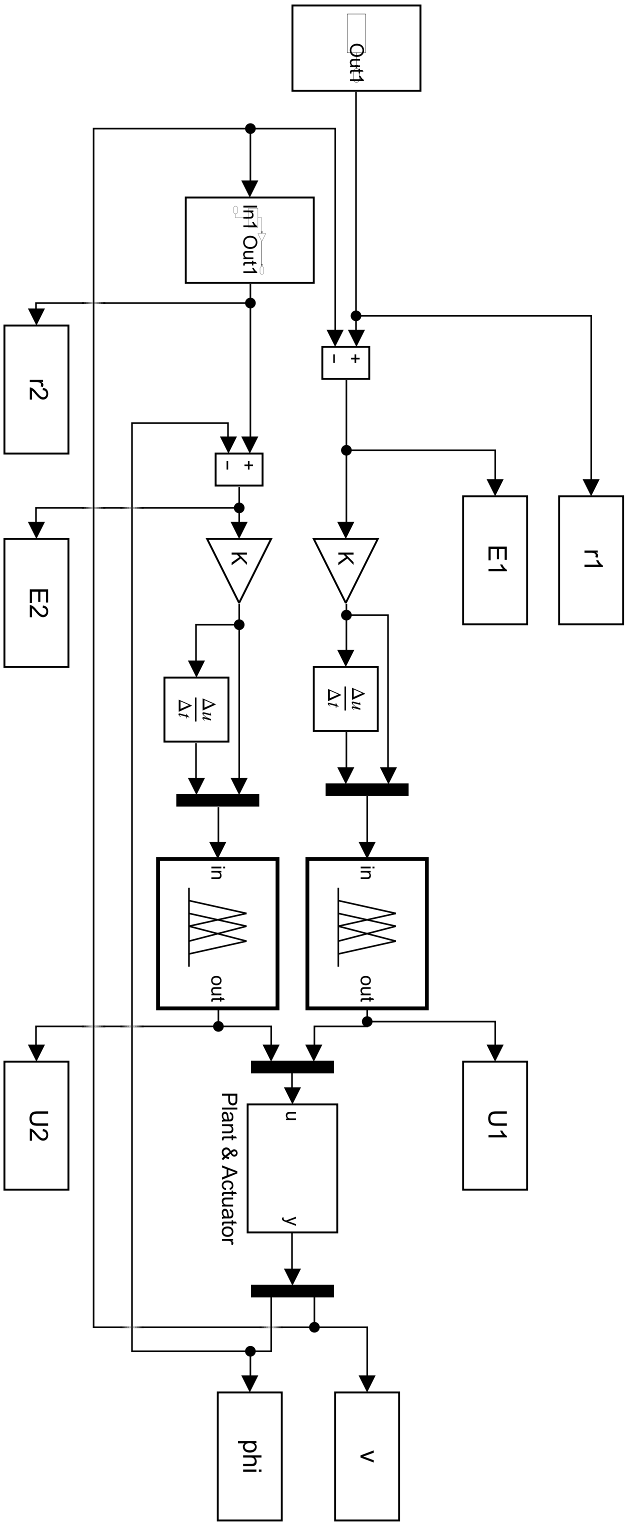


Figure 5.8: System with fuzzy controller on Simulink.

The previous figure Fig. 5.8 shows the Simulink model of the unified input structure for *Fuzzy* logic control, in this simulation we use two *Fuzzy* controllers every one of them take two inputs (error $\ll e \gg$ and variation of error $\ll \Delta e \gg$). This control strategy is applied to our model (lateral motion of the B747) and the simulation results is shows below.

$e/\Delta e$	**N**	**Z**	**P**
N	NG	N	Z
Z	N	Z	P
P	Z	P	GP

Table 5.3: Rule base.

The previous table represents the corresponding fuzzy logic control strategy. N represents 'negative', Z represents 'zero', and P represents 'positive'

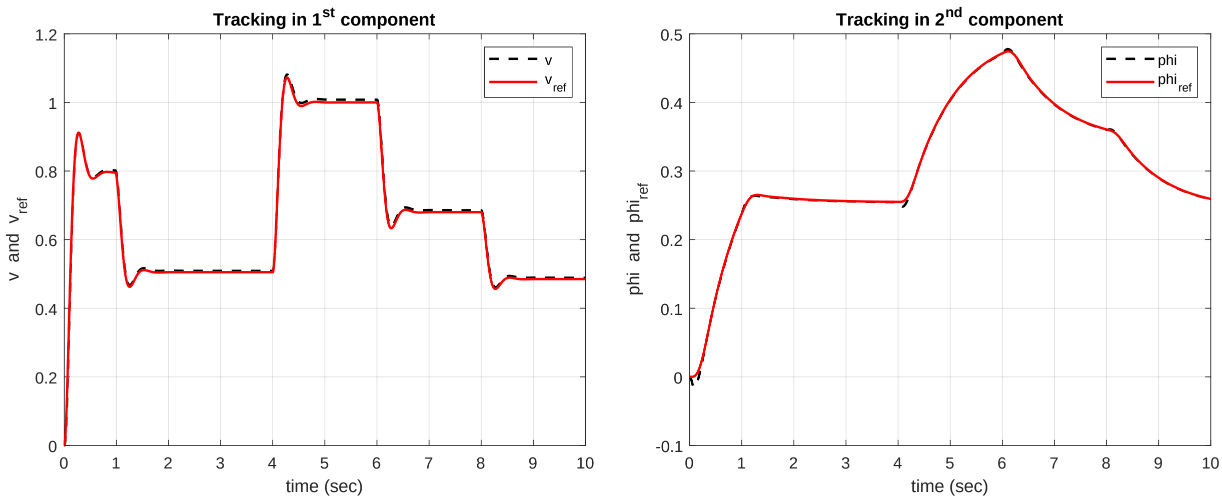


Figure 5.9: Tracking trajectory with *Fuzzy* logic control.

The previous figure Fig. 5.9 shows a good tracking to the reference control in both transient and steady state phase but with some overshoots and short response time to reach the set point for the first output.
The second output shows a good tracking to the reference control in both transient and steady state phase, but we have a spike in times (0.1, 4, 6 and 8 second) although the controller correct's.

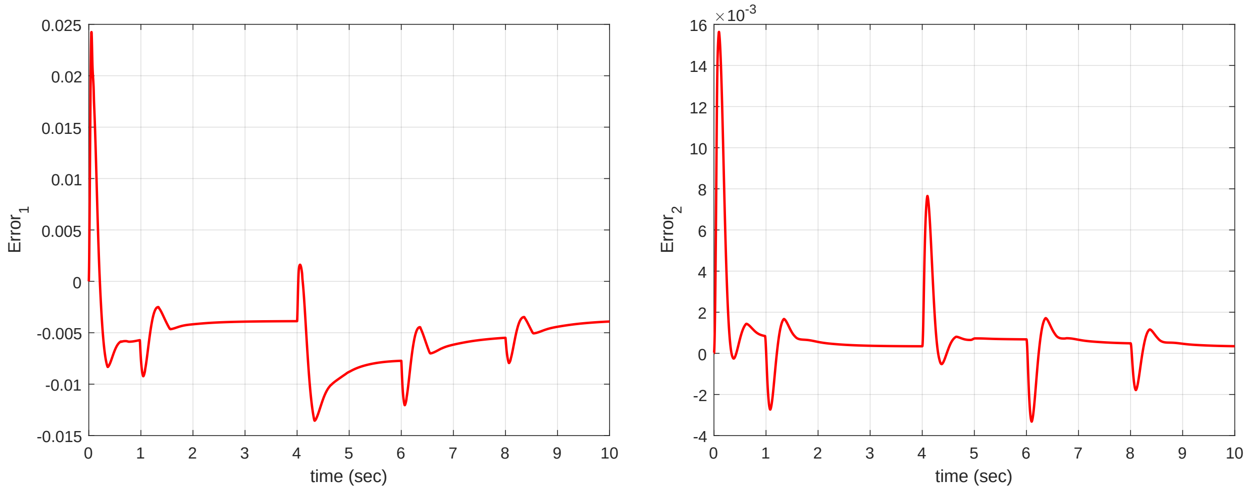


Figure 5.10: Error signals with *Fuzzy* logic control.

The previous figure Fig. 5.10 shows that the error $(E1)$ is between two maximal values 0.003 and -0.015 and the error $(E2)$ is between 0.008 and -0.003.

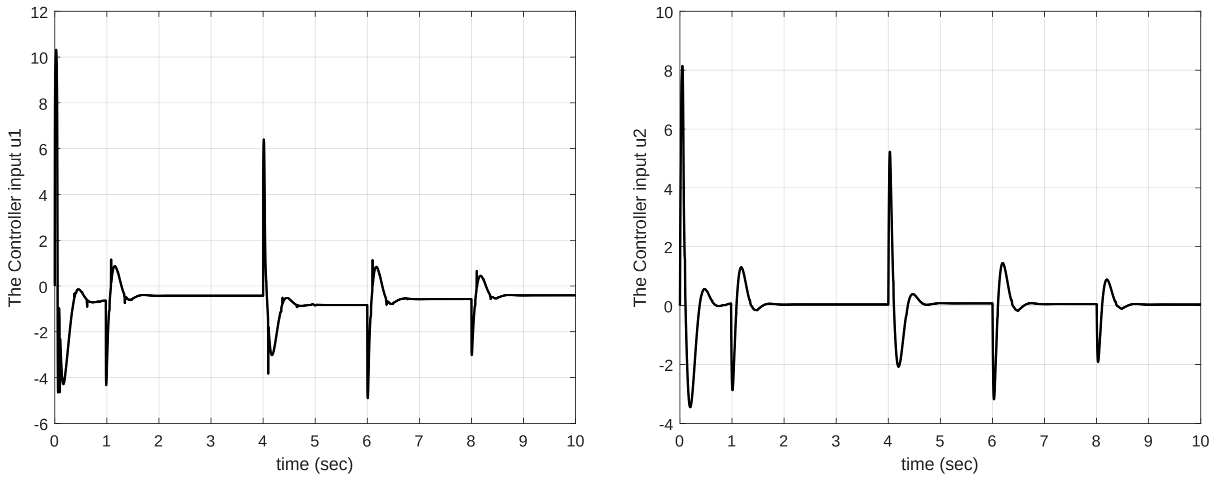


Figure 5.11: Rudder and aileron response with *Fuzzy* logic control.

The previous Fig. 5.11 shows smooth convergence of the error signals and the deflection of rudder and aileron that do not go over the limit ± 5 so its accepted, elsewhere we have undesired picks for *Fuzzy* controllers (for U_1, U_2 see Fig. 5.11).

5.5 The Comparison Between *PID* and *Fuzzy* Controllers

⋇ One of the difficulties of working with *PID* controllers is computing the gains K_p, K_I, K_D.

⋇ When the system is interconnected (for example MIMO coupled system), then first the system need to be decoupled, and genetic algorithms are used to handle the problem.

⋇ *Fuzzy* controllers are more robust for small changes in the system then *PID* ones.

⋇ *Fuzzy* controllers are preferred to *PID* for handling non-linear problems (the inverse is true for linear ones).

⋇ Conception of *Fuzzy* controllers is easier than *PID* ones, although there is a lack of Directives on how to design them.

⋇ Tracking is better with *Fuzzy* controllers than with *PID*, with a smaller settling time.

⋇ We observe in the simulations (see Fig. 5.9) undesired pulses that appear in the output, fortunately *Fuzzy* controller corrects it.

⋇ Comparison table :

Variable	**Value of PID controller**	**Value of *Fuzzy* controller**
RiseTime	0.0761	0.0749
SettlingTime	8.3854	8.3814
SettlingMin	0.4374	0.4446
SettlingMax	1.0734	1.0812
Overshoot	122.6360	121.1806
Undershoot	0	0
Peak	1.0734	1.0812
PeakTime	4.2750	4.2770

Table 5.4: The comparison between the two command.

- From the comparison table we observe that The values of *RiseTime* and *SettlingTime* in *Fuzzy* controller are smaller than values of *PID* controller.
- The *Fuzzy* controller decrease the overshoot compared with *PID* controller.

- ⋇ We observe some undesired chattering for *PID* controllers (for U_1, U_2, see Fig. 5.7), which is accepted theoretically, but in reality the rudder and aileron cannot perform that kind of motion, in contrast with *Fuzzy* controllers, which have a smooth motion (see Fig. 5.11), which makes it realizable.

5.6 Conclusion

In this chapter, we studied the application of the *PID* and *Fuzzy* logic controllers to the aircraft Boeing 747. All the simulation results presented in this chapter were carried out in MATLAB-SIMULINK software. We started with applying a decoupled *SISO PID* controller to the dynamics of B747, where the results showed an acceptable tracking behavior in the steady state regimen compared to the transient phase that suffers from an overshoot. Second, we talk about the new method which is an enhanced *PID* controller followed with a system. We then tested the performance of the *Fuzzy* logic controller, where the tracking performance was better in the transient phase with relatively less overshoot. Overall, from the conducted comparative study, we concluded that the synthesized *Fuzzy* logic controller provides better tracking performances compared to the designed *PID* controller.

General Conclusion

In this dissertation, we have presented a comparative study between two control techniques, namely the *PID* controller (classical) and the *Fuzzy* logic controller (intelligent), when applied to the dynamics of Boeing 747 aircraft.

After an introduction of the aerodynamic concepts, aircraft's modeling parts, and theoretical foundation of different control strategies with a focus on the *MIMO PID* and *Fuzzy* logic controller, this work was divided mainly into two parts. The first part addresses the mathematical modelling problem of an aircraft, where the non-linear problem and its linearized version were derived. The second part, however, contributed to the design of control strategies to track and stabilize the lateral motion of the Boeing 747 airplane. The two synthesized strategies, *PID* and *Fuzzy* logic controllers, were compared in terms of their performance to achieve the control objective and their characteristics as well. Overall, we conclude from this study that the tested intelligent controller outperforms the classical *PID* controller in stabilizing the lateral motion of B747 plane.

The work conducted in this thesis can be extended in different directions. For instance, one may reconsider the aircraft's mathematical modelling dynamics in which the non-linear parts will be kept and also the modelling of disturbances and faults will be included. Moreover, for such type of developed models more sophisticated robust non-linear control strategies could be applied and tested, for example H^{∞}, sliding mode, or adaptive controllers. Moreover, the problem of the identified parameters based on available measurements (Velocity, Pression) could be considered as well. Furthermore, fault detection and isolation (FDI) techniques, and hence, fault tolerant control (FTC) methods could be synthesized to detect and isolate aircraft's faults and accounts for these faults with the appropriate control strategy.

Appendix A

Appendix

A.1 Introduction

The control surfaces for a conventional aircraft are the elevator, ailerons and the rudders. The elevator is used to control the pitch angle, the aileron is used to control the roll angle, and the rudder is used to control the yaw angle. However, it is generally, known that by acting on one of this control elements, motion about one axis produces motion about other axes, which is known as coupling.

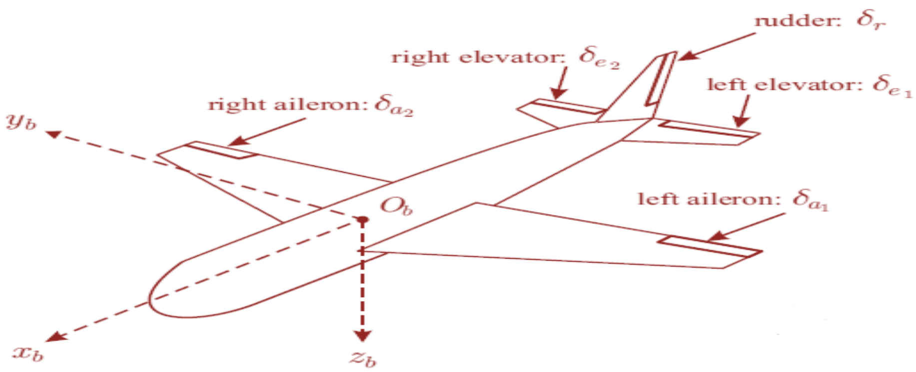


Figure A.1: The aerodynamic control surfaces.

A.2 Aerodynamic terms

Assume that the aerodynamic forces and moments are dependent on the disturbed motion variables and their derivatives only, and for small perturbation we get the following linearization [22]:

$$
\begin{cases}
X_a = X_{a0} + \dfrac{\partial X}{\partial u}u + \dfrac{\partial X}{\partial v}v + \dfrac{\partial X}{\partial w}w + \dfrac{\partial X}{\partial p}p + \dfrac{\partial X}{\partial q}q + \dfrac{\partial X}{\partial r}r + \dfrac{\partial X}{\partial \dot{w}}\dot{w} \\
Y_a = Y_{a0} + \dfrac{\partial Y}{\partial u}u + \dfrac{\partial Y}{\partial v}v + \dfrac{\partial Y}{\partial w}w + \dfrac{\partial Y}{\partial p}p + \dfrac{\partial Y}{\partial q}q + \dfrac{\partial Y}{\partial r}r + \dfrac{\partial Y}{\partial \dot{w}}\dot{w} \\
Z_a = Z_{a0} + \dfrac{\partial Z}{\partial u}u + \dfrac{\partial Z}{\partial v}v + \dfrac{\partial Z}{\partial w}w + \dfrac{\partial Z}{\partial p}p + \dfrac{\partial Z}{\partial q}q + \dfrac{\partial Z}{\partial r}r + \dfrac{\partial Z}{\partial \dot{w}}\dot{w} \\
L_a = L_{a0} + \dfrac{\partial L}{\partial u}u + \dfrac{\partial L}{\partial v}v + \dfrac{\partial L}{\partial w}w + \dfrac{\partial L}{\partial p}p + \dfrac{\partial L}{\partial q}q + \dfrac{\partial L}{\partial r}r + \dfrac{\partial L}{\partial \dot{w}}\dot{w} \\
M_a = M_{a0} + \dfrac{\partial M}{\partial u}u + \dfrac{\partial M}{\partial v}v + \dfrac{\partial M}{\partial w}w + \dfrac{\partial M}{\partial p}p + \dfrac{\partial M}{\partial q}q + \dfrac{\partial M}{\partial r}r + \dfrac{\partial M}{\partial \dot{w}}\dot{w} \\
N_a = N_{a0} + \dfrac{\partial N}{\partial u}u + \dfrac{\partial N}{\partial v}v + \dfrac{\partial N}{\partial w}w + \dfrac{\partial N}{\partial p}p + \dfrac{\partial N}{\partial q}q + \dfrac{\partial N}{\partial r}r + \dfrac{\partial N}{\partial \dot{w}}\dot{w}
\end{cases}
\tag{A.1}
$$

Remark A.1. *These equations are developed by using Taylor series expansion. The coefficients* $\dfrac{\partial X}{\partial u}$, $\dfrac{\partial X}{\partial v}$, $\dfrac{\partial X}{\partial w}$ *etc.are called aerodynamic stability derivatives.*

A.3 Aerodynamic control terms

The primary aerodynamic controls are the elevator δ_e, ailerons δ_a and rudder δ_r. Taking the first derivatives about these variables we get [22]:

$$X_c = \frac{\partial X}{\partial \delta_e}\delta_e + \frac{\partial X}{\partial \delta_a}\delta_a + \frac{\partial X}{\partial \delta_r}\delta_r \tag{A.2}$$

As a shorthand notation we use: $X_c = \dot{X}_{\delta_e}\delta_e + \dot{X}_{\delta_a}\delta_a + \dot{X}_{\delta_r}\delta_r$, the aerodynamic control terms in the remaining equations of motion are assembled in a similar way.$(i.eY_c, Z_c, L_c, M_c, N_c)$

A.4 Gravitational terms

The weight force mg acting on the aero-plane may be resolved into components acting in each of the three aero-plane axes. When the aero-plane is disturbed these components will vary according to the perturbations in attitude thereby making a contribution to the disturbed motion [22]. Thus the gravitational contribution is obtained by resolving the aero-plane weight into the disturbed body axes. Since the origin of the aero-plane body axes is coincident with the cg there is no weight moment about any of the axes, therefore, $L_g = M_g = N_g = 0$. The forces produced by the gravitational actions are

$$(\vec{F}_g)^l = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \text{ and } (\vec{F}_g)^B = T_1(\phi)T_2(\theta)T_3(\psi) \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = \begin{pmatrix} -\sin(\theta) \\ \sin(\phi)\cos(\theta) \\ \cos(\phi)\cos(\theta) \end{pmatrix} mg$$

For symmetric steady state equilibrium, we assume that $\theta = \theta_0$ and $\phi = \phi_0 = 0$

$$(\vec{F}_g)^B = \begin{pmatrix} -mg\sin(\theta_0) \\ 0 \\ mg\cos(\theta_0) \end{pmatrix}$$

After small perturbation in the Euler-angles, the gravitational term becomes:

$$\vec{F}_g = \begin{pmatrix} X_g \\ Y_g \\ Z_g \end{pmatrix} \{T_1(\Delta\phi)T_2(\Delta\theta)T_3\Delta(\psi)\}(\vec{F}_g)^B$$

$$\vec{F}_g = \begin{pmatrix} 1 & \Delta\psi & -\Delta\theta \\ -\Delta\psi & 1 & \Delta\phi \\ \Delta\theta & -\Delta\phi & 1 \end{pmatrix} \begin{pmatrix} -mg\sin(\theta_0) \\ 0 \\ mg\cos(\theta_0) \end{pmatrix}$$

$$\vec{F}_g = \begin{pmatrix} X_g \\ Y_g \\ Z_g \end{pmatrix} = mg \begin{pmatrix} -\sin(\theta_0) - \Delta\theta\cos(\theta_0) \\ \Delta\psi\sin(\theta_0) - \Delta\phi\cos(\theta_0) \\ \cos(\theta_0) - \Delta\theta\sin(\theta_0) \end{pmatrix}$$

A.5 Power terms

Power is controlled by the change in thrust engine so we have [22].

$$X_p = \frac{\partial X}{\partial \tau}\tau, Y_p = \frac{\partial Y}{\partial \tau}\tau, Z_p = \frac{\partial Z}{\partial \tau}\tau, L_p = \frac{\partial L}{\partial \tau}\tau, M_p = \frac{\partial M}{\partial \tau}\tau, N_p = \frac{\partial N}{\partial \tau}\tau$$

Therefore τ quantifies the thrust perturbation relative to the trim setting τ_e.

Appendix B

Appendix

B.1 Introduction

This appendix contains the definitions of the parameters used by the $B747$ non-linear dynamic model and some general data on this aircraft. The dynamic model itself has been described in Chapter 2

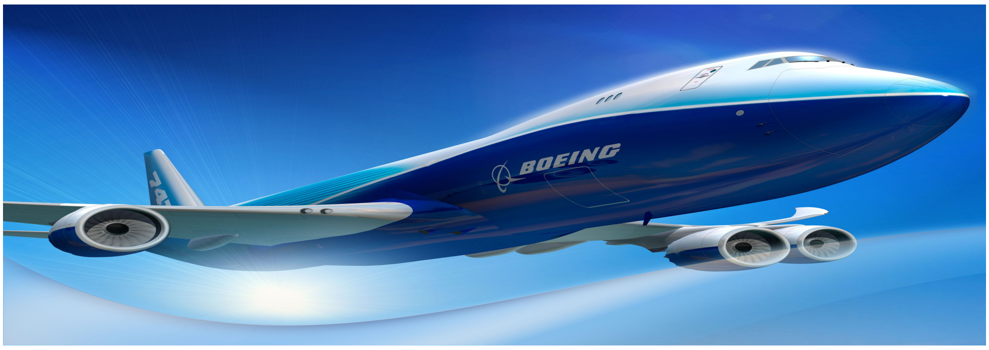


Figure B.1: The B747 aircraft

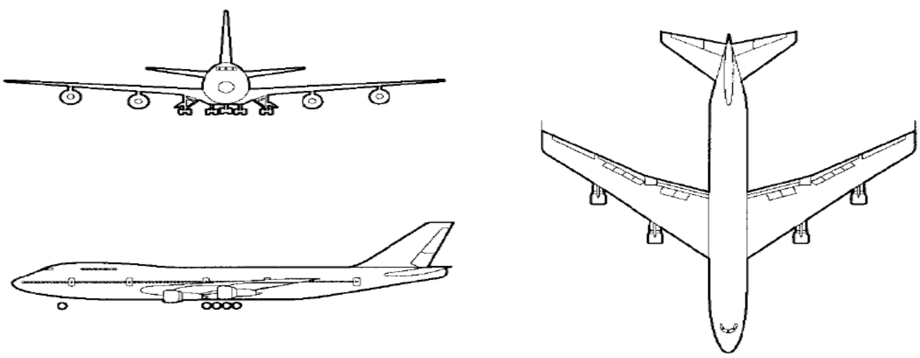


Figure B.2: The three views of the B747 aircraft.

B.2 Technical character of the aircraft B747 :

Constructor	Boeing Commercial Airplanes
Type	commercial jet airliner and cargo aircraft
Serial number	19658 LN:47
Year	1969
Engine	4 turbofans Pratt Whitney PW4000 or General Electric or Rolls-Royce
Wingspan	59,60 m
Wing surface	524,9 m^2
Medium rope	27.3 ft
Length	70,60 m
Height	19,30 m
Take-off weight	396 900 Kg
Cruising speed	$939 Km/H$
Autonomy	13 480 Km
Crew	4
Payload	568 in 1 class, 430 in 2 classes, 416 in 3 classes (10 rows of seats)
Propulsion diameter	1.80 m

Table B.1: The general parameters of B747

B.3 System matrix

We present above the different matrices of the simulation:

$$A = \begin{bmatrix} -0.0558 & 0 & -235.9 & 9.81 \\ -0.0127 & -0.4351 & 0.4143 & 0 \\ 0.0036 & -0.0061 & -0.1458 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.0729 & 0.0000 \\ -4.75 & 0.00775 \\ 0.153 & 0.143 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 0.000 \end{bmatrix}$$

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