TURRET GUN STUDY AND REALIZATION OF A REDUCED MODEL

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TURRET GUN STUDY AND REALIZATION OF A REDUCED MODEL

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Aknowledgment

Writing thanks can seem like an easy task, because at this point, most of the writing has already been done. The most delicate task is to remember all those who contributed to it from far or near by putting us on the rails

In the name of Allah, the Most Beneficent and Most Merciful. We thank Allah for all His blessing and strength that He give us in completing this humble work.

I would never be eloquent enough to express my deep gratitude and my sincere thankfulness to the people and organizations whose collaboration was effective, dedication and material, technical or psychological support, notably enabled the realization of the research project and the finalization of this memory.

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I would like to dedicate this thesis to my parents who have always been there for me in all circumstances, for their unconditional love, to my brothers for their support. I would also like to dedicate this modest work, fruit of my studies, to my homeland and to the memory of those who fell so that Algeria could live upright and proud.

A *Aknowledgment*

To you Mother
To you who is no longer but who is still
To you who is and always will be
To you Father
To you Brothers

"If you want to build a ship, don't drum up people together to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea"

Antoine de Saint-Exupéry

Abstract

The use of robotic technology is essentially very helpful in every area. Robotics and autonomous systems are already extensively used in many civil domains, especially since technological progress render them less expensive and bulky while, at the same time, more flexible and easier to interact with. As such, robots and multi-robots systems also have the potential to play, in the foreseeable future, a disruptive role in military operations in the sense that they will allow to perform tasks that today are considered too risky, complex or even impossible for humans. But these systems not only have the potential to perform conventional dirty, dull and dangerous military tasks like surveillance and counter mining; they are also likely to change the way military operations are conducted in the future, and even to make it possible to envisage new type of missions. In our project we will obtain the dynamic model of massive gun applying robotic laws, therfore using classical control theory based on feedback control including a study of different feedback configurations and the development of the associated compensator equations, and then optimizing the controller, we will be able to control the obtained model. In the last chapter we will show all the steps to design a fully functionnal scale model of a turret cannon, equipped with a laser as an aiming device, controlled by computer, and precise instructions will be given as well as tests in clear images.

إن استخدام التكنولوجيا الآلية مفيد للغاية في كل مجال. تستخدم الروبوتات والأنظمة المستقلة بالفعل على نطاق واسع في العديد من المجالات المدنية ، خاصة وأن التقدم التكنولوجي يجعلها أقل تكلفة وضخمة بينما في نفس الوقت أكثر مرونة وأسهل في التفاعل معها. على هذا النحو ، فإن الروبوتات وأنظمة الروبوتات المتعددة لديها أيضًا القدرة على لعب دور مدمر في العمليات العسكرية في المستقبل المنظور ، بمعنى أنها ستسمح بأداء المهام التي تعتبر اليوم محفوفة بالمخاطر أو معقدة أو حتى مستحيلة للبشر . لكن هذه الأنظمة لا تملك فقط القدرة على أداء المهام العسكرية التقليدية القذرة والباهتة والخطيرة مثل المراقبة والتعدين المضاد ؛ من المحتمل أيضًا أن يغيروا طريقة إجراء العمليات العسكرية في المستقبل ، وحتى ليتمكنوا من تصور نوع جديد من المهام. في مشروعنا ، سنحصل على النموذج الديناميكي للبندقية الضخمة التي تطبق القوانين الروبوتية ، ثم باستخدام نظرية التحكم المختلفة وتطوير الكلاسيكية القائمة على التحكم في التغذية الراجعة بما في ذلك دراسة تكوينات التغذية الراجعة معادلات المعادلة المرتبطة بها ، ثم تحسين وحدة التحكم ، سيكون قادراً على التحكم في النموذج الذي تم الحصول عليه. في الفصل الأخير سنعرض جميع الخطوات لتصميم نموذج مقياس كامل الوظائف لمدفع برج ، مجهز بالليزر كجهاز تصويب ، يتم التحكم فيه بواسطة الكمبيوتر ، وسيتم إعطاء تعليمات دقيقة بالإضافة إلى الاختبارات في صور واضحة.

Nomenclature

$q_1,...,q_n$: generalized coordinates that completely locate the n-Degree of freedom (DOF) manipulator arm

K: The Kinetic Energy

U: The Total Potential Energy

L : Lagrangian

$C(q,\dot{q})$: The Coriolis/Centripetal vector

$G(q)$: The Gravity Vector.

τ: The Generalized Applied Torque Vector

$F(\dot{q})$: The Additional Friction Term

F_v: The Coefficient Matrix of Viscous Friction

F_d: Dynamical Friction Term

τ_d: Disturbance Term

I_n: The $n \times n$ Identity Matrix.

$x \in \mathbb{R}^n$: The State Vector

H: The Hamiltonian Matrix

m_1: The Mass of The Turret Load

m_2: The Mass of The Barrel Load

R_1: The Radius of The Turret Load

R_2: The Length of The Barrel Load

θ_1: The Angular Position of The Turret Load (Horizontal Rotation)

θ_2: The Angular position of The Barrel Load (Vertical Rotation)

C: Center of The Rod

d: The perpendicular distance from turret's axis of rotation and The Center of The Rod

I: The Moment of Inertia of the Rod About an Axis that Passes Through its End

$I_{parallel}$: The moment of inertia about any axis parallel to that axis through the center of mass

U_0: The Potential Energy at Some Reference Level

Γ: Vector of the External Torques

V_i: Voltages applied to the motor

q_d: Desired Position

$\dot{q}_d$: Desired Velocity

$\ddot{q}_d$: Desired Acceleration

$\tilde{q}$: Position Tracking error

$\dot{\tilde{q}}$: Velocity Tracking error

$\ddot{\tilde{q}}$: Acceleration Tracking error

v_i^k: Speed of the agent i at the iteration k .

w : weighting function.

c_j : weighting factor.

$rand$: random number between 0 and 1.

s_i^k: Position of the agent i at the iteration k.

$pbest_i$: Best position of the agent i.

$gbest$: Best global value of the group.

$w_{\max}$: Final weight.

$w_{\min}$: Initial weight.

$iter_{\max}$: Maximum number of iterations.

$iter$: Current number of iteration.

s_i^k: Current position of the agent.

s_i^{k+1}: Actualized position of the agent.

v_i^{k+1}: Actualized speed of the agent.

q_d: desired position

τ_0: an auxiliary control input to be designed.

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GENERAL CONCLUSION

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General Introduction

The weapons are the contemporaries of the man, his closest companions, they were born grew up together and discovered the world, They forged a destiny and charted the way which leads to today. Whether bronze or iron, they are the ones who have always given their names to the ages of man.

Before spouting in the concrete, they spurt out of the mind, they are an idea, which takes strength in the imagination and the memory, a sparkle of the mists of time, which is rooted deeply in human's nature, and suddenly appears in the light this form is part of gestures and shapes the material in its image.
It is an alliance of incandescence and patience. It is the whole life of a country, the history and the present of a nation, which converges, concentrates and merges in weapons. Dressed in old materials they reconnect with their ancestors, dressed in carbon fiber they embody the present. Alloy of the forge and the laser, of the hand and the machine, of the know-how and the technology these weapons are projected towards the future, they have a stem that crosses time a symbol of freedom a witness that is passed on from generation to generation a heritage of the future and a sign of recognition.

In the dawn of history, war engines were performing the function of artillery (which may be loosely defined as a means of hurling missiles too heavy to be thrown by hand), and with these crude weapons the basic principles of artillery were laid down. Eight centuries B.C.--machines that were probably predecessors of the catapult and ballista, getting power from twisted ropes made of hair, hide or sinew. The ballista had horizontal arms like a bow. The arms were set in rope; a cord, fastened to the arms like a bowstring, fired arrows, darts, and stones. Like a modern field gun, the ballista shot low and directly toward the enemy.
The catapult was the howitzer, or mortar, of its day and could throw a hundred-pound stone 600 yards in a high arc to strike the enemy behind his wall or batter down his defenses.

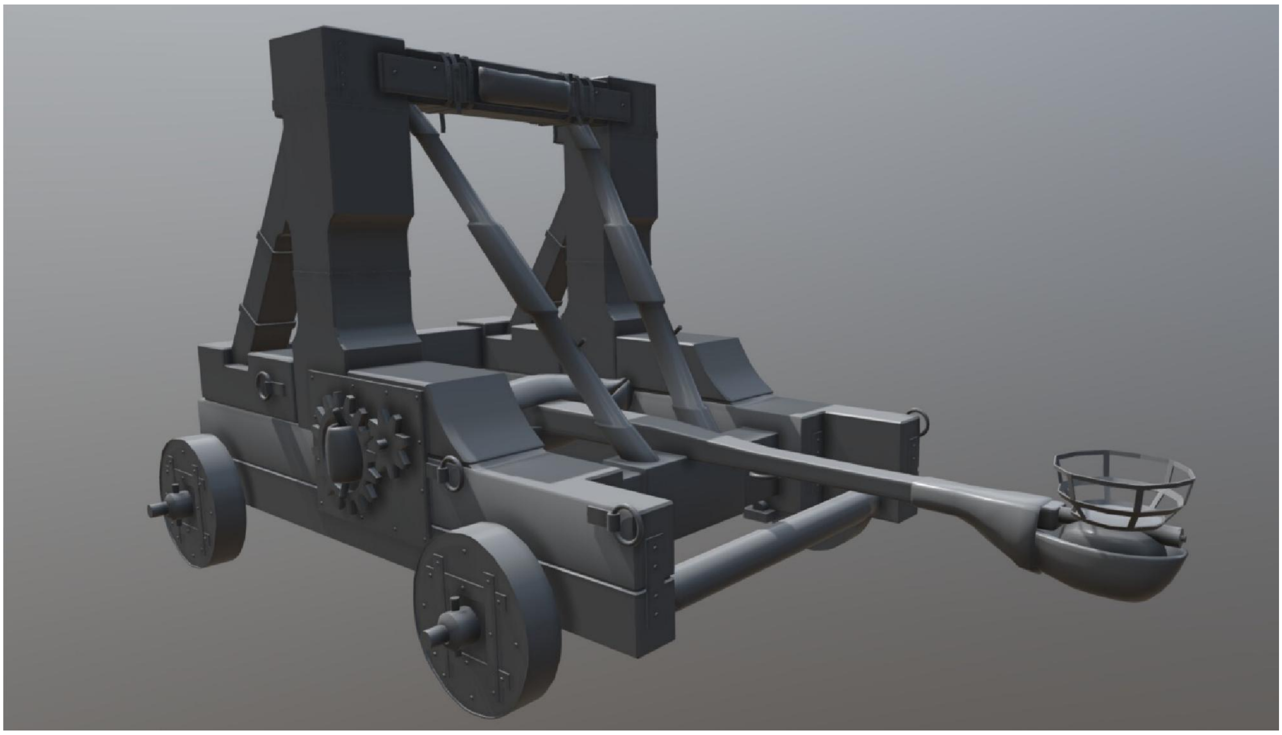

Figure 0.1 Roman Catapult

In early times the weapon was called a "scorpion," for like this dreaded insect it bore its "sting" erect. These weapons could be used with telling effect, as the Romans learned from Archimedes in the siege of Syracuse (214-212 B.C.). As Plutarch relates, "Archimedes soon began to play his engines upon the Romans and their ships, and shot stones of such an enormous size and with so incredible a noise and velocity that nothing could stand before them. At length the Romans were so terrified that, if they saw but a rope or a beam projecting over the walls of Syracuse, they cried out that Archimedes was leveling some machine at them, and turned their backs and fled." Long after the introduction of gunpowder, the old engines of war continued in use. Often they were side by side with cannon.

Chinese "thunder of the earth" (an effect produced by filling a large bombshell with a gunpowder mixture) sounded faint reverberations amongst the philosophers of the western world as early as A.D. 300. Though the Chinese were first instructed in the scientific casting of cannon by missionaries during the 1600's, crude cannon seem to have existed in China during the twelfth century and even earlier.

The Arabian madfaa, which in turn had doubtless descended from an eastern predecessor, was the original cannon brought to western civilization. This strange weapon seems to have been a small, mortar-like instrument of wood. Like an egg in an egg cup, the ball rested on the muzzle end until firing of the charge tossed it in the general direction of the enemy.Arabic accounts report that Muslims introduced firearms into Islamic Spain, from where they passed to Italy, going from there to France, and finally Germany. Muslims also developed and refined gunpowder and acquired rocket-making technology. In the 13th century a Syrian scholar, Hassan Al-Rammah (d. 1294-1295), wrote a remarkable book on military technology, which became very famous in the west. The first documented rocket is included in the book, a model of which is exhibited at the National Air and Space Museum in Washington D.C (BEKHITI B. Dynamic Modeling & control of Large Space Structures : Part 1 Missiles & Aircrafts).

At the beginning of the 1400's cast-iron balls had made an appearance. The greater efficiency of the iron ball, together with an improvement in gunpowder, further encouraged the building of smaller and stronger guns.By the middle 1400's the little popguns that tossed one-or two-pound pellets had grown into enormous bombards After 1470 the art of casting greatly improved in Europe. Lighter cannon began to replace the bombards. Throughout the 1500's improvement was mainly toward lightening the enormous weights of guns and projectiles, as well as finding better ways to move the artillery.

Before 1500 the siege gun had been the predominant piece. Now forged-iron cannon for field, garrison, and naval service--and later, cast-iron pieces--were steadily developed along with cast-bronze guns, some of which were beautifully ornamented with Renaissance workmanship and straight grooving of musket bores was extensively practiced. It was during the sixteenth century that the science of ballistics had its beginning. In 1537, Niccolo Tartaglia published the first scientific treatise on gunnery. Principles of construction were tried and sometimes abandoned, only to reappear for successful application in later centuries. Breech-loading guns, for instance, had already been invented. They were unsatisfactory because the breech could not be sealed against escape of the powder gases, and the crude, chambered breechblocks, jammed against the bore with a wedge, often cracked under the shock of firing.

Many of the vital changes took place during the latter years of the 1800's, as rifles replaced the smoothbores. Steel came into universal use for gun founding; breech and recoil mechanisms were perfected; smokeless powder and high explosives came into the picture. Hardly less important was the invention of more efficient sighting and laying mechanisms. The changes did not come overnight. In Britain, after breechloaders had been in use almost a decade, the ordnance men went back to muzzle-loading rifles; faulty breech mechanisms caused too many accidents.

Prior to 1800, there was no need for elaborate gun sighting systems, because the guns themselves were inaccurate except at close range. Guns were simply pointed at the target by eye. Gun sights introduced early in the nineteenth century consisted of fixed front and rear sights mounted so that the line of sight across their tips was parallel to the bore of the gun. Toward the end of the nineteenth century, a simple sight telescope was developed by a Navy Lieutenant.

Despite their post-Civil War development, modern machine guns did not begin to exhibit their full potential in battle until World War I. The effects on employment of these new weapons systems altered the doctrinal way of waging war for both Allied and Axis powers. Properly employed machine guns proved to be devastating to massed infantry formations and paved the way for the creation of a completely new methodology of war fighting. The machine gun became the keystone of the infantry defense and a major supplier of organic firepower in the offense. New tactics were developed by both sides to not only exploit the effects of the machine gun, but to counter the enemy's machine gun employment capabilities. The machine gun changed the face of modern warfare just as surely as the development of aircraft and precision indirect fire artillery. The impact of this weapon can be seen not only in military writings of that period, but also in the principles of employment still in use today (Carlucci, Donald E. Ballistics : theory and of guns and ammunition / Authors, Donald E. Carlucci, Sidney S. Jacobson. --2nd edition).

Guns must be mounted on tank or aboard ship in such a manner that they can be rotated horizontally (trained) and vertically (elevated). By means of these two motions, a gun can be pointed in any direction. The mechanism which supports the gun and moves it in elevation and train is called the gun mount. The major components of a typical gun mount are the elevating mechanism, the traversing mechanism, the recoil mechanism, the trunnions, the carriage and the stand. The elevating mechanism may be power or hand driven and moves the gun in elevation; the traversing mechanism, also power or manually operated, traverses (trains) the weapon. The recoil mechanism absorbs the forces resulting from the explosion of the propelling charge and allows the gun to recoil (move to the rear). The stand supports the entire gun, mounts, and is rigidly attached to the deck; the carriage rests and rotates on the stand so that the weapon can be traversed. The trunnions provide a pivot support between gun and carriage so that the gun can be elevated (BEKHITI B. Dynamic Modeling & control of Large Space Structures : Part 1 Missiles & Aircrafts).

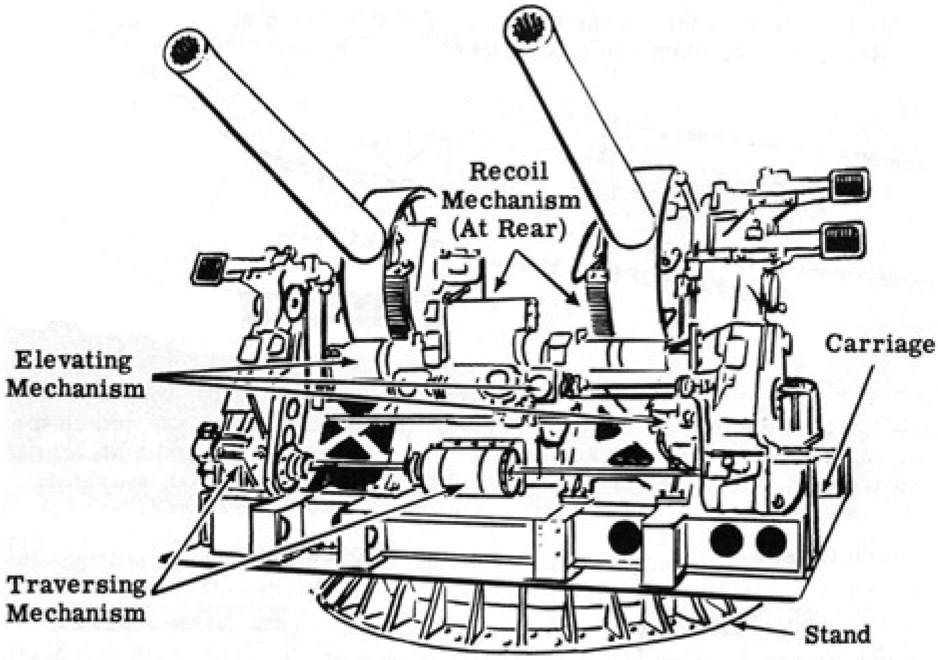

Figure 0.2 Major Gun Mount Components

Most major-caliber guns (8" and up) are mounted in heavily armored structures called "turrets." Intermediate caliber guns (over 4" and less than 8") are mounted in unarmored gun houses, or are provided with shields; minor-caliber guns (over 0.60" to 4") are shielded or simply mounted in the open.

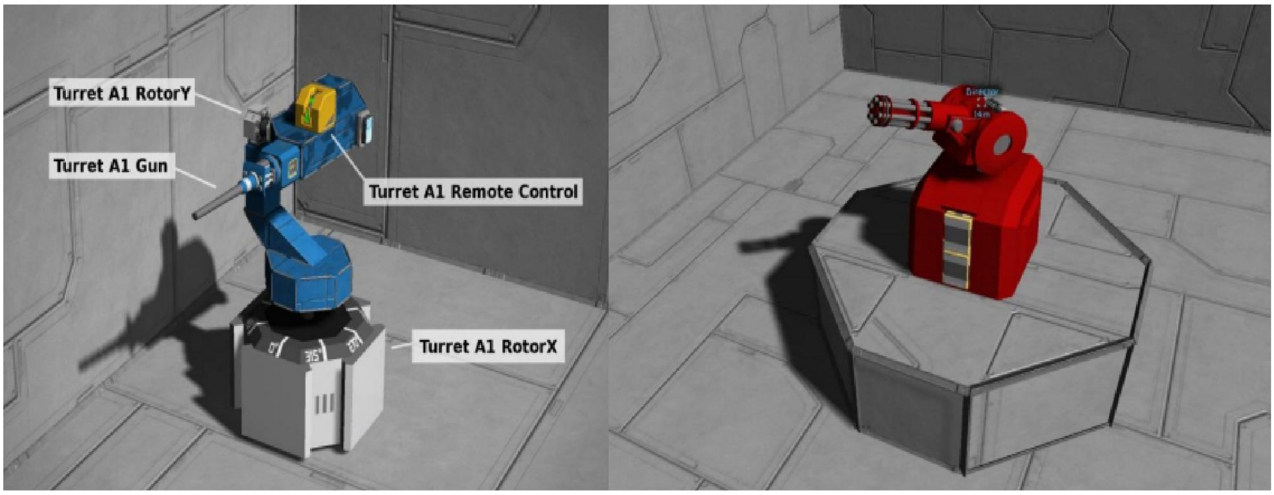

Figure 0.3 Some Configurations Of Turret Guns

In accordance with the program of Algerian Army Force commander, who has performed modernization and rejuvenation of the main tools of Algerian defense system, on both of Army, Navy and Air Force, so that a few examples of such numerous defense equipments bought by Algerian from abroad, are tanks, warships, military aircraft and so forth. This made researchers and scientists encouraged to make examination, exploration and developments in major defense system appliance technologies, especially on its gun turret used on military vehicles. The military vehicle itself is composed of ground vehicles, navy vehicles and air military vehicle that has its own characteristics, so the gun turret itself has flexibility because it can be placed on all military vehicles on land, sea or air. Gun turret can be manned by humans can also be controlled remotely, can then also move automatically (BEKHITI B. Dynamic Modeling & control of Large Space Structures : Part 1 Missiles & Aircrafts).

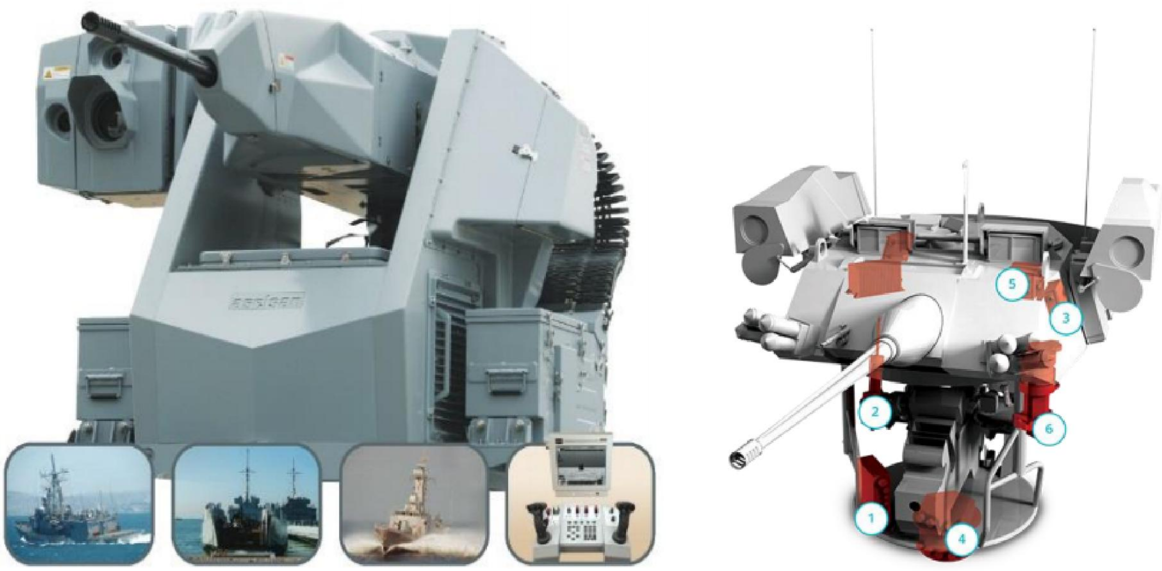

Figure 0.4 Battleship Guns

Chapter 1

Mathematical Preliminaries

In this chapter, we try to review the necessary mathematical tools that are needed in robotic systems. We review some mathematical results on norms, matrices and function that are necessary in analysis of such systems. Then we present the Lyapunov stability concept that will be used in this thesis to design nonlinear control systems for Turret Dynamics. Additional stability results that are particular cases of the Lyapounov tchnique or are based on input-output concepts are discussed.

1.1 Introduction :

The major interest of control engineers is to design a control system to guarantee that a specific plant present a desired performance objective. It is with no doubt that, the most important objective is the stability of the system in question.

Intuitively, we think of stability as; suppose that a system is operating under some conditions, what effect a slight change of conditions has on the system's operation. The answer to this is complex because there exist many different variants of the basic problem.

The main objective of this chapter is to present the Lyapunov stability theory with all the necessary mathematical tools that permit us to perform the analysis and design of control systems for Turret guns.

Since the stability analysis leads to perform some operations on the size of some vectors and matrices, we start by review on definitions and properties of norms matrices.

Then in order to introduce the Lyapunov stability theory, some fundamental definitions are given. The stability theory is presented next through several definitions and theorems. All proofs are omitted but reference are made to books that are more specialized where proofs are provided.

Finally, we gives some important advanced stability results that are based on input-output stability concept and will be used to design robust controllers.

1.2 Norms:

A "norm" is a generalization of the idea of distance and length. As we are to perform a stability analysis on the size of some vectors and matrices, we give a brief description of some useful norms found in the literature and will be used in this work.

1.2.1. Vector Norms

Definition 1.1 -(Lewis et al.,1993) A norm f(.): $\mathbb{R}^n \rightarrow \mathbb{R}$ of a vector x is a real-valued function defined on the vector X such that

(a) $\| X \| \geq 0$ for all $x \in X$ with $\| x \| = 0$ if and only if $x = 0$

(b) $\| ax \| = | a | . \| x \| \neq 0$ **,** for all $x \in X$ and any scalar a

(c) $\| x + y \| \leq \| x \| + \| y \|$ **,** for all $x, y \in X$

Note that $|a|$ denotes the absolute value of a for a real a, or the magnitude of a if it is complex.

In case where $X \in \mathbb{R}^n$,the following are important norms on $\mathbb{R}^n$;

1-norm: $$\| x \|_1 = \sum_{j=1}^{n} | x_i |, \tag{1.2.1}$$

2-norm: $$\| x \|_2 = (\sum_{j=1}^{n} x_j^{\,2})^{1/2}, \tag{1.2.2}$$

This norm is also known as Euclidian norm.

p-norm: $$\| x \|_p = (\sum_{j=1}^{n} x_j^{\,p})^{1/p}, \tag{1.2.3}$$

∞-norm: $$\| x \|_\infty = \max_{1<j<n} | x_j |, \tag{1.2.4}$$

1.2.2. Matrix Induced Norms

In robotic application, a particular vector x may be operated by a matrix **A** to obtain another vector $y = Ax$ **.** To relate the sizes x of y, we define the induced matrix norms as follows;

Definition 1.1 -(Lewis et al.,1993) let $\|x\|$ be a given norm of the vector $x \in \mathbb{R}^n$.

Then, each $n \times n$ matrix **A** has an induced norm defined by

$$\| A \|_i = \max_{\|x\|=1} \| Ax \| \tag{1.2.5}$$

For this, we can show that these induced matrix norms satisfy the condition of definition1.1. It is also possible to show that the matrix induced norm satisfy

$$\| AB \|_i \leq \| A \|_i \| B \|_i \tag{1.2.6}$$

For all **A**, **B** matrices of appropriate dimensions.

Important norms in $\mathbb{R}^n$ are then given as, with $A = (a_{ij})$,

$$\| A \|_{i,\infty} = \max_i \sum_j | a_{ij} | \tag{1.2.7}$$

$$\| A \|_{i,1} = \max_j \sum_i | a_{ij} | \tag{1.2.8}$$

$$\| A \|_{i,2} = \sqrt{\lambda_{\max}(A^T A)} \tag{1.2.9}$$

With $\lambda_{\max}(.)$ is the maximum eigenvalue of (.).

1.2.2. Function Norms

Next, we consider an important class of signals that will be encountered in this work, which are the time dependent functions and vector functions.

Definition 1.3-(Lewis et al.,1993) A function $f(.):\mathbb{R}_+ \to \mathbb{R}$ is uniformly continuous if for any $\varepsilon > 0$, there is a $\delta(\varepsilon)$ such that

$$|t - t_0| < \delta(\varepsilon) \text{ implies that } |f(t) - f(t_0)| < \varepsilon \tag{1.2.10}$$

Then, f is said to belong to L_p if for $p \in [1, \infty)$,

$$\int_0^{\infty} |f(t)|^p \, dt < \infty \tag{1.2.11}$$

f is said to belong L_∞ if it is bounded, means that, if

$$\sup_{t\in[1,\infty)} (|f(t)|) \le B \tag{1.2.12}$$

Note that $\sup(|f(t)|)$ is the smallest number that is larger than or equal to the maximum value of $f(t)$. The following definition of the norm of vector function is not unique.

Definition 1.4-(Lewis et al.,1993) let L_p denote the set of $n\times 1$ vectors of functions, f_i, each of which belong to L_p. The norm of $f \in L_p$ is

$$\| f(.) \|_p = \left(\int_0^{\infty} \sum_0^n |f_i(t)|^p \, dt \right)^{1/p} \tag{1.2.13}$$

1.3 Matrix properties:

In this section, we collect some matrix properties that play an important role in the study of the stability of robot systems. Let **A** be a real $n\times n$ matrix of elements (a_{ij}).

Consider the following definitions.

Definition 1.5-(Lewis et al.,1993)

Positive Definite: Matrix **A** is positive definite if $x^T A x > 0$ for all $x \in \mathbb{R}^n, x \ne 0$.
Positive Semi-definite: Matrix **A** is positive semi-definite if $x^T A x \ge 0$ for all $x \in \mathbb{R}^n$.
Negative Definite: Matrix **A** is negative definite if $x^T A x < 0$ for all $x \in \mathbb{R}^n, x \ne 0$.
Negative Semi-definite: Matrix **A** is negative semi-definite if $x^T A x \le 0$ for all $x \in \mathbb{R}^n$.
Non-Definite: A matrix **A** is not definite if is none of the above definitions.
Note that any $n\times n$ matrix **A** can be composed into two parts: symmetric and asymmetric (skew-symmetric) part.

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = A_1 + A_2 \tag{1.3.1}$$

Where

$$A_1 = \frac{1}{2}(A + A^T) = A_1^T \text{ and } A_2 = \frac{1}{2}(A - A^T) = -A_2^T$$

Also, we can study the definiteness of a matrix by considering its symmetric part.

Notice that quadratic forms (energy like function) are scalars means that

$$E_{1\times 1} = (x^T Ax) = (x^T Ax)^T = (x^T A^T x) \Leftrightarrow x^T (A_1 + A_2)x = x^T (A_1 + A_2)x = x^T (A_1 + A_2)^T x$$

$$\Leftrightarrow x^T (A_1 + A_2)x = x^T (A_1^T + A_2^T)x$$

$$\Leftrightarrow x^T Ax = x^T A_1^T x + x^T A_2^T x \tag{1.3.2}$$

Also, we know from (1.3.1)

$$E = x^T Ax = x^T A_1 x + x^T A_2 x = x^T A_1^T x - x^T A_2^T x \tag{1.3.3}$$

Now, from equations (1.3.2) and (1.3.3) we can deduce that:

$$E = x^T Ax = x^T A_1 x + x^T A_2 x = x^T A_1^T x - x^T A_2^T x \Leftrightarrow \begin{cases} E = x^T Ax = x^T A_1^T x \\ x^T A_2^T x = 0 \end{cases} \tag{1.3.4}$$

Now, as a conclusion, we can study the definiteness of a matrix by considering its symmetric part.

$$x^T Ax = x^T \left(\frac{A + A^T}{2} \right) x = x^T A_s x \tag{1.3.5}$$

With A_s is the symmetric part of **A**.

In case where **A** is a symmetric $n \times n$ matrix, the following theorem states some results relating the definiteness of **A** with its eigenvalues.

Theorem 1.1-(Lewis et al.,1993) let **A** be a symmetric $n \times n$ matrix, we have the following:

Positive Definite: Matrix **A** is positive definite if all eigenvalues are positive.
Positive Semi-definite: Matrix **A** is positive semi-definite if all eigenvalues are non-negative.
Negative Definite: Matrix **A** is negative definite if all eigenvalues are negative.
Negative Semi-definite: Matrix **A** is negative semi-definite if all eigenvalues are non-positive.

Theorem 1.2 (Rayleigh-Ritz)-(Lewis et al.,1993) let **A** be a real symmetric $n \times n$ positive definite matrix, let $\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ denote the minimum and maximum eigenvalues of the associated variable respectively. Then for any $x \in \mathbb{R}^n$, we have

$$\lambda_{\min}(A)\|x\|^2 \leq x^T Ax \leq \lambda_{\max}(A)\|x\|^2 \qquad (1.3.6)$$

Theorem 1.3 (Gershgorin)-(Lewis et al.,1993) let **A** be a real symmetric $n \times n$ matrix of elements (a_{ij}). Suppose that

$$a_{ij} > \sum_{j=1}^{n} |a_{ij}|, \qquad \text{for } i = 1,..,n \quad j \neq i \qquad (1.3.7)$$

If all the diagonal elements are positive, then the matrix **A** is positive definite.

1.4 Function properties:

We review certain classes of functions, and some concepts, that are used in the proof of the fundamental results exposed in this work.

Consider a continuous function $f(.) : \mathbb{R}^n \to \mathbb{R}$, the following lemma tells about definiteness of the function f.

Lemma 1.1-(Vidyasagar, 1992; Hariche, lectures 2008 INELEC) the continuous function $f(.) : \mathbb{R}^n \to \mathbb{R}$ is locally positive definite if the following two conditions are verified;

(a) $f(0) = 0$;

(b) There exists a constant $r > 0$ such that $f(x) > 0$ for all x belong to some ball

$$B_r = \{x : \|x\| \leq r\}, \quad x \neq 0 \qquad (1.4.1)$$

f is positive definite function if

(a) $f(0) = 0$;

(b) $f(x) > 0$ for all $x \in \mathbb{R}^n - \{0\}$;

(c) f there exists a constant $r > 0$ such that $\inf_{\|x\| \geq r} f(x) > 0$

f is radially unbounded if and only if

(a) $f(x) \to \infty$ as $\|x\| \to \infty$, uniformly in x

Definition 1.6-(Lewis et al.,1993) let the function $f(.):\mathbb{R}^n \to \mathbb{R}$ be a continuous function, f isa class k function if

(a) $f(0)=0$**;**

(b) $f(x)>0$ for all $x>0$;

(c) f nondecreasing, i.e. $f(x_1)>f(x_2)$ for all $x_1>x_2$.

Note that some authors replace (c) above by the more stringent requirements that $f(.)$ is strictly increasing. It turns that both definitions are equally good in proving stability theorems.

Definition 1.7-(Lewis et al.,1993) A continuous function $f(.):\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ is locally decrescent if there exists a class k function $\beta(.)$ and a neighborhood N of origin of $\mathbb{R}^n$ such that

$$f(t,x)\le\beta(\|x\|) \tag{1.4.2}$$

For $t>0$ and all $x\in N$ if $N=\mathbb{R}^n$, then we say that f is decrescent.
It is important to note that the above definitions are stated in terms of continuous nonlinear systems, keeping in mind that discrete and nonlinear systems admit similar results and linear systems are no more than a special case of nonlinear systems.

1.5 Stability Definitions

In this section, various types of stability are defined. Through this section, we consider the unforced system

$$\dot{x}=f(t,x) \tag{1.5.1}$$

Where $x(t)\in\mathbb{R}^n$ and $f(.):\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ is continuous. Then the vector x_e is an equilibrium of the system (1.5.1) if

$$f(t,x_e)=0\,,\quad \forall t\ge 0 \tag{1.5.2}$$

Definition 1.8-(Lewis et al.,1993) the vector x_e is stable at t_0, if starting close enough to x_e at t_0, the state will always stay close to x_e at $t\ge t_0$. More precisely, x_e is stable at t_0 if $\forall\varepsilon>0$, $\exists\delta(\varepsilon,t_0)$ such that if

$$\|x_0-x_e\|\le\delta(\varepsilon,t_0)\,,\quad \text{then } \|x(t)-x_e\|\le\varepsilon \text{ for all } t\ge t_0 \tag{1.5.3}$$

x_e is stable, if it is stable for any t_0. Moreover, x_e is uniformly stable if $\delta(\varepsilon,t_0)$ does not depend on t_0.

Definition 1.9-(Vidyasagar, 1992) x_e is attractive if for each $t_0 \in \mathbb{R}_+$, there exist an $\eta(t_0) > 0$ such that

$$\| x_0 - x_e \| \leq \eta, \text{ then } \| x(t) - x_e \| \to 0 \text{ as } t \to \infty \quad (1.5.4)$$

x_e is uniformly attractive if there is a number $\eta > 0$ such that

$$\| x_0 - x_e \| \leq \eta, \quad t_0 > 0, \text{ then } \| x(t) - x_e \| \to 0 \quad (1.5.5)$$

As $t \to \infty$ uniformly in x_0 and t_0.

Definition 1.10-(Vidyasagar, 1992) The equilibrium x_e is asymptotically stable if states starting sufficiently close to x_e will eventually converge to it. In other words, if it is stable and attractive.
x_e is uniformly asymptotically stable if it is uniformly stable and uniformly attractive.

Definition 1.11-(Vidyasagar, 1992) the equilibrium is exponentially stable if there exist $r, \alpha, \beta > 0$ such that

$$\| x(t) - x_e \| < \alpha \| x_0 \| \exp(-\beta t), \forall t \geq t_0, \forall x_0 \in B_r \quad (1.5.6)$$

Definition 1.12-(Vidyasagar, 1992) x_e is globally asymptotically stable if any initial state x_0 will stay close to x_e and will eventually converge to it. In other words, x_e is globally asymptotically stable if it is stable and if every $x(t)$ converges to x_e as time grows to infinity.

Definition 1.13-(Vidyasagar, 1992) x_e is globally uniformly asymptotically stable if
(a) It is uniformly stable
(b) if for each pair of positive numbers M, ε with M arbitrary large and ε arbitrary small, there exist a finite number $T = T(M, \varepsilon)$ such that if $\| x(t) - x_e \| < M \quad t_0 > 0$, then

$$\| x(t) - x_e \| \leq \varepsilon, \forall t \geq T(M, \varepsilon) \quad (1.5.7)$$

Definition 1.13-(Vidyasagar, 1992) x_e is globally exponentially stable if there exist constant $\alpha, \beta > 0$ such that

$$\| x(t) - x_e \| < \alpha \| x_0 \| \exp(-\beta t), \forall t \geq t_0, \forall x_0 \in \mathbb{R}^n \quad (1.5.8)$$

1.6 Lyapunov Direct Method

Lyapounov stability theory deals with the behavior of unforced nonlinear system described by the differential equations

$$\dot{x} = f(t, x(t)), \tag{1.6.1}$$

$t \geq 0, x \in \mathbb{R}^n, x(t) \in \mathbb{R}^n$ and $f(.,.): \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}$ is continuous

The idea behind this theory is that if we consider an "isolated" system in the sense that there are no external forces acting on it, as equation (1.6.1), where without loss of generality, the origin is an equilibrium point. Then we suppose it is possible to define a function, so that it is zero at the origin and positive everywhere else. And describes, in some sense, the total energy of the system. If the system, originally at equilibrium, is perturbed to a new nonzero initial state, then there are several possibilities. If the system dynamics are such that the energy of the system is non-increasing with time, then depending on the energy function, this may be sufficient to conclude that the origin is stable. If the dynamics are such that the energy reduces to zero with time, then it may be sufficient to decide that the equilibrium point is asymptotically stable. Finally, if the dynamics are such that the energy increases beyond its initial values, then it is possible to conclude that the system is unstable.

From this reasoning, Lyapunov was able to extract a general theory that is applicable to any differential equation. This theory requires one to search for a function that satisfies some prespecified properties. This function is a generalization of the energy of mechanical systems, and is now commonly known as Lyapunov function.

Lyapunov theory will allow us to determine the stability of particular equilibrium point without actually solving the differential equation (1.6.1). Moreover, it will provide us with qualitative results of the stability questions, which may be used in designing stabilizing controllers for nonlinear systems.

In this section, we shall give the basic Lyapunov's theorems that deal mainly with stability, asymptotic stability and exponential stability.

Theorem 1.4-(Vidyasagar, 1992) the origin of the system (1.6.1) is stable, if there exist a continuously differential function (C^1 function) and locally positive definite (1pdf), $V: \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}$ and a constant $r > 0$ such that

$$\dot{V}(t,x) \leq 0, \forall t \geq t_0 \text{ and } \forall x \in B_r \tag{1.6.2}$$

Where is x is evaluated along the trajectories of (1.6.1)

Theorem 1.5-(Vidyasagar, 1992) the origin of the system (1.6.1) is uniformly stable, if there exist a C^1, decrescent, locally positive definite function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ and a constant $r>0$ such that

$$\dot{V}(t,x)\le 0, \forall t\ge t_0 \text{ and } \forall x\in B_r \tag{1.6.3}$$

Theorem 1.6-(Vidyasagar, 1992) the origin of the system (1.6.1) is asymptotically stable, if there exist a scalar function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ such that

(a) $V(t,x)$ is positive definite

(b) $\dot{V}(t,x)$ is negative definite

In addition, if is decrescent, then the origin is uniformly asymptotically stable.

Theorem 1.7-(Vidyasagar, 1992) the origin of the system (1.6.1) is globally uniformly asymptotically stable, if there exist a C^1, scalar function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ such that

(a) $V(t,x)$ is positive definite

(b) $V(t,x)$ is decrescent and radially unbounded

(c) $\dot{V}(t,x)$ is negative definite

Theorem 1.8-(Vidyasagar, 1992) Suppose there exist constant α,β,η and $r>0$, $p\ge 1$ and a C^1 function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ such that

$$\alpha\,\|x\|^p\le V(t,x)\le\beta\,\|x\|^p \tag{1.6.4}$$

$$\dot{V}(t,x)\le-\eta\,\|x\|^p\ \forall t\ge 0\forall x\in B_r \tag{1.6.5}$$

Then the equilibrium point is exponentially stable

Theorem 1.9-(Vidyasagar, 1992) the origin of the system (1.6.1) is globally uniformly exponentially stable, if there exist constant α,β,η and $r>0$, $p\ge 1$ and a C^1 function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}$ such that

$$\alpha\,\|x\|^p\le V(t,x)\le\beta\,\|x\|^p \tag{1.6.6}$$

$$\dot{V}(t,x)\le-\eta\,\|x\|^p\ \forall t\ge 0\forall x\in R_r \tag{1.6.7}$$

Remark 1.1 -the function is commonly known as Lyapunov function or a candidate if it satisfies the hypothesis in theorem 1, i.e. V is C^1 and locally positive definite, if for a particular system (1.6.1) the condition imposed to $\dot{V}$ is also satisfied, then V is referred to as a Lyapunov function.

Remark 1.2 -from the above theorems, one is able to draw conclusion about the stability status of the equilibrium without solving the system equations.
Moreover, the Lyapunov function V has an intuitive appeal as the total energy of the system. On the other hand, they represent only sufficient conditions for the various forms of stability. Thus if a particular Lyapunov function candidate V fails to satisfy the hypothesis on $\dot{V}$, then no conclusion can be made, and one has to find another Lyapunov function candidate.

Remark 1.3 -the above theorems may be used to design controller that will stabilize a nonlinear system such as a Turret manipulator. In fact, if one select a Lyapunov function candidate $V(t,x)$, then finding its total derivative $\dot{V}(t,x)$ will exhibit an explicit dependence on the control signal. By choosing the control signal to make $\dot{V}(t,x)$ negative definite, stability of the closed loop system is guaranteed.
Unfortunately, in some cases $\dot{V}(t,x)$ may be shown to be negative, but not necessarily negative definite. In this case a useful theorem, in case the open loop system is autonomous, may be used to guarantee the global stability results as stated in the following theorem.

Theorem 1.10-(LaSalle's theorem)-(Lewis et al.,1993) given the autonomous system

$$\dot{x} = f(t), \tag{1.6.8}$$

With and let the origin be an equilibrium point. Suppose that a Lyapunov function $V(x)$, has been found such that $V(x) > 0$ and $\dot{V}(x) \le 0, \forall x \in \mathbb{N} \subset \mathbb{R}^n$, then the origin is asymptotically stable if and only if $\dot{V}(x) = 0$ only at $x = 0$.
A variant and a more general theorem is given in (Vidyasagar, 1992) and is stated as follows.

Theorem 1.11-(Vidyasagar, 1992) given the autonomous system(1.6.8). Suppose that there exists a positive definite, radially bounded Lyapunov function such that

$$\dot{V}(x) \le 0, \forall x \in \mathbb{R}^n \tag{1.6.9}$$

Define the set $S = \{x \in \mathbb{R}^n : \dot{V}(x) = 0\}$ and suppose that the only trajectories contained in S are trivial trajectories. Then, the equilibrium point is globally asymptotically stable.
In case of non-autonomous systems, a useful lemma that leads to results similar to those of LaSalle's theorem can be used.

Lemma 1.2 (Barballat's Lemma)- (Lewis et al.,1993) let $f(t)$be a differentiable function of t,

First version: if $\dot{f}(t)=\dfrac{df(t)}{dt}$ is uniformly continuous and $\lim_{t\to\infty} f(t)<\infty$ then $\lim_{t\to\infty} \dot{f}(t)<\infty$

Second version: if $f(t)>0$,$\dot{f}(t)$ bounded, then $\lim_{t\to\infty} \dot{f}(t)=0$

For linear system, we consider the following stability theorem.

Theorem 1.12- (Lewis et al.,1993) consider the linear autonomous system

$$\dot{x} = Ax(t) \tag{1.6.10}$$

Given a matrix $A\in\mathbb{R}^{n\times n}$, the following three statements are equivalent;

1) A is a Hurwitz matrix

2) There exist some positive definite matrix $Q\in\mathbb{R}^{n\times n}$ such that the Lyapunov matrix equation

$$A^T P + PA = -Q \tag{1.6.11}$$

Has a unique positive solution $P\in\mathbb{R}^{n\times n}$

3) For every positive definite matrix $Q\in\mathbb{R}^{n\times n}$,(1.6.11) has a unique solution P ,and this solution is positive definite.

1.7 Additional Stability Results

In this section, we present additional stability definitions and theorems that are useful in the design of particular types of controllers. We give first other results on local and global exponential stability found in the literature that can be used directly and easily. These are presented hereafter.

Lemma 1.3-(Dawsen et al., 1992) given a continuous system

$$\dot{x} = f(t, x(t)), \tag{1.7.1}$$

Let $V(t,x)$ be the associated Lyapunov function with the following properties

$$\lambda_1 \| x \|^2 \le V(t,x) \le \lambda_2 \| x \|^2$$
$$\dot{V}(t,x) \le -\lambda_3 \| x \|^2 + \varepsilon \exp(-\beta t)$$

For $(x,t) \in \mathbb{R}^n \times \mathbb{R}$ all where $\lambda_1, \lambda_2, \lambda_3, \varepsilon$ and β are positive scalar constants. Then, the state $x(t)$ is globally exponentially stable in the sense that

$$\| x(t) \| \leq \begin{cases} \left(\dfrac{\lambda_2}{\lambda_1} \| x(t_0) \|^2 \exp(-\lambda t) + \dfrac{\varepsilon}{\lambda_1} \exp(-\lambda t) \right)^{\frac{1}{2}}, if\ \beta = \lambda \\ \left(\dfrac{\lambda_2}{\lambda_1} \| x(t_0) \|^2 \exp(-\lambda t) + \dfrac{\varepsilon}{\lambda_1(\lambda - \beta)} (\exp(-\beta t) - \exp(-\lambda t)) \right)^{\frac{1}{2}}, if\ \beta \neq \lambda \end{cases}$$

Where $\lambda = \dfrac{\lambda_3}{\lambda_1}$

Theorem 1.13-(Berghuis, 1993a) let be a Lyapunov function of a given continuous time system satisfying

$$\lambda_1 \| x \|^2 \leq V(t,x) \leq \lambda_2 \| x \|^2 \tag{1.7.2}$$

$$\dot{V}(t,x) \leq -\alpha_1 \| x \|^2 - \alpha_2 \| x \|^3 \text{ for all } \| x \| \leq \frac{\alpha_1}{\alpha_2} \tag{1.7.3}$$

Then, if
$$\| x(t_0) \| \leq \sqrt{\frac{\lambda_3}{\lambda_1} \frac{\alpha_1}{\alpha_2}} \tag{1.7.4}$$

Then
$$\| x(t) \|^2 \leq m \exp(-\rho t) \| x(t_0) \|^2 \tag{1.7.5}$$

For some $\rho, m > 0$ that is $x(t)$ converges exponentially to zero.

Definition 1.15-(Lewis et al., 1993) x_e is bounded at t_0 if states starting close to x_e will never get too far. In other words x_e is bounded at t_0 if for each $\delta > 0$ such that

$$\| x(t) - x_e \| < \delta \tag{1.7.6}$$

There exist a positive $\varepsilon(r, t_0)$ such that for all $t > t_0$

$$\| x(t) - x_e \| < \varepsilon(r, t_0) \tag{1.7.7}$$

x_e is bounded if it is bounded for any t_0 . x_e is uniformly bounded over $[t, \infty)$ if $\varepsilon(r, t_0)$ can be made independent of t_0 .

Definition 1.16-(Lewis et al., 1993) x_e is uniformly ultimately bounded if for any δ ,$\varepsilon > 0$, there exist a finite $T(\delta,\varepsilon)$ such that whenever $\| x(t) - x_e \| < \delta$

$$\| x(t) - x_e \| < \varepsilon(r, t_0) \tag{1.7.8}$$

For all $t > T(\delta,\varepsilon)$. If $T(\delta,\varepsilon)$ can be made independent of δ , then x_e is said to be globally uniformly bounded.

In the sequel, we present few results discussed in (Dawson et al., 1990), (Spong and Vidyasagar, 1989) and (Xu, 1995), and will be used in the design of robust controllers. The following theorems present uniform boundedness and uniform ultimate boundedness results based on the properties of a Lyapunov function (Dawson et al., 1990).

Theorem 1.14-(Dawson et al., 1990) if $V(.)$ is a Lyapounov function for a given continuous time system with the properties

$$\lambda_1 \| x(t) \|^2 \leq V(t,x) \leq \lambda_2 \| x(t) \|^2 \tag{1.7.9}$$

$$\dot{V}(t,x) \leq 0 \quad \text{if} \quad \eta_1 < \| x(t) \| < \eta_2 \tag{1.7.10}$$

Where
$$\eta_2 > \left(\frac{\lambda_2}{\lambda_1} \right)^{\frac{1}{2}} \eta_1 \tag{1.7.11}$$

With λ_1, λ_2 some positive scalars, then

$$\| x(t) \| < \left(\frac{\lambda_2}{\lambda_1} \right)^{\frac{1}{2}} (\eta_1 + \varepsilon), \forall t \in [t_0, t_1] \tag{1.7.12}$$

Where ε is some arbitrary small positive constant.

Theorem1.15-(Dawsen et al., 1990) let $V(.)$ be a Lyapunov function for a given continuous time system with the properties

$$\gamma_1(\| x(t) \|) \leq V(t,x) \leq \gamma_2(\| x(t) \|) \tag{1.7.13}$$

$$\dot{V}(t,x) \leq \gamma_3(\| x(t) \|) + \gamma_3(\eta) \tag{1.7.14}$$

Where η is a positive constant, $\gamma_1(.)$ and $\gamma_2(.)$ are continuous strictly increasing function, γ_3 is a continuous non-decreasing function. If $\dot{V}(t,x) \leq 0$ for all $\| x(t) \| > \eta$, then the given system has uniform ultimate boundedness property; that is a solution to the system with initial states, then given the quantity

We have $$d_s > (\gamma_1 \circ \gamma_2)(\eta) \quad (1.7.15)$$
For every $t \in [t_0 + T, \infty)$, where $$\| x(t) \| < d_s \quad (1.7.16)$$

$$T = \begin{cases} 0, if \, \| x(t) \| \leq \eta_s \\ \dfrac{\gamma_2(\| x(t_0) \|) - \gamma_1(\eta_s)}{\gamma_3(\eta_s) - \gamma_3(\eta)}, if \, \| x(t) \| > \eta_s \end{cases} \quad (1.7.17)$$

And $$\eta_s = (\gamma_2^{-1} \circ \gamma_1)(d_s) \quad (1.7.18)$$
In many other results, equation (1.7.14) can be found as

$$\dot{V}(t, x) \leq g(\| x(t) \|) < 0 \quad 1.7.19)$$

Where $g(\| x(t) \|)$ is a second order polynomial of its arguments (Berghuis, 1993a).

Finally, an important aspect that should be discussed the passivity theorem of nonlinear systems, instead of using the unforced system; consider the nonlinear input-output system

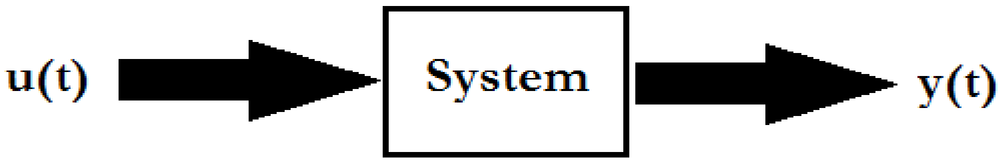

Figure 1.1 Input-Output description of nonlinear system

The stability analysis of such a system is studied based on input-output measurements only.

Definition 1.17-(Berghuis, 1993a) Assume that system of Fig.1.1 has the same number of inputs and outputs. Then the system is said to be passive if

$$\int_0^T y^T(t) u(t) d\tau \leq \gamma \quad (1.7.20)$$

For all $T > 0$ and $\gamma > -\infty$. It is strictly passive if there exist a $\delta > 0$ and $\gamma > -\infty$, such that

$$\int_0^T y^T(t) u(t) d\tau \geq \delta \int_0^T u^T(t) u(t) d\tau + \gamma \quad (1.7.21)$$

For all $T > 0$
A passivity system is in effect one that does not create energy. If the system under

consideration is linear and time invariant, then the passivity is equivalent to positivity (Lewis et al., 1993).

In the following theorem, we give a simplified from of the passivity theorem, which finds its application to robot manipulators control, as when we deal with feedback systems described by

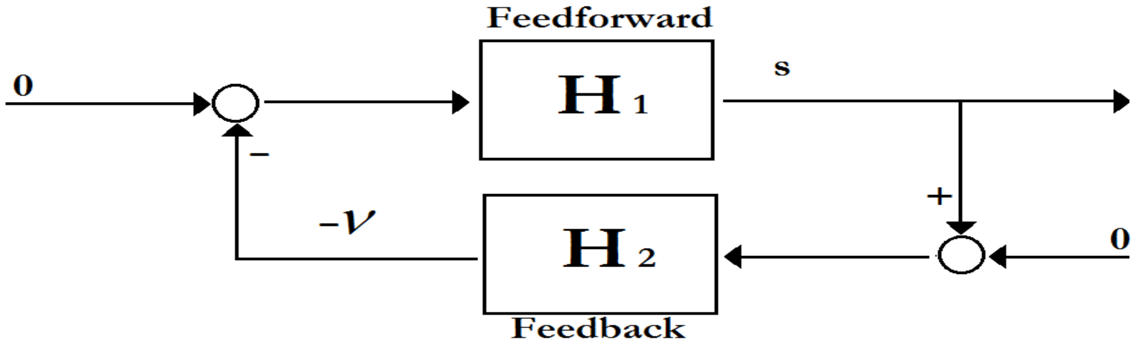

Figure 1.2 Feedback Interconnection of two systems

Theorem 1.16-(Berghuis, 1993a) Consider the feedback system of Fig.1.2, where

$$s_1 = H_1 v \quad \text{and} \quad v = -H_2 s \tag{1.7.22}$$

Where H_1 and H_2 map L_{1e} into L_{2e}, and suppose there exist solutions s and v in L_{2e}. Assume that

$$\langle s, v \rangle \equiv \int_0^T s^T(\sigma) v(\sigma) d\sigma \geq -\alpha s \tag{1.7.23}$$

$$\langle -v, s \rangle \equiv \int_0^T v^T(\sigma) s(\sigma) d\sigma \geq \beta \| s \|_{2T}^2 \tag{1.7.24}$$

$\alpha > 0, \beta > 0$. Then, $s \in L_2$.

The passivity theorem gives conditions under which the L_2 stability of the interconnected closed loop system is guaranteed. In other words, the passivity theorem guarantees that a certain system does not create energy by making sure that its interconnecting parts are either dissipating or not creating energy. This special case of the passivity to divide a nonlinear closed loop system into two blocks. If the feedforward block defines a passive mapping from input to output (according to definition 1.17), then the challenge is to make the feedback system strictly passive the output signal is guaranteed to belong to L_2.

1.8 Conclusion

In this chapter, we have reviewed some results from control theory that are useful in robot manipulators control design problem. The interest has been to include enough material to serve as a background to reader in order to follow the analysis performed in this thesis. We have given the basic properties of norms, matrices, and functions and presented the Lyapunov stability concept, which is the basic stability tool used in this work. Some additional stability results have also been given. These results will be used in showing the closed loop stability of robots when robust controller s are considered.

To complete this part of preliminaries, in the following chapter, we introduce the dynamical description of Turret gun manipulators, and derive the dynamical model that will be used in the subsequent chapte

Chapter 2

Turret Dynamics

In this chapter, we introduce the general model that can be used to describe the mothion of Turret gun systems. First, we present a description of the geometric structure of simple open loop Turret Gun. Then, the dynamical model of Turret Gun manipulators is described. This model has a number of structural properties that are useful for control system design. These properties are presented next. Finally, the patricular Turret Gun system in the study of our work is described.

2.1. Introduction

For Turret design purposes, it is necessary to have a mathematical model that reveals the dynamic behavior of the manipulator. This mathematical model is derived using the Lagrangian mechanics (W. Khalil 1990, Lewis et al. 1993).

In this section, we analyze the dynamical behavior of Turret Guns. The dynamic behavior is described in terms of the time rate of change of the arm configuration in relation to the joint torque exerted by the actuators. This relationship can be expressed by a set of differential equations, called equations of motion, that govern the dynamic response of the arm linkage to input joint torque.

Two methods can be used in order to obtain the equations of motion (W. Khalil); the Newton-Euler formulation, and the Lagrangian formulation. The Newton-Euler formulation is derived by the direct interpolation of Newton's second law of motion, which describes dynamic system in terms of forces and momentum as well as torque and angular momentum. In the Lagrange formulation, on the other hand, the system's dynamical behavior is described in term of work and energy using generalized coordinates. All the workless forces and constraint forces are automatically eliminated in this method. Further, the derivation is simpler and more systematic than in the Newton-Euler formulation. The Lagrangian formulation is used in this chapter to derive the dynamical model of Turret Gun manipulators.

2.2. Lagrangian Formulation

The Lagrangian formulation describes the behavior of a dynamic system in term of work and energy stored in the system using generalized coordinates. All the constraint forces are automatically eliminated in the formulation of this approach. The closed form dynamic equations can be derived systematically in any coordinate system.

Let $q_1,...,q_n$ be generalized coordinates that completely locate the n-Degree of freedom (DOF) manipulator arm, and let $K(q,\dot{q})$ and $U(q)$ be the total kinetic energy and potential energy stored in the dynamic system respectively. The Lagrangian $L(q,\dot{q})$ is defined as

$$L(q,\dot{q}) = K(q,\dot{q}) - U(q) \tag{2.2.1}$$

Note that, since the kinetic and potential energies are function of q_i and $\dot{q}_i, i=1,..,n$ so is the Lagrangianc.

Using the Lagrangian (2.2.1), equations of motion of the manipulator are given by:

$$\frac{d}{dt}\left\{\frac{\partial}{\partial \dot{q}}(L)\right\}-\frac{\partial}{\partial q}(L)=\tau \qquad (2.2.2)$$

Where τ is the generalized force corresponding to the generalized coordinates q.

To obtain the general arm dynamical equation, we determine the arm kinetic and potential energies, the Lagrangian, and then substitute into the Lagrange's equation (2.2.2) to obtain the final result.

2.2.1 Arm kinetic Energy

Given a point on link i with coordinates of ${}^{i}r$,with respect to frame i attached to the link. The base coordinate of this point are:

$$r={}^{0}T_{i}\,{}^{i}r \qquad (2.2.3)$$

Where ${}^{0}T_{i}={}^{0}T_{1}\,{}^{1}T_{2}\ldots{}^{j-1}T_{i}$ is a 4×4homogenous transformation matrix.

And
$${}^{i}T_{j}=\begin{cases}{}^{i}T_{i+1}\,{}^{i+1}T_{i+2}\ldots{}^{j-1}T_{j} & \text{for } i<j\\ I & \text{for } i=j\\ ({}^{i}T_{j})^{-1} & \text{for } i>j\end{cases}$$

Note that ${}^{0}T_{i}$ is a function of the joint variables $q_1, q_2, \ldots, q_n$, consequently, the velocity of the point in the base coordinates is:

$$v=\left(\sum_{j=1}^{i}\left(\frac{\partial}{\partial q_j}{}^{0}T_{i}\right)\dot{q}_j\right){}^{i}r+{}^{0}T_{i}\left(\sum_{j=1}^{i}\left(\frac{\partial}{\partial q_j}{}^{i}r\right)\dot{q}_j\right) \qquad (2.2.4)$$

Remark 2.1

1. Since $\left(\frac{\partial}{\partial q_j}{}^{0}T_{i}\right)=0$ for $j>i$; we may replace the upper summation limit by n-the number of links.

$\left(\frac{\partial}{\partial q_j}{}^{i}r\right)=0$ Because ${}^{i}r$ is constant with respect to frame i attached to the link.

The kinetic energy of an infinitesimal mass dm at ${}^{i}r$ that has velocity vector of $v = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$ is defined as

$$dK_i = \frac{1}{2}(v_x{}^2 + v_y{}^2 + v_z{}^2)dm = \frac{1}{2}trace(vv^T)dm \tag{2.2.5.a}$$

Using equation (2.2.4),(2.2.5.a) can be written as

$$dK_i = \frac{1}{2}trace\left(\sum_{j=1}^{n}\sum_{k=1}^{n}\left(\frac{\partial {}^0T_i}{\partial q_j}\right)({}^{i}r\,{}^{i}r^T dm)\left(\frac{\partial {}^0T^T{}_i}{\partial q_k}\right)\dot{q}_j\dot{q}_k\right) \tag{2.2.5.b}$$

Thus, the total kinetic energy for link I is given by

$$K_i = \int_{Link\ i} dK_i \tag{2.2.6}$$

Substituting for dK_i from (2.2.5.b), we can move the integral inside summations. Then defining the 4×4pseudo-inertia matrix for link i as

$$I_i = \int_{Link\ i} {}^{i}r\,{}^{i}r^T dm \tag{2.2.7.a}$$

Which is equal to

$$I_i = \begin{bmatrix} \int x^2dm & \int yxdm & \int zxdm & \int xdm \\ \int ydm & \int y^2dm & \int zydm & \int ydm \\ \int xzdm & \int yzdm & \int z^2dm & \int zdm \\ \int xdm & \int ydm & \int zdm & \int dm \end{bmatrix} \tag{2.2.7.b}$$

With ${}^{i}r = [x\ y\ z\ 1]^T$, and the integrals are taken over the volume of link i. This is a constant matrix that is evaluated once for each link. It depends on the geometry and mass distribution link i.

After defining I_i, we may write the kinetic energy of link i as

$$K_i = \frac{1}{2}trace\left(\sum_{k=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial {}^0T_i}{\partial q_j}\right)I_i\left(\frac{\partial {}^0T_i^T}{\partial q_k}\right)\dot{q}_j\dot{q}_k\right) \tag{2.2.8}$$

The total arm kinetic energy is then written as

$$K = \sum_{i=n}^{n} K_i = \frac{1}{2}\sum_{i=1}^{n} trace\left(\sum_{k=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial\, {}^0T_i}{\partial q_j}\right) I_i \left(\frac{\partial\, {}^0T_i^T}{\partial q_k}\right)\dot{q}_j \dot{q}_k\right) \quad (2.2.9.a)$$

Since the trace of a sum of matrices is the sum of individual traces, we may interchange the summation and the trace operator to obtain

$$K = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}(q)\dot{q}_j \dot{q}_k \quad (2.2.9.b)$$

Or

$$K = \frac{1}{2}\dot{q}^T M(q)\dot{q} \quad (2.2.9.c)$$

Where the arm inertia matrix $M(q)$ has the elements defined as

$$m_{jk}(q) = \sum_{i=1}^{n} trace\left(\frac{\partial\, {}^0T_i}{\partial q_j}\right) I_i \left(\frac{\partial\, {}^0T_i^T}{\partial q_k}\right) \quad (2.2.9.d)$$

Remark 2.2:

Since the kinetic energy is scalar quantity then $K = \frac{1}{2}\dot{q}^T M(q)\dot{q} = K^T = \left(\frac{1}{2}\dot{q}^T M(q)\dot{q}\right)^T$

$$K = K^T \quad \Rightarrow M(q) = M(q)^T \;\; or \;\; m_{jk}(q) = m_{kj}(q)$$

2.2.2 Arm Potential Energy

If link i has a mass m_i and a center of gravity ${}^i\overline{r}$ expressed in the coordinates of its frame i, the potential energy of the link is given by:

$$U_i = -m_i\, {}^0g^T\, {}^0T_i\, {}^i\overline{r} \quad (2.2.10)$$

Letting

$${}^0T_i\, {}^i\overline{r} = {}^0P_i \quad (2.2.11)$$

Representing the coordinates of the center of gravity in the coordinates, we have the total potential energy

$$U = \sum_{i=1}^{n} U_i = -\sum_{i=1}^{n} m_i\, {}^0g^T\, {}^0P_i \quad (2.2.12)$$

2.2.3 Equation of Motion

From equation (2.2.1) and (2.2.2), and using the fact that the potential energy does not depend on the joint velocity of the manipulator arm, we have

$$\frac{d}{dt}\left\{\frac{\partial K(q,\dot{q})}{\partial \dot{q}_i}\right\} + \frac{\partial U(q)}{\partial \dot{q}_i} - \frac{\partial K(q,\dot{q})}{\partial \dot{q}_i} = \tau_i \quad (2.2.13)$$

$$\frac{\partial K(q,\dot{q})}{\partial \dot{q}} = \frac{1}{2}\left(\frac{\partial(\dot{q}^T M(q)\dot{q})}{\partial \dot{q}}\right) = \frac{1}{2}(M(q)\dot{q} + M(q)^T\dot{q}) = M(q)\dot{q} \Rightarrow \frac{\partial K(q,\dot{q})}{\partial \dot{q}_i} = \sum_{j=1}^{n} m_{ij}(q)\dot{q}_j$$

$$\frac{d}{dt}\left\{\frac{\partial K(q,\dot{q})}{\partial \dot{q}_i}\right\} = \sum_{j=1}^{n} m_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\underbrace{\sum_{k=1}^{n}\left(\frac{\partial m_{ij}(q)}{\partial q_k}\dot{q}_k\right)}_{\frac{dm_{ij}(q)}{dt}}\dot{q}_j \qquad (2.2.14)$$

$$\tau_i = \sum_{j=1}^{n} m_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\underbrace{\sum_{k=1}^{n}\left(\frac{\partial m_{ij}(q)}{\partial q_k}\dot{q}_k\right)}_{\frac{dm_{ij}(q)}{dt}}\dot{q}_j - \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial m_{ij}(q)}{\partial q_i}\dot{q}_j\dot{q}_k + \frac{\partial U(q)}{\partial q_i} \qquad (2.2.15)$$

Using the symmetry property of the matrix $M(q)$ we have:

$$\begin{aligned}\frac{\partial m_{ij}(q)}{\partial q_k}\dot{q}_k\dot{q}_j - \frac{1}{2}\frac{\partial m_{kj}(q)}{\partial q_i}\dot{q}_k\dot{q}_j &= \frac{1}{2}\left\{\frac{\partial m_{ij}(q)}{\partial q_k}\dot{q}_k\dot{q}_j + \frac{\partial m_{ik}(q)}{\partial q_j}\dot{q}_k\dot{q}_j - \frac{\partial m_{kj}(q)}{\partial q_i}\dot{q}_k\dot{q}_j\right\} \\ &= \frac{1}{2}\left\{\frac{\partial m_{ij}(q)}{\partial q_k} + \frac{\partial m_{ik}(q)}{\partial q_j} - \frac{\partial m_{kj}(q)}{\partial q_i}\right\}\dot{q}_k\dot{q}_j \qquad (2.2.16)\end{aligned}$$

The use of Christoffel symbols, allows to write

$$C_{i,j}(q,\dot{q}) = \sum_{k=1}^{n} C_{i,j,k}\dot{q}_k \qquad \text{where} \qquad C_{i,j,k} = \frac{1}{2}\left\{\frac{\partial m_{ij}(q)}{\partial q_k} + \frac{\partial m_{ik}(q)}{\partial q_j} - \frac{\partial m_{kj}(q)}{\partial q_i}\right\} \qquad (2.2.17)$$

Substitute it into the equation of motion we get:

$$\tau_i = \sum_{j=1}^{n} m_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\left(\sum_{k=1}^{n} C_{i,j,k}\dot{q}_k\right)\dot{q}_j + \frac{\partial U(q)}{\partial q_i} \qquad (2.2.18)$$

Let now $G(q)$ be the vector whose i^{th} coordinates are given by $\frac{\partial U(q)}{\partial q_i}$. We can write equation (2.2.18) in a compact form:

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) \qquad (2.2.19)$$

Where $C(q,\dot{q}) \in \mathbb{R}^n$ is the Coriolis/Centripetal vector and $G(q) \in \mathbb{R}^n$ represent the gravity vector. Equation(2.2.19) is the final form of the robot dynamical equation.

The dynamical model of manipulator can be obtained by another method as follows; we have the arm Lagrangian is:

$$L(q,\dot{q}) = K(q,\dot{q}) - U(q) = \frac{1}{2}\dot{q}^T M(q)\dot{q} - U(q) \qquad (2.2.20)$$

Then, we have

$$\frac{\partial}{\partial \dot{q}}(L) = \frac{\partial K(q,\dot{q})}{\partial \dot{q}} = M(q)\dot{q} \Rightarrow \frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}}(L)\right) = M(q)\ddot{q} + \dot{M}(q)\dot{q} \qquad (2.2.21)$$

And

$$\frac{\partial}{\partial q}(L) = \frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M(q)\dot{q}) - \frac{\partial U(q)}{\partial q} \qquad (2.2.22)$$

Therefore, replacing (2.2.21) and (2.2.22) into (2.2.2) the arm dynamical equation will be

$$\tau = \frac{\partial}{\partial \dot{q}}(L) - \frac{\partial}{\partial q}(L) = M(q)\ddot{q} + \dot{M}(q)\dot{q} + \frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M(q)\dot{q}) - \frac{\partial U(q)}{\partial q} \qquad (2.2.23)$$

Where τ is the generalized applied torque vector, then

$$M(q)\ddot{q} + \left\{ \dot{M}(q)\dot{q} + \frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M(q)\dot{q}) \right\} + G(q) = \tau \qquad (2.2.24)$$

Finally, considering the equality of the Coriolis/Centripetal term

$$C(q,\dot{q})\dot{q} = \left\{ \dot{M}(q)\dot{q} + \frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M(q)\dot{q}) \right\} \qquad (2.2.25)$$

Let us now introduce the structural properties of the Turret Dynamics

2.3. Structural properties of the Turret Dynamics

In this section, we investigate the detailed structure and the properties of the different terms found in the Turret dynamical equation given by equation (2.2.19) of an n-Degree Of Freedom (DOF) Turret Gun. These properties are of great utility in the design of controllers and observers presented in this work.

In reality, a Turret Gun is always affected by friction and disturbances. Therefore, we shall generalize the arm model given by (2.2.19) to the form

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d \qquad (2.3.1)$$

Where the additional friction term is given as

$$F(\dot{q}) = F_v\dot{q} + F_d(q) \qquad (2.3.2)$$

With F_v the coefficient matrix of viscous friction and F_d a dynamical friction term. Also, a disturbance term, τ_d, is added.

2.3.1 Properties of the Inertia Matrix

The $n \times n$ inertia matrix $M(q)$,of the manipulator is symmetric and positive definite. In addition, $M(q)$ is bounded above and bounded below as follow

$$\mu_1 I_n \le M(q) \le \mu_2 I_n \qquad (2.3.3)$$

With μ_1 and μ_2 being positive scalars that may be computed for any arm, and I_n is the $n\times n$ identity matrix.
Likewise, the inverse of the inertia matrix is bounded, since

$$\frac{1}{\mu_2}I_n \le M^{-1}(q) \le \frac{1}{\mu_1}I_n \tag{2.3.4}$$

If the joints are revolute, the bounds μ_1 and μ_2 can be scalar functions of the joint variable q.
The boundedness property of the inertia matrix may also be expressed as

$$M_m \le \| M(q) \| \le M_M \tag{2.3.5}$$

Where any induced norm can be used to define the positive scalars M_m and M_M.

2.3.2 Properties of the Coriolis/Centripetal Term

The Coriolis/Centripetal matrix is characterized by the following three properties.

Property 01: the matrix $N(q,\dot q)=\dot M(q)-2C(q,\dot q)$ is skew-symmetric, so that

$$x^T N(q,\dot q)x = 0 \qquad \text{for all } x\in\mathbb{R}^n \tag{2.3.6}$$

$$N_{ij} = \dot m_{ij}(q) - 2\sum_{k=1}^{n} C_{k,j,i}(q,\dot q)\dot q_k = \sum_{k=1}^{n}\frac{\partial m_{ij}(q)}{\partial q_k}\dot q_k - 2\sum_{k=1}^{n} C_{k,j,i}(q,\dot q)\dot q_k \tag{2.3.7}$$

Substituting the expression of $C_{k,j,i}(q,\dot q)\dot q_k$ given in equation (2.2.17), we get

$$N_{ij} = \sum_{k=1}^{n}\left\{\frac{\partial m_{ij}(q)}{\partial q_k} - \frac{\partial m_{ij}(q)}{\partial q_k} - \frac{\partial m_{ik}(q)}{\partial q_j} + \frac{\partial m_{kj}(q)}{\partial q_i}\right\}\dot q_k \tag{2.3.8.a}$$

$$N_{ij} = \sum_{k=1}^{n}\left\{- \frac{\partial m_{ik}(q)}{\partial q_j} + \frac{\partial m_{kj}(q)}{\partial q_i}\right\}\dot q_k \tag{2.3.8.b}$$

Because it is symmetric, we can also write

$$N_{ij} = -\sum_{k=1}^{n}\left\{\frac{\partial m_{ik}(q)}{\partial q_j} - \frac{\partial m_{kj}(q)}{\partial q_i}\right\}\dot q_k = -\sum_{k=1}^{n}\left\{\frac{\partial m_{ik}(q)}{\partial q_j} + \frac{\partial m_{kj}(q)}{\partial q_i}\right\}\dot q_k = -N_{ij} \tag{2.3.8.c}$$

We can interpret this property using the statement of conservation of energy. Note that the Turret Dynamics can be written in terms of the skew-symmetric matrix as

$$M(q)\ddot{q} + \tfrac{1}{2}\left\{\dot{M}(q) - N(q,\dot{q})\right\}\dot{q} = \tau - G(q) \qquad (2.3.9)$$

Where friction and disturbance are ignored. Now, with K being the kinetic erergy, we have

$$\frac{\partial K}{dt} = \frac{1}{2}\frac{d}{dt}(\dot{q}^T M(q)\dot{q}) = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} \qquad (2.3.10)$$

Hence, equation (2.3.9) yields

$$\dot{K} = \frac{1}{2}\dot{q}^T N(q,\dot{q})\dot{q} - \dot{q}^T(\tau - G(q)) \qquad (2.3.11)$$

Or $$\dot{K} = -\dot{q}^T(\tau - G(q)) \qquad (2.3.12)$$

This is a statement of the conservation of the conservation of energy, with the right hand side representing the power input from the net external forces. The skew-symmetry of $N(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is nothing than a statement that fictitious forces $N(q,\dot{q})$ do no work.

Property 02: The matrix $C(q,\dot{q})$ verifies the following relation

$$C(q,x)y = C(q,y)x \qquad \text{for all } x, y \in \mathbb{R}^n \qquad (2.3.13)$$

Actually, the i^{th} coordinate of the vector $C(q,x)y$ is

$$C_{ij}(q,x)y_j = \sum_{k=1}^{n} C_{ijk}(q)x_k y_j = \sum_{k=1}^{n} C_{ijk}(q)y_k x_j = C_{ij}(q,y)x_j \qquad (2.3.14)$$

Since the Christoffel symbols are symmetric with respect to the two last indices.

Property 03: The norm of $C(q,\dot{q})$ verifies the relation

$$\| C(q,x) \| \leq C_M \| x \| \qquad \text{for all } x \in \mathbb{R}^n \qquad (2.3.15)$$

Actually, from equation (2.2.17) we can write

$$C_{i,j}(q,x) = \frac{1}{2}\sum_{k=1}^{n}\left\{\frac{\partial m_{ij}(q)}{\partial q_k} + \frac{\partial m_{ik}(q)}{\partial q_j} - \frac{\partial m_{kj}(q)}{\partial q_i}\right\} x_k = \sum_{k=1}^{n} C_{ijk}(q)x_k \qquad (2.3.16)$$

In the case of an n-degree of freedom manipulator with revolute links, we have

$$\max_{i,j,k} \sup_{q\in\mathbb{R}^n} \left\{ \frac{\partial m_{ij}(q)}{\partial q_k} + \frac{\partial m_{ik}(q)}{\partial q_j} - \frac{\partial m_{kj}(q)}{\partial q_i} \right\} \leq \alpha \qquad (2.317)$$

Which yields

$$\| C_{ij}(q,x) \| \leq \frac{n}{2}\alpha \qquad (2.3.18)$$

In prismatic joints, α is a scalar function of q.

2.3.3 Properties of the Gravity, Friction and Disturbance Terms

2.3.3.1 Properties of the Gravity Term

For revolute Turret Gun, the joint variable appears only in $G(q)$ through sine or cosine function. Consequently, we can have the following bound

$$\| G(q) \| \leq G_M \qquad (2.3.19)$$

Where $\| . \|$ is any appropriate vector norm and is a scalar function.

2.3.3.2 Properties of the Friction Term

The friction term described by equation (2.3.2) is composed of the viscous friction and the dynamic friction. Assuming that the friction on each joint depends, uniquely, on the considered joint velocity. We have then for $i = 1, \ldots, n$.

$$F_v = diag(v_i) \qquad (2.3.20)$$

$$F_d(\dot{q}) = [k_1 \operatorname{sgn}(\dot{q}_1) \ \ldots \ k_i \operatorname{sgn}(\dot{q}_i) \ \ldots \ k_n \operatorname{sgn}(\dot{q}_n)] \qquad (2.3.21)$$

With k_i being known constant coefficient, and the sign function defined by

$$\operatorname{sgn}(x) = \begin{cases} +1 & if \ x > 0 \\ -1 & if \ x < 0 \\ 0 & if \ x = 0 \end{cases} \qquad (2.3.22)$$

A bound on the friction terms may be assumed of the form

$$\| F_v \dot{q} + F_d(q) \| \leq F_{1.M} + F_{2.M} \| \dot{q} \| \qquad (2.3.23)$$

With $F_{1.M}$ and F_{2M} are known for a specific arm and $\| . \|$ a suitable norm.

2.3.3.3 Properties of the Disturbance Term

The Turret Gun dynamical equation (2.3.1) has a disturbance error τ_d, witch could represent inaccurately modeled dynamics, external perturbations and so on.
We assume that

$$\|\tau_d\| \leq T_M \tag{2.3.24}$$

Where T_M is a scalar constant that may be computed for a given Turret and $\|.\|$ is any suitable norm.

2.3.4 Linearity in the Parameters

The Turret Gun dynamical equation (2.2.19) enjoys an additional property that is often used in the design of adaptive controllers (Slotine and Li,1987; Craig,1988; Ortega and Spong,1988; Spong and Ortega, 1990; Lewis et al,1993). Namely, equation (2.2.19) is linear in the parameters, a property first exploited by Craig (1988) in adaptive control. This is important, since some or the entire robot dynamical equation may be unknown, thus the dynamics are linear in the unknown terms.
This property is expressed as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta \tag{2.3.25}$$

With θ is the vector containing all the parameters of the manipulator, and $Y(q,\dot{q},\ddot{q})$ is a matrix of robot functions depending on the joint variables (position, velocity, and acceleration). This matrix may be computed for any robot manipulator or in our case any robotic Turret and so is known.

2.3.5 State Space Representation

From the Turret Dynamics given by (2.2.19), we can give a state space formulation obtained by defining the state vector $x \in \mathbb{R}^n$ as

$$x = (q^T \ \ \dot{q}^T)^T \tag{2.3.26}$$

For simplicity, we neglect the disturbances and the friction terms and note that according to (2.2.19), we can write

$$\frac{d}{dt}\dot{q} = -M^{-1}(q)\big(C(q,\dot{q})\dot{q} + G(q)\big) + M^{-1}(q)\tau \tag{2.3.27}$$

Now, we can directly write the state representation

$$\dot{x} = \begin{pmatrix} \dot{q} \\ -M^{-1}(q)\left(C(q,\dot{q})\dot{q} + G(q)\right) \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}(q) \end{pmatrix}\tau \tag{2.3.28}$$

This last equation may be rewritten as

$$\dot{x} = \begin{pmatrix} 0 & I_n \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ I_n \end{pmatrix} u \tag{2.3.29}$$

With the nonlinear control input $u$is defined as

$$u = -M^{-1}(q)\left(C(q,\dot{q})\dot{q} + G(q) - \tau\right) \tag{2.3.30}$$

2.3.6 Passivity Property

The class of Turret manipulators systems described by (2.2.19), is passive from τ to $\dot{q}$ that is, there exist a constant β such that

$$\langle \dot{q}, \tau \rangle \equiv \int_0^T \dot{q}^T(\sigma)\tau(\sigma)d\sigma \geq -\beta \tag{2.3.31}$$

This property cannot be directly shown from the Turret equation as given by (2.2.19). For this, another way to derive the dynamics of rigid robot systems in general, is the Hamiltonian formalism (Berghuis, 1993a, Goldstein, 1980).
We introduce the Hamiltonian matrix c given by

$$H(q,p) = p^T\dot{q} - L(q,\dot{q}) \tag{2.3.32}$$

Where, as before $L(q,\dot{q})$ is the Lagrangian, and p is the n×1 vector of generalized momenta, defined as

$$p = \frac{\partial L(q,\dot{q})}{\partial \dot{q}} = M(q)\dot{q} \tag{2.3.33}$$

Now, from the definition of the momenta we can deduce that the Hamiltonian is just the summation of kinetic and potential energies as

$$H(q,p) = p^T\dot{q} - L(q,\dot{q}) = K(q,\dot{q}) + U(q) \tag{2.3.34}$$

To this point, by taking the time derivate of the Hamiltonian $H(q,p)$ in (2.3.34)

$$\frac{dH(q,p)}{dt} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} + \dot{q}^T\frac{\partial U(q)}{\partial q} \tag{2.3.35}$$

$$= \dot{q}^T(\tau - C(q,\dot{q})\dot{q} - G(q)) + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} + \dot{q}^T\frac{\partial U(q)}{\partial q} \tag{2.3.36}$$

Where we have substituted for $M(q)\ddot{q}$ using the equation of motion. Collecting terms and using the fact that $G(q) = \dfrac{\partial U(q)}{\partial q}$ yields

$$\dot{H}(q,p) = \dot{q}^T \tau + \frac{1}{2}\dot{q}^T \left\{ \dot{M}(q) - 2C(q,\dot{q}) \right\} \dot{q} = \dot{q}^T \tau \qquad (2.3.37)$$

The latter equality following from the skew-symmetry property. Integrating both sides of (2.3.36) with respect to time gives

$$\int_0^T \dot{q}^T(\sigma)\tau(\sigma)d\sigma = H(T) - H(0) \geq -H(0) \qquad (2.3.38)$$

Since the total energy is non-negative, and the passivity property therefore follows with $\beta = H(0)$.

Remark

This passivity property merely states that the Turret cannot create energy. From a control point of view, a passive system cannot go unstable. Then, if the controller is designed in such a way to preserve this property, the asymptotic stability of the closed loop system is guaranteed. A class of controllers that is based on this property is detailed in next chapter.

2.4. Dynamic Model of Massive Gun

Consider a two-load tracking system (Fig. 2.1) which consists of a rotating base (turret) with a tilting arm (barrel). This tracking system has two loads: the turret load and the barrel load, with each being driven by motor through a gear train which has backlash, flexibility and damping.

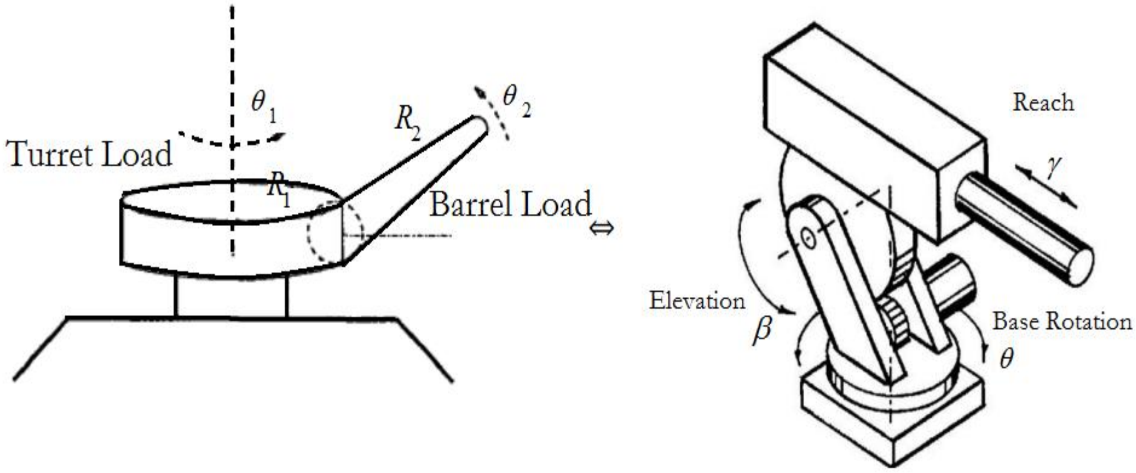

Figure 2.1 Two-load Tracking System Studied

Let m_1 be the mass,cbe the radius, and θ_1 be the angular position, of the turret load,

m_2 be the mass, R_2 be the length, and θ_2 be the angular position, of the barrel load, as shown in Fig. 2.1. (BEKHITI B. (2019) "Dynamic Modeling & Control of Large Space Structures: Part1 Missiles & Space-Crafts")

Remark: Gun's barrel is a tube which the bullet is projected from, in other word, it is a caved arm that contains gun's muzzle. Gun's turret is the basis on which the barrel stands.

Hypothesis:

-We have assumed that the turret base is a uniform (density and shape) thin disk of mass m_1 rotates around axis "O".

-We have assumed that a barrel is a uniform (density and shape) thin rod of mass m_2 concentrated at the center "C" of the rod (i.e. $L = R_2/2$). The perpendicular distance from turret's axis of rotation and "C" is given by

$$d = \left(R_1 + \frac{1}{2} R_2 \cos(\theta_2) \right) \tag{2.4.1}$$

-The moment of inertia of the rod about an axis that passes through its end is:

$$I = \frac{1}{3} m_2 \left(\frac{R_2}{2} \cos(\theta_2) \right)^2 \tag{2.4.2}$$

From classical mechanics it is very well-known that the moment of inertia of any object about an axis through its center of mass is the minimum moment of inertia for an axis in that direction in space. The moment of inertia about any axis parallel to that axis through the center of mass is given by:

$$I_{parallel} = I_{cm} + md^2 \tag{2.4.3}$$

The moment of inertia of a composite object can be obtained by superposition of the moments of its constituent parts. The parallel axis theorem is an important part of this process. For example, in our case:

$$\begin{aligned} I = I_{Turret} + I_{Barrel} &= \frac{1}{2} m_1 R_1^2 + \left\{ \frac{1}{3} m_2 \left(\frac{R_2}{2} \cos(\theta_2) \right)^2 + m_2 \left(R_1 + \frac{R_2}{2} \cos(\theta_2) \right)^2 \right\} \\ &= \frac{1}{2} m_1 R_1^2 + \frac{1}{3} m_2 R_2^2 \cos^2(\theta_2) + m_2 R_1^2 + m_2 R_1 R_2 \cos(\theta_2) \end{aligned} \tag{2.4.4}$$

The kinetic energy of an object has two components: one that is due to its linear motion and the second is due to its rotational motion. If the linear velocity of the center of mass of a link is v_{ci}, and if the angular velocity of the link is ω_i, the kinetic energy, K_i of the object is:

$$K_i = \frac{1}{2}(m_i\, v_{ci}{}^T v_{ci} + \omega_i^T I_{ci}\omega_i) \quad \& \, K = \sum K_i \tag{2.4.5}$$

Where I_{ci} is the inertia tensor of an object i computed with respect to the object's center of mass, C_i. In our case study only a rotation motion is included therefore:

$$K_i = \frac{1}{2}\omega_i^T I_{ci}\omega_i \tag{2.4.6}$$

$$\begin{aligned} K &= \sum K_i \\ &= \frac{1}{2}\left\{\left(\frac{1}{2}m_1 R_1{}^2 + \frac{1}{3}m_2 R_2{}^2 \cos^2(\theta_2) + m_2 R_1{}^2 + m_2 R_1 R_2 \cos(\theta_2)\right)\dot{\theta}_1{}^2 + \left(\frac{1}{3}m_2 R_2{}^2\right)\dot{\theta}_2{}^2\right\} \end{aligned} \tag{2.4.7}$$

The gravity forces are the gradient of the potential energy of the mechanism. The potential energy of link i increases with the elevation of its center of mass C_i. This energy is proportional to the mass, the gravity constant, and to the height of the center of mass.

$$U_i = m_i g h_0 + U_0 \tag{2.4.8}$$

Where U_0 represents the potential energy at some reference level. The height is given as the projection of the position vector $\vec{r}_{ci}$ along the gravity direction,

$$U_i = -m_i g^T \vec{r}_{ci} \tag{2.4.9}$$

In our case, we have:

$$U = \sum U_i = -\frac{1}{2}m_2 g R_2 \sin(\theta_2) \tag{2.4.10}$$

Lagrange's equation involve a scalar quantity L, the Lagrangian, which represents the difference between the two scalars corresponding to the kinetic energy K and the potential U of the mechanism, see (2.2.1).

For an n-DOF mechanism, the Lagrange formulation provides the n equations of motion in the form (2.2.2).

Following the Lagrangian method, we can derive the system dynamic equations from (2.3.25) :

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+G(\theta)=\Gamma \tag{2.4.11}$$

Or

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & I\\ M(\theta) & C(\theta,\dot{\theta})\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ G(\theta)\end{bmatrix}=\begin{bmatrix}0\\ \Gamma\end{bmatrix} \tag{2.4.12}$$

Where Γ is the vector of the external torques, $M(\theta)$ is the matrix of inertia, $C(\theta,\dot{\theta})$ is coriolis and centrifugal force, and $G(\theta)$ is gravity loading force

With

$$\theta=[\theta_1,\theta_2]^T,\Gamma=[\Gamma_1,\Gamma_2]^T,$$

$$M(\theta)=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix},C(\theta)=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix},$$

$$G(\theta)=\left[0,\frac{1}{2}m_2gR_2\cos(\theta_2)\right]^T,$$

$$M_{11}=\frac{1}{2}m_1R_1^{\ 2}+\frac{1}{3}m_2R_2^{\ 2}\cos^2(\theta_2)+m_2R_1^{\ 2}+m_2R_1R_2\cos(\theta_2),$$

$$M_{22}=\frac{1}{3}m_2R_2^{\ 2},$$

$$M_{21}=M_{12}=0,$$

$$C_{11}=-m_2R_1R_2\sin(\theta_2)\dot{\theta}_2\ ,\ C_{12}=\frac{1}{3}m_2R_2^{\ 2}\sin(2\theta_2)\dot{\theta}_1\ ,$$

$$C_{21}=\left[\frac{1}{2}m_2R_1R_2\sin(\theta_2)+\frac{1}{6}m_2R_2^{\ 2}\sin(2\theta_2)\right]\dot{\theta}_1,\ C_{22}=0.$$

Electromechanical rotary actuators are designed to improve stabilization accuracy and slew rates for some relatively new turret applications. This series of rotary actuators with high torque DC stepper motor (i.e. brushless DC motors) are designed to meet and exceed the demanding requirement of many combat vehicle turret applications. The used DC motors equations are:

$$\Gamma_i=k_i\left(\frac{s+b}{s^2+a_1s+a_2}\right)V_i \tag{2.4.12}$$

Where $(V_i,i=1,2)$ are voltages applied to the motor, and k_i,a_1,a_2,b are motor constants.

The stepper will be able to hold the gun in place horizontally/vertically when the CG is not aligned directly with the rotational axis. In the next table, we introduce some of private parameters:

Electromechanical actuators Specification		**Mechanical Specifications**
Peak Torque	1stmotor 910 in-lbs (103 Nm) 2ndmotor 227.5 in-lbs (25.75 Nm)	
Continuous Torque	1stmotor 300 in-lbs (34 Nm) 2ndmotor 75 in-lbs (8.5 Nm)	
No Load Speed	390 rpm	$m_1 = 145\,\text{Kg}$
Rated Power	1stmotor 1.25 hp (932 W) 2ndmotor 0.75 hp (232 W)	$m_2 = 25\,\text{Kg}$
Supply Voltage	24 VDC, 48 VDC, 270 VDC, or 650 VDC	$R_1 = 75\,cm$
Weight	1stmotor 90 in-lbs (41 Kg) 2ndmotor 22 in-lbs (10 Kg)	$R_2 = 2.3\,cm$

Table 2.1 Parameters of the Motors

Chapter 3

Classical Control of Turret Gun Manipulators

The purpose of this chapter is to describe different technique to classical control of rigid robot manipulators in general, to apply it to Turret Guns. Both regulation and trajectory tracking problems are considered. Before talking on the controller design we could start by the motion planning and trajectory generation, then for the control point of view we show first that PD-type controllers are useful in solving the regulation problem, wheras the tracking problem needs some sophisticated control algorithms. Simulation study is performed on 2 DOF Turret Gun.

3.1. Introduction

Over the last two decades, the control of robot manipulators has taken the interest of many researchers, who developed, through several survey works (Zodiac, 1992; Spong et al., 1994) many different control law for such systems based on different approaches.

Traditionally, control design in industrial robot manipulators is understood as simple fact of using a simple Proportional and Derivative (PD) regulator at the level of each motor driving the manipulator joints. As simple PD controllers are generally good in stabilizing second order systems, the control of n-link manipulator can be interpreted as the control of n-independent chains of double integrators, for which a PD controller can be designed. This known as the independent joint control (Lewis et al., 1993). This scheme totally ignores the system dynamics and attempts to control the manipulator by using the locally measured variables of each joint.

Actually, the first experiments conducted with real robot were performed with simple PD compensators and have been, in general, satisfactory as far as stability and middle range performance are concerned. The main reason that the experiments were successfully conducted is the **"local"** equivalence of the complete industrial robot dynamics to a linear model described by a set of second order systems therefore, is locally stabilizable. This local domain enlarges as the nonlinearities and the coupling terms become less important. In addition, and as we will see, this attraction region can be arbitrarily enlarged by increasing the controller gains.

In this chapter, we shall discuss the some control schemes applied to robot manipulators as they are used to solve both regulation and trajectory tracking problems. The regulation problem also called "point to point", the control objective is to regulate the joint angles about the desired position in spite of torque disturbances. On the other hand, the trajectory tracking consists in following a time varying reference trajectory specified within the manipulator's workspace.

First, the regulation problem is treated using simple PD control. This control law will ensure the global stability of the closed loop system with the price of using high proportional gains. This fact will cause a steady state error and destroys the regulation. To improve this performance, a PD controller with gravity compensation is considered.

Next, we discuss the trajectory tracking problem, in this case, we may expect that the same PD controller will still work well provided that the magnitude of the velocity component a given desired trajectory is relatively small. However, it is claimed much of the literature (Arimoto, 1990; Khelfi, 1995; Berghuis, 1993a;

Spong et al., 1994; Zodiac, 1992; Asada and Slotine, 1986) that in the case of fast movement for trajectory tracking the performance of such a classical servo control becomes unsatisfactory.

According to this argument, several advanced control technique have been proposed so far (Asada and Slotine, 1986; Slotine and Li, 1987a; Paden and Panja, 1988; Criage, 1988). In our case, we selected the most widely considered control laws. Among these techniques, we will see a method that attempt to partially compensate the robot dynamics nonlinearities and guarantee that the global asymptotic stability. Then, a method that uses the complete robot dynamics in the purpose of completely linearized and decouple the system will be discussed. In this case, the closed loop system is reduced to n-decoupled linear second order systems, for which simple linear control law can be designed to guarantee the global exponential convergence of the tracking error. Finally, we consider the non-adaptive version of the well-known Slotine and Li algorithm (1987a). The benefic contribution of this control law is that it uses a new variable that will yield to the global stability result sing simple stability arguments.

Finally, simulation study on 2-DOF Turret Gun manipulator discussed in section 2.5 is performed. Before all this, we shall start this chapter by study the problem of generating the reference trajectory used as an input to the robot controller.

3.2 Motion Planning

3.2.1 Introduction

In a workbench, a robot is concerned in performing three big classes of tasks: pure displacement, pure static force and complaint tasks combining displacements and forces. In this part, we discuss the problem of generating the reference point used as an input to the robot controller in the case of pure displacements.

During a displacement task, all what we wait from robot is to follow, with prescribed time law, a trajectory defined by a series of frames, referred to as points, corresponding to successive positions of the robot's end-effector. The problem of motion planning, then is to compute the reference for the control law that guarantee that the robot passes through these points for the control purpose considered in this work, we consider as a reference input the joint coordinates corresponding to the desired joint positions. The problem of motion planning is deeply discussed in Yoshikawa (1990), Dombre et al. (1988) and Lewis et al. (1993).

3.2.2 Motion Planning between Two Points

We consider an n-degree of freedom manipulator. Let q^i and q^f be the joint coordinate vector corresponding to the initial and final configurations respectively. We note by V_M and A_M the maximal velocity and acceleration vectors respectively.

The interpolated motion between q^i and q^f is function of time t, is described by the following equation:

$$q(t)=q^i+r(t)\times D \quad 0\leq t\leq t_f \tag{3.2.1}$$

With

$$D=q^f-q^i \tag{3.2.2}$$

Where the value at the limits of the interpolation function $r(t)$ are given by:

$$r(0)=0, \qquad r(t_f) \tag{3.2.3}$$

Expression (3.2.1) can also be written as

$$q(t)=q^f+\left(1-r(t)\right)\times D \qquad\qquad 0\leq t\leq t_f \tag{3.2.4}$$

For which is a suitable formulation in the case the target tracking, when term q^f varies.

Many functions permit so satisfy the passage through q^f at $t=0$and through q^f at $t=t_f$. The most used interpolation methods in robotics are polynomial interpolation, Bang-Bang law, and are both discussed in this part.

3.2.2.1 Polynomial Interpolation

The most frequently encountered polynomial interpolation techniques are linear interpolation and interpolation by three and five order polynomials.

3.2.2.1.a Linear Interpolation

This method is the simplest interpolation method. The motion of each joint is described by a linear equation of time. The motion equation is written as

$$q(t)=q^i+\frac{t}{t_f}\times D \qquad\qquad 0\leq t\leq t_f \tag{3.2.5}$$

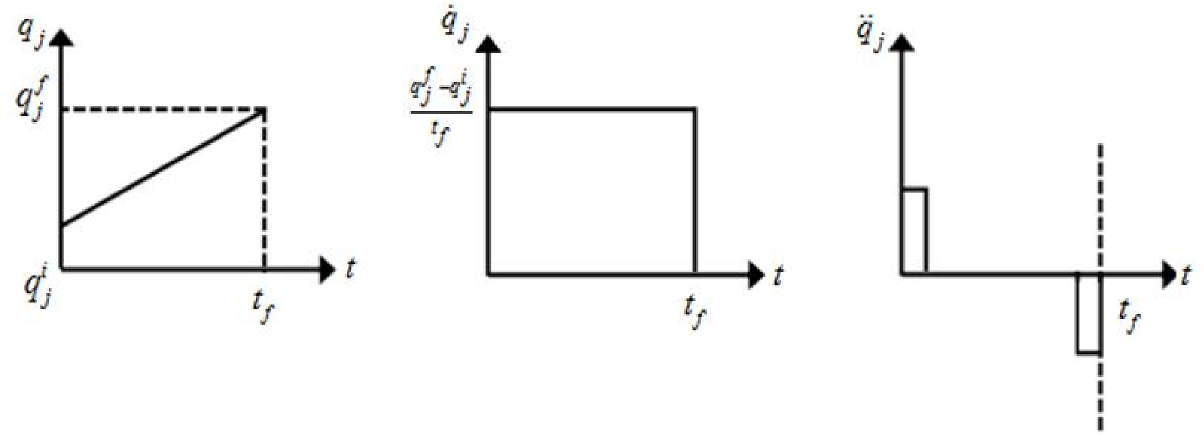

Figure 3.1 Linear interpolation for given joint j

3.2.2.1.b Three degree Polynomial Interpolation

If we impose a null velocity to the start and end points, we will add two additional constants to the above two constraints on the position in the linear interpolation method; The polynomial that satisfies these constraints is of minimal degree of three and has the general form

$$q(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 \tag{3.2.6}$$

With an initial and final conditions

$$q(0) = q^i, \quad q(t_f) = q^f, \quad \dot{q}(0) = 0, \quad \dot{q}(t_f) = 0 \tag{3.2.7}$$

Using these initial and final conditions, we can find the coefficients of (3.2.6) as

$$\alpha_0 = q^i, \quad \alpha_1 = 0, \quad \alpha_2 = \frac{3}{t_f^{\,2}} \times D, \quad \alpha_3 = \frac{2}{t_f^{\,3}} \times D \tag{3.2.8}$$

Expression (3.2.6) can also be rewritten under the form of (3.2.1), (3.2.4) taking

$$r(t) = 3 \times \left(\frac{t}{t_f}\right)^2 + 2 \times \left(\frac{t}{t_f}\right)^3 \tag{3.2.9}$$

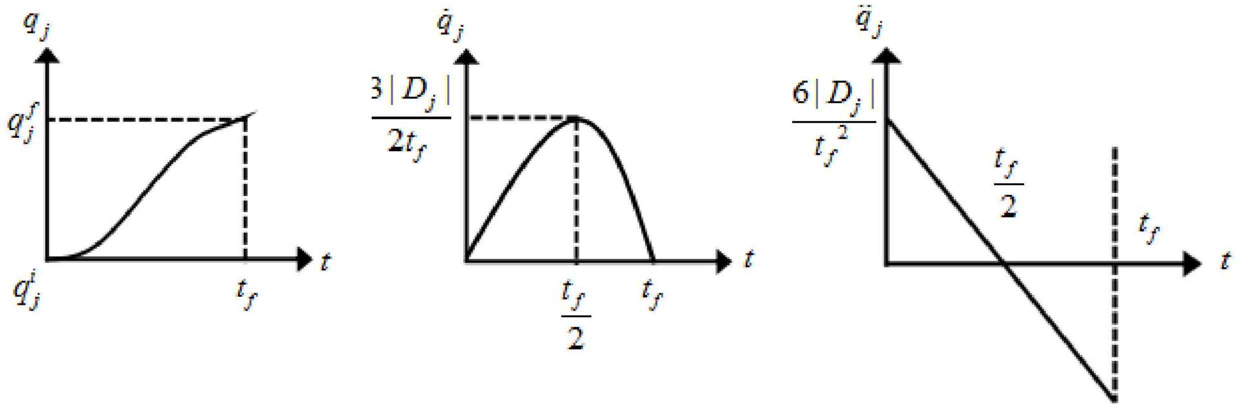

Figure 3.2 Third order polynomial law for a given joint j

3.2.2.1.c Five degree Polynomial Interpolation

If we seek the continuity of the acceleration, we should satisfy six constraints and the interpolation polynomial should be of order five. Choosing in addition to (3.2.7), the conditions

$$\ddot{q}_j(0)=0, \qquad \ddot{q}_j(t_f)=0 \tag{3.2.10}$$

Using the same parameterization as before, we can easily show that the motion in this case is described by (3.2.1) with

$$r(t)=10\times\left(\frac{t}{t_f}\right)^3-15\times\left(\frac{t}{t_f}\right)^4+6\times\left(\frac{t}{t_f}\right)^5 \tag{3.2.11}$$

The evolution of position, velocity and acceleration for joint j with a five-degree polynomial are represented in fig. 3.3.

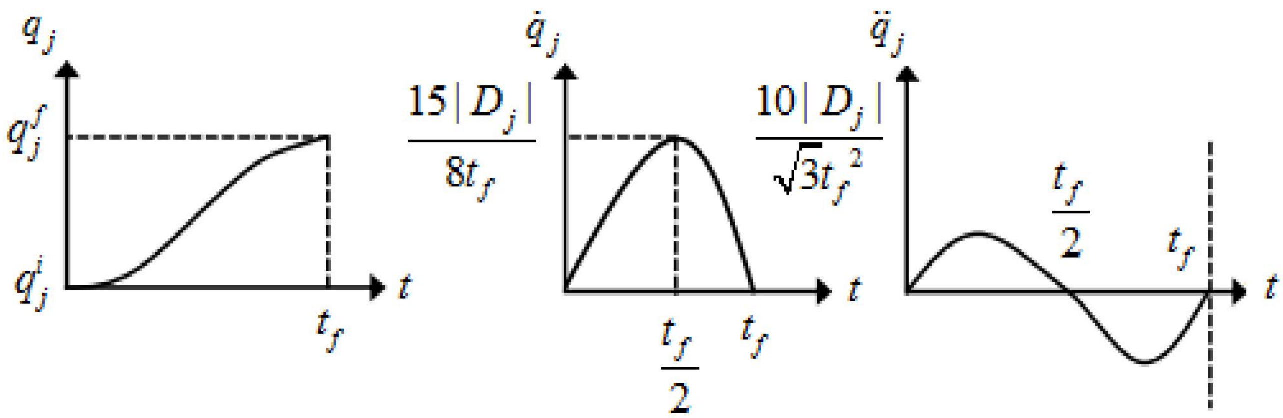

Figure 3.3 Fifth order polynomial law for a given joint j

3.2.2.2 Bang-Bang law

The motion in this case is constrained by a constant acceleration phase until $t_f/2$ and a constant deceleration phase until the point (see fig. 3.4). The initial and final velocities are zero. The motion is then continuous in position and velocity but not in acceleration.

The position is given by

$$\begin{cases} q(t) = q^i + 2\times\left(\dfrac{t}{t_f}\right)^2 \times D & 0 \le t \le \dfrac{t_f}{2} \\ q(t) = q^i + \left[-1 + 4\times\left(\dfrac{t}{t_f}\right) - 2\times\left(\dfrac{t}{t_f}\right)^2\right]\times D & \dfrac{t_f}{2} \le t \le t_f \end{cases} \tag{3.2.12}$$

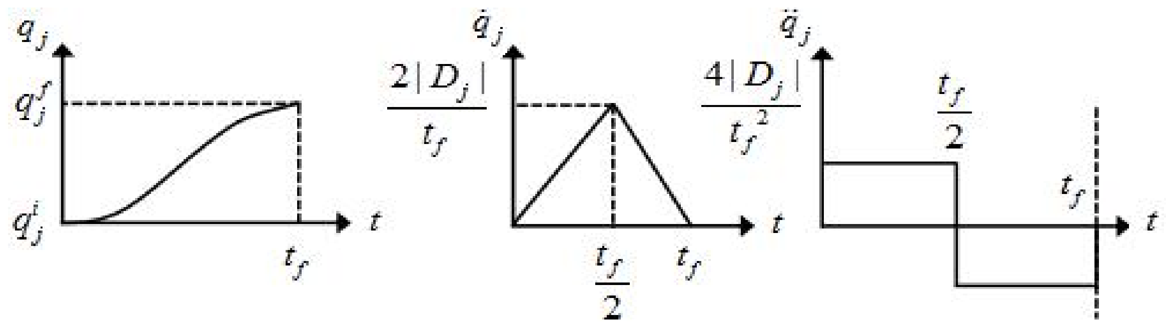

Figure 3.4 Bang-Bang law with a given joint j

3.3 Robot Control Design

The objective of rigid robot manipulator control is to make the robot manipulator respond in a predictable and desirable fashion to a set of input signals. In this section, we try to present a large synthesis on the classical controllers applied to robot manipulators in general and thus to Turret Gun and study the stability of the closed loop system in each case.
We consider first controllers that deal with the regulation problem. A simple PD controller is shown to guarantee global uniform asymptotic stability with the condition of using high proportional gains. To cope with this condition, an extra nonlinear term is added to the PD controller in order to ensure the asymptotic stability of the closed loop system with no steady state error.

The trajectory tracking problem is treated using different nonlinear control laws, all of which, are based on using the complete or a part of the Turret dynamics in order to compensate for the nonlinearities present in the Turret equation of motion. The stability of the closed loop system is shown using Lyapunov's second method arguments.

Based on the structure of the proposed controllers, they are classified into three min classes. The first class contains the feedback linearization controllers that seek to completely linearize and decouple the robot dynamics. The second class contains the passivity-based controllers that neither attempt to linearize nor decouple the system, but they exploit the passivity property of the robot manipulator (cf. section 2.4.6) to search for the global asymptotic stability of the closed loop system (Zodiac, 1992; Spong et al., 1994; Ortega and Spong, 1989; Spong and Ortega, 1990; Slotine and Li, 1987a; Lewis et al., 1993; Berghuis, 1993a). The third class is variable structure system (VSS) based controllers that force the nonlinear dynamical system to converge exponentially to the desired trajectories via discontinuous control law.
These controllers are analyzed and commented through simulation on the 2 DOF Turret Gun presented in section 2.4.

3.3.1 Regulation Problem

4.3.1.1 PD Controller

In this first step, we use a simple PD controller to solve the position problem for robot manipulators and so in our case for the Turret Gun position problem. Neglecting friction and external disturbances, we consider the Turret dynamics described by the following equation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (3.3.1)$$

The regulation objective is to maintain the position of the manipulator joint variables constant around some desired constant position given by q_d. The PD compensator is given by

$$\tau = -K_p(q-q_d) - K_v\dot{q} \tag{3.3.2}$$

Let $\tilde{q} = q - q_d$ denote the position error variable. Then, we have $\dot{\tilde{q}} = \dot{q}$ and $\ddot{\tilde{q}} = \ddot{q}$ since $\dot{q}_d = 0$. Applying this control law to the robot dynamics gives the closed loop system

$$M(q)\ddot{\tilde{q}} + C(q,\dot{\tilde{q}})\dot{\tilde{q}} + G(q) + K_p\tilde{q} + K_v\dot{\tilde{q}} = 0 \tag{3.3.3}$$

The equilibrium points of the closed loop system are defined by the following set

$$S = \left\{(\tilde{q},\dot{\tilde{q}}) \ / \ G(\tilde{q}+q_d) + K_p\tilde{q} = 0, \qquad \dot{\tilde{q}} = 0\right\} \tag{3.3.4}$$

The stability analysis of the closed loop system is performed using the Lyapunov function candidate given by

$$V(q,\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p\tilde{q} + P(q) + P_0 \tag{3.3.5}$$

Where: $P(q)$ is the potential energy of the system P_0 is some positive constant such that the Lyapunov function V is positive definite.

The time derivative of V is obtained as

$$\dot{V}(q,\dot{\tilde{q}}) = \dot{\tilde{q}}^T M(q)\ddot{\tilde{q}} + \frac{1}{2}\dot{\tilde{q}}^T \dot{M}(q)\dot{\tilde{q}} + \dot{\tilde{q}}^T K_p\tilde{q} + \dot{P}(q) \tag{3.3.6}$$

The time derivative of the potential energy is calculated as

$$\dot{P}(q) = \left(\frac{\partial q}{\partial t}\right)^T \frac{\partial P}{\partial q} = \dot{q}^T G(q) \tag{3.3.7}$$

Since

$$\frac{\partial P}{\partial q} = G(q) \tag{3.3.8}$$

Then, evaluating (3.3.6) along (3.3.3) yields

$$\dot{V}(q,\dot{\tilde{q}}) = -\dot{\tilde{q}}^T C(q,\dot{q}) - \dot{\tilde{q}}^T K_v\dot{\tilde{q}} - \dot{\tilde{q}}^T K_p\tilde{q} - \dot{\tilde{q}}^T G(q) + \frac{1}{2}\dot{\tilde{q}}^T \dot{M}(q)\dot{\tilde{q}} + \dot{\tilde{q}}^T K_p\tilde{q} + \dot{\tilde{q}}^T G(q) \tag{3.3.9}$$

Exploiting property 2.1 (cf. section 2.4.2) of the matrix $\dot{M}(q)-2C(q,\dot{q})$ we get directly the result

$$\dot{V}(q,\dot{\tilde{q}})=-\dot{\tilde{q}}^T K_v \dot{\tilde{q}} \tag{3.3.10}$$

Which is a negative semi-definite function. Then the equilibrium point $(q,\dot{\tilde{q}})=(0,0)$ is stable in the sense of Lyapunov (cf. section 1.6).

This result shows that $\dot{V}$ is negative semi-definite, which is not sufficient to demonstrate that the equilibrium point $(\tilde{q},\dot{\tilde{q}})=(0,0)$ is asymptotically stable. We have now to prove that as $\dot{\tilde{q}}=0$, the robot does not reach a configuration $q \neq q_d$. This can be done, thanks to the La Salle invariant set theorem (theorem 1.10 section 1.6).

The set $\mathcal{R}$ of points in the neighborhood of the equilibrium point that satisfies $\dot{V}=0$ is such that $\dot{\tilde{q}}=0$ and thus $\ddot{\tilde{q}}=0$. From equation (3.3.3), we conclude that necessarily

$$G(q)+K_p\tilde{q}=0 \tag{3.3.11}$$

Which result true if

$$\tilde{q}=-K_p^{-1}G(q) \tag{3.3.12}$$

Then,

$$\|\tilde{q}\| \leq -\left\|K_p^{-1}\right\|\left\|G(q)\right\| \leq -G_M\left\|K_p^{-1}\right\| \tag{3.3.13}$$

Where $\|G(q)\|$ is an upper bound of the gravity vector given in equation (2.4.19) (cf. section 2.4.3). Thus, increasing the proportional gain can in principle arbitrarily reduce the steady state error given in (3.3.13). Unfortunately, measurement noise and other unmolded dynamics will limit the use of high gains. To overcome this problem, PID controllers may be used (Zodiac, 1992). It is shown that when adding an integral term to the PD type control law (3.3.2) the constraint on the controller proportional gains is removed and local asymptotic stability is ensured. A difficulty in using the integral action in the control law is that oscillations may appear due to the interaction between integral gain and present friction nonlinearities. Another solution proposed in Takegaki and Arimoto (1981) is discussed in the following section.

3.3.1.2 PD Controller with Gravity Compensation

We can see from the control law of a simple PD controller that the system dynamics are used. Thus, the nonlinearities of the robot system are not compensated for. Then, including some of nonlinear dynamic terms in the PD controller will probably give better results. Indeed, Takegaki and Arimoto (1981) modified the simple controller for desired set point control by inserting the nonlinear gravity term in the control law. We shall now study this type of control generally referred to as 'point-to-point' control (Zodiac, 1992).

Consider the robot dynamics given in equation (3.3.1), neglecting friction and external disturbances. The proposed control law is given as

$$\tau = G(q) - K_p\tilde{q} - K_v\dot{\tilde{q}} \tag{3.3.14}$$

Where $\tilde{q}$ and $\dot{\tilde{q}}$are defined as above.
This controller can be implemented according to the following figure.

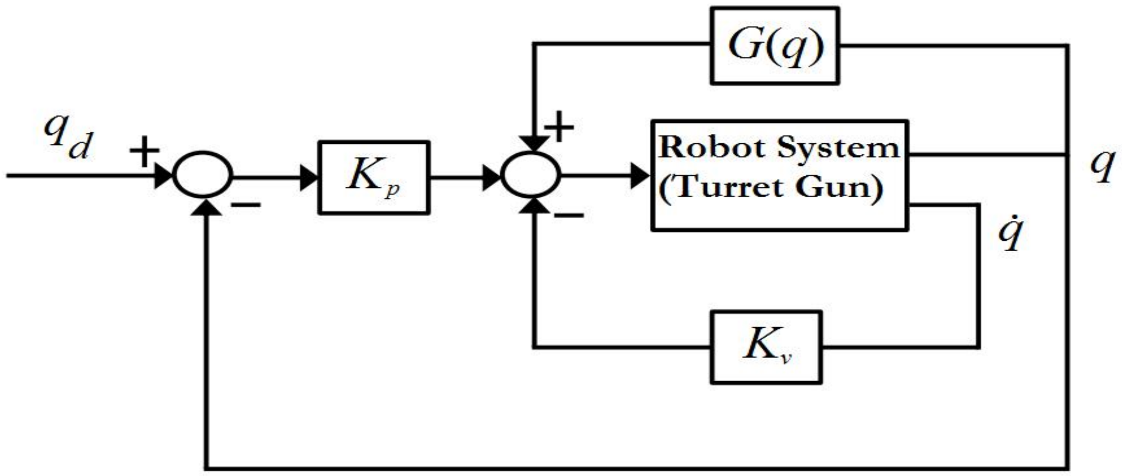

Figure 3.5 PD with Gravity Compensator Controller

Applying this control law to the Turret Dynamics, we obtain the closed loop system governed by

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + K_p\tilde{q} + K_v\dot{\tilde{q}} = 0 \tag{3.3.15}$$

To show the stability of the closed loop system, let us consider the following Lyapunov function candidate

$$V(q,\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p\tilde{q} \tag{3.3.16}$$

Which is composed of the kinetic energy and a potential energy introduced by the control law. The time derivative of this Lyapunov function is given by

$$\dot{V} = \dot{\tilde{q}}^T M(q)\ddot{\tilde{q}} + \frac{1}{2}\dot{\tilde{q}}^T \dot{M}(q)\dot{\tilde{q}} + \dot{\tilde{q}}^T K_p\tilde{q} \tag{3.3.17}$$

The arguments of $\dot{V}$ are omitted for sake of notation simplicity. Evaluating (3.3.17) along the error dynamics (3.3.15) yields

$$\dot{V} = -\dot{\tilde{q}}^T K_v \dot{\tilde{q}} + \frac{1}{2}\dot{\tilde{q}}^T\left(\dot{M}(q) - 2C(q,\dot{q})\right)\dot{\tilde{q}} \tag{3.3.18}$$

Since $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric (property 2.1, cf. section 2.4.2), we get finally

$$\dot{V} = -\dot{\tilde{q}}^T K_v \dot{\tilde{q}} \leq 0 \tag{3.3.19}$$

Which is negative function for $\dot{\tilde{q}} \neq 0$. This tells about the convergence of the velocity tracking error. To show the asymptotic convergence of the entire equilibrium point $(\tilde{q},\dot{\tilde{q}}) = (0,0)$, we have to invoke the LaSalle's invariant set theorem. Consider the set

$$S = \left\{(\tilde{q},\dot{\tilde{q}}) \;\; / \;\; \dot{V}(q,\dot{\tilde{q}}) = 0\right\} \tag{3.3.20}$$

If we have $\dot{\tilde{q}} = 0$, from equation (3.3.15), we will necessarily have $\tilde{q}$. Then, the only trajectory contained S in is the equilibrium point of the manipulator, and from LaSalle's theorem (theorem, 1.10), the global asymptotic stability of the equilibrium point $(\tilde{q},\dot{\tilde{q}}) = (0,0)$ is shown.

3.3.2 Trajectory Tracking Control

In the previous section, we have seen that the linear PD controller ensures the asymptotic stability when the proportional gains are high. Whereas, PD with gravity compensation controller will not require much higher gains to give better results as far as middle range performance is concerned. In this section, we consider the trajectory-tracing problem. Dawson et al. (1990) examined the stability of the PD controller for trajectory following problem of a robot manipulator. With some algebraic manipulations and judicious choice of a Lyapunov function, they have shown that the tracking error is Uniformly Bounded (U.B) (cf. theorem 1.14, section 1.7), if the PD gains are chosen greater than a specific bound and if initial tracking error is zero. Hence, when the application requires some rapid displacements of the manipulator and a big dynamical precision, it is necessary to design a more sophisticated control law that takes into account all part of the dynamical interaction forces.

3.3.2.1 Paden and Panja Controller

The control objective is to asymptotically track a desired trajectory predefines by the variable $q_d(t)$ and its successive derivatives $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ that present the desired velocity and acceleration respectively. The control law is given by

$$\tau = M(q)\ddot{q}_d(t) + C(q,\dot{q})\dot{q}_d(t) + G(q) - K_p(q - q_d) - K_v(\dot{q} - \dot{q}_d) \qquad (3.3.21)$$

Applying this law to the Turret Dynamics (3.3.1), we obtain the closed loop system as

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + K_p\tilde{q} + K_v\dot{\tilde{q}} = 0 \qquad (3.3.22)$$

Where $\ddot{\tilde{q}}, \dot{\tilde{q}}$ and $\tilde{q}$ define the acceleration, velocity and position tracking errors respectively.

The stability of the closed loop system is studied using the following Lyapunov function candidate

$$V(q,\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M\left(\tilde{q} + q_d(t)\right)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p \tilde{q} \qquad (3.3.23)$$

And its time derivative evaluated along the trajectory of the error dynamics (3.3.22) is directly obtained as

$$\dot{V} = -\dot{\tilde{q}}^T(t)K_v\dot{\tilde{q}}(t) + \frac{1}{2}\dot{\tilde{q}}^T(t)\left(\dot{M}\left(\tilde{q} + q_d(t)\right) - 2C(q,\dot{q})\right)\dot{\tilde{q}}(t) \qquad (3.3.24)$$

Using the fact that the matrix is skew symmetric, we obtain

$$\dot{V} = -\dot{\tilde{q}}^T(t)K_v\dot{\tilde{q}}(t) \qquad (3.3.25)$$

We can see the right hand side of (3.3.25) is negative, and $\dot{V} = 0$ for $\dot{\tilde{q}}(t) = 0$.

Using Barballat's lemma (cf. Lemma 1.2, section 1.6), we can prove that the equilibrium point $(\tilde{q},\dot{\tilde{q}}) = (0,0)$ is globally asymptotically stable. From Barballat's lemma it is clear that $\dot{V} \to 0$ as $t \to \infty$ then $\dot{\tilde{q}}(t) \to 0$ as $t \to \infty$ and $\ddot{\tilde{q}}(t) \to 0$ as $t \to \infty$, we should only show that $\tilde{q}(t) \to 0$ as $t \to \infty$. Considering the result $\dot{\tilde{q}} \to 0$, $\ddot{\tilde{q}} \to 0$ as $t \to \infty$ into the closed loop equation (3.3.23) we can obtain the result that $\tilde{q}(t) \to 0$ as $t \to \infty$; hence, the equilibrium point $(\tilde{q},\dot{\tilde{q}}) = (0,0)$ is globally asymptotically stable.

3.3.2.2 State Feedback Linearization Based Controller

Another trajectory tracking control law inspired from literature of nonlinear control system is considered in this section, in the design of a controller for nonlinear systems, it seems logical to investigate whether there exists a transformation and a regular static state feedback that transforms the nonlinear system into a linear one. This is known as the feedback linearization problem (Nijmeijer and Van der Schaft, 1990). In robotics, a method based on this theoretic technique is known as the inverse dynamics or the computed torque method. It is shown (Dombre et al., 1988; Lewis et al., 1993; Berghuis, 1993a) that this method relies on the cancellation of nonlinear terms present in the robot dynamics (3.3.1), and consequently, requires exact system knowledge. This method is presented here.

Firstly, let we search for the relative degree of the robot dynamical system, where the output of the robot is considered be the state itself.

$$y(t) = h(q(t)) = [q_1 \quad q_2 \quad q_3]^T = q(t) \tag{3.3.26}$$

The time derivative of the output is the speed of the joints

$$\dot{y}(t) = \left(\frac{\partial h(q)}{\partial q}\right)^T \dot{q}(t) = \dot{q}(t) \tag{3.3.27}$$

The input of the system does not appear yet, we continue derivation

$$\ddot{y}(t) = \left(\frac{d}{dt}\left(\frac{\partial h(q)}{\partial q}\right)^T\right)\dot{q}(t) + \left(\frac{\partial h(q)}{\partial q}\right)^T \ddot{q}(t) = \ddot{q}(t) \tag{3.3.28}$$

Now from the Turret dynamical equation (3.3.1) the acceleration can be written as

$$\ddot{y}(t) = \ddot{q}(t) = M(q)^{-1}\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right) \tag{3.3.29}$$

Now the relative degree of the robot manipulator is two, hence the dynamics is reduced to n-decoupled linear second order systems.

$$y^{(\rho)}(t) = \tau_0 \Rightarrow \tau_0 = M(q)^{-1}\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right) \tag{3.3.30}$$

$$\Rightarrow \tau = M(q)\tau_0 + C(q,\dot{q})\dot{q} + G(q) \tag{3.3.31}$$

Applying this law to the idealized Turret dynamics written in equation (3.3.1) yields to the n-decoupled linear system described by

$$\ddot{q} = \tau_0 \tag{3.3.32}$$

Where τ_0 is an auxiliary control input to be designed. Typically, τ_0 is chosen as

$$\tau_0 = \ddot{q}_d - K_p(q-q_d) - K_v(\dot{q}-\dot{q}_d) \tag{3.3.33}$$

This yields to the error equation

$$\ddot{q} + K_v\dot{\tilde{q}} + K_p\tilde{q} = 0 \tag{3.3.34}$$

Equation (3.3.34) shows that the error dynamics are governed by a linear second order system. Furthermore, if the gains are chosen to be diagonal, then the system will be perfectly decoupled.

By suitably choosing the matrices $K_p$and K_v, the tracking error will converge asymptotically to zero. To illustrate this, we can write the error equation (3.3.34) in state space representation, i.e. we define the state variable $x = \left(\tilde{q}^T, \dot{\tilde{q}}^T\right)^T$, and we can write

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -K_p & -K_v \end{bmatrix} x = Ax \tag{3.3.35}$$

In order to investigate the stability of the equilibrium point, $x = 0$, we choose the Lyapunov function candidate

$$V(x) = x^T P x \tag{3.3.36}$$

Where $P = P^T > 0$ is the solution to Lyapunov equation

$$A^T P + PA = -Q \tag{3.3.37}$$

With $Q > 0$. Then, the time derivative of $V(x)$ is

$$\begin{aligned} \dot{V}(x) &= \dot{x}^T Px + xP\dot{x} = x^T A^T Px + x^T PAx \\ &= x^T (A^T P + PA)x = -x^T Qx \end{aligned} \tag{3.3.38}$$

Which is the negative definite function, from which and according to the theorem 1.8 (cf. section 1.6) global uniform exponential stability of the equilibrium point follows. Fig 3.6 shows the implementation of this feedback linearization.

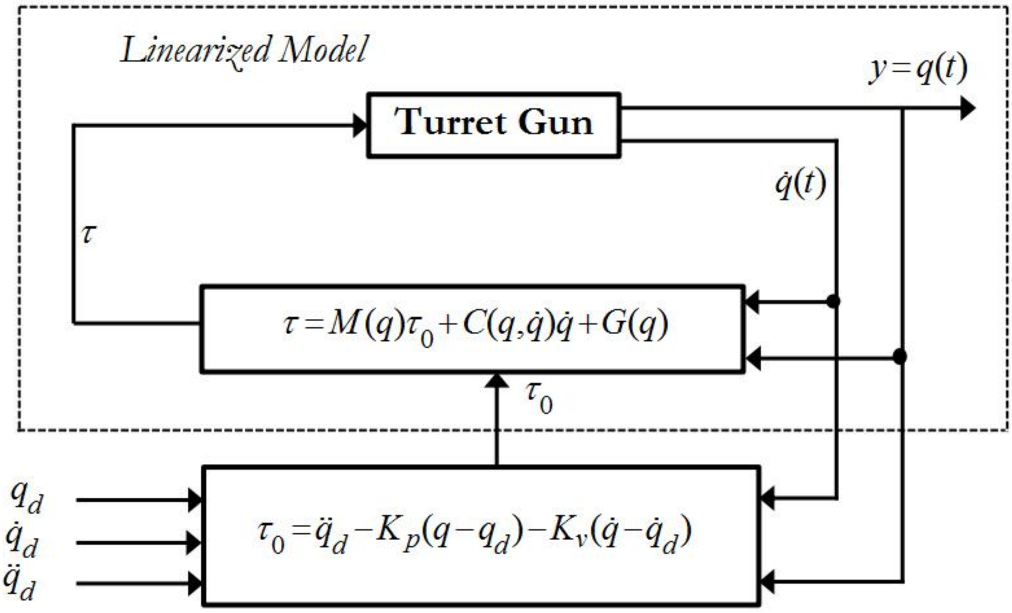

Figure 3.6 Computed Torque Control Implementation

In this Figure, it is illustrated that the computed torque law attempt to cancel the system nonlinearities in order to achieve what is known in nonlinear control theory Feedback linearization.

We notice that this type of control need exact knowledge of the model parameters to exact linearize and decouple the manipulator dynamics. If it were the case, computed torque controller would be the best controller for the robot manipulators. In practice, this is not the case, and at least some of the dynamics are not known exactly. Moreover, calculation time has been so far the main restrictive factor that has prevented this method from having a large impact. Most of practical experiments have been carried out in research laboratories at universities (Dombre et al., 1988; Lewis et al., 1993; Zodiac, 1992). Nowadays, there is no commercially available industrial manipulator equipped with this type of controller (Zodiac, 1992). A possible reason for this (apart from the calculation time limitation) is that computed torque needs the analytic derivation of the robot model and the identification of the associated parameters. These efforts can be important when thinking in term of six degree of freedom robot manipulators. However, methods seeking to simplify the dynamics model do exist (Arimoto and Miyazaki, 1984; Dombre et al., 1988; Arimoto, 1990); they allow to obtain, using minimization methods, a simplified inertia matrix, hence a simplified Lagrange dynamics and finally a simplified computed torque (Zodiac, 1992).

A variant of the computed torque controller that can be implemented is the **predictive controller** (Dombre et al., 1988). In this method, instead of using the nonlinear terms $M(q)$, $C(q,\dot{q})$ and $G(q)$ calculated at actual values of q and $\dot{q}$ we rather evaluate them in terms of the desired joint variables q_d and $\dot{q}_d$. In this case, the control law has the form

$$\tau = M(q_d)\tau_0 + C(q_d,\dot{q}_d)\dot{q}_d + G(q_d) \tag{3.3.39}$$

Where

$$\tau_0 = \ddot{q}_d + K_p(q_d - q) + K_v(\dot{q}_d - \dot{q}) \tag{3.3.40}$$

If we suppose that the tracking is perfect, we can assume that $M(q) = M(q_d)$. If the Turret model is free of errors and for zero initial tracking error and decouple the system just as do the computed torque law.

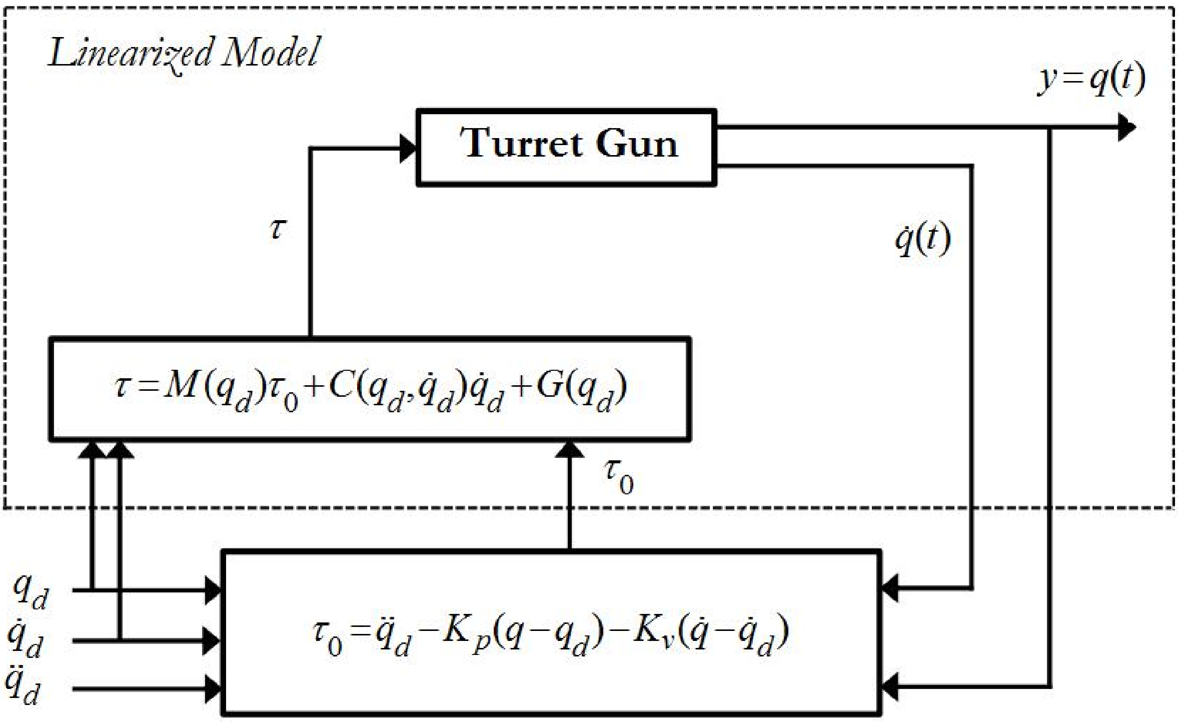

Figure 3.7 Predictive Computed Torque Control Implementation

Another approach, which seeks neither to linearize nor decouple the nonlinear system but only looks for the asymptotic stability, is also used in robot control. This scheme was proposed in the work of Slotine and Li in adaptive control of robot manipulators (Slotine and Li, 1987a).

3.3.3 Passivity Interpretation

From the above analysis, we can see that various solution to the control of rigid manipulators, are provided. Conceptually, these techniques can be classified into two classes, the class of invers dynamics controllers, and the class of passivity- based controllers.

We have seen in section 3.3.2.2. That computed torque controllers are based on the feedback linearization technique. This control law reduces the control of nonlinear robot system to the control of n-decoupled second order linear systems (as seen from equation 3.3.32). This law can be implemented in the so-called inner loop/outer loop architecture as shown in Fig 3.8

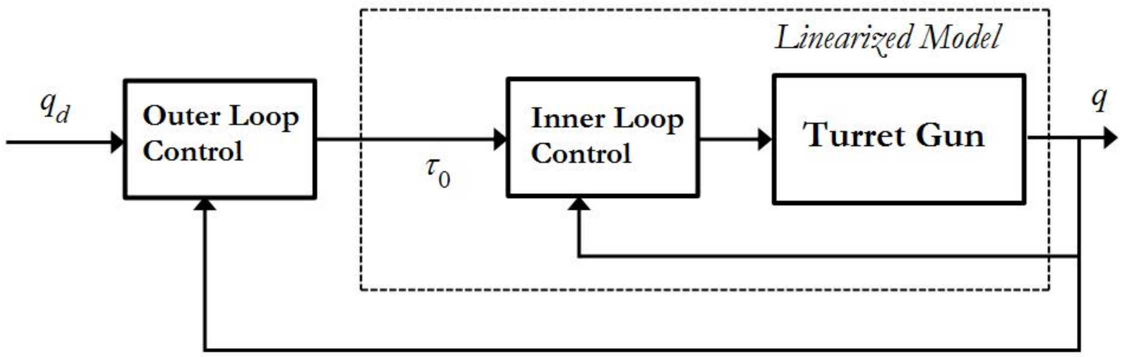

Figure 3.8 Inner loop/outer loop architecture implementation

The inner loop control contains the nonlinear elements responsible of linearizing the system and it has a fixed structure as defined by Lagrange's equations. Since Turret dynamics are complex, we expect that high computations are required at this level. The additional term τ_0 is computed in the outer loop.

In reality, once the inner loop is fixed, which is the case of the inverse dynamics control, the design of any outer loop controller to stabilize the linearized system is possible. In fact, any linear control design can be used to design the outer loop controller, which is the main future of this technique. The outer loop controller given by (3.3.31) is merely the simplest choice of outer loop control, and achieves asymptotic tracking of the joint space trajectories in the ideal case of perfect knowledge of the system dynamics (3.3.1). However, one has complete freedom to modify the outer loop to achieve various other goals such as to enhance the robustness to a parametric uncertainties, unmolded dynamics and external disturbances.

The fact that computed torque control method stems from a general system theoretic methodology, feedback linearization; it disregards the natural structure imposed by the physical character of the robot system. In the following discussion, we will show that the other methods presented above (PD with gravity compensation controller, Paden and Panja controller) do exploit the physical structure of the robot system, especially its passivity property discussed in (2.4.6). These methods are classified into passivity-based controllers. The idea of this philosophy is to reshape the robot system's natural energy in such a way that the tracking objective is attained.

If we assume that the control objective is to regulate the robot manipulator at some desired position q_d. It is intuitively clear that the control law should be constructed such that the strict energy minimum obtained at $(q,\dot{q})=(0,0)$ is shifted to $(\tilde{q},\dot{q})=(0,0)$ for the closed loop system. According to the Hamiltonian dynamic equation for the class of rigid robot manipulators given by (2.4.36)-(2.4.37) (cf. section 2.4.6), this can be realized by shifting the potential energy of the system such that it attains its minimum at $\tilde{q}=0$. To do this, consider a desired energy function of closed loop system be $P_0(p)$. From this, define the control law

$$\tau_0=\frac{\partial p}{\partial q}-\frac{\partial p_0}{\partial q}+\upsilon \tag{3.3.41}$$

Where υ is the new $n\times 1$ input. The original Hamiltonoan equation defined by (2.4.36)-(2.4.37) are modified to

$$H_0=T(q,p)+P_0(p) \tag{3.3.42}$$

It can be easily verified that

$$\dot{H}_0=\dot{q}^T\upsilon \tag{3.3.43}$$

This implies that a marginally stable closed loop system is obtained (Van der Schaft, 1990; Berghuis, 1993a). That is passive from the new input υ to $\dot{q}$, since

$$\langle\dot{q},\upsilon\rangle=\int_0^T\dot{q}^T(\sigma)\upsilon(\sigma)d\sigma\leq -H_0(0) \tag{3.3.44}$$

To asymptotically stabilize the system, damping, that is velocity error feedback, should be injected in the loop, so define

$$\upsilon=-K_d\dot{q} \tag{3.3.45}$$

These yields

$$\dot{H}_0=-\dot{q}^TK_d\dot{q} \tag{3.3.46}$$

Invoking Barballat's Lemma (cf. lemma 1.2) the entire equilibrium point $(\tilde{q},\dot{q})=(0,0)$ can be shown to be uniformly asymptotically stable. To this point, the control law has not yet been defined. A choice that ensures a strict minimum of the desired potential energy as $\tilde{q}=0$ can be defined as

$$P_0(q)=\tilde{q}^T K_p \tilde{q} \tag{3.3.47}$$

For this choice, and using the control expression (3.3.41), we obtain that

$$\tau_0 = G(q) - K_p\tilde{q} - K_d\dot{q} \tag{3.3.48}$$

Note that we have obtained the controller proposed by Takegaki and Arimoto (1981), and is given by equation (3.3.14) (cf. section 3.3.1.2).

For tracking purpose, it is clear that the control law should be constructed to shift the energy minimum at $(q,\dot{q})=(0,0)$ of the open loop towards $(\tilde{q},\dot{\tilde{q}})=(0,0)$ for the closed loop system. To attain this objective, both kinetic energy and potential energy should be modified in a desired manner. This can be achieved by chooding the control law to be

$$\tau_0 = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) - K_p\tilde{q} + \upsilon \tag{3.3.49}$$

This controller structure establishes a passive mapping from ι to $\dot{\tilde{q}}$ as can be verified using the energy function

$$H(\tilde{q},\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \dot{\tilde{q}}^T K_p \tilde{q} \tag{3.3.50}$$

Which time derivative evaluated along the dynamics of the error gives

$$\dot{H} = -\dot{\tilde{q}}^T K_d \dot{q} \tag{3.3.51}$$

From this, we can write

$$\left\langle \dot{\tilde{q}},\upsilon \right\rangle \equiv \int_0^T \dot{\tilde{q}}^T(\sigma)\upsilon(\sigma)d\sigma \leq -H(0) \tag{3.3.52}$$

Again, damping should be inserted to guarantee the asymptotic stability of the closed loop system, then defining

$$\upsilon = -K_d\dot{\tilde{q}} \tag{3.3.53}$$

And using LaSalle's theorem, we can show that the convergence of the equilibrium point $(\tilde{q},\dot{\tilde{q}})=(0,0)$ is asymptotically stable.

The controller as defined by (3.3.49) and (3.3.53) is no more than the Paden and Panja controller given by (3.3.22) (cf. section 3.3.2.1).

From this analysis, we conclude that this approach consists of designing a controller such that the closed loop system matches a desired energy function that resembles the natural energy contents of the open loop system. In this way, passivity of the robot system can be preserved in the closed loop. For this reason, these controllers are said to be passivity-based controllers. Moreover, the inclusion of the damping in the loop via velocity feedback, asymptotic stability can be obtained.

3.5. Implementation results

In this part we will show and discuss the results obtained after implementing the last methods

3.5.1 Implementing the PD controller with Gravity Compensator

With pulse reference and random coefficients K_p and K_v

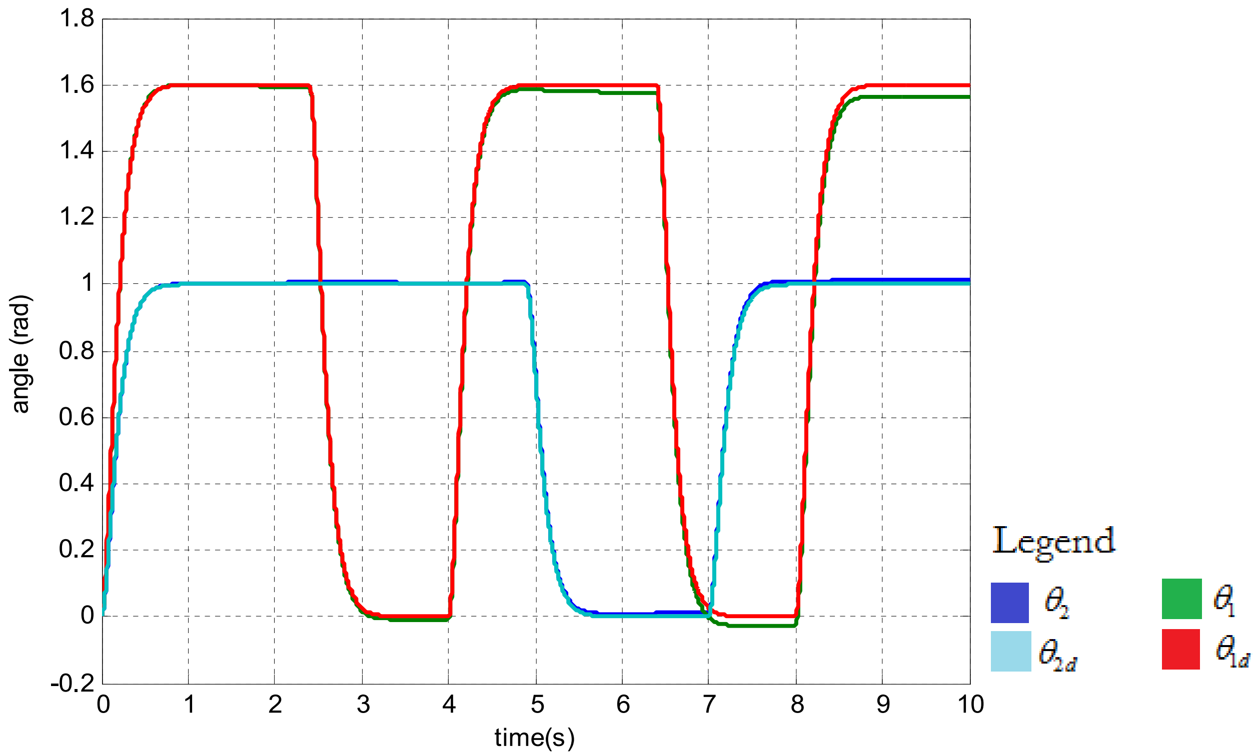

Figure 3.9 PD Controller with Gravity Compensator with random coefficients K_p & K_v pulse reference, inputs/outputs

Now with step reference random coefficients K_p and K_v

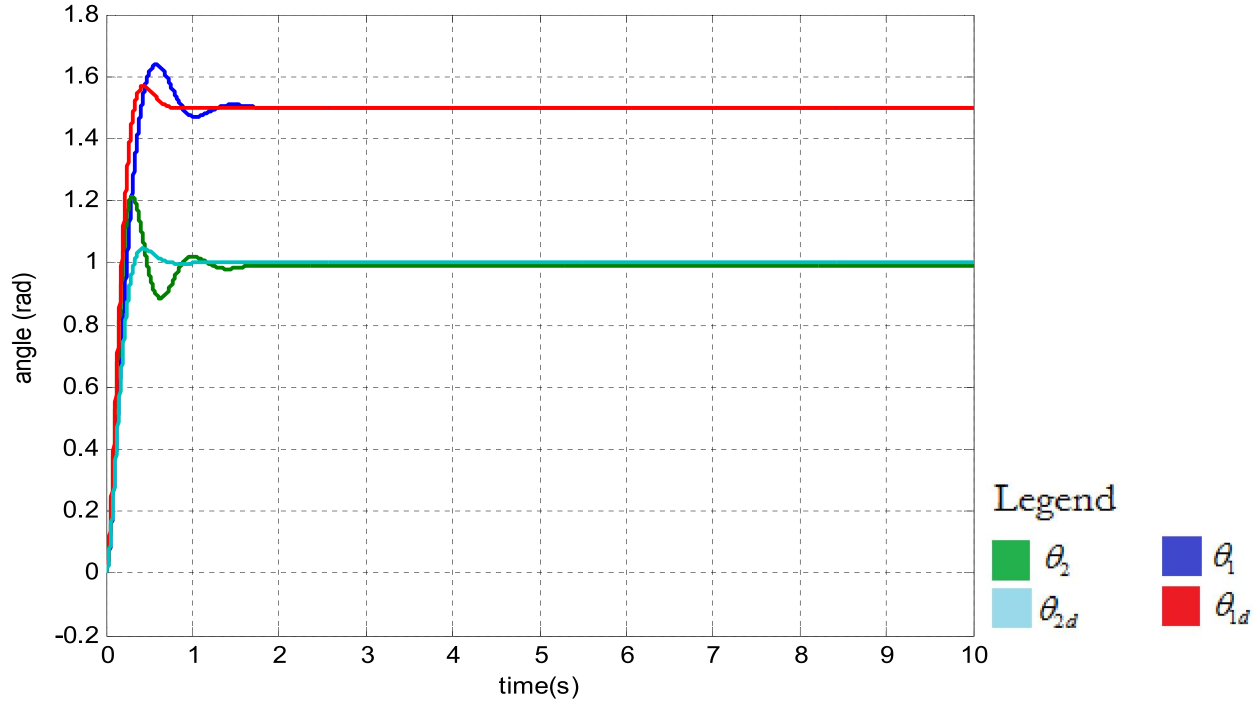

Figure 3.10 PD Controller with Gravity Compensator with random coefficients K_p & K_v step reference, inputs/outputs

Now with sinusoidal reference random coefficients K_p and K_v

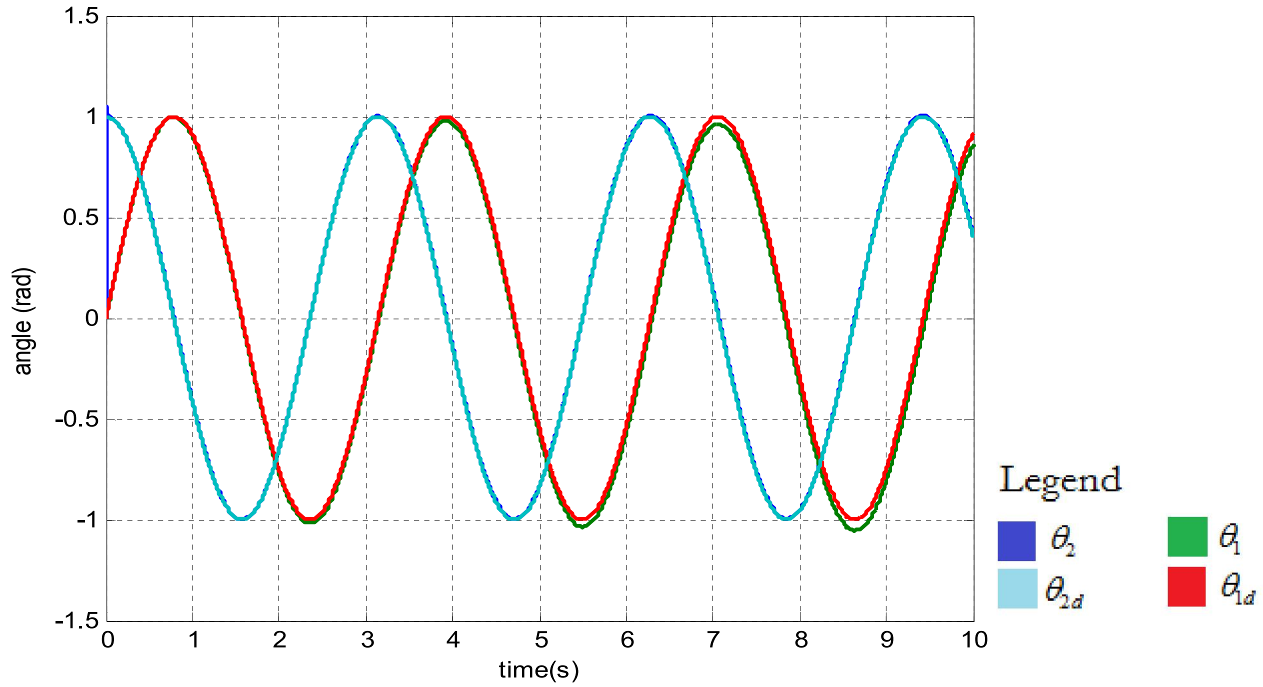

Figure 3.11 PD Controller with Gravity Compensator with random coefficients K_p & K_v sinusoidal reference, inputs/outputs

3.5.2 Implementing the Paden & Panja Controller

With pulse reference and random coefficients K_p and K_v

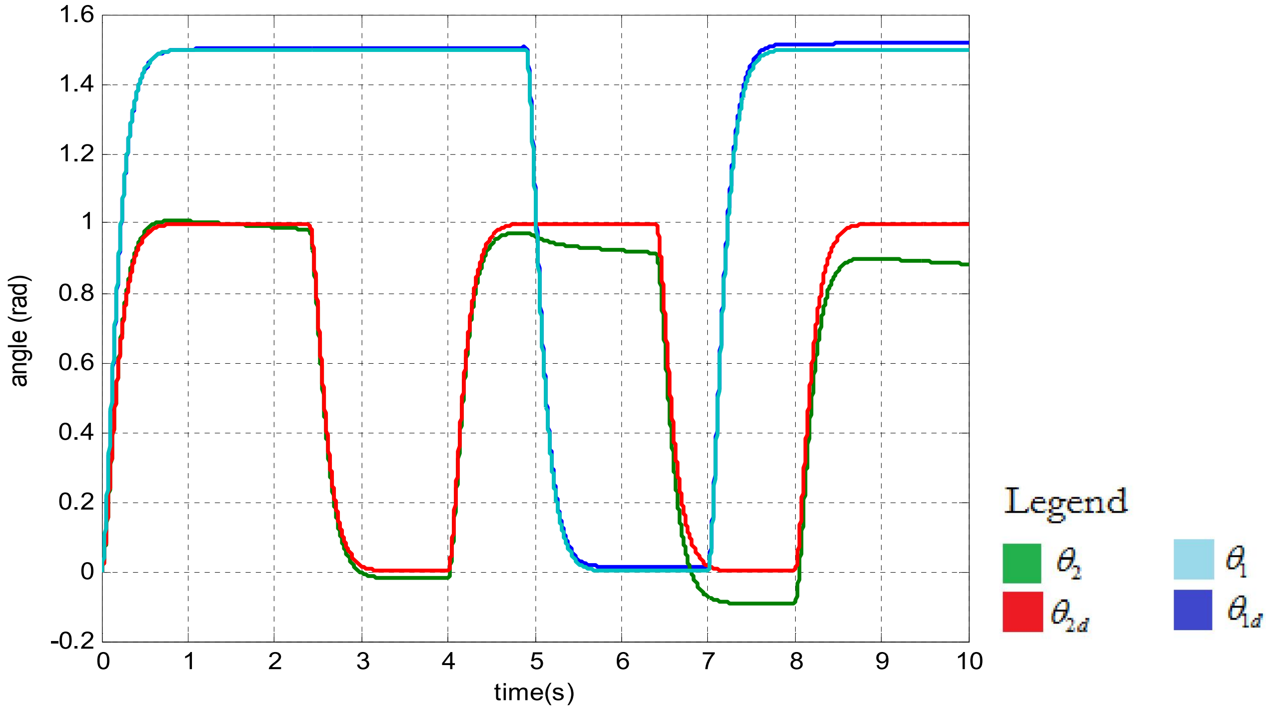

Figure 3.12 Paden & Panja controller with random coefficients K_p & K_v pulse reference, inputs/outputs

Now with step reference random coefficients K_p and K_v

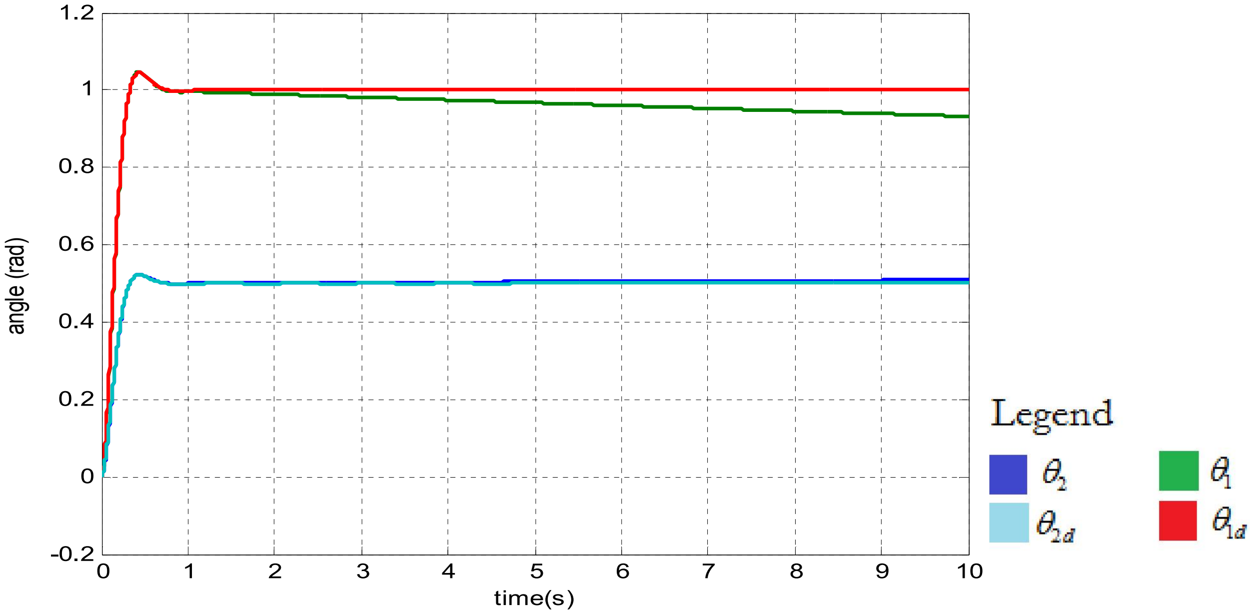

Figure 3.13 Paden & Panja controller with random coefficients K_p & K_v step reference, inputs/outputs

Finally, with sinusoidal reference random coefficients K_p and K_v

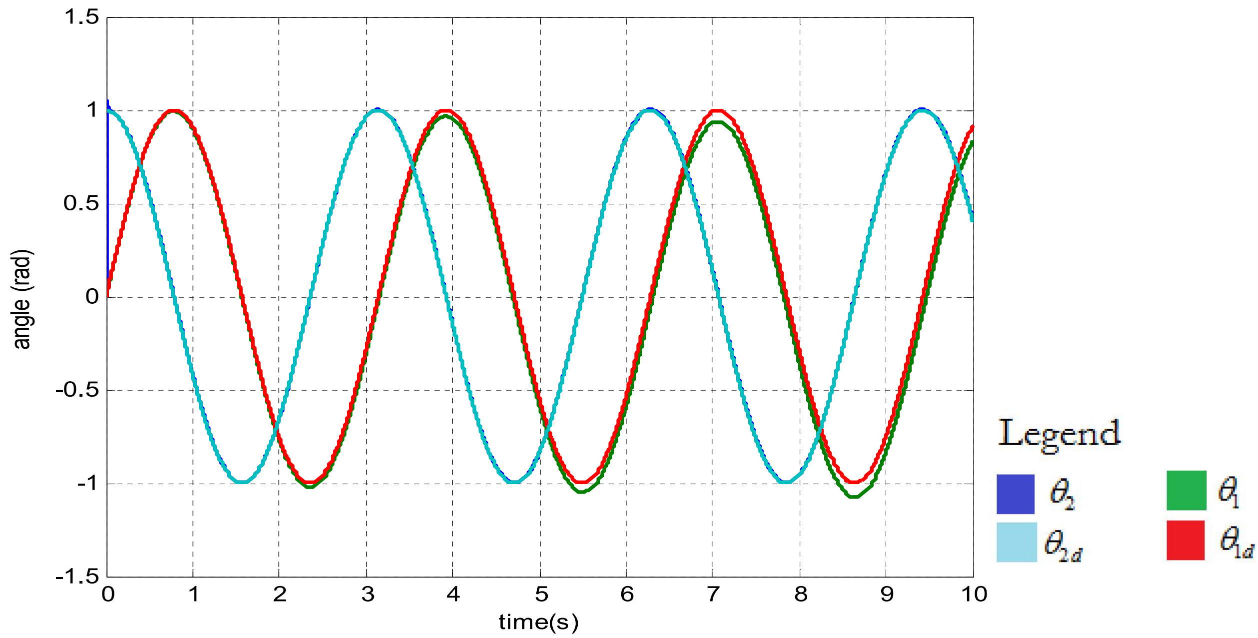

Figure 3.14 Paden & Panja controller with random coefficients K_p & K_v sinusoidal reference, inputs/outputs

3.5.3 Implementing the State Feedback Linearization Based Controller

With pulse reference and random coefficients K_p and K_v

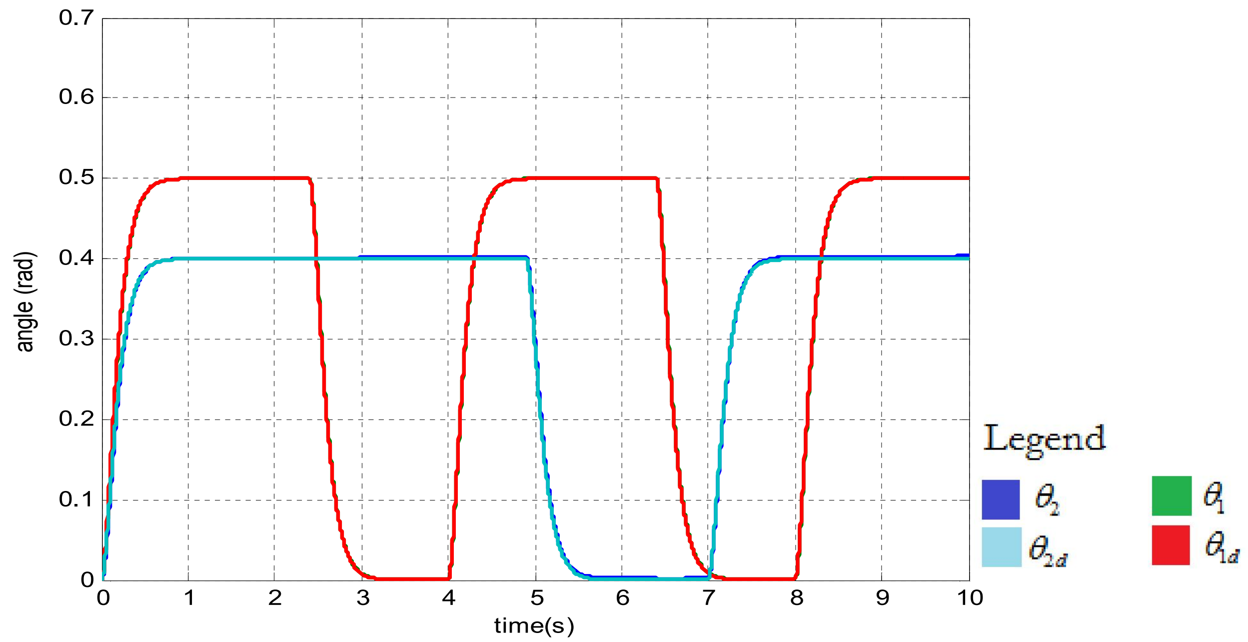

Figure 3.15 State Feedback Linearization Based Controller pulse reference and random coefficients K_p and K_v, inputs/outputs

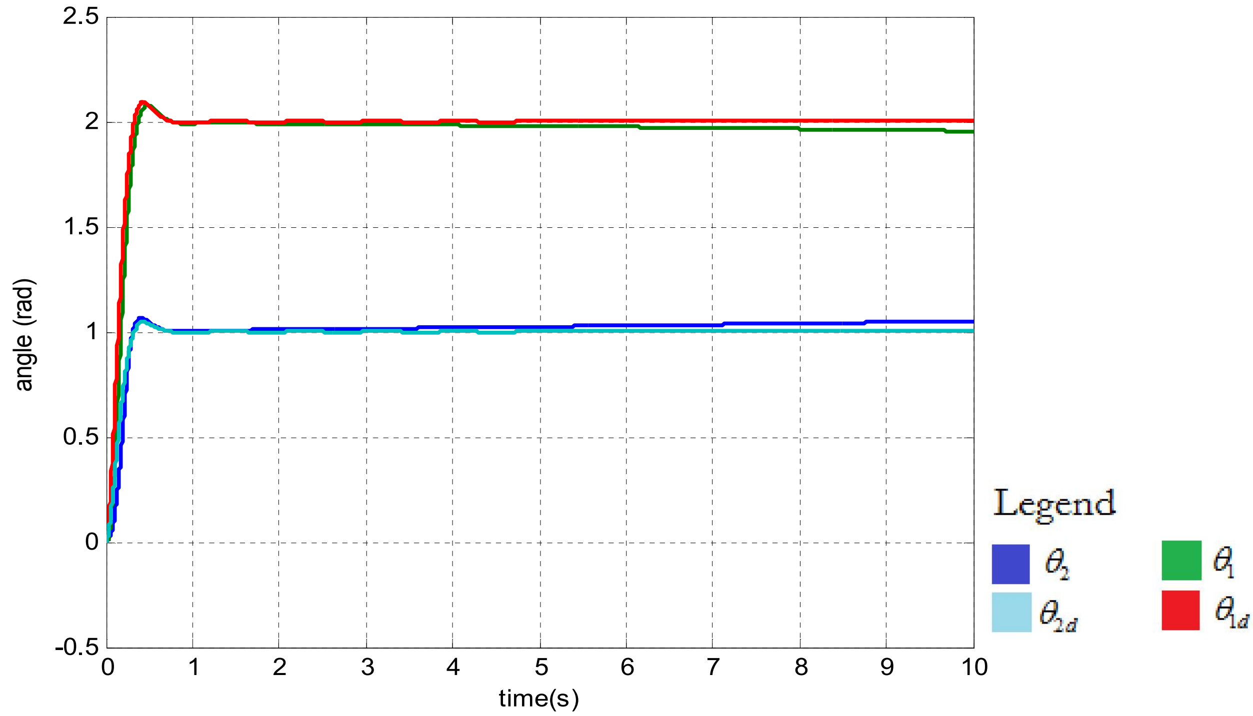

Figure 3.16 State Feedback Linearization Based Controller step reference and random coefficients K_p and K_v, inputs/outputs

Finally, with sinusoidal reference random coefficients K_p and K_v

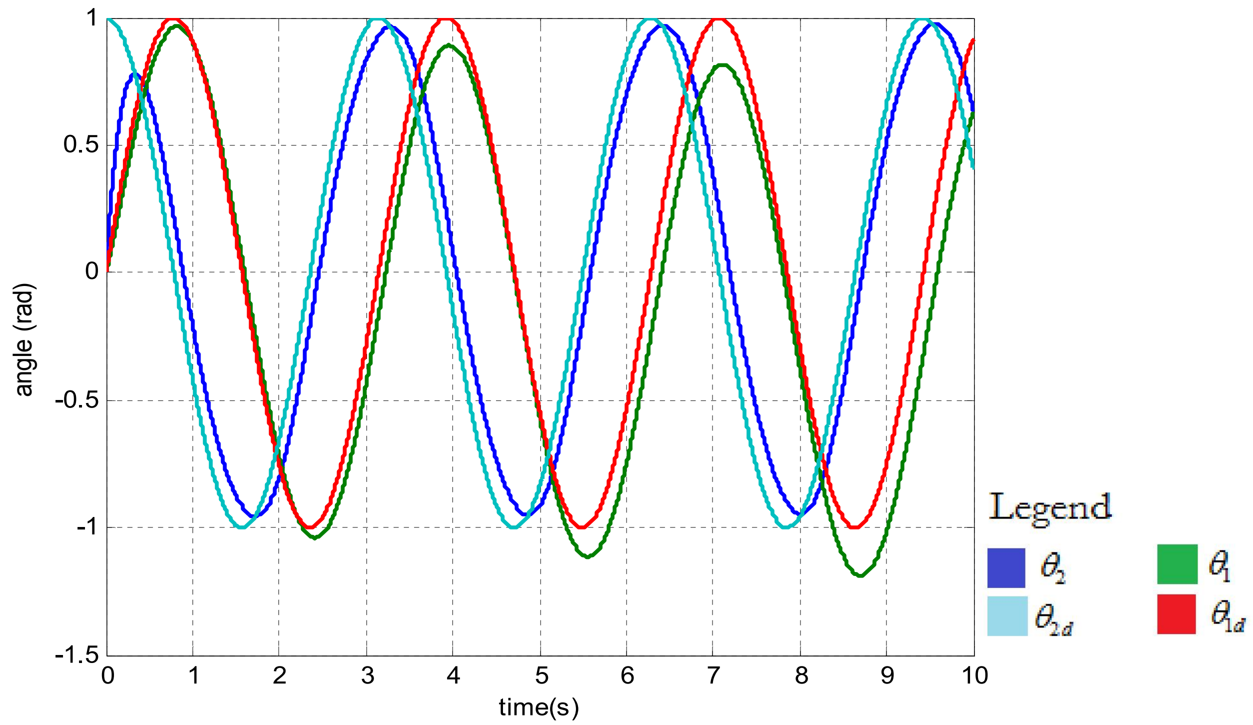

Figure 3.17 State Feedback Linearization Based Controller sinusoidal reference and random coefficients K_p and K_v, inputs/outputs

3.5.4 Discussion of the obtained results

From the results obtained, it seems that the first two controllers: PD Controller with Gravity Compensator and the Paden and Panja, seem to offer fairly satisfactory performance with low errors, the State Feedback Linearization Based Controller is not precise enough.

After trying these three controllers, we have to make a choice, which due to the errors of the third is reduced to the first two. By examining the coefficients used to obtain the results, it turns out that the coefficients of the first are much lower and therefore more in line with reality because of noise amplification. In addition, the time of response is lower than with the second controller.

3.5. Conclusion

In this chapter, some of the classical control schemes applied to robotic systems were presented. A simple PD controller was studied first for the regulation problem. We have seen that the condition of implementing big proportional gains is vital for achieving the asymptotic stability of the equilibrium point. Then a gravitation compensation term was added to the linear PD controller, where we have seen that this modified controller ensures the global uniform asymptotic stability of the closed loop system with no conditions on the controller gains. With this control law, the regulation problem was then solved using only simple linear techniques. When the application requires some rapid displacements of the manipulator and a high dynamical precision, it is necessary to design a more sophisticated control law that takes into account all or a part of the dynamical interaction forces. For tracking purposes, PD with gravity compensation was shown via simulation, to guarantee only bounded tracking errors that be decreased arbitrarily by using high controller gains.

A solution to this, and in order to obtain a perfect tracking, all the nonlinear terms on the system dynamics were used in the controller law. First Paden and Panja controller was shown to give perfect tracking and ensures the asymptotic stability of the tracking error. Then, the computed torque controller was discussed and we have shown that this controller, if the dynamics of the robot are exactly known, perfectly linearizes and decouples the Turret dynamics resulting on decoupled second order linear system that we can arbitrarily place its pols by properly selecting its controller gains. A variant of this controller is the predictive controller, where instead of calculating the controller nonlinear terms at the actual position and velocity signals, they are calculated any desired ones. This controller will the n linearize and decouple the system just as do the computed torque controller assuming perfect tracking and zero initial tracking errors.

Through simulation study, we have shown the convergence of the proposed control schemes for regulation and trajectory tracking purposes. A comparative study has shown that the proposed trajectory tracking controllers provide a slightly difference on their response provided that the controller's gains are selected according to equivalence principle.

It is important to notice that the basic future of the control strategies presented in this chapter is assumption that full state information, that is position and velocity, is available for feedback. In practice, this assumption can only be partially fulfilled.

Finally, we will work with the first controller: PD Controller with Gravity Compensator in the following chapters, because in our case it has shown better performances. We will optimize the coefficients as best as possible using the algorithms provided for this purpose.

Chapter 4

Advanced Optimized Control Strategies

In optimization of a design, the design objective could be simply to minimize the cost of production or to maximize the efficiency of production. An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till an optimum or a satisfactory solution is found.With the advent of computers, optimization has become a part of computer-aided design activities. There are two distinct types of optimization algorithms widely used today. In this chapter, we will provide you some of these methods,

4.1 Introduction

The main goal of optimization is to try to get the best possible result for a given system. However, depending on the context or the environment, the optimal result can be represented either by a maximum or by a minimum of aptitude of a system, represented by a quality function. While optimization to find a minimum output is often called minimum skill, while minimizing the costs of a system. For this, quality is the negative of the cost.

The common challenge is to find the global minima / maxima with a certain number of local minima / maxima. This is more easily visualized using the concept of a cost surface for which there may be a number of small peaks and small valleys.

Multivariate systems are more complex than single variable systems, and are more difficult to model and solve mathematically. The number of variables can be used to express the dimensions of the system. Dynamic systems are systems whose output is a function of time and static systems that are invariant in time.

System variables can be classified as discrete or continuous. Continuous variables can take an infinite number of values, while discrete variables can only be assigned for a finite number of possible values. A common approach for optimization using digital processes. Constrained systems are systems for which variables can only take values within fixed limits. Variables in unconstrained systems have no such limits applied. Mathematical optimization works best on systems without constraints.

The optimization methods used to have the minimum use a single set of inputs in order to numerically find the optimal results. These methods are challenged by the problem of local minima/maxima. Unlike these methods, the random search methods use probabilistic calculations to find the sets of variables in which the optimization is carried out, so these latter methods do not have the problem of local minima/maxima. This is why deterministic calculation methods are faster than random methods.

Most of the classical optimization methods can be described as minimum search algorithms seeking the cost surface for a minimum cost, and therefore suffer from the challenge of local minima. These classical calculation methods are often based and solved numerically.

More recently, natural optimization methods known as meta-heuristics have been developed in order to remedy the inherent limitations of optimization based on numerical calculations. We will use two (2) meta-heuristic methods of natural optimization, which are:

a) The algorithm simulating the movement of swarms of particles (PSO)

b) The ant colony algorithm (ACO)

Modern Meta-heuristic methods are a set of stochastic optimization techniques inspired by natural and biological phenomena. These techniques can be classified into two groups:

1. Population solution methods known as evolutionary algorithms.
2. Single solution methods.

These methods provide an intelligent search for the space of solutions using statistical approaches and therefore do not require the computation of the derivatives of the cost function; therefore, natural methods can deal with non-continuous and discrete multivariate systems (SALMI S. (2019) "Introduction to the Meta-Heuristic Optimization").

4.2 Particle Swarm Optimization

Particle swarm optimization algorithms were introduced in 1995 by Kennedy and Eberhart as an alternative to standard genetic algorithms. These algorithms are inspired by swarms of insects (or schools of fish or flocks of birds) and their coordinated movements, in fact, just as these animals move in groups to find food or avoid predators, particle swarm algorithms seek solutions for an optimization problem. The individuals in the algorithm are called particles and the population is called a swarm.

In this algorithm, a particle decides its next movement based on its own experience, which in this case is the memory of the best position it has encountered, and based on its best neighbor. The new speed and direction of the particle will be defined according to three trends: the propensity to follow its own path, its tendency to return to its best position reached and its tendency to go towards its best neighbor.

4.2.1 Principle

The principle of PSO developed by Kennedy and Eberhart is based on the behavior of flocks of birds. Thus, PSO was fundamentally developed through the simulation of the behavior of flocks of birds in two-dimensional space. The position of each agent is represented by its coordinates along the two axes X and Y , to which we associate the speeds expressed by V_x (speed along the axis X) and V_y (speed along the axis Y). The modification of the behavior of each agent is based on the position and speed information.

At each iteration the agent proceeds via an objective function to evaluate its best value up to $(pbest)$ and its position along the two axes X and Y .

This information is obtained from an analysis of the personal experiences of each agent. In addition, each agent knows the best global value in the group $(gbest)$ among $(pbests)$. This information represents the value around which other agents are performing. Thus, each agent tries to modify his position based on the following information:

- Current position (x, y),
- Current speed (V_x, V_y),
- Distance between the current position and $pbest$
- Distance between the current position and $gbest$

This modification can be represented by the concept of speed. The modified speed of each agent will be written as follows:

$$v_i^{k+1} = wv_i^k + c_1 rand_1 \times (pbest_i - s_i^k) + c_2 rand_2 \times (gbest - s_i^k) \tag{4.2.1}$$

Where

v_i^k : Speed of the agent i at the iteration k .

w : weighting function.

c_j : weighting factor.

$rand$: random number between 0 and 1.

s_i^k : Position of the agent i at the iteration k .

$pbest_i$: Best position of the agent i .

$gbest$: Best global value of the group.

The weighting function usually used in the equation (4.2.1) is given as

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \tag{4.2.2}$$

Where

$w_{\max}$: Final weight.

$w_{\min}$: Initial weight.

$iter_{\max}$: Maximum number of iterations.

$iter$: Current number of iteration.

The model used in the equation (4.2.2) is known as "Inertia Weights Approach (IWA)". The current position (searching the point in the solutions area) is actualized according to the equation

$$s_i^{k+1} = s_i^k + v_i^{k+1} \tag{4.2.3}$$

Where

s_i^k : Current position of the agent.

s_i^{k+1} : Actualized position of the agent.

v_i^{k+1} : Actualized speed of the agent.

The concepts of PSO already presented allow us to build the following general Flowchart fig.4.1

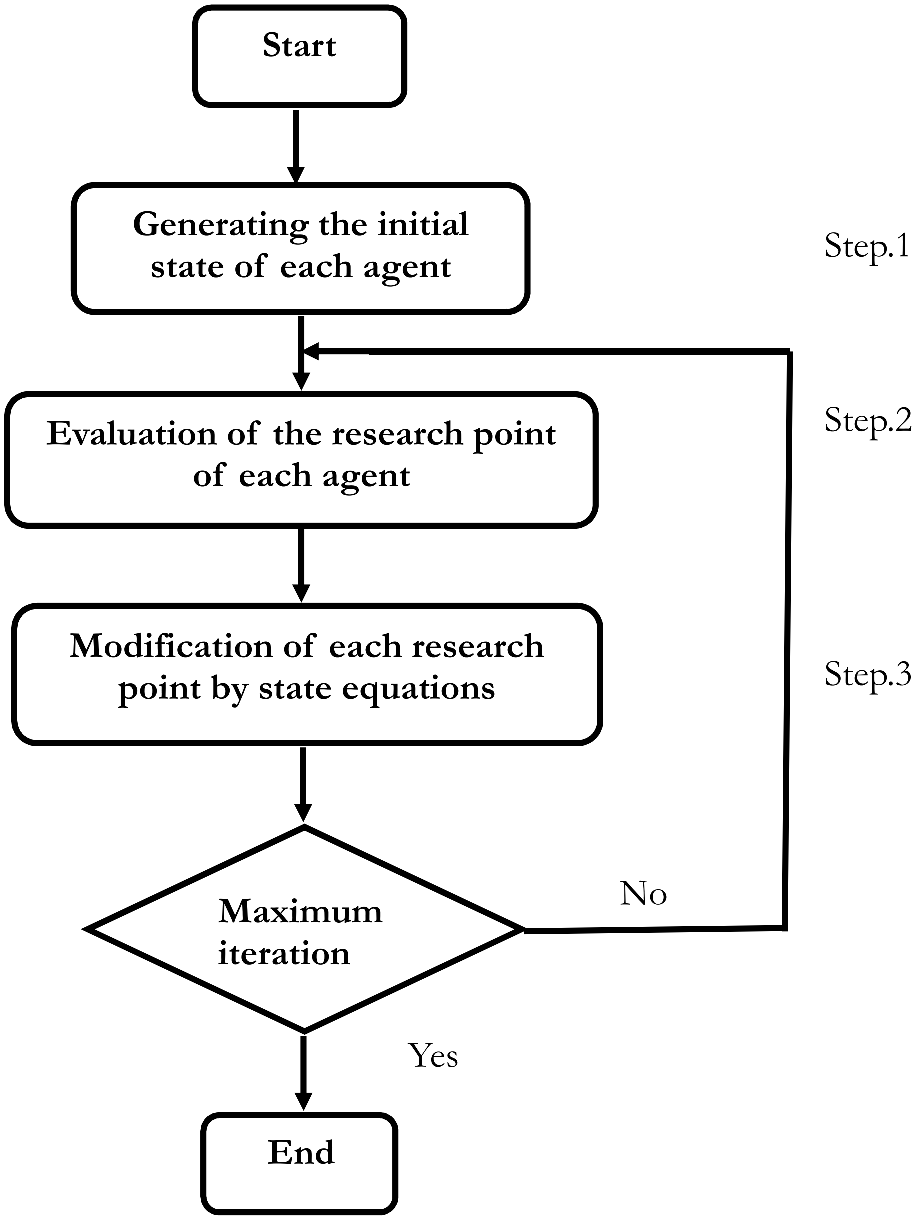

Figure 4.1 PSO general Flowchart

This algorithm is particularly simple to implement. As we can see, the space of solutions is explored by multiple particles, the best areas discovered by a particle being communicated to a given neighborhood in order to pass on the information.

However, in general the neighborhood is not complete, which prevents the algorithm from falling into local optima.

PSO is a particularly efficient optimization technique (even for noisy objective functions), which often seems to work better than genetic algorithms which fail to seem sufficiently guided.

Indeed the crossing of two genetic codes evaluated as "good" or "adapted" does not in any way provide the certainty that the result will also be adapted. In other words, PSO uses a velocity vector that genetic algorithms do not have.

The PSO algorithm can be applied to solve different optimization problems in various fields of application. You just have to adapt the variables and parameters of the algorithm to those of the problem under consideration.

4.2.2 PSO Algorithm

For each particle i = 1,…,s do

Randomly initialize xi

Randomly initialize vi (or just set vi to zero)

Set yi=xi

endfor

Repeat

For each particle i = 1,…,s do

Evaluate the fitness of particle i , f(xi)

Update y_i *using equation*

Update $\hat{y}$ *using equation*

For each dimension j = 1,…, Nd do

Apply velocity update using equation

endloop

Apply position update using equation

endloop

Until some convergence criteria is satisfied

4.3 Ant Colony Optimization

As already mentioned, swarm intelligence is a relatively new approach to problem solving that takes inspiration from social behaviors of insects and of other animals. In particular, ants have inspired a number of methods and techniques among which the most studied and the most successful is the general-purpose optimization technique known as ant colony optimization. Ant colony optimization (ACO) takes inspiration from the foraging behavior of some ant species. These ants deposit pheromone on the ground in order to mark some favorable path that should be followed by other members of the colony. Ant colony optimization exploits a similar mechanism for solving optimization problems.

From the early nineties, when the first ant colony optimization algorithm was proposed, ACO attracted the attention of increasing numbers of researchers and many successful applications are now available.

Moreover, a substantial corpus of theoretical results is becoming available that provides useful guidelines to researchers and practitioners in further applications of ACO (Doringo M., Stutzle T. Birattari M. (2006) "Ant Colony Optimization") .

4.3.1 Biological Inspiration

In the forties and fifties of the twentieth century, the French entomologist Pierre-Paul Grasse´ observed that some species of termites react to what he called "significant stimuli". He observed that the effects of these reactions could act as new significant stimuli for both the insect that produced them and for the other insects in the colony. Grasse´ used the term "stigmergy" to describe this particular type of communication in which the "workers are stimulated by the performance they have achieved".

The two main characteristics of "stigmergy" that differentiate it from other forms of communication are the following.

❑ Stigmergy is an indirect, non-symbolic form of communication mediated by the environment: insects exchange information by modifying their environment; and

❑ Stigmergic information is local: it can only be accessed by those insects that visit the locus in which it was released (or its immediate neighborhood).

Examples of stigmergy can be observed in colonies of ants. In many ant species, ants walking to and from a food source deposit on the ground a substance called pheromone. Other ants perceive the presence of pheromone and tend to follow paths where pheromone concentration is higher. Through this mechanism, ants are able to transport food to their nest in a remarkably effective way.

Deneubourg et al. thoroughly investigated the pheromone laying and following behavior of ants. In an experiment known as the "double bridge experiment", the nest of a colony of Argentine ants was connected to a food source by two bridges of equal lengths Fig.4.2(a).

In such a setting, ants start to explore the surroundings of the nest and eventually reach the food source. Along their path between food source and nest, Argentine ants deposit pheromone.

Initially, each ant randomly chooses one of the two bridges. However, due to random fluctuations, after some time one of the two bridges presents a higher concentration of pheromone than the other and, therefore, attracts more ants. This brings a further amount of pheromone on that bridge making it more attractive with the result that after some time the whole colony converges toward the use of the same bridge.1

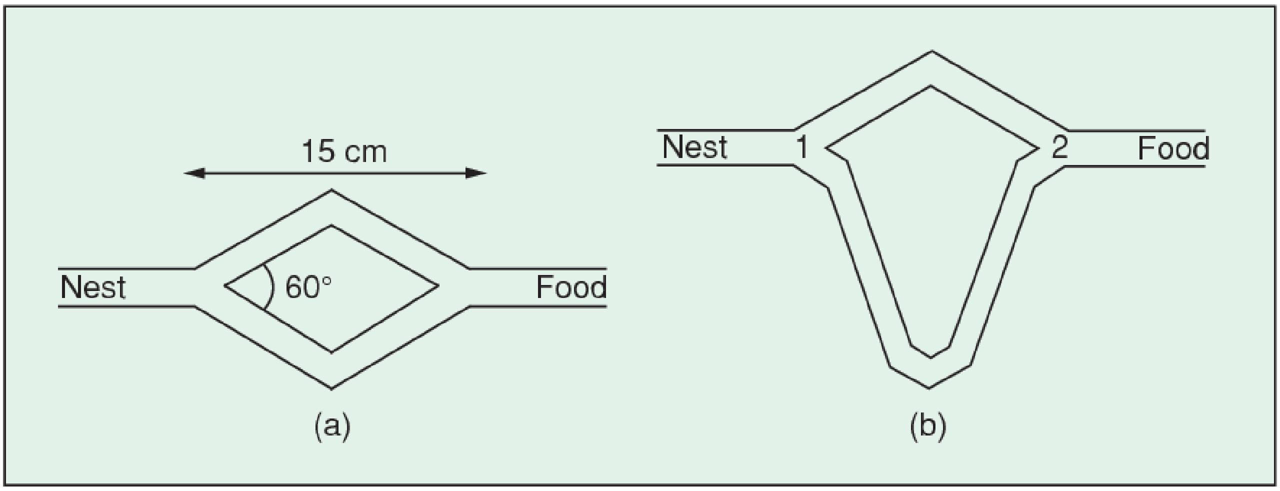

Figure 4.2 Experimental setup for the double bridge experiment

(a) Branches have equal lengths; (b) Branches have different lengths

This colony-level behavior, based on autocatalysis, that is, on the exploitation of positive feedback, can be used by ants to find the shortest path between a food source and their nest. Goss et al. considered a variant of the double bridge experiment in which one bridge is significantly longer than the other see Figure 4.2(b). In this case, the stochastic fluctuations in the initial choice of a bridge are much reduced and a second mechanism plays an important role: the ants choosing by chance the short bridge are the first to reach the nest. The short bridge receives, therefore, pheromone earlier than the long one and this fact increases the probability that further ants select it rather than the long one. Goss et al. developed a model of the observed behavior: assuming that at a given moment in time m_1 ants have used the first bridge and m_2 the second one, the probability p_1 for an ant to choose the first bridge is:

$$p_1 = \frac{(m_1 + k)^h}{(m_1 + k)^h + (m_2 + k)^h} \tag{4.3.1}$$

Where parameters k and h are to be fitted to the experimental data, obviously $p_2 = 1 - p_1$. Monte Carlo simulations showed a very good fit for $k \approx 20$ and $h \approx 2$.

The model proposed by Deneubourg and co-workers for explaining the foraging behavior of ants was the main source of inspiration for the development of ant colony optimization. In ACO, a number of artificial ants build solutions to the considered optimization problem at hand and exchange information on the quality of these solutions via a communication scheme that is reminiscent of the one adopted by real ants.

Different ant colony optimization algorithms have been proposed. The original ant colony optimization algorithm is known as Ant System and was proposed in the early nineties. Since then, a number of other ACO algorithms were introduced.

All ant colony optimization algorithms share the same idea discussed above; In the traveling salesman problem, a set of cities is given and the distance between each of them is known. The goal is to find the shortest tour that allows each city to be visited once and only once. In more formal terms, the goal is to find a Hamiltonian tour of minimal length on a fully connected graph.

In ant colony optimization, the problem is tackled by simulating a number of artificial ants moving on a graph that encodes the problem itself: each vertex represents a city and each edge represents a connection between two cities. A variable called pheromone is associated with each edge and can be read and modified by ants.

Ant colony optimization is an iterative algorithm. At each iteration, a number of artificial ants are considered. Each of them builds a solution by walking from vertex to vertex on the graph with the constraint of not visiting any vertex that she has already visited in her walk. At each step of the solution construction, an ant selects the following vertex to be visited according to a stochastic mechanism that is biased by the pheromone: when in vertex i, the following vertex is selected stochastically among the previously unvisited ones (see Fig.4.3)

In particular, if j has not been previously visited, it can be selected with a probability that is proportional to the pheromone associated with edge (i, j).

At the end of an iteration, on the basis of the quality of the solutions constructed by the ants, the pheromone values are modified in order to bias ants in future iterations to construct solutions similar to the best ones previously constructed.

(Doringo M., Stutzle T. Birattari M. (2006) "Ant Colony Optimization")

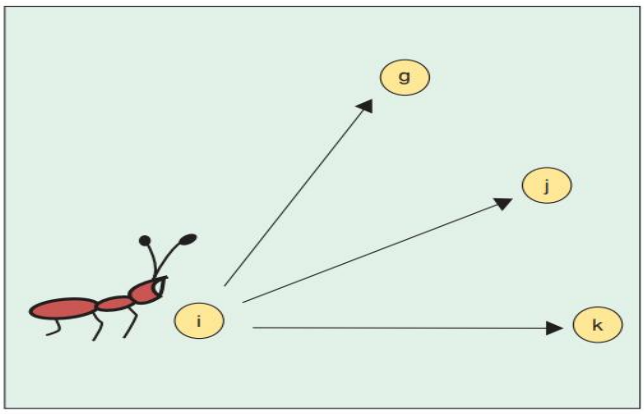

Figure 4.3 An ant in city i chooses the next city to visit via a stochastic mechanism

The concepts of PSO already presented allow us to build the following general Flowchart fig.4.4

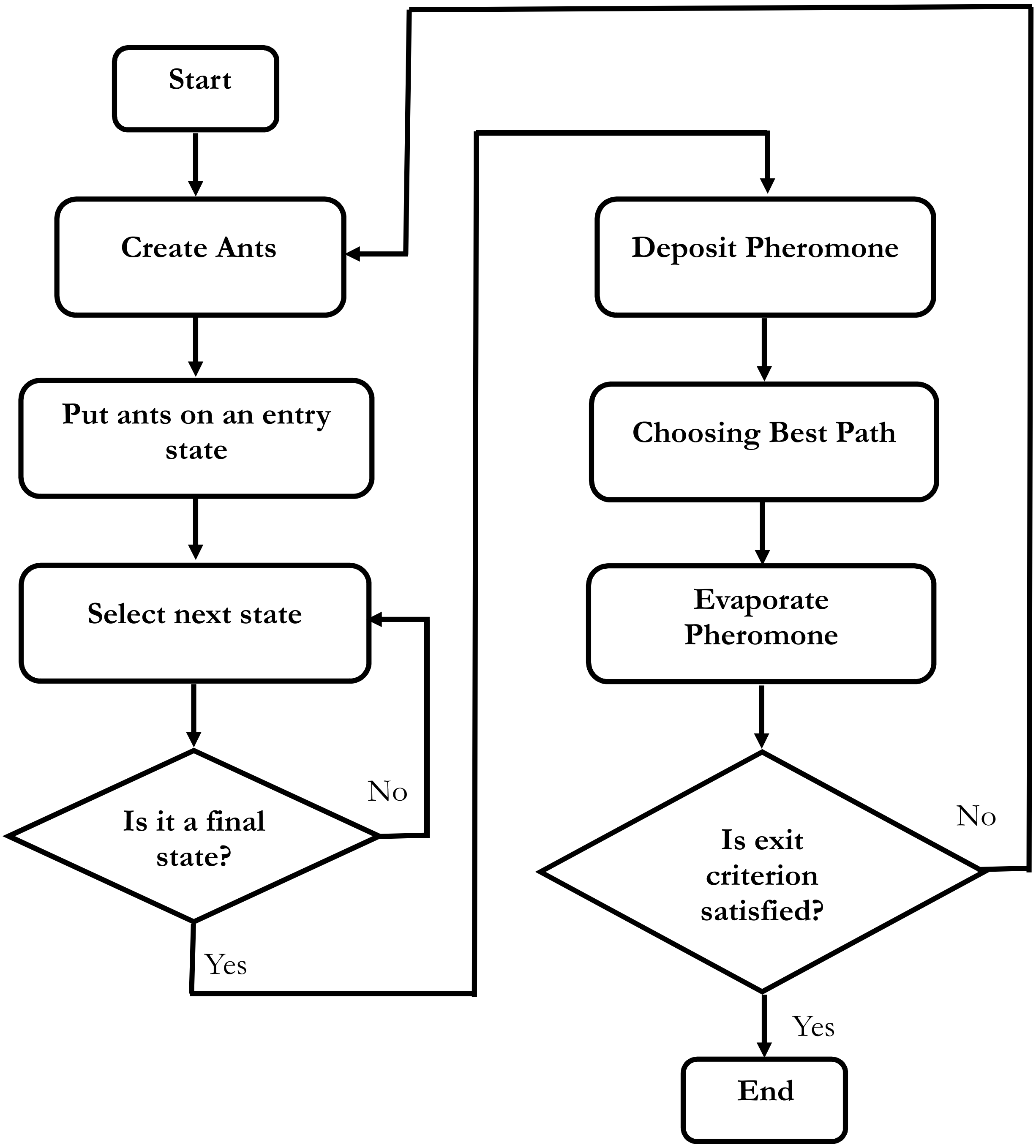

Figure 4.4 Flowchart of ant colony optimization

4.3.2 ACO Simplified Algorithm

Initialize the system parameters

While termination condition not met do

Construct Solutions

Apply Path Search

Update Pheromones

end

4.4 Application in the Studied Case

Let us now apply the algorithms seen above on our PD controller with gravity compensator

4.4.1 PSO Application

First, with an pulse reference let's calculate the two coefficients K_p and K_v in 10 iterations

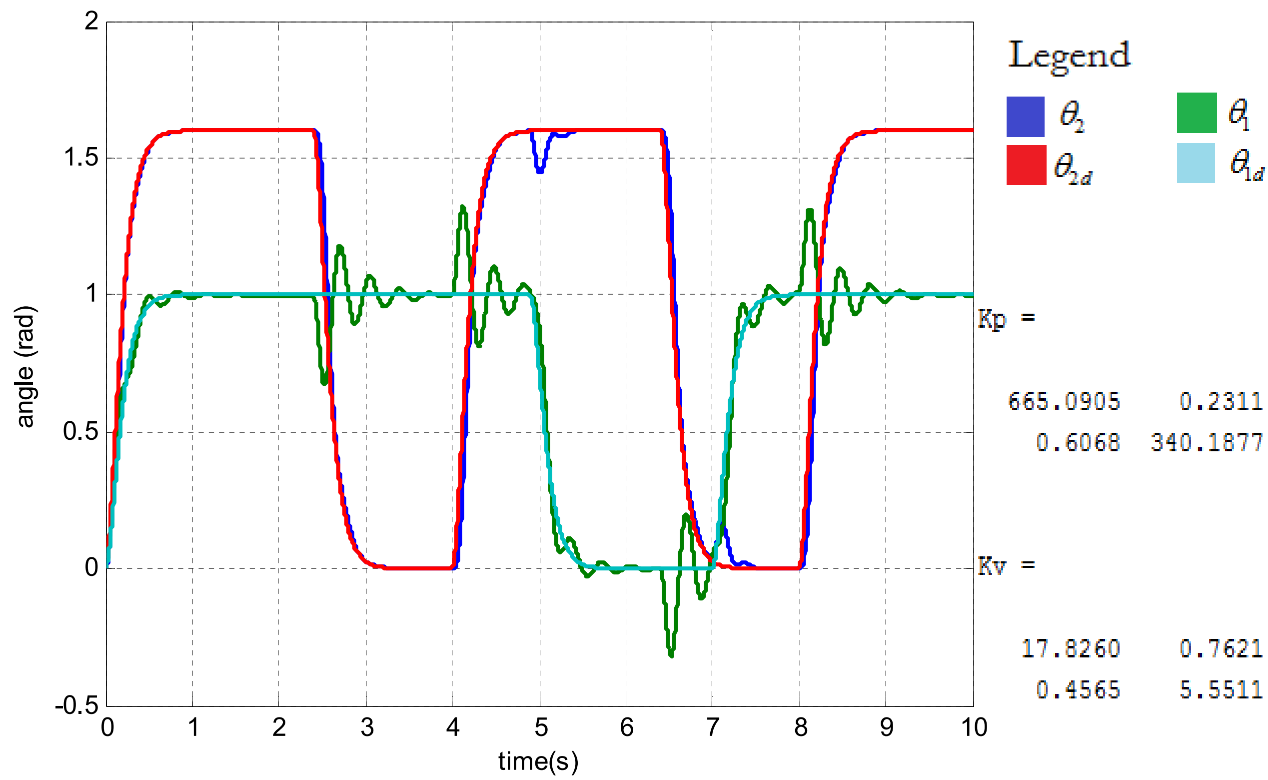

Figure 4.5 Pulse reference Input/Output With optimized Coefficients K_p and K_v Using PSO

Secondly, with an step reference let's calculate the two coefficients K_p and K_v in 10 iterations

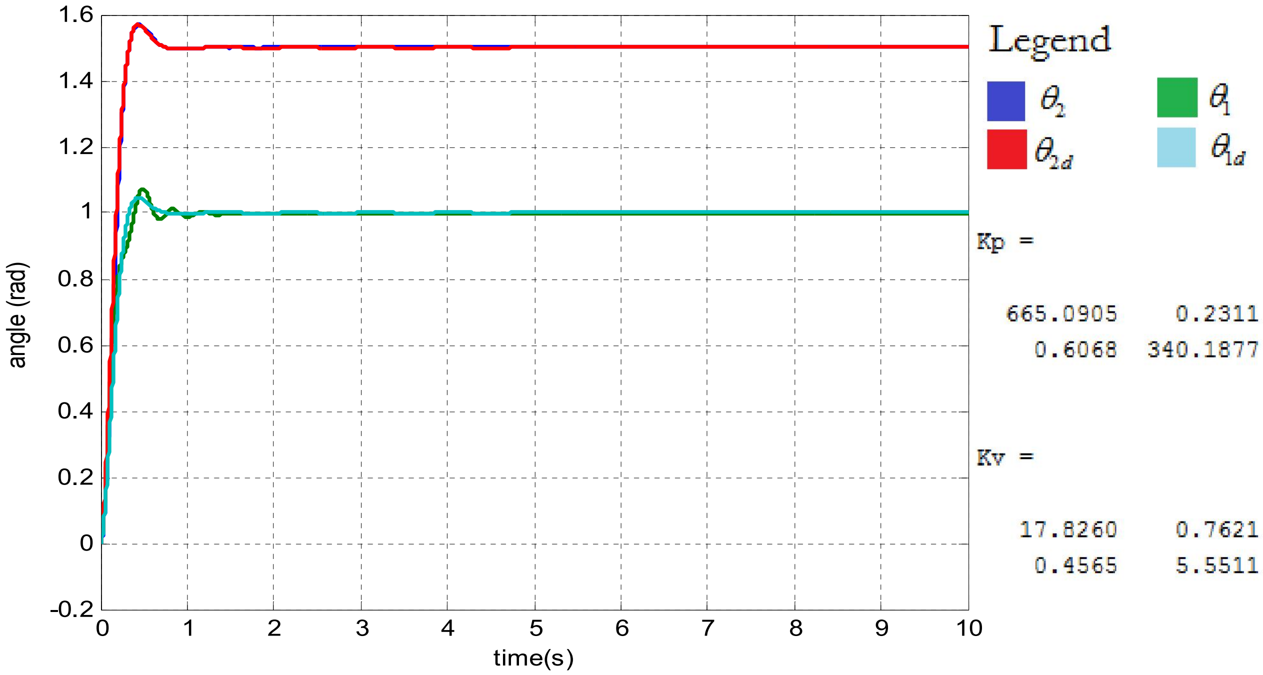

Figure 4.6 Step reference Input/Output With optimized Coefficients K_p and K_v Using PSO

Finally, with a sinusoidal reference let's calculate the two coefficients K_p and K_v in 10 iterations

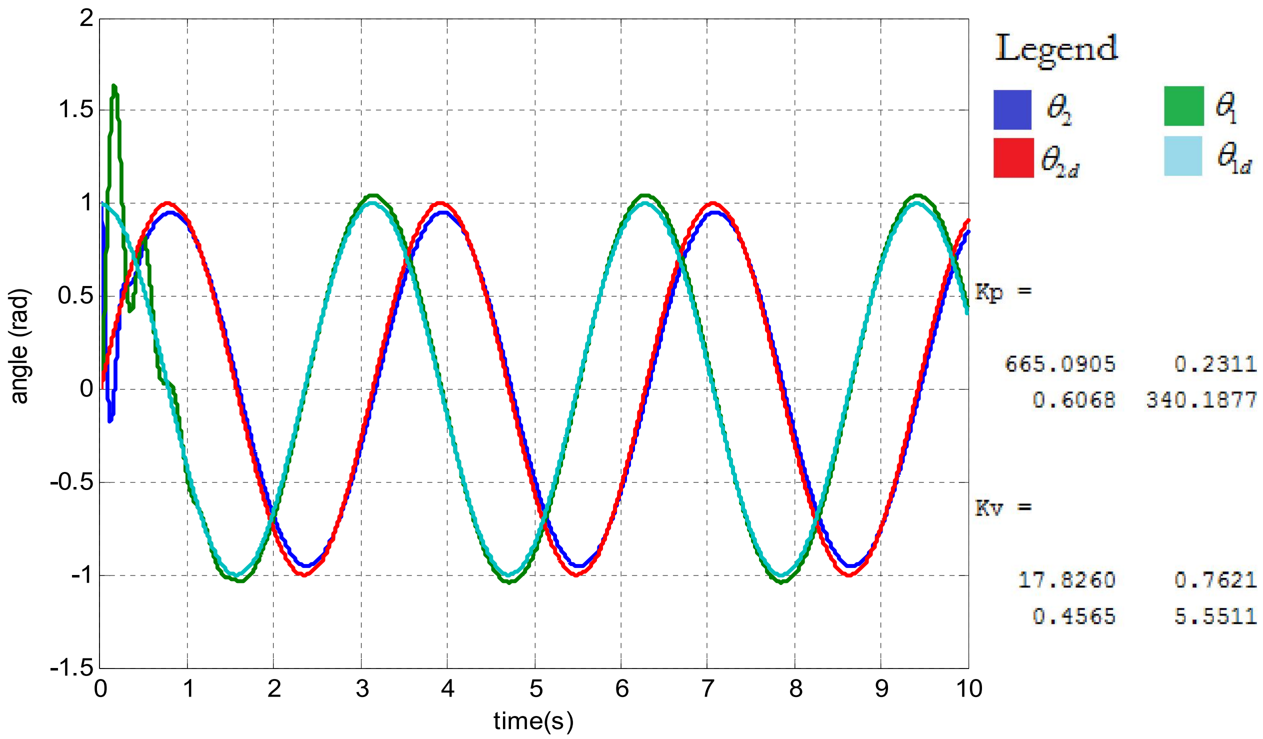

Figure 4.7 Sinusoidal reference Input/Output With optimized Coefficients K_p and K_v Using PSO

4.4.2 ACO Application

First, with an pulse reference let's calculate the two coefficients K_p and K_v in 10 iterations

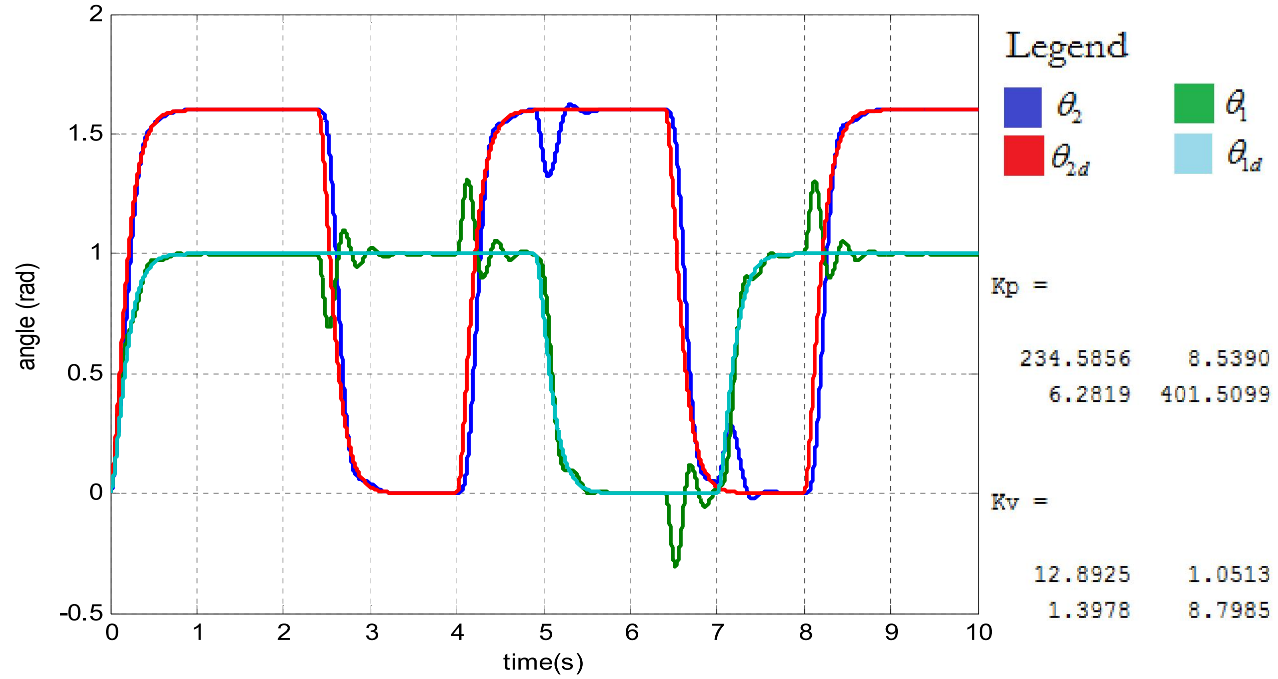

Figure 4.8 Pulse reference Input/Output With optimized Coefficients K_p and K_v Using ACO

Secondly, with an step reference let's calculate the two coefficients K_p and K_v in 10 iterations

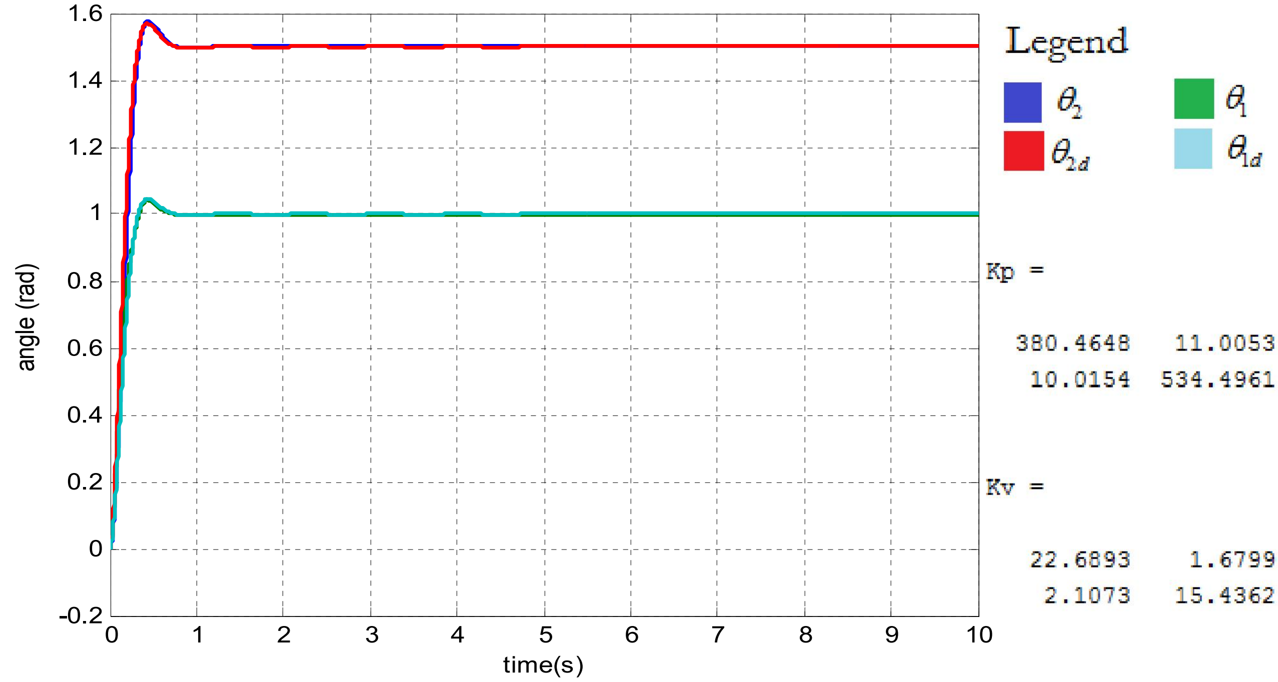

Figure 4.9 Step reference Input/Output With optimized Coefficients K_p and K_v Using ACO

Finally, with a sinusoidal reference let's calculate the two coefficients K_p and K_v in 10 iterations

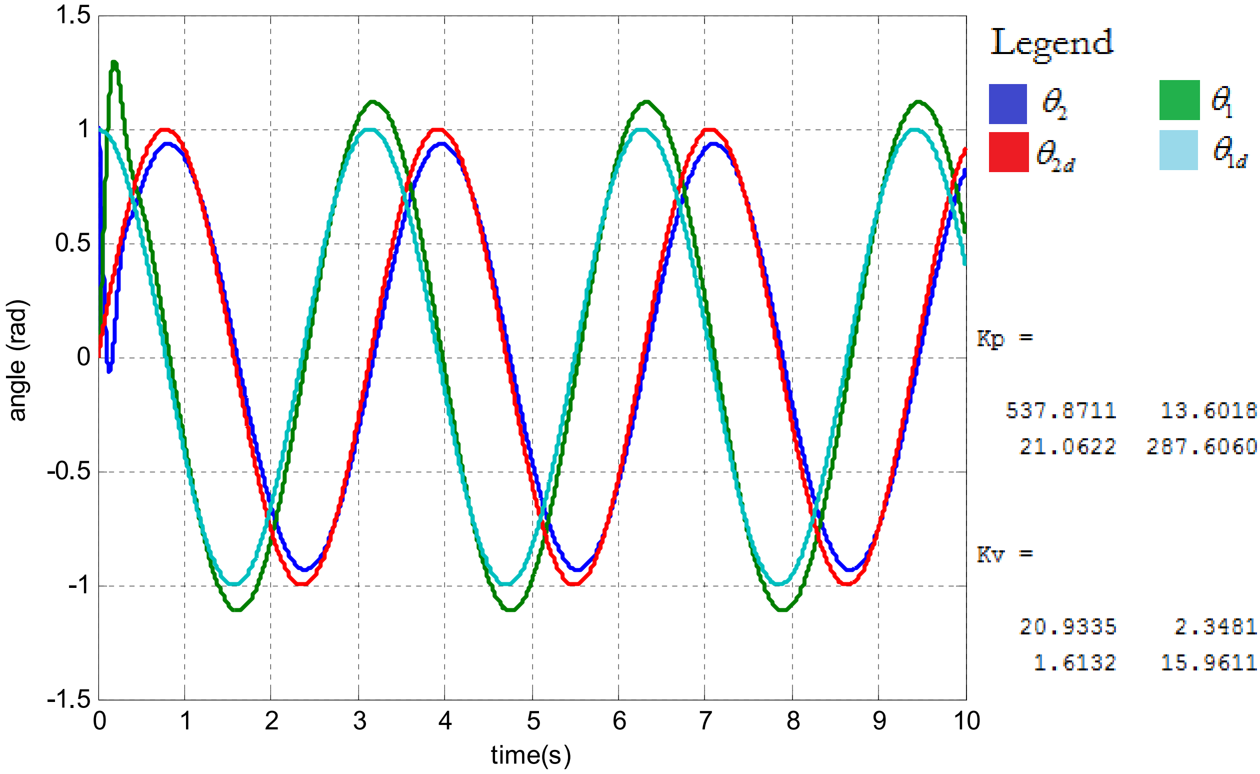

Figure 4.10 Sinusoidal reference Input/Output With optimized Coefficients K_p and K_v Using ACO

4.5 Discussion of the Obtained Results

Now that the results have been obtained, it is time to compare them. Both algorithms have given promising outputs, especially in the case of a Step input where the response time is almost zero, the two I / O signals are superimposed almost instantaneously. In the case of the Pulse, it seems that the two suffer slight interference possibly due to the coupling of the two outputs; in the case of a Sinusoidal input, the response time obtained thanks to ACO is more interesting. It should be emphasized however that ACO in general requires more iterations than the PSO of because it advances with a fixed step; it is then preferable to reuse the results obtained by decreasing the step for increased precision.

Overall, for a rather low number of common iterations (10), the ACO gave slightly more satisfactory results than the PSO. Note the capital importance of the response time for a Turret Gun, especially in the case of moving targets and taking into account the ignition time of the ammunition (s).

Chapter 5

Turret Gun Realization

In this chapter, we try to provide you all the necessary knowledge to realize a Tuuret Gun with a laser pointer, in order to clarify the work performed, it will be structered and decoupled into four (4) major steps: Engineering, Procurement then Construction and Commissioning, at each step more details are fusnished in addition to explanations and tricks to overcome the difficulties in moving from theory to practice, to gain time and materials.

5.1 Introduction

The life of a project starts with the gleam in the eye, an abstract idea; to concretize it we need to create a project planning; its tasks are the design and organization of a system, which fulfills certain requirements under given restrictions at lowest costs. This includes the selection and dimensioning of equipment, resources and other elements, the connection of these elements to performance chains and the design of logistic networks.

The tasks of project realization are the scheduling of the implementation, the construction and manufacturing of the system elements, the build-up of the whole system, and finally the start-up and tests. Both, planning as well as realization need qualified project management.

Even for projects with reduced volume, it is important and even essential to develop a course of action to optimize the efforts to provide, and the costs for optimal results while respecting the time constraints.

In this context, we cut this realization into four main parts: engineering or abstract conception, procurement, construction and finally commissioning, and each part will be discussed and explained in order to expose all the problems and difficulties encountered with their detailed solutions.

5.2 Engineering and Abstract Conception

The first step is essential, it consists in inventing or innovating an idea, a concept or even a product, at this stage, the sketches and corrections are progressively made, taking into account the constraints, and possibly the materials and resources available. Everything must be meticulously planned, any error during this phase are reflected throughout the realization and can even prove to be uncorrectable, and everything must be redone.

For this first step and using a 3D modeling software to have an idea about the global steps. We have two (2) degrees of freedom, we will need two servomotors, and we have chosen the following basic configuration in the Fig.5.1

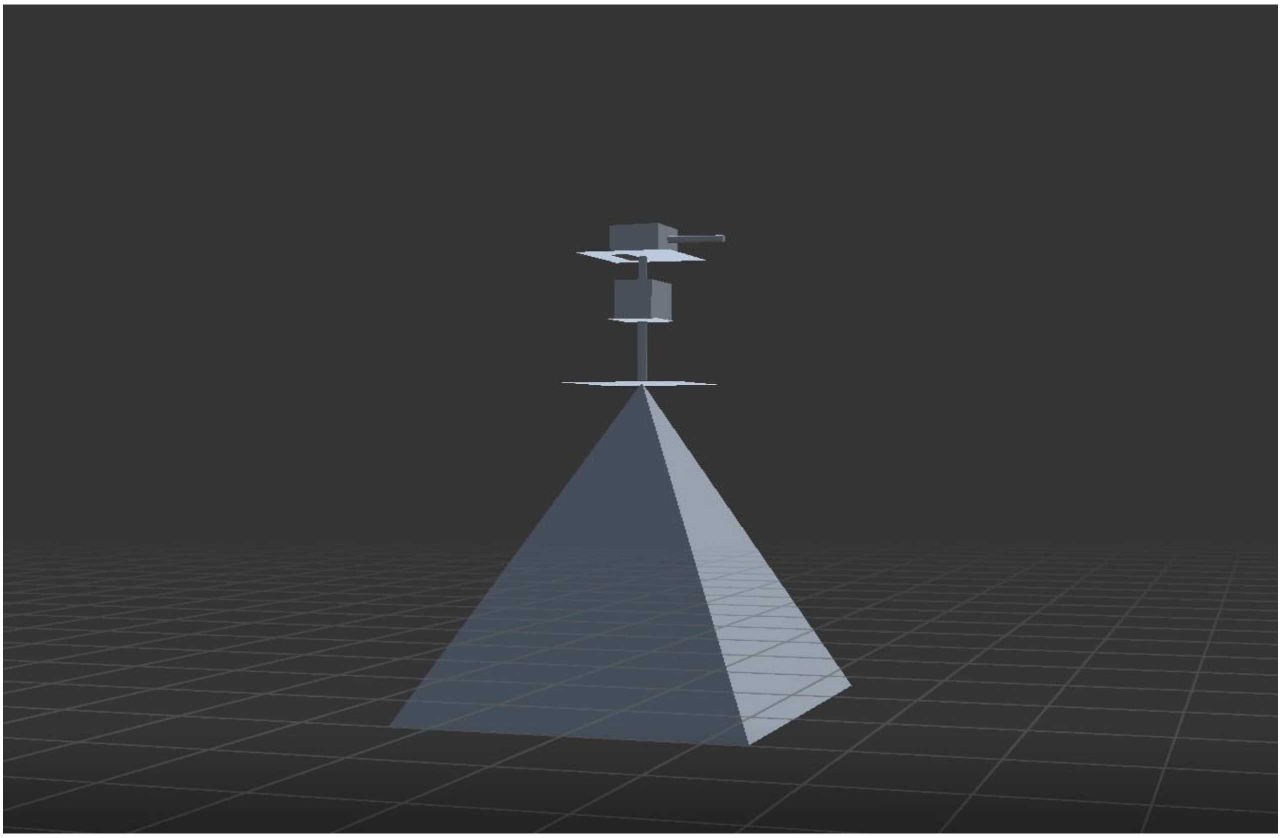

Figure 5.1 The basic configuration chosen

Then, we will opt for the following cannon fitted with a laser

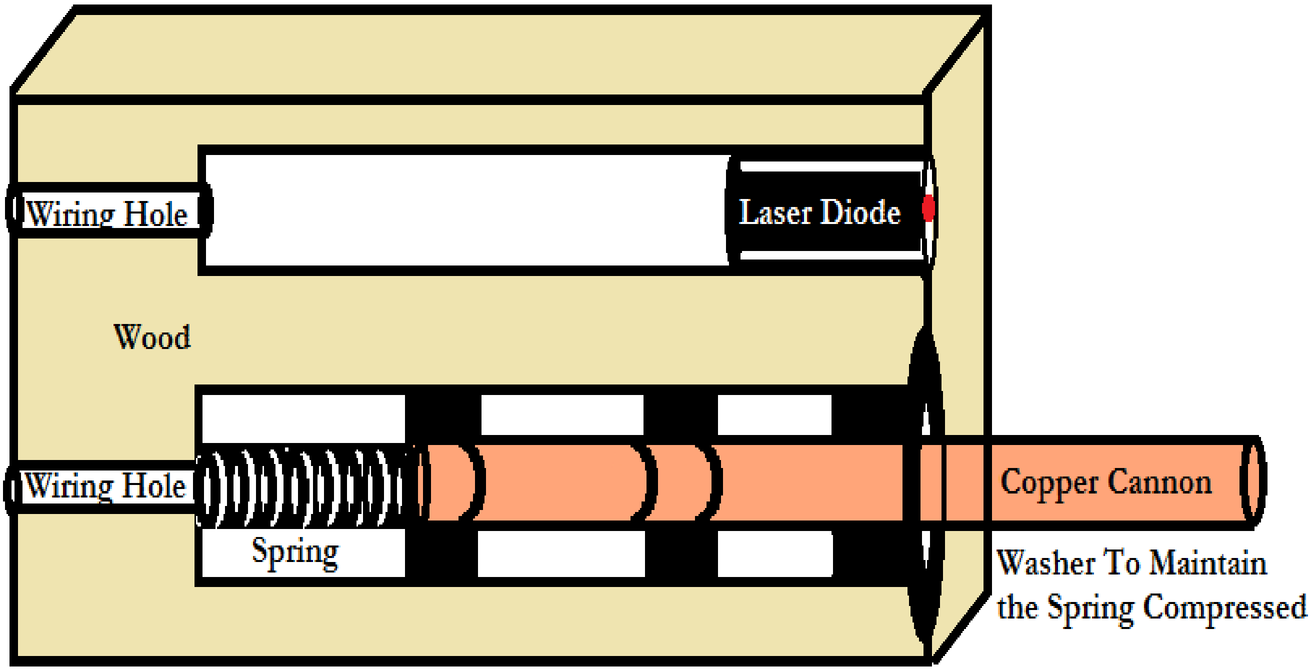

Figure 5.2.a Simplified 2D Cut of The Cannon

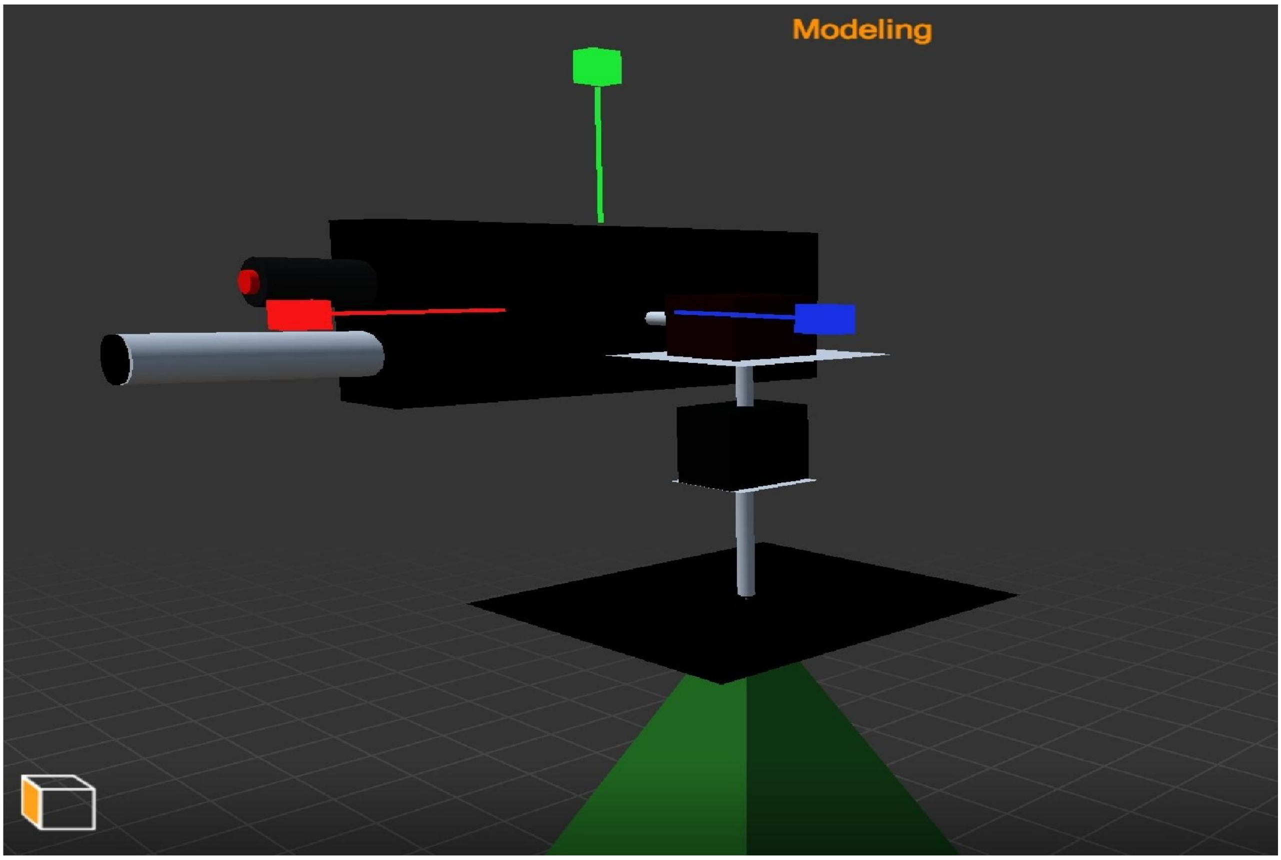

Figure 5.2.b The 3D Cannon Configuration

The final result must then have the following form

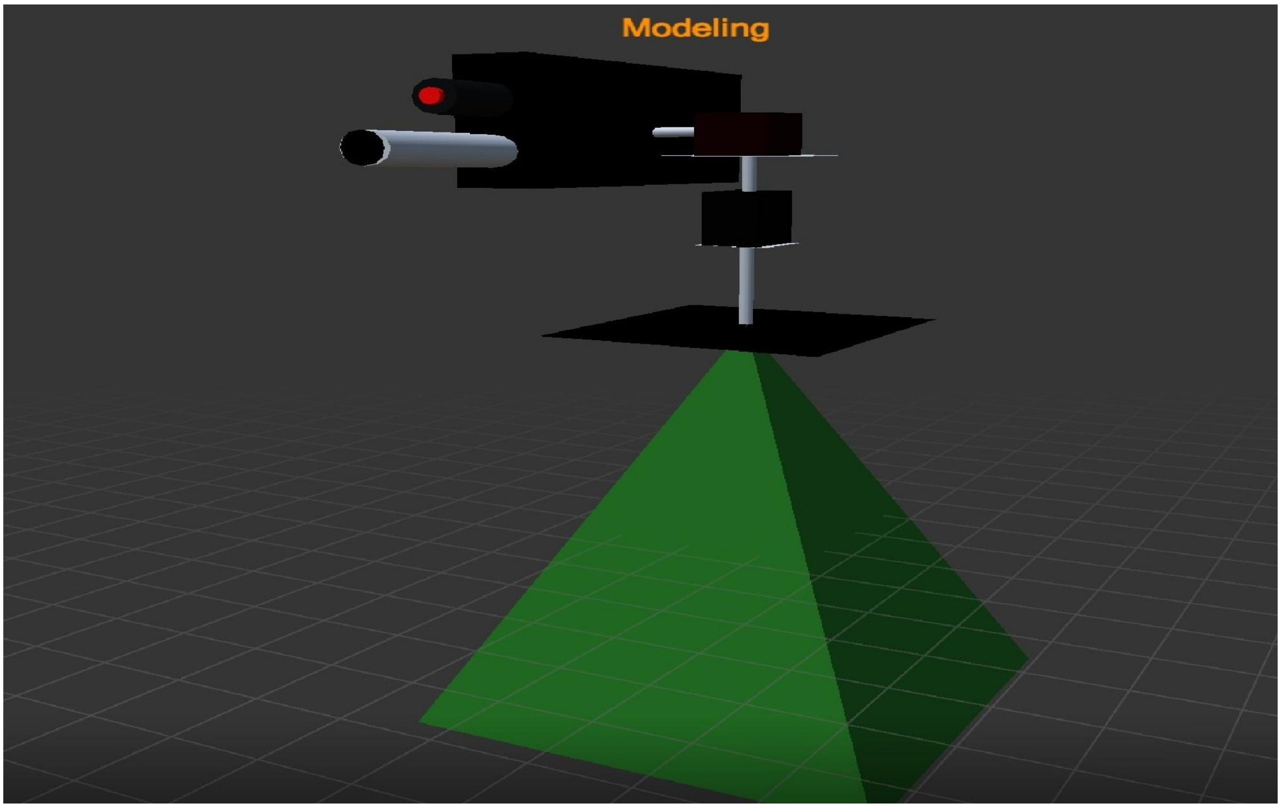

Figure 5.3 The Final 3D Model of The Turret Gun

Let us now illustrate the two degrees of freedom :

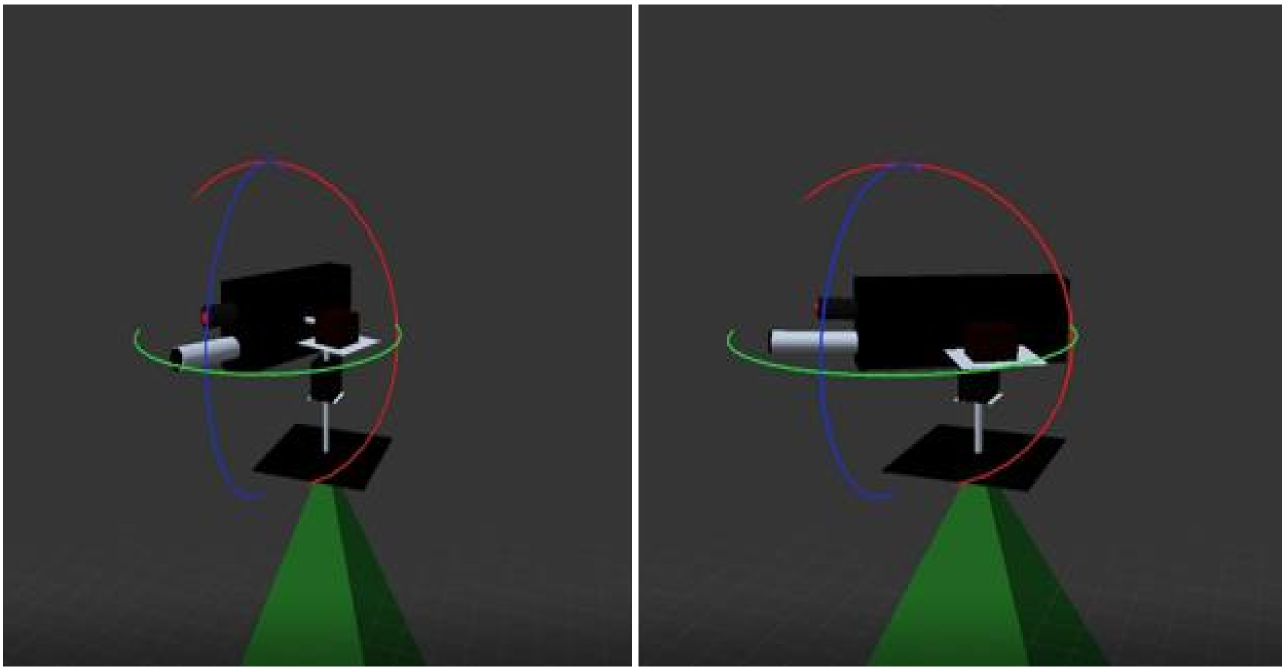

Figure 5.4 The Illustration of the First Degree of Freedom

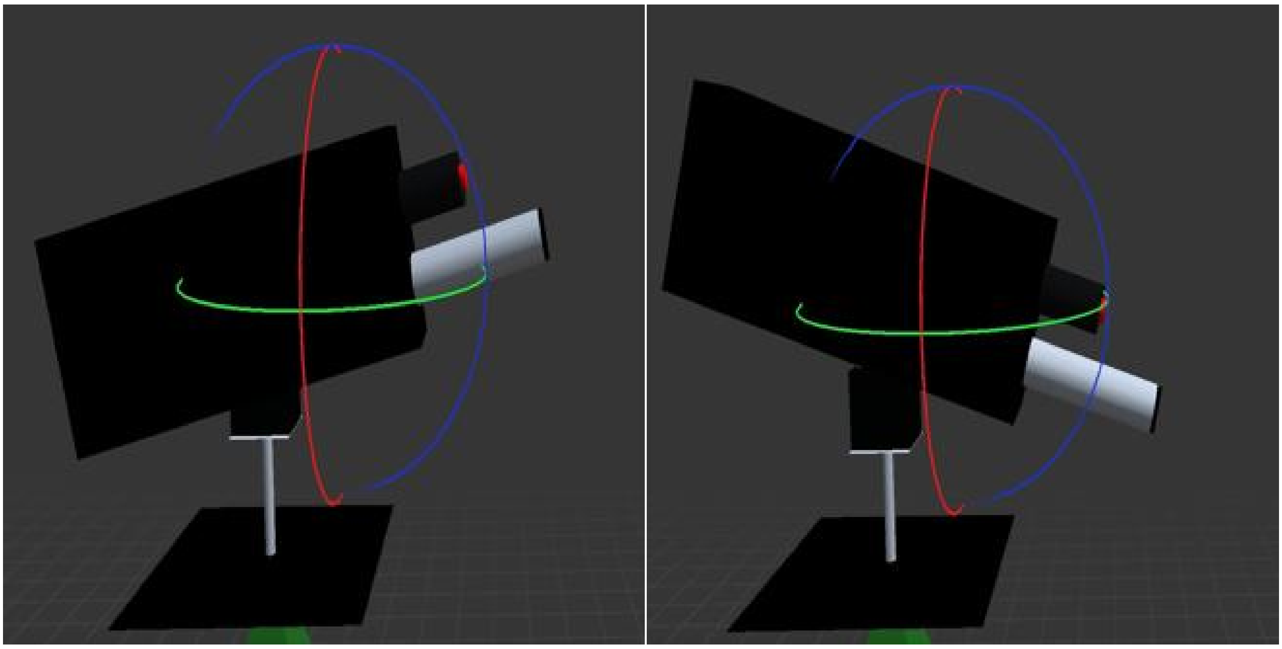

Figure 5.5 The Illustration of The Second Degree of Freedom

The above illustrations show that the final product will be fully functional, and the performance will be amply satisfying.

5.3 Procurement :

Now that we have a more precise idea, we must find the adequate materials filling the specificities related to the physical constraints, but also monitoring the overall budget.

5.3.1 The servos

We will use the servo MG996R

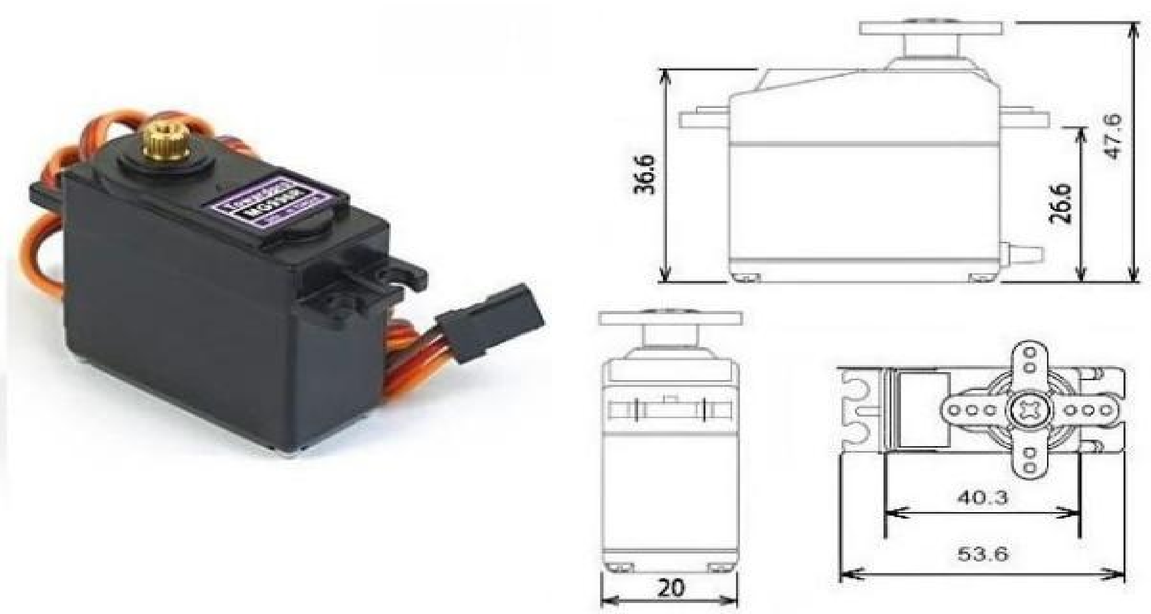

Figure 5.6 The Dimensions of the MG996R Servo in Millimeters (mm)

This High-Torque MG996R Digital Servo features metal gearing resulting in extra high 10kg stalling torque in a tiny package. The MG996R is essentially an upgraded version of the famous MG995 servo, and features upgraded shock-proofing and a redesigned PCB and IC control system that make it much more accurate than its predecessor. The gearing and motor have also been upgraded to improve dead bandwith and centering. The unit comes complete with 30cm wire and 3 pin 'S' type female header connector that fits most receivers, including Futaba, JR, GWS, Cirrus, Blue Bird, Blue Arrow, Corona, Berg, Spektrum and Hitec.

This high-torque standard servo can rotate approximately 120 degrees (60 in each direction).
We can use any servo code, hardware or library to control these servos, so it's great for beginners who want to make stuff move without building a motor controller with feedback & gear box, especially since it will fit in small places. The MG996R Metal Gear Servo also comes with a selection of arms and hardware.

<u>**Specifications**</u>

- Weight: 55 g
- Dimension: 40.7 x 19.7 x 42.9 mm approx.
- Stall torque: 9.4 kgf·cm (4.8 V), 11 kgf·cm (6 V)
- Operating speed: 0.17 s/60º (4.8 V), 0.14 s/60º (6 V)
- Operating voltage: 4.8 V a 7.2 V
- Running Current 500 mA — 900 mA (6V)
- Stall Current 2.5 A (6V)
- Dead band width: 5 µs
- Stable and shock proof double ball bearing design
- Temperature range: 0 ºC – 55 ºC

Wiring

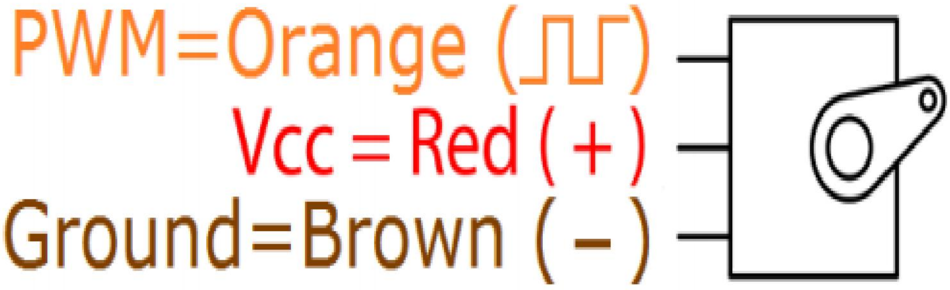

Figure 5.7 Servo's Wiring Color Code

5.3.2 The Laser Diode

Laser Output Type: Red CROSS-LINE Laser
Dimensions: Diameter: 12mm, Lenght: 35mm.
Weight: 15 gm.
Wavelength: 650nm.
Voltage: 3~6V.
Output Power: Min 2.5mW, Typical 3.0mW, Max 5.0mW.
Working current: Min 10mA, Typical 20mA, Max 25mA.
Focus point Intensity can be adjusted with help of lens ring head.

Figure 5.8 The 5.0mW Laser Diode

5.3.3 The Microcontroller

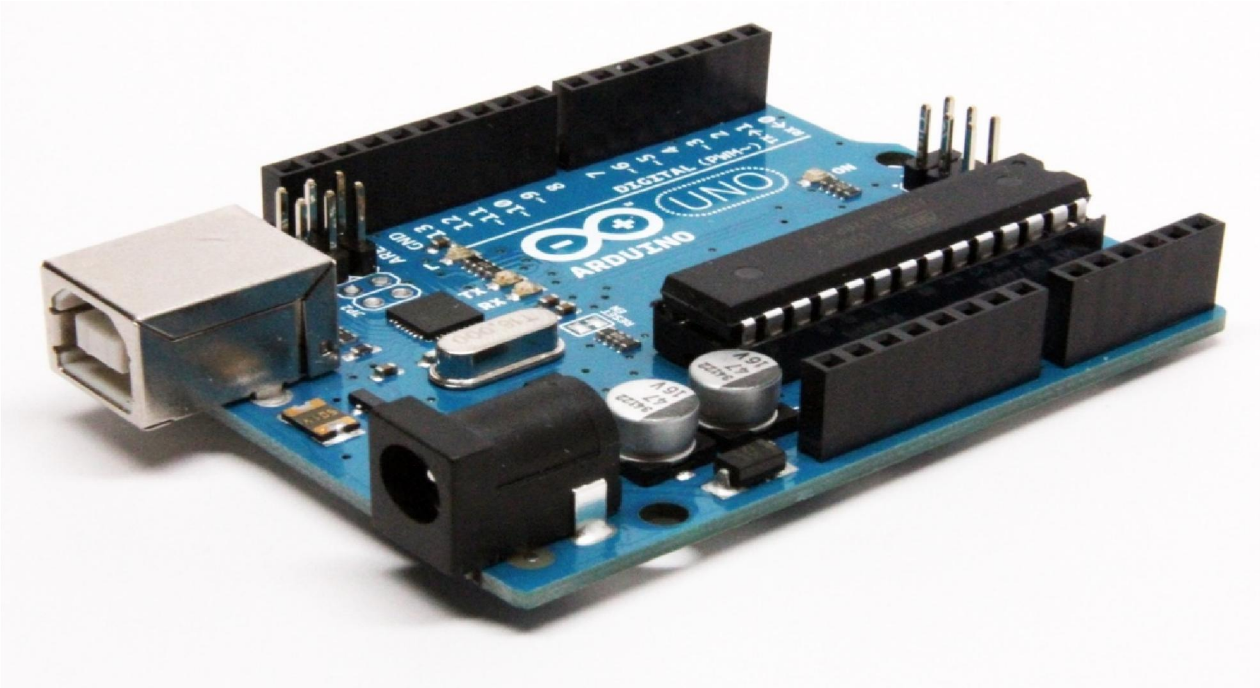

Figure 5.9 Arduino Uno

The Arduino Uno is a microcontroller board based on the ATmega328 (datasheet). It has 14 digital input/output pins (of which 6 can be used as PWM outputs), 6 analog inputs, a 16 MHz crystal oscillator, a USB connection, a power jack, an ICSP header, and a reset button. It contains everything needed to support the microcontroller; simply connect it to a computer with a USB cable or power it with a AC-to-DC adapter or battery to get started. The Uno differs from all preceding boards in that it does not use the FTDI USB-to-serial driver chip. Instead, it features the Atmega8U2 programmed as a USB-to-serial converter.

Specifications:

Microcontroller	ATmega328
Operating Voltage	5V
Input Voltage (recommended)	7-12V
Input Voltage (limits)	6-20V
Digital I/O Pins	14 (of which 6 provide PWM output)
Analog Input Pins	6
DC Current per I/O Pin	40 mA
DC Current for 3.3V Pin	50 mA
Flash Memory	32 KB of which 0.5 KB used by bootloader
SRAM 2 KB EEPROM	1 KB
Clock Speed	16 MHz

5.3.4 The Solderless breadboards

A modern solderless breadboard socket consists of a perforated block of plastic with numerous tin plated phosphor bronze or nickel silver alloy spring clips under the perforations. The clips are often called tie points or contact points. The number of tie points is often given in the specification of the breadboard.
We will use it because it is reusable, and easier to manipulate then soldering directly the wires.

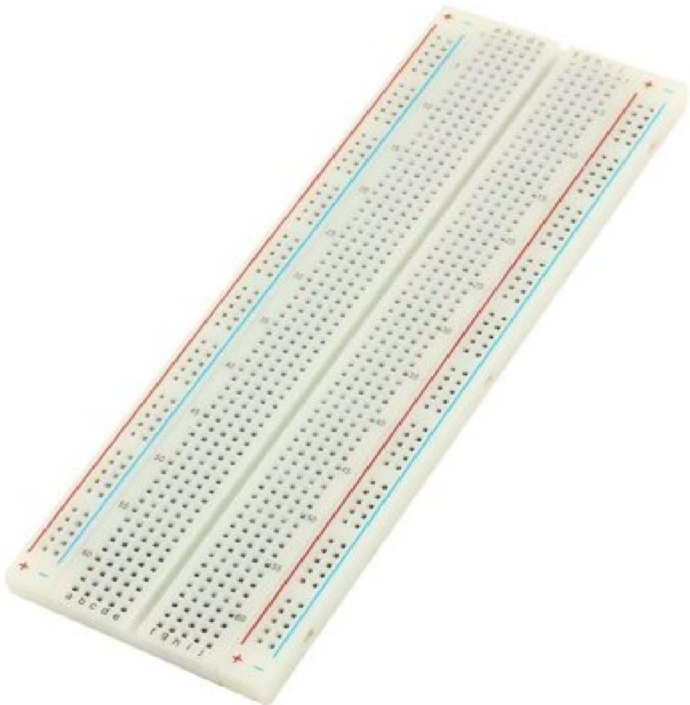

Figure 5.10 Solderless breadboards

5.3.5 Wires Alimentation and Materials

We use jumper wires male to male

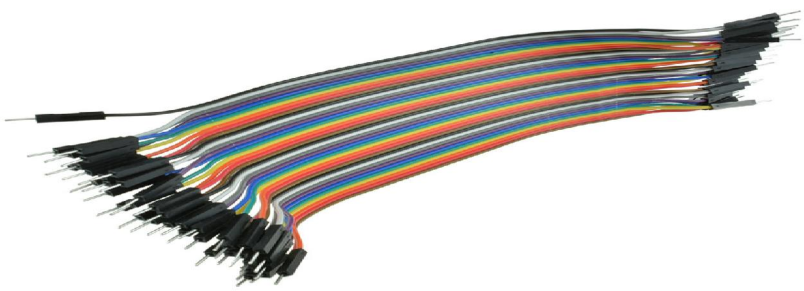

Figure 5.11 Jumper Wires Male to Male

We will need three (3) 5v transformers for the both servos and the laser diode and a 9v transformer for the firing mechanism

Figure 5.12.a 220v to 5v Alimentation

We will also use a metallic Tripod for the frame

Figure 5.12.b Metallic Tripod

In addition, wood will be used for the platforms and the body of the copper tube cannon on which will be mounted the laser diode.

5.4 Construction

This section is the outcome of the two precedent steps; We will use the materials to create concretely from the abstract plans.

In order to do that correctly, we must adapt our project to the reality to overcome the various constraints that physics, materials and costs impose, and we will clearly state all the difficulties encountered, and solutions developed.

Finally, we will mount every parts together to check that everything fits perfectly before the first tests, ensuring that there are no major faults, because that would make the commissioning phase risky.

5.4.1 Mounting the Servos

First will fix the servos with epoxy glue and screws to wooden boards, then using a longer screw, we will fix those boards to the Tripod

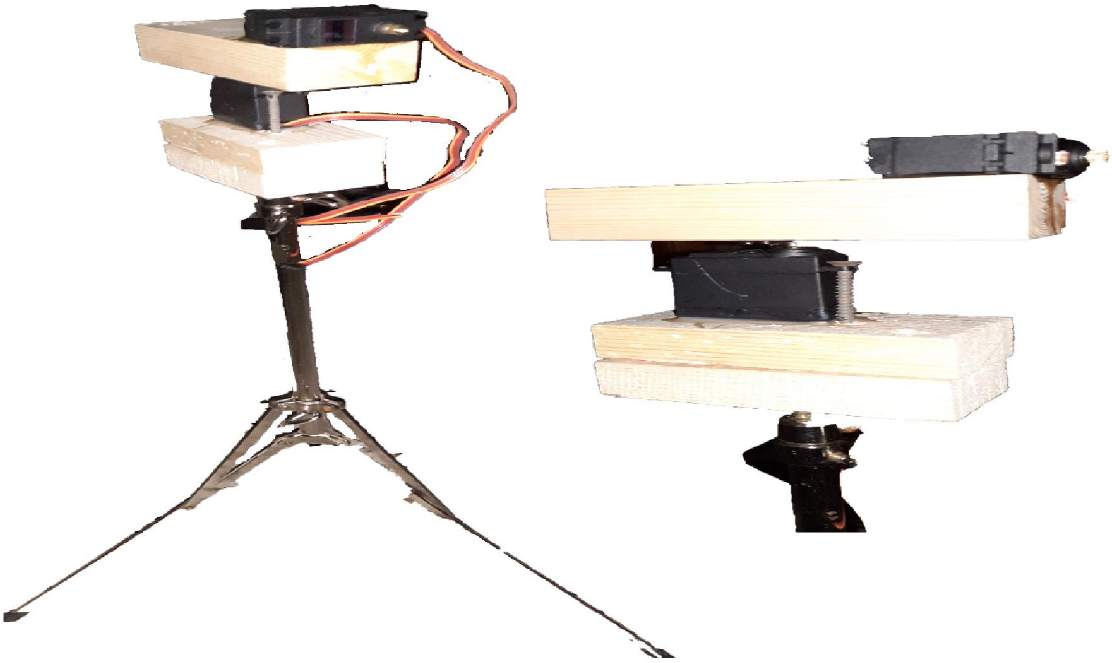

Figure 5.13 The Assembly of the Servos on Their Wooden Supports

5.4.2 Cannon's Wooden Body

In this section, we will build the wooden block that will host the cannon ,.It must be drilled with precautions to prevent the wood from splitting

Figure 5.14 The Wooden Body of The Cannon

5.4.3 Barrel Construction

First, let’s cut the copper tube with the right dimensions, and we will solder to it two copper rings with the right diameter to allow the barrel to slide while absorbing the recoil force

Figure 5.15 The Copper Tube with Rings Soldered into it

Then, we will build the firing system on which the spring will be stuck strongly using epoxy glue and then to the copper tube.

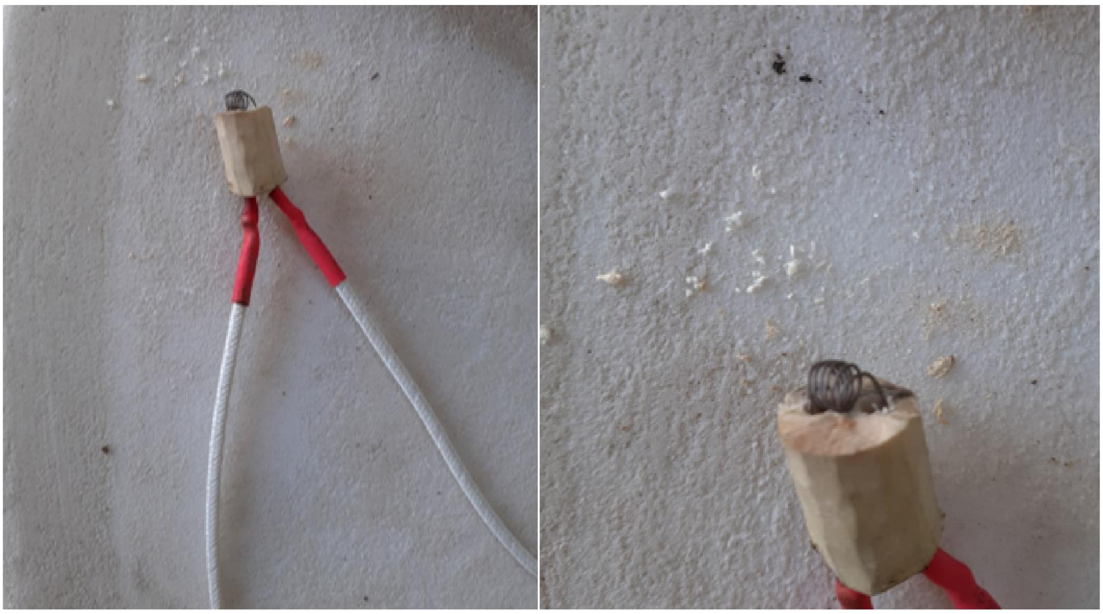

Figure 5.16 The Electric Firing System

We get the following result

Figure 5.17 The Barrel with The Firing System and Recoil Absorption System

5.4.4 First Assembly and Paint Job

We reach the end of the construction stage; it only remains for us to assemble all the parts to make sure they match perfectly.

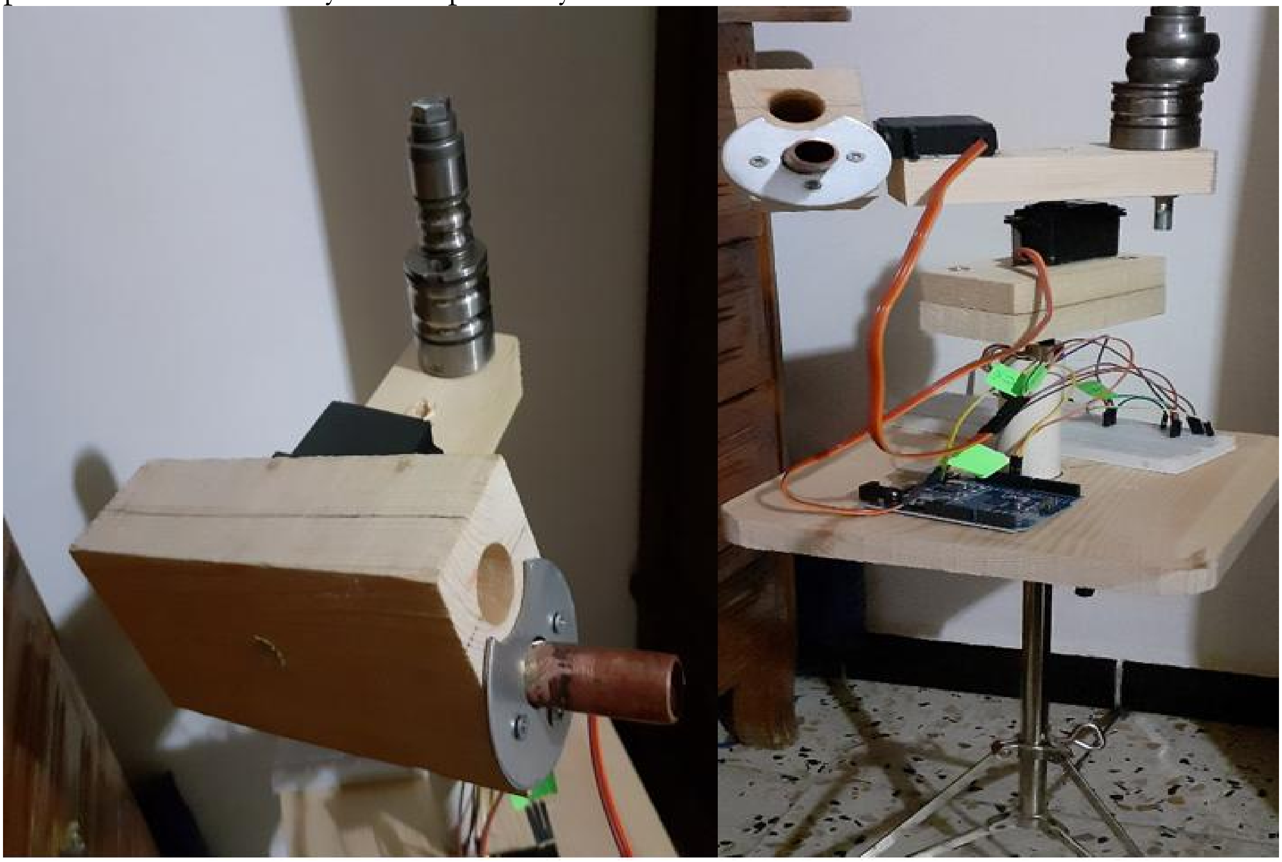

Figure 5.18 First Assembly of The Project

Then we will disassemble them again, cover with adhesive the parts that should not be painted.

Figure 5.19 All The Parts of The Turret Gun Ready for Painting

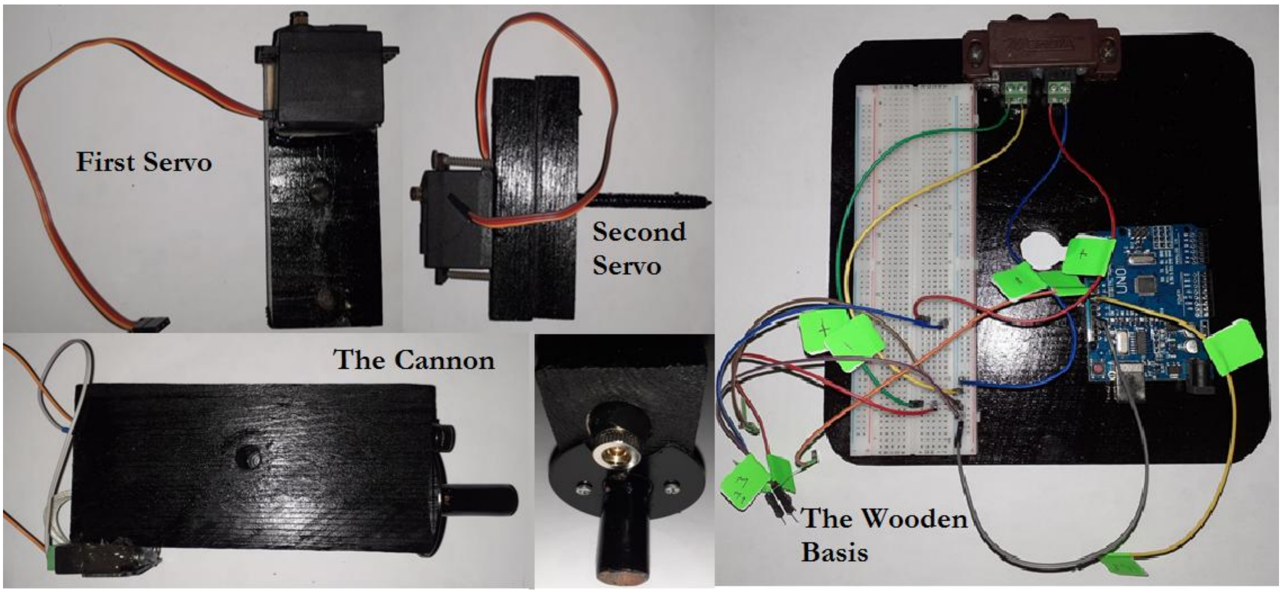

Figure 5.20 All The Parts of The Turret Gun Painted

After the applied paint has dried, we assemble all the painted components for the tests that will be performed in the next phase.

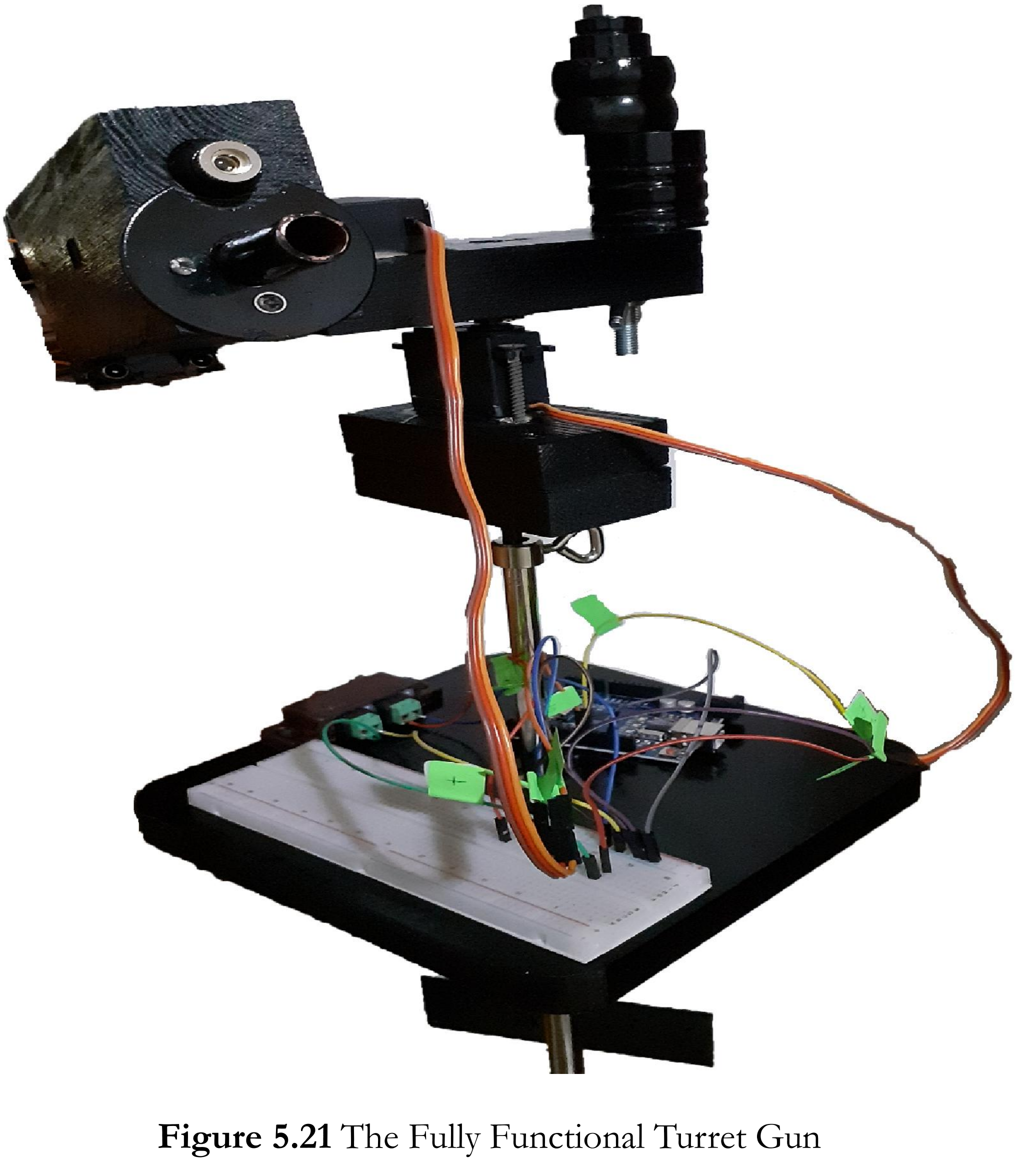

Figure 5.21 The Fully Functional Turret Gun

5.4 Commissioning

Now that the construction phase is complete, it is time to move on to testing, this step marks the end of the realization, and the culmination of the previous phases.

The commissioning is the process that ensures the proper functioning of the final product, that all systems and subsystems are in perfect harmony in their operation and that none of them disturbs another, to finally be able to say that the project is finished.

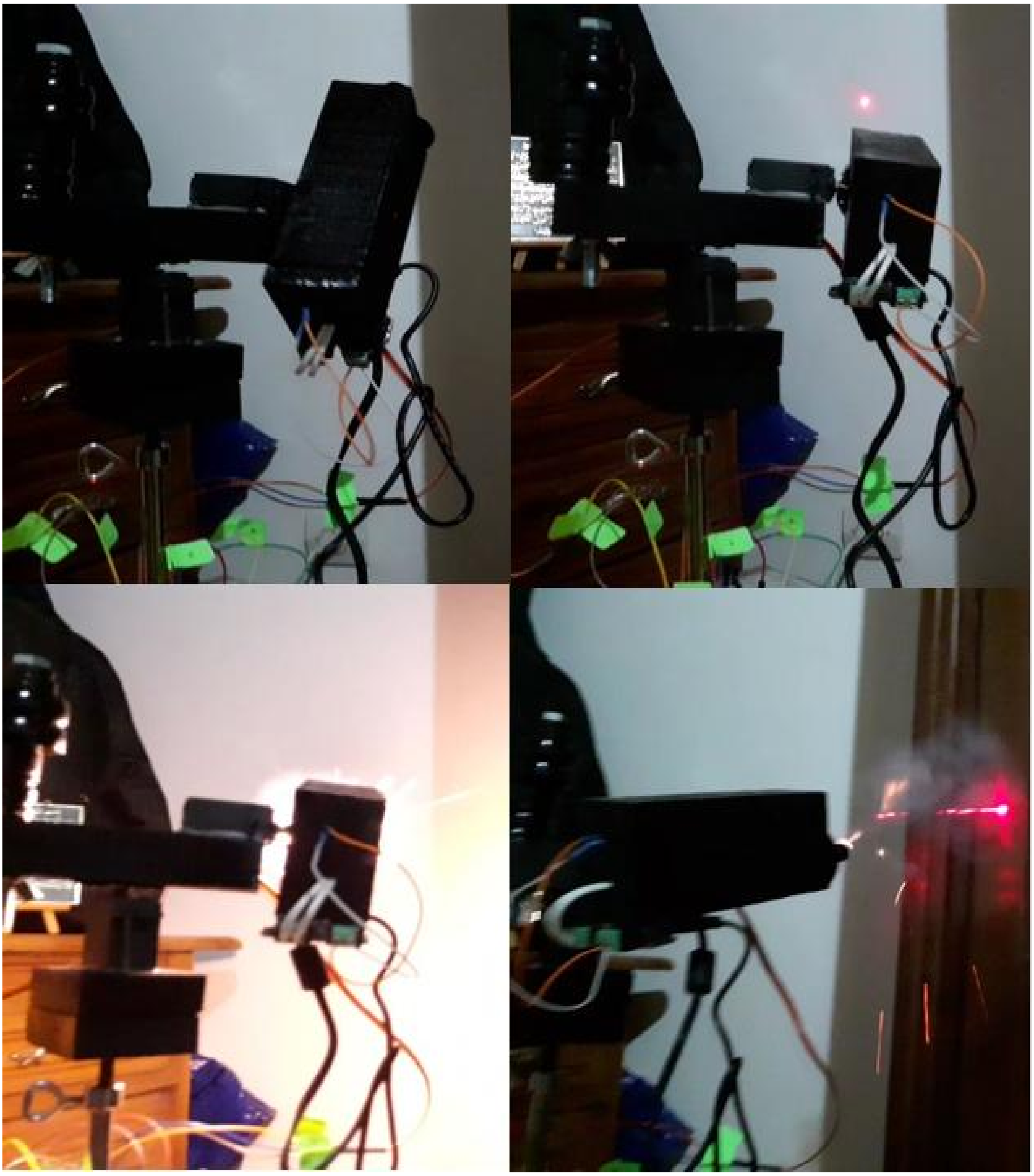

Figure 5.22 The Turret Gun Firing Tests

General Conclusion

In this humble work on the study of the Turret Guns; based on an already existing dynamic model of which we have best shown the stages of obtaining it, and relying on Classical Control of Rigid Robot manipulators, we were able to obtain the expected results, in other words realizing a controller capable of forcing the studied system to give the desired outputs.

Going further still to optimize this controller to the maximum, so as to leave no margin for error due to the strategic sensitivity represented by the setting of such a powerful and modern weapon.

Regarding the use of ACO and PSO for the adaptation of parameters of a PD controller with Gravity Compensator, an introduction to the basics of these two algorithms was presented.

Study of the behavior of the Turret Gun and its performance helped to retain the following results:

• Evidence of the effectiveness of the feedback linearization.
• PD controllers with Gravity compensator have shown more than satisfactory performance for the control of our dynamic model;
• The optimization of the controller coefficients thanks to the two algorithms PSO and ACO made it possible to minimize the errors and a clear improvement of the response time of the system to the command.

This memory will allow to have a more oriented view and to imbue the precepts of the Theory of Control, especially in such interesting domain, because of its vital importance for any nation worthy of the name.

It is a question of going beyond the abstract of fascinating mathematical formulas to an even more breathtaking and dizzying reality that modern life is, due to all the dazzling advances.

The bibliographic study of Fire Control allowed us to present a brief history on the use of the different forms that we had with conventional shooting weapons, and to determine the path they took to reach their modern forms

Having been able to see this synergy more closely, the outcome of the mind when it meets matter, giving it life, making it bend, submitting to its will, is a privilege.

After having chosen the most suitable approach, the controller adapted to our case study and followed the progress of these stages until the result; it was even more enriching to develop our prototype, which piece by piece assembled like a puzzle where each part played its role.

Experience, or rather the translation of theory and practice, creates sparks as they complement each other, thwarting unexpected and mishaps. In the end, with a minimum of material we put within reach of all this achievement.

Arimoto S. (1990) "Design of robot control systems" Advanced Robotics, Vol. 4, pp. 79-97.

Arimoto S. and F. Miyazaki, (1984) "Stability and Robustness of PID Feedback Control for Robot Manipulators for Sensory Capability", in 'Robotics Research', eds. M. Brady and R. P. Paul, pp.783-799, Cambridge, MIT Press.

Asada H. and J. J. E. Slotine (1986) "Robot analysis and control" N. Y., John Wiley and sons

Bekhiti Belkacem (2019) "Dynamic Modeling & Control of Large Space Structures: Part1 Missiles & Space-Crafts"

Bekhiti Belkacem, Abdelhakim Dahimene ,Kamel Hariche (2017) "Advanced nonlinear control and state observation in robotics" Scholars' Press

Bekhiti Belkacem(2017) "Multivarible Control System Design Using the Théory of Matrix Polynomials". PhD Thesis University M'Hamed Bougara of Boumerdes, Algeria

Belkacem Bekhiti, Abdelhakim Dahimene ,Kamel Hariche (2017) "Intelligent Guidance and Control of Quadrotor Unmanned Aerial Vehicl" LAP LAMBERT Academic Publishing

Berghuis H., (1993a) "Model Based Control: From Theory to Practice". PhD Thesis, University of Twente, Holland.

Carlucci, Donald E.(2014) "Ballistics: theory and of guns and ammunition"/ Authors, Donald E. Carlucci, Sidney S. Jacobson. --2nd edition.

Dawson D. M., Z. Qu and J. C. Caroll (1992) "On the observation and output feedback problems for nonlinear uncertain dynamic systems" Systems & control letters 18.

Dombre E. and W. Khalil (1988) "Modelisation et commande des robots", Hermes, Paris.

Doringo M. , Stutzle T. Birattari M. (2006) "Ant Colony Optimization" Article in IEEE Computational Magazine

Khelfi M. F., (1995a) " Observateurs non lineaires, Application a la commande des robots manipulateurs" Ph.D Thesis, Univ. Henri Poincare, Nancy I, France

Khelfi M. F., M. Zasadzinski, M. Darouach and E. Richard (1995b) "Reduced order H observer for robot manipulators" Proc. Of the 34th Conf. On Decision and Control. New Orleans, pp. 1011-1013.

Lewis F. L., C. T. Abdallah and D. M. Dawson, (1993) "Control of robot manipulators" N. Y., Macmillan

Nijmeijer H. and A. J. Van der Schaft, (1990) "Nonlinear Dynamical Control Systems", Berlin, Springer-Verlag.

Ortega R. and M. Spong, (1989) "Adaptive motion control of rigid robots: A Tutorial". Automatica, vol.25 'pp. 877-888.

Paden B. and R. Panja, (1988) "Globally Asymptotically Stable 'PD +' Controller for Robot Manipulators", Int. Journal on Control, Vol. 47, pp. 1697-1712

SALMI Saadi (2019) "Introduction to the Meta-Heuristic Optimization" DAR KIFAYA, Bab-Ezzouar, Algeria, pp 11-20

Slotine J. J. E., W. Li, (1987a) "On the Adaptive Control of Robot Manipulators", Int. Journal of Robotics Researches, Vol. 6, pp. 49-59.

Spong. M. W. and R. Ortega, (1990) "On adaptive inverse Dynamic control of rigid robots". IEEE. Trans. on automatic control, Vol.35, pp. 92-95.

Spong. M. W, F. L. Lewis and C. T. Abdallah, (1994) "Robot control, dynamics, motion planning, and analysis". IEEE. Press.

Takegaki M. and S. Arimoto, (1981) "A new Feedback method for Dynamic Control of Manipulators", ASEM Journal of Dynamic systems, Measurement and Control, Vol. 102, pp. 119-125.

Van der Schaft A. J., (1990) "System Theory and Mechanics", In 'Three Decades of Mathematical System Theory', Eds. H. Nijmeijer and J. M. Schumacher, pp. 426-452. Lecture Notes in Control and Information sciences, Vol. 135, Berlin, Springer-Verlag.

Vidyasagar M. (1992) "Nonlinear Systems Analysis - second edition" New Jersey: prentice Hall Inter

Yoshikawa T., (1990) "Foundation of Robotics: Analysis and Control", MIT Press.

Zodiac (1992) "Notes on: Theory of robot control", Edited for the European summer school of the laboratory of Automatic control of Grenoble, France.