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Faculty of Sciences

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Master Thesis

Fermion mass mixing and Higgs boson width at Next-to-Leading Order

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Abstract

In this master's thesis, we study two issues. The first is the fermion mass mixing and the second is the Higgs boson decay. All of this is within the framework of the Electro-Weak theory of the Standard Model. We review both the mixing and the Higgs mechanism to study the effect of the mixing on the Higgs boson decay. In addition for that purpose, It was necessary to go through the process of renormalization in order to get rid of any ultraviolet divergence. The study is executed at one loop level by using special calculation tools under the Mathematica program, that are FormCalc, FeynCalc and FeynArt.

Keywords: EWSM, Higgs Boson decay, Mass mixing, FormCalc, FeynCalc and FeynArt.

Dedication

First of all, I thank God, almighty, for having enlightened me on the good path to follow and for giving me the strength and daring to overcome all difficulties, then I dedicate my humble work to my loving parents, without whom I would not have been able to continue and persevere. My sister Farah and my brothers Adnan, Firas, and Acil, who were always supportive with their love. I extend my special thanks to both Dr. Bouayad Noureddine and Dr. Yanallah Abdelkader, and I am so grateful to them for their guidance and direction. Each of my loving friends Zina and Zola, and special thanks to Kemmane Mohammed who never left my side and who had great merit in completing this work.

Zghiche Zahra

I decicate this work to my parents who are the most important persons in my life, I also dedicate this work to Dr. Bouayad Noureddine and Dr. Yanallah Abdelkader and i am so thankful for thier help and a special dedicate for my husband Abdellah.

Bechou Dalila

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Introduction

During the past century, a major event changed the way we understand the nature: the merging of quantum mechanics with special relativity, leading to the birth of the so-called The Standard Model of physics particle [11], [12], [18], [19] which was experimentally confirmed with a great accuracy. A model through which we were able to give a simple explanation of the concept of mass, through the Higgs mechanism and symmetry breaking. According to this mechanism, the electromagnetic and weak interactions are unified into electroweak interaction at high energy. This interaction gives rise to a scalar boson with which the particles interact to acquire their masses. The search for the Higgs boson was the holy grail of all particle accelerators until July 2012 [28]. Finally, the missing single particle was found to complete the Standard Model. With this, the passion for searching for the Higgs boson was transformed into experiments to find out the effect of its mechanism which gives to remain particles their masses. More investigations are spent to study decaying of the Higgs particle, its production and properties.

One of the central ingredient of the SM is the Cabbibo-Kobayachi-Maskawa (CKM) matrix or the quark mixing matrix, which rules the charged current interactions of the quark mass eigenstates and enables the heavier ones to decay to the lighter ones [22]. In particular, it is the key to our understanding why certain laws of nature are not invariant under simultaneous charge conjugation and parity transformations.

In this master thesis, we are first interested in an overview of the Standard Model and Electroweak Theory to know what are all of its parameters, especially from the standpoint of group theory and in addition to its Lagrangian with all its parts. Then we talk about the Higgs mechanism to find the way that the mass generation of all the elementary particles happened. After we move to the mixing of the fermion masses by introducing the fermion mass mixing matrices. We will discuss the study of the renormalization of the particle masses, the quark mixing matrix element and the electric charge at one loop level. First, we must solve the ultraviolet divergence problem that appears in the particle self energies extracted from the propagators, and make it in the form of a convergent mathematical expression. After clearly showing the ultraviolet divergence, we remove it by the counter terms method. This focuses mainly on adding the counterterm to the lagrangian in order to reach a consistency between the the standard model parameters theoretically and the results obtained experimentally. One of the most important things we will renormalize is the quark mixing matrix element V_{ij} [5], then we will research the effect of this latter on the decay of the Higgs boson. After that, we will compare between this effect and the effect in the W boson decay to two quarks which is the only physical process containing the CKM matrix element.

The plane of this master thesis in order to reach our goal will be as follows

Chapter 01

In this chapter we will focus on the Standard Model, with all of its components. Starting from particles to the canonical lagrangian with its detailed parts. Among them is the Higgs part that will lead directly to the Higgs mechanism, which is responsible for the spontaneous breaking of the electroweak symmetry. Through this mechanism fermions and bosons obtain their masses.We end with the fermion mass mixing, where does the Yukawa couplings matrix appear, which we have to diagonalize. Then finally, we get the final expression of all the fermions generations masses.

Chapter 02

This chapter is about the renormalization of the mass mixing matrix, but firstly we must examine why we have to renomalize in the first place, and we will find ourselves face to a famous problem in quantum field theories: the ultraviolet divergences. We will solve the ultraviolet divergences problem by giving an example from QED (the fermionic propagator), where we will highlight the dimensional regularization method in order to show the ultraviolet divergence more clearly. We move to look for a method that enables us to remove the ultraviolet divergence. That is the renormalization procedure. Then we jump to choose a set of parameters and fields to apply the renormalization scheme for EWSM on them. We will renormalize the fermion sector, the gauge sector, the Higgs sector, the electric charge, and finally the quark mass mixing matrix element which is the aim of this chapter [5], [23]. We explicitly work at one loop, even we believe that our formalism is likely to carry out over to higher orders.

Chapter 03

In this chapter we study the Mass Mixing effect on Higgs Width at One Loop Order. The Higgs boson decay is into a particle and anti-particle. We use the automatic pakages $FeynArts$ and $FormCalc$ [14], [13], [18] to draw the Feynman digrams, generate the Fortran codes of the tree level, and the one loop level respectively. We choose a set of Higgs decay channels for which we calculate decay width through use $FormCalc$ package under $Mathematica$. Namely those decays are:

$H \longrightarrow b\bar{b}$, $H \longrightarrow c\bar{c}$,

$H \longrightarrow \tau\bar{\tau}$, $H \longrightarrow gg$,

$H \longrightarrow AA$ and $H \longrightarrow ZA$

With mentioning all steps to generate the Fortran codes we need to calculate the Higgs decay width and the branching ratios.

Finaly we move to the W boson decay into quark and anti-quark which is the only physical process containing the CKM matrix element [33], then we compare between the mass mixing matrix effect in both the Higgs and the W boson decay. The last step will be our conclusion. Some long calculations that concern what is studied in this chapter are put in the Annexes.

Chapter 1

Standard Model and Fermion Mass Mixing

1.1 Standard Model

1.1.1 Introduction

The arrival of the Standard Model to its prosperous state today was the result of many years of developments and progress, it was completed in early 1970s but not fully experimentally verified until the discovery of the Higgs boson at the Large Hadron Colider (LHC) in 2012. The SM is the theory that describes all elementary particles discovered so far and their interactions, and it summarizes all we know about fundamental forces of electromagnetic, weak and strong interaction (but excluding gravity). Those three fundamental interactions described by the SM are mediated by Spin-1 gauge bosons [21]

The photons mediating the electromagnetic interaction: Imagined in 1900 by Max Planck and confirmed in 1905 by Albert Einstein, it is massless and has zero electric charge. A quantum object, the photon manifests itself as a wave (electromagnetic wave) and as an assembly of corpuscles (gamma radiation). The correspondence between the two aspects is given by the Planck formula: $E = \hbar\nu$, where the corpuscular aspect is contained in the left hand side which represents the energy of each photon and on the right side ν represents the frequency of the wave associated with this photon. The electromagnetic interactions have an infinite range because their particles (photons) are massless.

The W and Z bosons mediating the weak interaction: W bosons were seen in January 1983 during a series of experiments made possible by Carlo Rubbia and Simon van der Meer. They found the Z boson a few months later, in May 1983. The respective symbols are W^+, W^-, and Z^0. The $W_\pm$ bosons have either a positive or negative electric charge of 1 elementary charge and are each other's antiparticles.

The gluon mediating the strong interaction: observed indirectly in 1979, it is a vector boson with a spin of 1. While massive spin 1 particles have three polarization states, massless gauge bosons like the gluon have only two polarization states because gauge invariance requires the polarization to be transverse to the gluon travelling direction. In quantum field theory, unbroken gauge invariance requires that gauge bosons have zero mass. There are eight remaining independent color states, which correspond to the eight types or colors of gluons.

In addition to bosons, the SM contains also spin 1/2 fermions (the matter particles) [18] which are divided into two subclasses :

1 Quarks : Elementary fermions which were discovered between 1968 and 1995. They are sensitive to all fundamental interactions and are the constituents of all hadrons. When associated three by three, they form the baryons. With quark-antiquark assemblages they form mesons. There are six types of quarks split in three generations (or families) [21].

Quark type :	Family 1	Family 2	Family 3	Electric charge
Up	up (u)	charm (c)	top (t)	+2/3.
Down	down (d)	strange (s)	bottom (b)	-1/3.

They carry a fractional electric charge and a color charge which they swap by exchanging a gluon with a neighboring quark. Free at short distance, quarks become strongly bounded if the distance increases between them. Quarks can only be detected indirectly because of confinement: one never observes isolated quarks or gluons. If at low energy they exist in the form of bound states called hadrons, at very high energies however quarks seems to form a plasma state of quarks and gluons.

2 Leptons: There are six types of leptons, grouped in three generations. The lightest charged leptons is the electron (e) which was discovered in 1897 by J.J.Thomson, followed by the muon (μ) observed by Carl D.Anderson in 1936 and the tau (τ) which was detected between 1974 and 1977. All these particles have antiparticles with opposite charge and are completed by the neutral neutrinos: which can take three forms (or flavors): The electron neutrino (ν_e) (1956), the muonic ((ν_μ) (1962) and tauic (ν_τ) (2000), Neutrinos have no electric charge and have a very low mass of which only one upper bound is known. They periodically transform into each other in a process called neutrino oscillations. They are sensitive only to weak interaction.

The SM is a gauge theory with the gauge group $SU(3_C) \otimes SU(2)_I \otimes U(1)_Y$ [21]. The first gauge group $SU(3)_C$ forms the underlying symmetry group of the strong interaction, the (C) stands for the color. Its corresponding eight gauge fields are the massless gluons that participate in the strong interaction. The second factor, the group $SU(2)_I \otimes U(1)_Y$ describes the elecroweak sector of the SM, the (I) stands for the weak isospin and the (Y) stands for the weak hypercharge.

$SU(2)_I \otimes U(1)_Y$ involves respectively two parameters g and g' which are the couplings of gauge bosons to fermions.

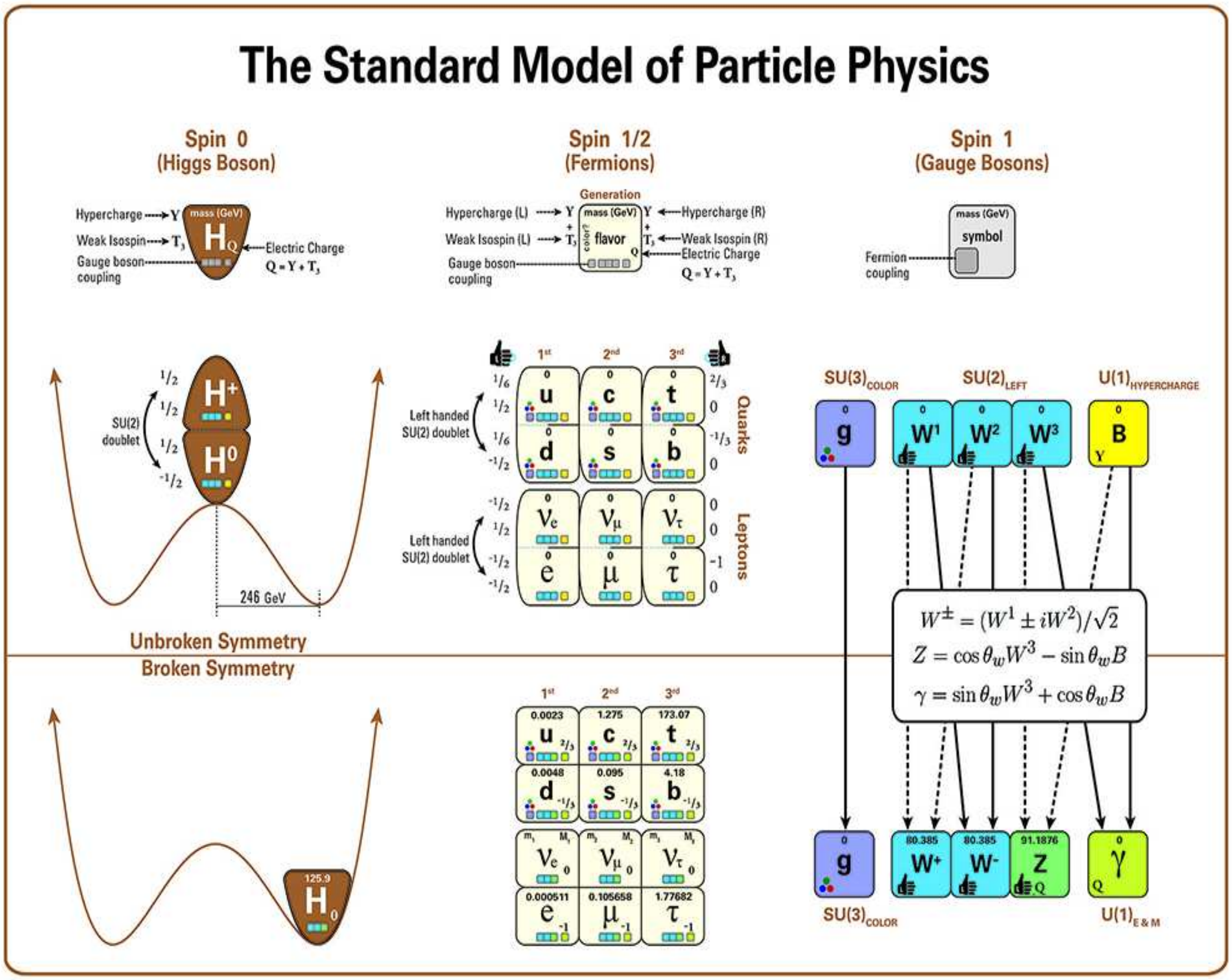


Figure 1.1: Standard Model elementary particles (WIKIMEDIA COMMONS)

1.1.2 The Standard Model Lagrangian

The SM is a quantum feild theory, and any quantum feild theory must be described by a Lagrangian which involves different feilds and contains kinetic and interaction termes.
The Lagrangian of the SM can be composed as follow [18], [21], [20], [5]:

$$\mathcal{L}_{SM} = \mathcal{L}_{kinetic} + \mathcal{L}_{GF} + \mathcal{L}_{Higgs} + +\mathcal{L}_{Yukawa} \tag{1.1}$$

Each of them is separately gauge invariant.

The Fermionic Part

Chiral Fermion Fields:

The SM contains three generation of a collection of chiral fermion fields [21]. There are two types of chirality, left and right. Each chiral fermion is described by a ψ field. Neutrinos are a Dirac or Majorana spinors and only their left part exists. So they are described in the SM by a single left chirality spinor. The left- and right-handed chiral fermion states are obtained from an unpolarized Dirac spinor using the projection operators [18]

$$P_R = \frac{1}{2}(1 + \gamma^5) \tag{1.2}$$

$$P_L = \frac{1}{2}(1 - \gamma^5) \tag{1.3}$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ the products of the Dirac matrices, in such a way that

$$P_R\psi = \psi_R \tag{1.4}$$

$$P_L\psi = \psi_L \tag{1.5}$$

Using the anticomutation relation $\{\gamma^\nu, \gamma^5\} = 0$ and the fact that γ^5 is Hermitian we also have:

$$\bar{\psi}P_R = \psi^\dagger\gamma^0 P_R = \psi^\dagger P_L\gamma^0 = (P_L\psi)^\dagger\gamma^0 = \bar{\psi}_L \tag{1.6}$$

similarly we get

$$\bar{\psi}P_L = \bar{\psi}_R \tag{1.7}$$

with $P_R + P_L = 1$ and $P_R^2 = P_R$, $P_L^2 = P_L$.

We can get a new lagrangian in term of chiral fermion fields by inserting the projection operators into the Dirac Lagrangian. Let us start by the Lagrangian for a generic fermion ψ with mass m_f [5]:

$$\mathcal{L} = \bar{\psi} i \partial_\nu \gamma^\nu \psi - m_f \bar{\psi}\psi \tag{1.8}$$

The first term splits into two terms involving left and right handed chiral fermion fields. By inserting a factor $(P_L^2 + P_R^2) = 1$ before the ψ and using the anticomutation relation to pull one factor of that projection operator through γ^ν in each term one obtain

$$\bar{\psi} i \partial_\nu \gamma^\nu \psi = \bar{\psi} P_R i \partial_\nu \gamma^\nu P_L \psi + \bar{\psi} P_L i \partial_\nu \gamma^\nu P_R \psi = \bar{\psi}_L i \partial_\nu \gamma^\nu \psi_L + \bar{\psi}_R i \partial_\nu \gamma^\nu \psi_R \tag{1.9}$$

We can then incorporate the gauge transformation properties by promoting the derivative ∂_ν to a covariante derivative D_ν and these two terms of (1.9) will be gauge invariante for any of the fermion fields given in table 1 [21], [20].

				$SU(2)_L$	$U(1)_Y$
L_L^i :	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$,	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$,	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$,	2 ,	$-1/2$
Q_L^i :	$\begin{pmatrix} u \\ d \end{pmatrix}_L$,	$\begin{pmatrix} c \\ s \end{pmatrix}_L$,	$\begin{pmatrix} t \\ b \end{pmatrix}_L$,	2 ,	$-1/6$
u_R^j :	u_R ,	c_R ,	t_R ,	1 ,	-1
d_R^j :	d_R ,	s_R ,	b_R ,	1 ,	$-2/3$
e_R^j :	e_R ,	μ_R ,	τ_R ,	1 ,	$-1/3$
ν_R^j :	ν_{eR} ,	$\nu_{\mu R}$,	$\nu_{R\tau}$,	1 ,	0

Table 1.1: The three families of chiral fermions of standard Model

Now for the mass term using a similar method we have:

$$-m\bar{\psi}\psi = -m\bar{\psi}P_L\psi - m\bar{\psi}P_R\psi = -m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R \tag{1.10}$$

Because the left-handed and right-handed fermions of the SM carry different $SU(2)_I \otimes U(1)_Y$ gauge charges, such mass terms are not gauge invariant and thus cannot be inserted by hand into the Lagrangian. Therefore, given the correct gauge charges of the SM fermions, (unbroken) gauge invariance implies that all the SM fermions are massless.

Kinetic terms for the fermions and gauge interactions:

The kinetic term for the chiral fermion fields, including the interaction with the gauge fields due to the covariante derivative, is written as [20], [5]:

$$\begin{aligned}\mathcal{L}_{kinetic} = &\sum_{\mathrm{i}=1}^{3}\left(\bar{L}_{L,i} i\gamma^{\mu} D^{l}_{\mu} L_{L,i} + \bar{Q}_{L,i} i\gamma^{\mu} D^{q}_{\mu} Q_{L,i}\right) \\ &+ \sum_{\mathrm{i}=1}^{3}\left(\bar{l}_{R,i} i\gamma^{\mu} D^{l}_{\mu} l_{R,i} + \bar{u}_{R,i} i\gamma^{\mu} D^{q}_{\mu} u_{R,i} + \bar{d}_{R,i} i\gamma^{\mu} D^{q}_{\mu} d_{R,i}\right)\end{aligned} \tag{1.11}$$

This term describes how to couple the gauge bosons to fermions, the gauge covariant derivatives are given by [20]:

$$D^{l}_{\mu} L_{L,i} = \left(\partial_{\mu} + \frac{1}{2} i g W^{a}_{\mu} \sigma^{a} - \frac{1}{2} i g' B_{\mu}\right) L_{L,i} \tag{1.12}$$

$$D^{l}_{\mu} l_{R,i} = \left(\partial_{\mu} - i g' B_{\mu}\right) l_{R,i} \tag{1.13}$$

$$D^{q}_{\mu} Q_{L,i} = \left(\partial_{\mu} + \frac{1}{2} i g_{s} G^{a}_{\mu} \lambda^{a} + \frac{1}{2} i g W^{a}_{\mu} \sigma^{a} + \frac{1}{6} i g' B_{\mu}\right) Q_{L,i} \tag{1.14}$$

$$D^{q}_{\mu} u_{R,i} = \left(\partial_{\mu} + \frac{1}{2} i g_{s} G^{a}_{\mu} \lambda^{a} + \frac{2}{3} i g' B_{\mu}\right) u_{R,i} \tag{1.15}$$

$$D^{q}_{\mu} d_{R,i} = \left(\partial_{\mu} + \frac{1}{2} i g_{s} G^{a}_{\mu} \lambda^{a} + \frac{1}{3} i g' B_{\mu}\right) d_{R,i} \tag{1.16}$$

Here, the Pauli matrices σ^{a} and the Gell-Mann matrices λ^{a} are the generators of the $SU(2)$ and $SU(3)$ Lie algebras respectively [20]. The SM carry different $SU(2)_{I} \otimes U(1)_{Y}$ gauge charges because of the chirality of fermions. So the explicit mass terms for the fermions are forbidden and thus cannot be inserted by hand in the lagrangian. The masses of the fermions are generated via their fields Yukawa couplings to the higgs field (spontaneous symmetry breaking).

The Gauge part

The $\mathcal{L}_{GF}$ describes the dynamics and self interactions of the gauge fields related to each of the three factors of the symmetry group. The pure gauge field Lagrangian reads [21], [20], [5]:

$$\mathcal{L}_{GF} = -\frac{1}{4}(\partial_{\mu} G^{A}_{\nu} - \partial_{\nu} G\mu^{A} + g_{s} f^{abc} G^{B}_{\mu} G^{C}_{\nu}) - \frac{1}{4}(\partial_{\mu} W^{a}_{\nu} - \partial_{\nu} W^{a}_{\mu} + g' \varepsilon^{abc} W^{b}_{\mu} W^{c}_{\nu})^{2} - \frac{1}{4}(\partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu})^{2} \tag{1.17}$$

where $G^{A}_{\mu\nu}$, $W^{a}_{\mu\nu}$ and B_{ν} are the kinetic tensors associated with the gauge fields of the groups $SU(3)_{C}$, $SU(2)_{L}$, and, $U(1)_{Y}$. The isotriplet $W^{a}_{\nu}, a = 1, 2, 3$ is associated with the generator I^{a}_{ν} of the weak isospin group $SU(2)_{W}$. The isosinglet B_{ν} with the weak hypercharge Y_{W} of the group $U(1)_{Y}$. $\varepsilon^{abc} = f^{abc}$ are the totally antisymmetric constante of SU(2). g_{S} is the strong interaction coupling strength.

The Higgs part

We add to the SM an $SU(2)$-doublet of complex scalar field denoted by $\Phi(x)$ and written as [18], [20], [5]:

$$\Phi(x) = \begin{pmatrix} \phi^\dagger(x) \\ \phi^0(x) \end{pmatrix}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}_L \tag{1.18}$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are properly normalized real scalar fields. The $\Phi(x)$'s hyper-charge is $Y = 1/2$. The new term in the lagrangian involving $\Phi(x)$ is given by:

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \tag{1.19}$$

where the first term contains the kinetic and gauge interaction terms via the covariante derivative:

$$D_\mu = \partial_\mu - ig_2 I_W^a W_\mu^a + i\frac{1}{2} g_1 Y_W B_\mu \tag{1.20}$$

The second term is a potential energie function involving Φ. The most general gauge invariant potential energie function, or scalar potential, is given by [18], [21], [20]

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \tag{1.21}$$

It is constructed in such a way that it gives rise to spontaneous symmetry breaking. This means that the parameters λ and μ are chosen such that the potential $V(\Phi)$ takes its minimum for a nonvanishing Higgs field, i.e. the vacuum expectation value $\langle \Phi \rangle$ of Higgs field is nonzero.

Yukawa part:

The construction of the lagrangian term that describe the Higgs couplings to fermions is simple. The most general gauge-invariante renormalizable lagrangian terms involving the Higgs doublet and fermions are, for all generations [21], [20], [5]:

$$\begin{aligned} \mathcal{L}_{Yukawa} = &- \sum_{\mathrm{i,j=1}}^{3} \left[\sum_{\mathrm{a=u},\nu} \bar{\Psi}_{L,i}^a \alpha_{ij}^a \tilde{H} \Psi_{R,j}^a + \sum_{\mathrm{b=d,e}} \bar{\Psi}_{L,i} \alpha_{ij}^b H \Psi_{R,j}^b \right] \\ &- \left(\sum_{\mathrm{i,j=1}}^{3} \left[\sum_{\mathrm{a=u},\nu} \bar{\Psi}_{L,i}^a \alpha_{ij}^a \tilde{H} \Psi_{R,j}^a + \sum_{\mathrm{b=d,e}} \bar{\Psi}_{L,i} \alpha_{ij}^b H \Psi_{R,j}^b \right] \right)^\dagger \end{aligned} \tag{1.22}$$

where the second term is the Hermitian conjugate of the first term, α_{ij}^a and α_{ij}^b are the Yukawa couplings matrices, $\tilde{H}$ is the charge conjugated Higgs field.

1.2 Higgs mechanism and mass generation

1.2.1 Introduction

The symmetry of the $SU(2)_I \otimes U(1)_Y$ describes the electroweak sector of the SM; and so far we had mainly considered the SM before electroweak symmetry breaking and SM lagrangian parts had been written in terms of gauge eigenstates only, the gauge invariance of the electroweak Lagrangian does not allow the invariance of the explicit mass terms for bosons and fermions. By this logic no particle is massive, which is against the experimental results. The only explanation for these results is that there is a symmetry breaking of $SU(2)_I \otimes U(1)_Y$.

After electroweak symmetry breaking, it is convenient to rewrite the lagrangian in terms eigenstates: the gauge eigenstates $W^{1,2,3}$ and B are effectively no longer physical eigenstates with mass zero. They become the charged gauge bosons $W^{\pm}_{\mu}$ with mass M_W and the neutral bosons Z_μ with mass M_Z and the photon A_μ with mass zero, this is called the *Higgs mechanism*.

1.2.2 The Spontaneous Breaking of The Electroweak Symmetry

We had already introduced the scalar field (1.18) in the form of a weak Isospin $I_\Phi = \frac{1}{2}$ doublet with a weak hypercharge $Y(\Phi) = 1$, and we had coupled it. The potential which contains the interaction terms of the Higgs field with itself has two extremums found by resolving [6]

$$V'(\Phi) = \frac{\partial V(\Phi)}{\partial |\Phi|} = 2(\mu^2|\Phi| + 2\lambda|\Phi|^3) = 0 \tag{1.23}$$

So this is true when:

$$|\Phi|_1 = 0 \tag{1.24}$$

or

$$|\Phi|_2 = \sqrt{\frac{-\mu^2}{\lambda}} \tag{1.25}$$

Considering the relative possible signs of the coefficients of the two terms in V we have two cases [6]:

-When $-\mu^2$ and λ are both positive, the potential energy function has a minimum at $|\Phi|_1 = 0$ and $|\Phi|_2$ are imaginary. In this case the potential is symmetric and the electroweak symmetry is unbroken in the vacuum, because the action of a gauge transformation does not change the vacuum state $\Phi = 0$, The right side of Fig 1.2.

-When $-\mu^2$ is negative and λ is positive, the potential energy function has a minimum $|\Phi|_2$ away from $|\Phi|_1 = 0$ which is a maximum. In this case the vacuum, or minimum energy state is not invariant under $SU(2)_I \otimes U(1)_Y$ transformations: the gauge symmetry is spontaneously broken in the vacuum,

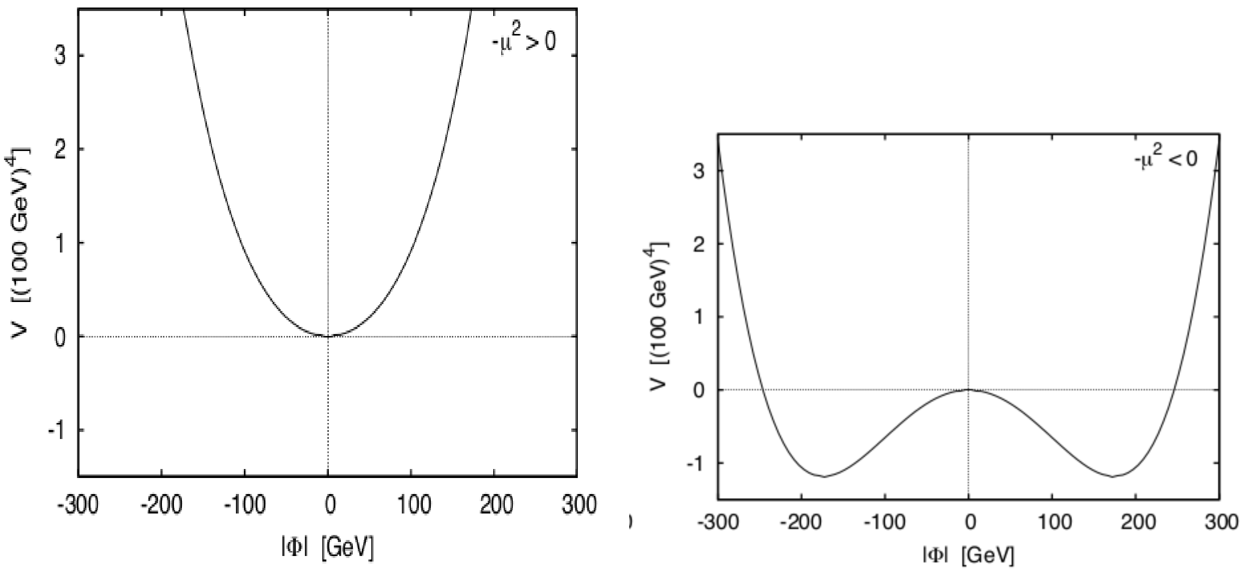


Figure 1.2: Plot of $V(\Phi)$ as a function of $|\Phi|$

The left side of Fig 1.2.

At this the case $-\mu^2 < 0$, the Higgs field must follow one direction to achieve the electroweak symmetry breaking. This means that the potential induces a Vacuum Expectation Value for H [6]:

$$|\langle 0|\Phi|0\rangle|^2 = \frac{\mu^2}{2\lambda} = \frac{v^2}{2} \neq 0 \tag{1.26}$$

And when this choice is made, the electroweak Lagrangian symmetry under the group $SU(2)_L \otimes U(1)_Y$ is broken spontaneously. we can parameterize the state (1.18) in this way:

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_2 + i\chi_i \\ v + h(x) - i\chi_3 \end{pmatrix} \tag{1.27}$$

where χ_i, χ_2, χ_3 and h(x) are small fluctuations so we can rewrite Φ in another convenient form:

$$\Phi(x) == \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_2 + i\chi_i \\ v + h(x) - i\chi_3 \end{pmatrix} = \frac{1}{\sqrt{2}} exp\left[i\frac{\overrightarrow{\sigma} \cdot \overrightarrow{\chi}}{v}\right] \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.28}$$

Now consider the gauge transformation of Φ:

$$U(1)_Y : \Phi \longrightarrow exp\left[\frac{i}{2}\lambda_Y^a(x)\right]\Phi \tag{1.29}$$

$$SU(2)_L : \Phi \longrightarrow exp\left[\frac{i}{2}\lambda_L^a(x)\frac{\sigma^a}{2}\right]\Phi \tag{1.30}$$

If we choose $\lambda^a(x) = -2\chi^a/v, a = 1, 2, 3$ at each point in spacetime we arrive at

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.31}$$

this gauge choice is known as unitarity gauge. We can notice that the fields $\chi_{1,2,3}(x)$ disappear under the effect of these transformations: they are therefore non-physical or unobservable. The exception is for the $h(x)$ field and that is what makes it "physical Higgs boson" with a mass:

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v} \tag{1.32}$$

1.2.3 Higgs-bosons coupling and mass generation

We returne now to the lagrangian (1.19) to examine the gauge-kinetic term:

$$\mathcal{L}_{gauge-kinetic} = (D_\mu \Phi^\dagger)(D^\mu \Phi) \tag{1.33}$$

By inserting (1.31) into $L_{gauge-kinetic}$, the vacuum expectation value (vev) introduces couplings in mass terms for the gauge bosons through applying the covariant derivative (1.20) to Φ:

$$\begin{aligned} D_\mu \Phi = & \quad \frac{1}{\sqrt{2}} \left(\partial_\mu - ig\frac{\sigma^a}{2} W^a_\mu - ig'\frac{Y}{2} B_\mu \right) \times \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \\ = & \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \partial_\mu - i\frac{g}{2} W^3_\mu - i\frac{g'}{2} B_\mu & -ig(W^1_\mu - iW^1_\mu) \\ -ig(W^1_\mu - iW^2_\mu) & \partial_\mu + i\frac{g}{2} W^3_\mu - i\frac{g'}{2} B_\mu \end{pmatrix} \times \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \\ = & \quad \begin{pmatrix} -ig(W^1_\mu - iW^1_\mu)(v + h) \\ (v + h)\partial_\mu h + \frac{i}{2}(gW^3_\mu - g' B_\mu (v + h) \end{pmatrix} \end{aligned} \tag{1.34}$$

Accordingly, the Hermitian conjugate term is:

$$(D_\mu \Phi)^\dagger = \frac{1}{\sqrt{2}} \left(ig(W^1_\mu + iW^2_\mu)(v + h), \partial_\mu h - \tfrac{i}{2}(gW^3_\mu - g' B_\mu (v + h) \right) \tag{1.35}$$

Multiplying (1.34) by (1.35) gives:

$$\begin{aligned} (D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{\sqrt{2}} \Big[& \partial_\mu h \partial^\mu h + \frac{g^2}{4}(v + h)^2 (W^1_\mu + iW^2_\mu)(W^{1\mu} - iW^{2\mu}) \\ & + \frac{1}{4}(v + h)^2 (gW^3_\mu - g' B_\mu)(gW^{3\mu} - g' B^\mu) \Big] \end{aligned} \tag{1.36}$$

The first term is the properly normalized kinetic term for the real scalar field h. To generate bosons masses, we extract the h independent terms. For the second and the third terms, we note that the combinations $W_\mu^1 \pm iW_\mu^2$ correspond to the charged W bosons [18]

$$W^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}} \tag{1.37}$$

$$W^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}} \tag{1.38}$$

For the remained parts we should define [5]

$$Z_\mu \equiv \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \tag{1.39}$$

$$A_\mu \equiv \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \tag{1.40}$$

θ_W is the Weinberg angle defined in term of the coupling constants of $SU(2)$ and $U(1)$ by [6], [5]

$$\cos\theta_W = C_W = \frac{g'}{\sqrt{g^2 + g'^2}} \tag{1.41}$$

$$\sin\theta_W = S_W = \frac{g}{\sqrt{g^2 + g'^2}} \tag{1.42}$$

The electromagetic coupling is expressed as:

$$e = g' S_W = g C_W = \frac{gg'}{\sqrt{g^2 + g'^2}} \tag{1.43}$$

Replacing (1.37) and (1.38) in the second term of (1.36) gives:

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v}{2} h W_\mu^+ W^{-\mu} + \frac{g^2}{4} hh W_\mu^+ W^{-\mu} + \cdots \tag{1.44}$$

The first term here is a mass term for the W bosons with:

$$M_W^2 = \frac{g^2 v^2}{4} \tag{1.45}$$

The Higgs vacuum expectation value has given the W boson a mass.

The second and the third terms give the interaction of one or two Higgs bosons with $W_\mu^+ W^{-\mu}$, They correspond to Feynman diagram vertices Fig 1.3 with the following rules:

$$hW_\mu^+ W_\nu^- : i\frac{g^2 v}{2} g_{\mu\nu} = igM_W g_{\mu\nu} = 2i\frac{M_W^2}{v} g_{\mu\nu} \tag{1.46}$$

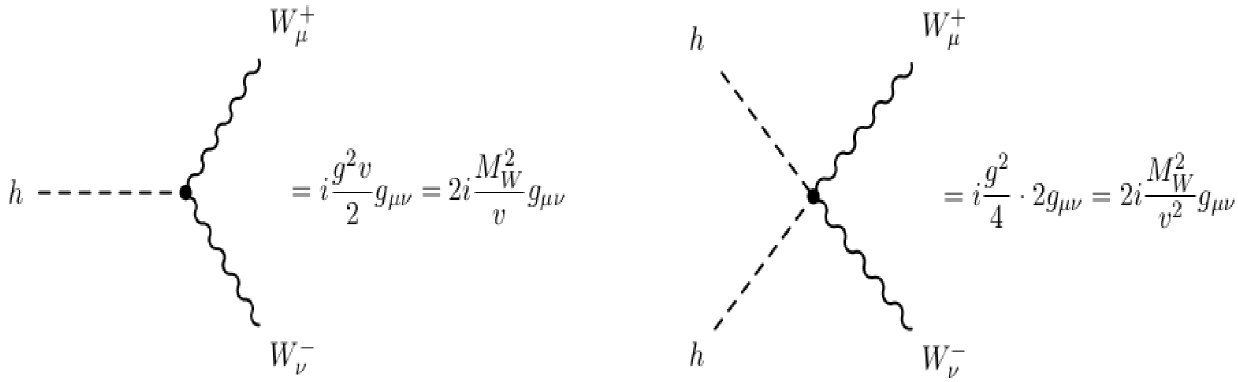


Figure 1.3: Feynman rules for the hWW and hhWW vertices

$$hhW_\mu^+W_\nu^- : i\frac{g^2}{4} \times 2! g_{\mu\nu} = 2i\frac{M_W^2}{v^2} g_{\mu\nu} \tag{1.47}$$

where the extra factor 2! in the second expression is a combinatorial factor coming from the two identical Higgs bosons in the Lagrangian term. We now consider the third term of (1.36). We take the linear combination of W_μ^3 and B_μ and according to (1.41) and (1.42) we find:

$$\begin{aligned}(gW_\mu^3 - g'B_\mu) &= \sqrt{g^2 + g'^2}\left(\frac{g}{\sqrt{g^2+g'^2}}W_\mu^3 - \frac{g}{\sqrt{g^2+g'^2}}B_\mu\right) \\ &= \sqrt{g^2+g'^2}(C_W W_\mu^3 - S_W B_\mu) \\ &= \sqrt{g^2+g'^2} Z_\mu\end{aligned} \tag{1.48}$$

The third term in (1.36) becomes:

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) = \frac{(g^2+g'^2)v^2}{8} Z_\mu Z^\mu + \frac{(g^2+g'^2)v}{4} hZ_\mu Z^\mu + \frac{(g^2+g'^2)}{8} hhZ_\mu Z^\mu + \cdots \tag{1.49}$$

The first term here is a mass term for the Z boson:[1]

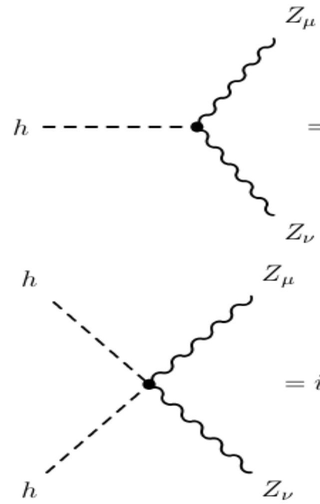
$$M_Z^2 = \frac{(g^2+g'^2)v^2}{4} \tag{1.50}$$

The second and the third terms in (1.49) give the interaction of one or two Higgs bosons with ZZ, the corresponding Feynman rules are:

$$hZ_\mu Z_\nu : i\frac{(g^2+g'^2)v}{4} \times 2! g_{\mu\nu} = i\sqrt{g^2+g'^2} M_Z g_{\mu\nu} = 2i\frac{M_Z^2}{v} g_{\mu\nu} \tag{1.51}$$

$$hhZ_\mu Z_\nu : i\frac{(g^2+g'^2)}{8} \times 2! \times 2! g_{\mu\nu} = 2i\frac{M_Z^2}{v^2} g_{\mu\nu} \tag{1.52}$$

[1]Remember that the mass term for a real vector field takes the form $\frac{1}{2}M_Z^2 Z_\mu Z^\mu$.



$$= i\frac{(g^2+g'^2)v}{4}\cdot 2g_{\mu\nu} = 2i\frac{M_Z^2}{v}g_{\mu\nu}$$

$$= i\frac{(g^2+g'^2)}{8}\cdot 2\cdot 2g_{\mu\nu} = 2i\frac{M_Z^2}{v^2}g_{\mu\nu}$$

Figure 1.4: Feynman rules for the hZZ and hhZZ vertices

where each coupling contains as $2!$ from the two identical Z bosons. We note that the orthogonal state (1.40) did not appear in the Lagrangian or rather does not exist. This means that it does not couple to the higgs field and thus does not acquire a mass through the Higgs mechanism. This state will be identified as the photon.

Finally, the gauge bosons masses are [6]:

$$M_{A_\nu} = M_\gamma = 0 \tag{1.53}$$

$$M_{W^+} = M_{W^-} = M_W = \frac{1}{2}vg \tag{1.54}$$

$$M_Z = \frac{1}{2}v\sqrt{(g^2+g'^2)} \tag{1.55}$$

1.3 Fermion mass mixing

1.3.1 Introduction

As we saw before, the electroweak symmetry of the SM is broken once we expand the Higgs doublet around a chosen vacuum state. After the spontaneous symmetry breaking, the interaction between fermions and the Higgs doublet could give them masses. This can be achieved by adding the Yukawa term by hand in the SM lagrangian which describes the Higgs fermion coupling.

In this section we will see how we can generate fermion masses, and we will introduce the mixing between the different fermions families and this leads us to define the fermion mixing matrices.

1.3.2 The construction of the lagrangian and fermion mass generation

Since the fermions field has mass dimension 3/2, the fermion pairs have mass dimension 3. Combining this pair with a single Higgs doublet (mass dimension 1) gives a mass dimension 4. Following this logic we can add a term which describes the coupling between the Higgs field and one of $SU(2)_L$ doublet fermion and one $SU(2)_L$ singlet (see table 1.1). The most general gauge-invariante Lagrangian terms involving The Higgs doublet and fermion are [5]:

$$\mathcal{L}_{down-quark} = -[\alpha_d \bar{Q}_L \Phi d_R + \alpha_d^* \bar{d}_R \Phi^\dagger Q_L] \tag{1.56}$$

$$\mathcal{L}_{electron} = -[\alpha_e \bar{L}_L \Phi e_R + \alpha_e^* \bar{e}_R \Phi^\dagger L_L] \tag{1.57}$$

For the up-type quarks and neutrinos (note that we are taking the neutrino massive here by including a right-handed neutrino ν_R) we use the anti-doublet or conjugate doublet[2]. The conjugate Higgs doublet is given by [5]

$$\tilde{\Phi} = i\sigma_2 \Phi^* = i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix}_L = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}_L \tag{1.58}$$

we can write another gauge-invariant Lagrangian terms

$$\mathcal{L}_{up-quark} = -[\alpha_u \bar{Q}_L \tilde{\Phi} u_R + \alpha_u^* \bar{u}_R \tilde{\Phi}^\dagger Q_L] \tag{1.59}$$

$$\mathcal{L}_{neutrinos} = -[\alpha_\nu \bar{L}_L \tilde{\Phi} \nu_R + \alpha_\nu^* \bar{\nu}_R \tilde{\Phi}^\dagger L_L] \tag{1.60}$$

where every second term in these ast four Lagrangian terms is the Hermitian conjugate of the first one.

The α_d, α_e, α_u and α_ν are the Yukawa couplings. They are complex in general (here we assume them as real dimensionless couplings). These constants have to be related in some way to the fermion masses. Substituting, after the symmetry breaking, the Higgs doublet form

$$\Phi = \begin{pmatrix} \phi^\dagger \\ \phi^0 \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.61}$$

and

$$\Phi^\dagger Q_L = \Big(0, v + h(x)\Big) \begin{pmatrix} u \\ d \end{pmatrix}_L = \frac{v + h(x)}{\sqrt{2}} d_L \tag{1.62}$$

[2]We take advantage of a useful property of $SU(2)$. This conjugate transforms in the same way as the doublet and it has hypercharge $Y = -1/2$

in the Yukawa quark terms we get

$$\mathcal{L}_{Yukawa} = -\frac{\alpha_d v}{\sqrt{2}}\bar{d}d + \frac{\alpha_d}{\sqrt{2}}h\bar{d}d \tag{1.63}$$

A simple comparison with the Lagrangian of Dirac (1.8) allows to deduce the mass expression. So the first term is a mass term for the down-type quark $m_d = \alpha_d v/\sqrt{2}$, and the second is an $h\bar{d}d$ coupling.

Following a similar calculation for the other lagrangian terms, it is possible to obtain the masses of up-type and down-type quarks and leptons:

$$\begin{cases} m_u = \alpha_u v/\sqrt{2}, m_c = \alpha_c v/\sqrt{2} \\ \\ m_t = \alpha_t v/\sqrt{2}, m_s = \alpha_s v/\sqrt{2} \\ \\ m_b = \alpha_b v/\sqrt{2} \\ \end{cases}$$

$$\begin{cases} m_e = \alpha_e v/\sqrt{2}, m_\mu = \alpha_\mu v/\sqrt{2} \\ \\ m_\tau = \alpha_\tau v/\sqrt{2}, m_\nu = \alpha_\nu v/\sqrt{2} \\ \end{cases}$$

As it has been discussed in the previous sections, the SM fermions come in three different families (or generations). In principle, as far as we do not take into account the mixing between the different fermion families, the last calculation to generate the fermion masses is sufficient if we want to describe each generation of fermions separately. But the most general Yukawa lagrangian including all left and right-handed fermion fields with a generation indices i, j is:

$$\begin{aligned} \mathcal{L}_{Yukawa} = &-\sum_{\text{i,j=1}}^{3}\left[\sum_{\text{a=u,}\nu}\bar{\Psi}^a_{L,i}\alpha^a_{ij}\tilde{\Phi}\Psi^a_{R,j} + \sum_{\text{b=d,e}}\bar{\Psi}_{L,i}\alpha^b_{ij}\Phi\Psi^b_{R,j}\right] \\ &-\left(\sum_{\text{i,j=1}}^{3}\left[\sum_{\text{a=u,}\nu}\bar{\Psi}^a_{L,i}\alpha^a_{ij}\tilde{\Phi}\Psi^a_{R,j} + \sum_{\text{b=d,e}}\bar{\Psi}_{L,i}\alpha^b_{ij}\Phi\Psi^b_{R,j}\right]\right)^\dagger \end{aligned} \tag{1.64}$$

The dimensionless coupling $\alpha^{a,b}_{ij}$ are now the (i, j) entries of 3×3 complex non-diagonal matrices.

The Yukawa lagrangian takes this new general form

$$
\begin{aligned}
\mathcal{L}_{Yukawa} = &-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{Q}^I_{L,i}\alpha^u_{ij}\tilde{\Phi}u^I_{R,j} + \bar{Q}^I_{L,i}\alpha^d_{i,j}\Phi d^I_{R,j} + \bar{L}^I_{L,i}\alpha^e_{ij}\Phi e^I_{R,j} + \bar{L}^I_{L,i}\alpha^\nu_{ij}\tilde{\Phi}\nu^I_{R,j}\right] \\
&-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{u}^I_{R,i}\alpha^{*u}_{ij}\tilde{\Phi}^\dagger Q^I_{L,j} + \bar{d}^I_{R,i}\alpha^{*d}_{i,j}\Phi^\dagger Q^I_{L,j} + \bar{e}^I_{R,i}\alpha^{*e}_{ij}\Phi^\dagger L^I_{L,j} + \bar{\nu}^I_{R,i}\alpha^{*\nu}_{ij}\tilde{\Phi}^\dagger L^I_{L,j}\right] \\
= &-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{Q}^I_{L,i}\alpha^d_{i,j}\Phi d^I_{R,j} + \bar{d}^I_{R,i}\alpha^{*d}_{i,j}\Phi^\dagger Q^I_{L,j} + \bar{Q}^I_{L,i}\alpha^u_{ij}\tilde{\Phi}u^I_{R,j} + \bar{u}^I_{R,i}\alpha^{*u}_{ij}\tilde{\Phi}^\dagger Q^I_{L,j}\right] \\
&-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{L}^I_{L,i}\alpha^e_{ij}\Phi e^I_{R,j} + \bar{e}^I_{R,i}\alpha^{*e}_{ij}\Phi^\dagger L^I_{L,j} + \bar{L}^I_{L,i}\alpha^\nu_{ij}\tilde{\Phi}\nu^I_{R,j} + \bar{\nu}^I_{R,i}\alpha^{*\nu}_{ij}\tilde{\Phi}^\dagger L^I_{L,j}\right]
\end{aligned}
\tag{1.65}
$$

The superscript I implies that the fermion fields are expressed in the interaction basis. At this point we can write

$$
\mathcal{L}_{Yukawa} = \mathcal{L}^{quark}_{yukawa} + \mathcal{L}^{lepton}_{yukawa} \tag{1.66}
$$

1.3.3 The diagonalization of Yukawa coupling Matrices

As the elements of the masses matrix of fermion flavors are proportional to those of Yukawa′s matrix the states of flavors have no defined mass. We have to diagonalize this mass matrix to find the eigenvalues of masses and their corresponding eigenstates called mass states [2]. After symmetry breaking, the fermion mass terms become:

$$
\begin{aligned}
\mathcal{L}_{mass} = &-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{d}^I_{L,i}M^d_{ij}d^I_{R,j} + \bar{d}^I_{R,j}M^{*d}_{ij}d^I_{L,i}\right] - \sum_{\mathrm{i,j=1}}^{3}\left[\bar{u}^I_{L,i}M^u_{ij}u^I_{R,j} + \bar{u}^I_{R,j}M^{*u}_{ij}u^I_{L,i}\right] \\
&-\sum_{\mathrm{i,j=1}}^{3}\left[\bar{e}^I_{L,i}M^e_{ij}e^I_{R,j} + \bar{e}^I_{R,j}M^{*e}_{ij}e^I_{L,i}\right] - \sum_{\mathrm{i,j=1}}^{3}\left[\bar{\nu}^I_{L,i}M^\nu_{ij}\nu^I_{R,j} + \bar{\nu}^I_{R,j}M^{*\nu}_{ij}\nu^I_{L,i}\right] + Interactionterms
\end{aligned}
\tag{1.67}
$$

The interaction terms of the fermion fields to the Higgs field $h(x)\bar{f}f$ are omitted. The expression (1.67) is in matrix form, where $M_{ij} = \frac{v}{\sqrt{2}}\alpha_{ij}$ and $M^*_{ij} = \frac{v}{\sqrt{2}}\alpha^*_{ij}$. We have to diagonalize the mass matrices M_{ij}, M^*_{ij} to obtain proper mass term by multiplying them on the left and the right by appropriate unitary transformation matrices: U, $(U^\dagger U) = \mathtt{1}$. The basis transformations that diagonalize the Yukawa coupling are defined by

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = U^d_{L,R} \begin{pmatrix} d \\ c \\ b \end{pmatrix}^I_{L,R}, \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U^u_{L,R} \begin{pmatrix} u \\ s \\ t \end{pmatrix}^I_{L,R}$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_{L,R} = U^e_{L,R} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}^I_{L,R}, \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{L,R} = U^\nu_{L,R} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}^I_{L,R} \tag{1.68}$$

Such that:

$$M^d_{diag} = U_L M^d_{ij} U^\dagger_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, M^u_{diag} = U_L M^u_{ij} U^\dagger_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{1.69}$$

$$M^e_{diag} = U_L M^e_{ij} U^\dagger_R = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, M^\nu_{diag} = U_L M^\nu_{ij} U^\dagger_R = \begin{pmatrix} m_{\nu_e} & 0 & 0 \\ 0 & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau} \end{pmatrix} \tag{1.70}$$

The $\mathcal{L}_{mass}$ bacomes:

$$\begin{aligned} \mathcal{L}_{mass} = &- \sum_{\mathrm{i,j=1}}^{3} \left[\bar{d}^I_{L,i} U^{d\dagger}_L U^d_L M^d_{ij} U^{d\dagger}_R U^d_R d^I_{R,j} + \bar{d}^I_{R,j} U^{d\dagger}_R U^d_R M^{d*}_{ij} U^{d\dagger}_L U^d_L d^I_{L,i} \right] \\ &- \sum_{\mathrm{i,j=1}}^{3} \left[\bar{u}^I_{L,i} U^{u\dagger}_L U^u_L M^u_{ij} U^{u\dagger}_R U^u_R u^I_{R,j} + \bar{u}^I_{R,j} U^{u\dagger}_R U^u_R M^{u*}_{ij} U^{u\dagger}_L U^u_L u^I_{L,i} \right] \\ &- \sum_{\mathrm{i,j=1}}^{3} \left[\bar{e}^I_{L,i} U^{e\dagger}_L U^e_L M^e_{ij} U^{e\dagger}_R U^e_R e^I_{R,j} + \bar{e}^I_{R,j} U^{e\dagger}_R U^e_R M^{e*}_{ij} U^{e\dagger}_L U^e_L e^I_{L,i} \right] \\ &- \sum_{\mathrm{i,j=1}}^{3} \left[\bar{\nu}^I_{L,i} U^{\nu\dagger}_L U^\nu_L M^\nu_{ij} U^{\nu\dagger}_R U^\nu_R \nu^I_{R,j} + \bar{\nu}^I_{R,j} U^{\nu\dagger}_R U^\nu_R M^{\nu*}_{ij} U^{\nu\dagger}_L U^\nu_L \nu^I_{L,i} \right] \end{aligned} \tag{1.71}$$

where U matrices could be absorbed in the fermion states. This removes all the U matrices from the $\mathcal{L}_{mass}$. This later is then expressed only in term of fermion mass eigenstates:

$$\begin{aligned} \mathcal{L}_{mass} = &- \sum_{\mathrm{i=1}}^{3} \left[\bar{d}_{L,i} M^d_i d_{R,i} + \bar{d}_{R,i} M^{d*}_i d_{L,i} \right] - \sum_{\mathrm{i=1}}^{3} \left[\bar{u}_{L,i} M^u_i u_{R,i} + \bar{u}_{R,j} M^{u*}_i u_{L,i} \right] \\ &- \sum_{\mathrm{i=1}}^{3} \left[\bar{e}_{L,i} M^e_i e_{R,i} + \bar{e}_{R,i} M^{e*}_i e_{L,i} \right] - \sum_{\mathrm{i=1}}^{3} \left[\bar{\nu}_{L,i} M^\nu_i \nu_{R,i} + \bar{\nu}_{R,i} M^{\nu*}_i \nu_{L,i} \right] \end{aligned} \tag{1.72}$$

Since $M_i^f = M_i^{f*}$ we can write

$$\begin{aligned}\mathcal{L}_{mass} = &-\sum_{\text{i}=1}^{3}\left[m_i^d(\bar{d}_{L,i}d_{R,i} + \bar{d}_{R,i}d_{L,i})\right] - \sum_{\text{i}=1}^{3}\left[m_i^u(\bar{u}_{L,i}u_{R,i} + \bar{u}_{R,i}u_{L,i})\right] \\ &- \sum_{\text{i}=1}^{3}\left[m_i^e(\bar{e}_{L,i}e_{R,i} + \bar{e}_{R,i}e_{L,i})\right] - \sum_{\text{i}=1}^{3}\left[m_i^\nu(\bar{\nu}_{L,i}\nu_{R,i} + \bar{\nu}_{R,i}\nu_{L,i})\right]\end{aligned} \tag{1.73}$$

Finally we have the general expression of fermion masses [2], [5]

$$m_i^f = \frac{v}{\sqrt{2}} U_{ij}^{L,f} \alpha_{jk}^f U_{ki}^{R,f\dagger} \tag{1.74}$$

After we have obtained the form of the mass for all fermions, we will study the mixing of quarks and leptons separately to obtain matrix mixing blocks for both of them.

1.3.4 The Fermion Mixing Matrix(CKM and PMNS)

The Yukawa couplings diagonalization leads to the mixing between the different families of fermions in off-diagonal matrix elements. The kinetic terms are also modified by the basis transformation (1.68). As a result, the fermion mixing between families appears in the charged current interaction [24]:

$$\begin{aligned}\mathcal{L}_f = &\begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}_i^I \left[i\gamma_\mu\partial_\mu + \gamma_\mu\begin{pmatrix}\frac{g'}{6}B_\mu + \frac{g}{2}W_\mu^3 & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & \frac{g'}{6}B_\mu - \frac{g}{2}W_\mu^3\end{pmatrix}\right]\begin{pmatrix}u_i^I \\ d_i^I\end{pmatrix}_i^I \\ &+ \begin{pmatrix}\bar{\nu} & \bar{e}\end{pmatrix}_i^I \left[i\gamma_\mu\partial_\mu + \gamma_\mu\begin{pmatrix}\frac{g'}{6}B_\mu + \frac{g}{2}W_\mu^3 & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & \frac{g'}{6}B_\mu - \frac{g}{2}W_\mu^3\end{pmatrix}\right]\begin{pmatrix}\nu_i^I \\ e_i^I\end{pmatrix}_i^I + \ldots\ldots \\ &- \frac{\sqrt{2}}{v}\sum_{\text{i}=1}^{3}\Big[\bar{Q}_{L,i}^I M_i^d d_{R,i}^I + \bar{d}_{R,i}^I M_i^{*d}\Phi^\dagger Q_{L,i}^I + \bar{Q}_{L,i}^I M_i^u\tilde{\Phi}u_{R,i}^I + \bar{u}_{R,i}^I M_i^{*u}\tilde{\Phi}^\dagger Q_{L,i}^I \\ &+ \bar{L}_{L,i}^I M_i^e\Phi e_{R,i}^I + \bar{e}_{R,i}^I M_i^{*e}\Phi^\dagger L_{L,i}^I + \bar{L}_{L,i}^I M_i^\nu\tilde{\Phi}\nu_{R,i}^I + \bar{\nu}_{R,i}^I M_i^{*\nu}\tilde{\Phi}^\dagger L_{L,i}^I\Big] + \ldots..\end{aligned} \tag{1.75}$$

Considering that the rest of the terms do not mix up- and down-type fermions, this lagrangien becomes as follow:

$$\begin{aligned}\mathcal{L}_f = &\frac{g}{\sqrt{2}}\bar{u}_{iL}^I\gamma_\mu W_\mu^- d_{iL}^I + \frac{g}{\sqrt{2}}\bar{d}_{iL}^I\gamma_\mu W_\mu^+ u_{iL}^I + \frac{g}{\sqrt{2}}\bar{\nu}_{iL}^I\gamma_\mu W_\mu^- e_{iL}^I + \frac{g}{\sqrt{2}}\bar{e}_{iL}^I\gamma_\mu W_\mu^+ \nu_{iL}^I \\ &- \frac{\sqrt{2}}{v}\sum_{\text{i}=1}^{3}\left[\bar{u}_{L,i}^I m_i^d d_{R,i}^I\phi^+ + \bar{d}_{L,i}^I m_i^u u_{R,i}^I\phi^- + \bar{u}_{R,i}^I m_i^{*d} d_{L,i}^I\phi^+ + \bar{d}_{R,i}^I m_i^{*u} u_{L,i}^I\phi^-\right. \\ &\left.+ \bar{\nu}_{L,i}^I m_i^e e_{R,i}^I\phi^+ + \bar{e}_{L,i}^I m_i^\nu \nu_{R,i}^I\phi^- + \bar{e}_{R,i}^I m_i^{*\nu}\nu_{L,i}^I\phi^+ + \bar{\nu}_{R,i}^I m_i^{*e} e_{L,i}^I\phi^-\right]\end{aligned} \tag{1.76}$$

We can now rewrite the lagranian (1.76) by expressing the interaction eigenstates $\{u^I, d^I, \nu^I, e^I\}$ as fermion mass eigenstates $\{u, d, \nu, e\}$

$$\begin{aligned}\mathcal{L}_f = & \frac{g}{\sqrt{2}}\bar{u}_{iL}(U_L^u U_L^{d\dagger})_{ij}\gamma_\mu W_\mu^- d_{iL} + \frac{g}{\sqrt{2}}\bar{d}_{iL}(U_L^d U_L^{u\dagger})_{ij}\gamma_\mu W_\mu^+ u_{iL} \\ & + \frac{g}{\sqrt{2}}\bar{\nu}_{iL}(U_L^\nu U_L^{e\dagger})_{ij}\gamma_\mu W_\mu^- e_{iL} + \frac{g}{\sqrt{2}}\bar{e}_{iL}(U_L^e U_L^{\nu\dagger})_{ij}\gamma_\mu W_\mu^+ \nu_{iL} + ... \\ & - \frac{\sqrt{2}}{v}\sum_{\mathrm{i}=1}^{3}\Big[\bar{u}_{L,i}(U_L^u U_L^{d\dagger})m_i^d d_{R,i}\phi^+ + \bar{d}_{L,i}(U_L^d U_L^{u\dagger})m_i^u u_{R,i}^I\phi^- \\ & + \bar{u}_{R,i}(U_L^u U_L^{d\dagger})m_i^{*d} d_{L,i}\phi^+ + \bar{d}_{R,i}(U_L^d U_L^{u\dagger})m_i^{*u} u_{L,i}\phi^- \\ & + \bar{\nu}_{L,i}(U_L^\nu U_L^{e\dagger})m_i^e e_{R,i}\phi^+ + \bar{e}_{L,i}(U_L^e U_L^{\nu\dagger})m_i^\nu \nu_{R,i}\phi^- \\ & + \bar{e}_{R,i}(U_L^e U_L^{\nu\dagger})m_i^{*\nu}\nu_{L,i}\phi^+ + \bar{\nu}_{R,i}(U_L^\nu U_L^{e\dagger})m_i^{*e} e_{L,i}\phi^-\Big] + ...\end{aligned} \tag{1.77}$$

where

$$V_{CKM} = U_L^u U_L^{d\dagger} \tag{1.78}$$

and

$$U_{PMNS} = U_L^\nu U_L^{e\dagger} \tag{1.79}$$

Thus, all of the interesting mixing effects are given by these two matrices [6]

$$V_{CKM} = U_L^u U_L^{d\dagger} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.80}$$

$$U_{PMNS} = U_L^\nu U_L^{e\dagger} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} = \begin{pmatrix} U_{\nu_e e} & U_{\nu_e \tau} & U_{\nu_e \mu} \\ U_{\nu_\tau e} & U_{\nu_\tau \tau} & U_{\nu_\tau \mu} \\ U_{\nu_\mu e} & U_{\nu_\mu \tau} & U_{\nu_\mu \mu} \end{pmatrix} \tag{1.81}$$

known as the Cabibbo-Kobayashi-Maskawa (CKM) (QuarkMixing Matrix) and the Pontecorvo- Maki-Nakagawa-Sakata (PMNS) (Lepton Mixing Matrix). They can be specified by three angles and a phase. and they can be written with an identical parametrization for each of them [24], [17], [21], [6]

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{13}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{13}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.82}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{13}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{13}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_{12}}{2}} & \\ & & e^{i\frac{\alpha_{13}}{2}} \end{pmatrix} \tag{1.83}$$

Note that 1, 2, 3 refer to mass eigenstates defined in terms of flavor eigenstates by these matrices.

Finally, these are CKM and PMNS Numerical values [6]:

$$V_{CKM} = \begin{pmatrix} 0.97427 & 0.22536 & 0.00355 \\ 0.22522 & 0.97343 & 0.0414 \\ 0.00886 & 0.0405 & 0.99914 \end{pmatrix} \pm \begin{pmatrix} 0.00014 & 0.00061 & 0.00015 \\ 0.00061 & 0.00015 & 0.0012 \\ 0.00032 & 0.0011 & 0.00005 \end{pmatrix} \tag{1.84}$$

$$U_{PMNS} = \begin{pmatrix} 0.788396077726927 & 0.408445470475221 & 0.152472950695434 \\ -0.468414463157376 & 0.744717881532313 & 0.427088737464917 \\ 0.102569223161843 & -0.488580313546053 & 0.778454422594894 \end{pmatrix} \tag{1.85}$$

Chapter 2

Renormalisation of Mass Mixing Matrix

2.1 Introduction

The ultraviolet divergences appear in intermediate steps of calculations in quantum field theories [1]. In the 1930's, when the ultraviolet divergences were first discovered in qunatum electrodynamics, many physicists believed that fundamental principles of physics have to be changed to eliminate the divergences [4]. In 1940's, Bethe, Feynman, Schwinger, Tomonaga, and Dyson, and others proposed a program of renormalization that gave finite and physically sensible results by absorbing the divergences into redefinitions of physical quantities. Some scalar or tensorial integral quantities diverge in the ultraviolet limit. We must, therfore, improve the theory to discard these divergences through performing what is called the renormalization prescription. The lagrangians, introduced in chapter 1 are called bare lagrangians, in that sense they couldn't predict the unbared physical constants as masses, charges ... ect, which are determined experimentally.

To cure these problems, we therefore add counter terms to those lagranians [4]. The counter terms should be chosen to cancel the ultraviolet divergences. Accordinglly, a set of diagrams and Feynman rules associated with these counter terms must be also added. The form of the counter terms are fixed by introducing renormalization conditions.

2.2 Ultraviolet divergences review through QED

The ultraviolet divergence is a serious problem in QED, and it accures in many different calculation up to high orders [1], [4]. Here in this section, we start giving an example to compute the correction of the fermionic propagator, as one of the QED processes in which the loop self energy couldn't be fixed experimentally because of the existence of the UV divergence. Then, we will treat this divergence using the counter term procedure [4].

2.2.1 Construction of the fermionic propagator

Getting the propagator from the vacuum expectation value of time-ordered products of field operators can be somewhat daunting. So, we apply an alternative practical method explained in below [32], [32], [20] and this directly on the following Dirac lagrangian [4]:

$$\mathcal{L}_f = \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi \tag{2.1}$$

Namely, we make the change $\overrightarrow{\partial}_\mu \longrightarrow -ip_\mu$, this leads to

$$\begin{aligned}\mathcal{L}_f &= \bar{\Psi}_f(i(-ip_\mu)\gamma^\mu - m_f)\Psi_f \\ &= \bar{\Psi}_f\left[\mathcal{D}(p)\right]\Psi_f\end{aligned} \tag{2.2}$$

Where the oparator $D(p)$ is define by the expression

$$\mathcal{D}(p) = \not{p} - m_f \tag{2.3}$$

The fermionic propagator in momentum space is simply obtained by inverting (2.3).

$$S_{0,f}(p) = i\mathcal{D}(p)^{-1} = \frac{i}{\not{p} - m_f + i\epsilon} \tag{2.4}$$

2.2.2 The one loop correction of the fermionic propagator

Let consider the full fermionic propagator in momentum space. It corresponds to a propagation of the fermion without interaction with external real photons, but rather with the emission and reabsorption of virtual photons. The full propagator can be written as a geometric series of graphs containing more and more insertions of the fermion self energy expression $-i\widehat{\Sigma}(p)$ (see Fig 2.1) (a similar example of the electron propagator is found in ref [20], [4]).

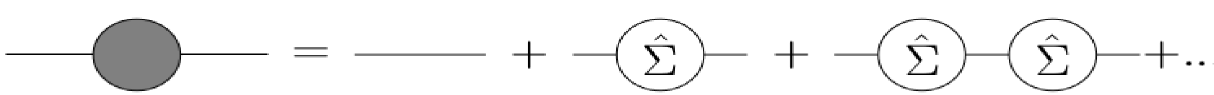


Figure 2.1: The Full Fermionic Propagator

We can link the exact fermionic propagator $S_f(p)$ with the free fermionic propagator $S_{f,0}(p)$ as

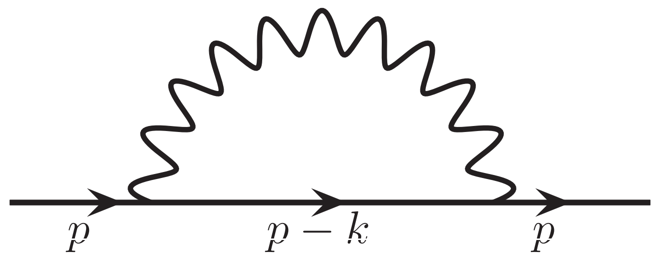


Figure 2.2: The fermion self energy

follow

$$
\begin{aligned}
S_f(p) &= S_{f,0}(p) + S_{f,0}(p)(-i\widehat{\Sigma})S_{f,0}(p) + S_{f,0}(p)(-i\widehat{\Sigma})S_{f,0}(p)(-i\widehat{\Sigma})S_{f,0}(p) + ... \\
&= S_{f,0}(p) + S_{f,0}(p)(-i\widehat{\Sigma})\left\{S_{f,0}(p) + S_{f,0}(p)(-i\widehat{\Sigma})S_{f,0}(p) + ...\right\} \\
&= S_{f,0}(p) + S_{f,0}(p)(-i\widehat{\Sigma})S_f(p)
\end{aligned}
\tag{2.5}
$$

where $-i\widehat{\Sigma}(p)$ is what we called fermion self energy. It is formed by An the sum represented by circles in the figure (2.1)
Multiplying on the left of (2.6) with $S_{f,0}^{-1}(p)$ and on the right with $S_f^{-1}(p)$ we get

$$
\begin{aligned}
S_{f,0}^{-1}(p) &= S_{f,0}^{-1}(p)S_{f,0}(p)S_f^{-1}(p) + S_{f,0}^{-1}(p)S_{f,0}(p)(-i\widehat{\Sigma})S_f(p)S_f^{-1}(p) \\
&= S_f^{-1}(p) - i\widehat{\Sigma}
\end{aligned}
\tag{2.6}
$$

which we can rewrite as

$$
S_f^{-1}(p) = S_{f,0}^{-1}(p) + i\widehat{\Sigma}(p) = -i\left[\not{p} - (m_f + \widehat{\Sigma}(p))\right] \tag{2.7}
$$

where (2.4) is used.
Then, by inverting (2.7), we obtain the exact fermionic propagator

$$
S_f(p) = \frac{i}{\not{p} - m_f + i\widehat{\Sigma}(p)} \tag{2.8}
$$

In lowest order there is only the diagram of Fig 2.2 contributing to $\widehat{\Sigma}(p)$. Applying Feynman's rules to this diagram makes it possible to write the mass operator at one loop level:

$$
-i\widehat{\Sigma}(p) = (ieQ_f)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{g^{\mu\nu}}{k^2 + i\epsilon} \gamma_\mu \frac{\not{p} - k^\mu\gamma_\mu + m_f}{(p-k)^2 - m_f^2 + i\epsilon} \gamma_\nu \tag{2.9}
$$

Each loop is associated with an integration on the quadri-momentum k^μ. We notice that this integration within the loop is linearly divergent. However, we know that in the real physical world, the fermions can propagate, which means that the exact fermionic propagator (2.8) is not zero. The emergence of the divergence at the level of the denominator of the fermionic propagator is only in the imaginary part, it must therefore be treated by renormalization. To do this, it is necessary to transform the above integral to a convergent expression plus a singular part. This goal is achieved under the dimensionally regularizing procedure.

The principle of dimensional regularization [4], [3], [35] introduced by t'Hooft and Veltman is based on the observation that the divergent loop integrals occur only when the integration measure is of dimension 4. The idea is to analytically extend this dimension to non-integer values, $d = 4 - \epsilon$ [4]. The divergences for $d = 4$ then appear as poles in $1/\epsilon$. Since the fields are no longer defined in dimension 4, it is necessary to introduce a renormalization scale μ in order to keep the dimensionless coupling constants. The results will depend on this scale. This regularization has the advantage of preserving gauge invariance.

So the expression (2.9) takes the form

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{4-D}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{g^{\mu\nu}}{k^2+i\varepsilon}\frac{\gamma_\mu(\not{p}-k^\mu\gamma_\mu+m_f)\gamma_\nu}{(p-k)^2-m_f^2+i\epsilon} \tag{2.10}$$

After doing steps required to regularize expression (2.10)[1], we find

$$\begin{aligned}\widehat{\Sigma}(p) = \frac{\alpha}{4\pi}Q_f^2(4\pi\mu^2)^{(2-\frac{D}{2})}\Gamma(2-\frac{D}{2})\Bigg\{&(\not{p}-m_f)\int_0^1 \mathrm{d}x\frac{(2-D)(1-x)}{\left[m_f^2x-x(1-x)p^2-i\epsilon\right]^{2-\frac{D}{2}}}\\ &+m_f^2\int_0^1 \mathrm{d}x\,\frac{2(1-x)+Dx}{\left[m_f^2x-x(1-x)p^2-i\epsilon\right]^{2-\frac{D}{2}}}\Bigg\}\end{aligned} \tag{2.11}$$

The singular part of the fermion self energy at the one loop level reads

$$\widehat{\Sigma}^{(1)}(p) = \left(\frac{\alpha}{4\pi}Q_f^2\right)\left[m_f\frac{4}{\varepsilon_{uv}}-\not{p}\frac{1}{\varepsilon_{uv}}\right]+\mathcal{O}(1), \tag{2.12}$$

where $\varepsilon_{uv} = 2 - D/2 = \epsilon/2$. Equation (2.12) shows that the ultraviolet divergences ($\varepsilon_{uv} \to 0$) appear twice: once with m_f coefficient and the other with $\not{p}$ coefficient. In order to cancel the divergences of the fermionic self energy, we introduce a counter term at the level of the fermionic Lagrangian.

2.3 Renormalisation constants and counter-terms

As we said previously, we have to choose a set of independents parameters and fields to renormalize [5]. In our work we choose the SM entities that we introduced in chapter 1, that is:

[1] which are listed in Appendix C

- all fermionic and bosonic fields: $f_{L,R}, Z, A_\nu, W^\pm, H$.

- SM model parameters: $e, M_W, M_Z, M_H, m_{f,i}, V_{i,j}$.

Then, we define the renormalization constants corresponding first to these parameters as[2]:

$$e_0 = (1+\delta Z_e)e \tag{2.13}$$
$$M^2_{W,0} = M^2_W + \delta M^2_W \tag{2.14}$$
$$M^2_{Z,0} = M^2_Z + \delta M^2_Z \tag{2.15}$$
$$M^2_{H,0} = M^2_H + \delta M^2_H \tag{2.16}$$
$$m_{f,i,0} = m_{f,i} + \delta m_{f,i} \tag{2.17}$$
$$V_{i,j,0} = V_{i,j} + \delta V_{i,j}, \tag{2.18}$$

and second to the fields as [4], [5]:

$$W_0^\pm = Z_W^{1/2} W = (1+\frac{1}{2}\delta Z_W)W^\pm \tag{2.19}$$

$$\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} Z_{ZZ}^{1/2} & Z_{ZA}^{1/2} \\ Z_{AZ}^{1/2} & Z_{AA}^{1/2} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & 1+\frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \tag{2.20}$$

$$H_0 = Z_H^{1/2} H = (1+\frac{1}{2}\delta Z_H)H \tag{2.21}$$

$$f^L_{i,0} = Z_{ij}^{1/2,f,L} f^L_j \tag{2.22}$$

$$f^R_{i,0} = Z_{ij}^{1/2,f,R} f^R_j \tag{2.23}$$

Since we treated the fermionic propagator in the last section, we continue with renormalization of the fermionic sector ($m_{f,i}$, $f^L_{i,0}$, $f^R_{i,0}$). Then after, we deal with the remains fields and parameters.

2.3.1 Renormalization of the fermionic propagator at one loop level

The aim of the renormalization of the fermionic propagator is to remove the ultraviolet divergences which appear during the loop calculation. Therefore, we introduce at the level of the fermionic lagrangian a new component known as counter term. The main idea is to separate the infinite part from the finite part of the original parameters in the lagrangian (2.1) [4]. The counter term of infinite contributions have, by definition, the same structure of the original terms in the lagrangian. The unbared parametres are fixed by experiments, and as consequence the singularity of the counter terms

[2]The bare quantities are indexed by 0

are absorbed in the bared parameters. These contraints lead to the renormalization conditons. Here, we introduce two renormalization constants Z_f^L and Z_{m_f}. The first constant links the mass eigenstates $f_{L,i}$ to the bare mass eigenstates $f_{0,i}^L$, and the second constant is in relation to the fermionic mass m_f, according to the following form [18], [4], [31]

$$\begin{aligned} f_{i,0}^L &= Z_{ij}^{1/2,f,L} f_j^L &(2.24)\\ f_{i,0}^R &= Z_{ij}^{1/2,f,R} f_j^R, &(2.25)\\ \bar{f}_{i,0}^L &= Z_{ij}^{1/2,f,L} \bar{f}_j^L &(2.26)\\ \bar{f}_{i,0}^R &= Z_{ij}^{1/2,f,R} \bar{f}_j^R, &(2.27)\end{aligned}$$

and

$$m_{0,f} = Z_{m_f} m_f. \tag{2.28}$$

where the renormalization constants can be written to the first order of perturbation as [4]

$$\begin{aligned} Z_f^L &= 1 + \delta Z_f^L + \cdots &(2.29)\\ Z_{m_f} &= 1 + \delta Z_{m_f} + \cdots &(2.30)\end{aligned}$$

This form of the renormalization constants enables us to split the bare Lagrangian $\mathcal{L}_{f,0}$ into the renormalized Lagrangian $\mathcal{L}_{f,R}$ and the counter terms lagrangian $\delta\mathcal{L}$. Neglecting the high orders, the bared fermionic Lagrangian (2.1) takes the form

$$\begin{aligned} \mathcal{L}_{f,0} &= \bar{f}_{0,i}^L (i\partial_\mu \gamma^\mu) f_{0,i}^L - m_f \bar{f}_{0,i}^L f_{0,i}^L \\ &= Z_f \bar{f}_i^L (i\partial_\mu \gamma^\mu) f_i^L - Z_f Z_{m_f} m_f \bar{f}_i^L f_i^L \\ &= (1+\delta Z_f^L) \bar{f}_i^L (i\partial_\mu \gamma^\mu) f_i^L - (1+\delta Z_f^L)(1+\delta Z_{m_f}) m_f \bar{f}_i^L f_i^L \\ &= \bar{f}_i^L (i\partial_\mu \gamma^\mu) f_i^L - m_f \bar{f}_i^L f_i^L + \delta Z_f^L \bar{f}_{L,i} (i\partial_\mu \gamma^\mu) f_i^L - (\delta Z_f^L + \delta Z_{m_f}) m_f \bar{f}_i^L f_i^L + \cdots \end{aligned} \tag{2.31}$$

$\mathcal{L}_{f,R}$ and $\delta\mathcal{L}_f$ read respectively [31]

$$\begin{aligned} \mathcal{L}_f &= \bar{f}_i^L (i\partial_\mu \gamma^\mu) f_i^L - m_f \bar{f}_i^L f_i^L &(2.32)\\ \delta\mathcal{L}_f^{(1)} &= \delta Z_f^L \bar{f}_i^L (i\partial_\mu \gamma^\mu) f_i^L + (\delta Z_f^L + \delta Z_{m_f}) m_f \bar{f}_i^L f_i^L &(2.33)\end{aligned}$$

Even, the fermionic Lagrangian is expressed, here, in terms of renormalized fields and renormalized masses, it has the same Feynman rules as those deduced from the initial Lagrangian. We have just to add the feynman rules corresponding to the counter term $\delta\mathcal{L}_f^{(1)}$. Therefore, we get for the self energy

$$\begin{aligned} -i\widehat{\Sigma}_{f,R}^{(1)}(p) &= -i\widehat{\Sigma}^{(1)} + i\delta Z_f^L \not{p} - i(\delta Z_f^L + \delta Z_{m_f})m_f \\ &= -i\widehat{\Sigma}^{(1)} + i(\not{p} - m_f)\delta Z_f^L - im_f\delta Z_{m_f} \end{aligned} \tag{2.34}$$

For convient presentation, this expression is ranged in the following form

$$\Sigma_{f,R}^{(1)}(p) = m_f(\widehat{\Sigma}_a^{(1)}(p) + \delta Z_{m_f}) + (\not{p} - m_f)(\widehat{\Sigma}_b^{(1)}(p) - \delta Z_f^L)\cdot \tag{2.35}$$

where

$$\begin{aligned} \widehat{\Sigma}_a^{(1)}(p) &= \left(\frac{\alpha}{4\pi}Q_f^2\right)\left\{3\left[\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln(\frac{\mu^2}{m_f^2}) - \gamma\right]\right. \\ &\qquad \left. -1 - 2\int_0^1 \mathrm{d}x(1+x)ln\left[x - \frac{p^2}{m_f^2}x(1-x) - i\epsilon\right]\right\}, \end{aligned} \tag{2.36}$$

and

$$\begin{aligned} \widehat{\Sigma}_b^{(1)}(p) = \left(\frac{\alpha}{4\pi}Q_f^2\right)&\left\{-\left[\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln(\frac{\mu^2}{m_f^2}) - \gamma\right]\right. \\ &\left. +1 + 2\int_0^1 \mathrm{d}x(1+x)\,ln\left[x - \frac{p^2}{m_f^2}x(1-x) - i\epsilon\right]\right\}. \end{aligned} \tag{2.37}$$

The purpose of the next paragraph is to determine the renormalization constants Z_{m_f} and Z_f using renormalization conditions.

Renormalization Conditions for the fermionic Sector:

We require two renormalization conditions related to $S_{f,0}$ (2.4), and S_f (2.8) [5] [18], [20], [4], [31]

- The first condition:
  The pole of S_f: ($\not{p} = m_{f,r} + \widehat{\Sigma}_f(p)$) must be equal to the pole of $S_{f,0}$: ($\not{p} = m_f$), as consequence

$$\left[\widehat{\Sigma}_f(p)\right]_{\not{p}=m_f} = 0 \tag{2.38}$$

- The second condition:
  The residue at the pole of the renormalized fermionic propagator must be equal to the residue

at pole of the free fermionic propagator:

$$Res\left[S_f\right]_{\not{p}=m_f} = Res\left[S_{f,0}\right]_{\not{p}=m_f} \Rightarrow \lim_{\not{p}\longrightarrow m_f} \frac{\not{p}-m_f}{\not{p}-m_f-\widehat{\Sigma}_f(p)} = 1 \tag{2.39}$$

Taking into account the Taylor expansion of $\widehat{\Sigma}_f(p)$ in the vicinity of m_f and of the first condition, we can write:

$$\widehat{\Sigma}_f(p) = \widehat{\Sigma}_f(m_f) + (\not{p}-m_f)\left[\frac{\partial\widehat{\Sigma}_f(p)}{\partial\not{p}}\right]_{\not{p}=m_f} + \frac{1}{2!}\left[\frac{\partial^2\widehat{\Sigma}_f(p)}{\partial\not{p}^2}\right]_{\not{p}=m_f} + \cdots \tag{2.40}$$

Then, the second condition for the fierst order takes the form:

$$\left[\frac{\partial\widehat{\Sigma}_f(p)}{\partial\not{p}}\right]_{\not{p}=m_f} = 0 \tag{2.41}$$

By applying these renormalization conditions on the relation (2.35), we have first for the constant δZ_{m_f}

$$\left[\widehat{\Sigma}^{(1)}_{f,R}(p)\right]_{\not{p}=m_{f,R}} = m_{f,R}\left(\widehat{\Sigma}^{(1)}_a(p) + \delta Z^{OS}_{m_f}\right)|_{\not{p}=m_{f,R}} = 0 \tag{2.42}$$

$$\Rightarrow \delta Z^{OS}_{m_f} = -\left[\widehat{\Sigma}^{(1)}_a(p)\right]_{\not{p}=m_{f,R}} = \frac{\alpha Q_f^2}{4\pi}3\left(-\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln\left(\frac{\mu^2}{m_f^2}\right) - \gamma - \frac{4}{3}\right) \tag{2.43}$$

and secondly for the constant δZ_f we have

$$\left[\frac{\partial}{\partial\not{p}}\left\{m_{f,R}(\widehat{\Sigma}^{(1)}_a(p) + \delta Z_{m_f}) + (\not{p}-m_{f,R})(\widehat{\Sigma}^{(1)}_b(p) - \delta Z^L_f)\right\}\right]_{\not{p}=m_{f,R}} \tag{2.44}$$

$$\Rightarrow \delta Z^{OS}_f = m_{f,R}\left[\frac{\partial\widehat{\Sigma}^{(1)}_a(p)}{\partial\not{p}}\right]_{\not{p}=m_f} + \left[\widehat{\Sigma}^{(1)}_b(p)\right]_{\not{p}=m_f}. \tag{2.45}$$

where

$$\left[\frac{\partial\widehat{\Sigma}^{(1)}_a(p)}{\partial\not{p}}\right]_{\not{p}=m_f} = \frac{\alpha Q_f^2}{4\pi}\left\{-\frac{6}{m_f} - 4\int_0^1 \mathrm{d}x\frac{(1+x)}{m_f^3}\left(\frac{p^2x(1-x)}{x\frac{p^2}{m_f^2}x(1-x) - i\epsilon}\right)\right\} \tag{2.46}$$

Substituting (2.46) and (2.37) in (2.45) we get

$$\delta Z_f^{OS} = \frac{\alpha Q_f^2}{4\pi}\left\{-\frac{6}{m_f} - 4\int_0^1 \mathrm{d}x \frac{(1+x)}{m_f^3}\left(\frac{p^2 x(1-x)}{x\frac{p^2}{m_f^2}x(1-x)-i\epsilon}\right)\right\} \quad (2.47)$$

$$+\left(\frac{\alpha}{4\pi}Q_f^2\right)\left\{-\left[\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln(\frac{\mu^2}{m_f^2}) - \gamma\right]\right. \quad (2.48)$$

$$\left. +1 + 2\int_0^1 \mathrm{d}x(1+x)\, ln\left[x - \frac{p^2}{m_f^2}x(1-x) - i\epsilon\right]\right\}.$$

$$\Rightarrow \delta Z_f^{OS} = \frac{\alpha Q_f^2}{4\pi}\left(-\frac{1}{\varepsilon_{uv}} + \frac{2}{\varepsilon_{uv}} + ln(4\pi) + 3ln\left(\frac{\mu^2}{m_f^2}\right) - \gamma - 4\right) \quad (2.49)$$

Note that we can write the fermion self energy in the following form [7]

$$\widehat{\Sigma}(p) = \not{p}P_L\widehat{\Sigma}_L^f(p^2) + \not{p}P_R\widehat{\Sigma}_R^f(p^2) + m_f\widehat{\Sigma}_S^f(p^2), \quad (2.50)$$

where P_L (1.3) and P_R (1.2) are the projector operators. $\widehat{\Sigma}_L$ and $\widehat{\Sigma}_R$ are the left and right-handed fermion self energy respectivly, and $\widehat{\Sigma}_S$ is the scalar part. According to the renormalization condition corresponding to this last form of $\widehat{\Sigma}$ [5], we rewrite the renormalization constants as follow

$$\delta Z_{ij}^{f,L} = \frac{2}{m_{f,i}^2 - m_{f,j}^2}\left[m_{f,j}^2\widehat{\Sigma}_{ij}^{f,L}(m_{f,j}^2) + m_{f,i}m_{f,j}\widehat{\Sigma}_{ij}^{f,R}(m_{f,j}^2)\right. \quad (2.51)$$

$$\left. + \quad (m_{f,i}^2 + m_{f,j}^2)\widehat{\Sigma}_{ij}^{f,S}(m_{f,j}^2)\right] \qquad i \neq j,$$

$$\delta Z_{ij}^{f,R} = \frac{2}{m_{f,i}^2 - m_{f,j}^2}\left[m_{f,j}^2\widehat{\Sigma}_{ij}^{f,R}(m_{f,j}^2) + m_{f,i}m_{f,j}\widehat{\Sigma}_{ij}^{f,L}(m_{f,j}^2)\right. \quad (2.52)$$

$$\left. + \quad 2m_{f,i}m_{f,j}\widehat{\Sigma}_{ij}^{f,S}(m_{f,j}^2)\right] \qquad i \neq j,$$

$$\delta Z_{ii}^{f,L} = -\widehat{\Sigma}_{ii}^{f,L}(m_{f,i}^2) - m_{f,i}^2\frac{\partial}{\partial p^2}\left[\widehat{\Sigma}_{ii}^{f,L}(p^2) + \widehat{\Sigma}_{ii}^{f,R}(p^2) + 2\widehat{\Sigma}_{ii}^{f,S}(p^2)\right]|_{p^2=m_{f,i}^2}, \quad (2.53)$$

$$\delta Z_{ii}^{f,R} = -\widehat{\Sigma}_{ii}^{f,R}(m_{f,i}^2) - m_{f,i}^2\frac{\partial}{\partial p^2}\left[\widehat{\Sigma}_{ii}^{f,L}(p^2) + \widehat{\Sigma}_{ii}^{f,R}(p^2) + 2\widehat{\Sigma}_{ii}^{f,S}(p^2)\right]|_{p^2=m_{f,i}^2}. \quad (2.54)$$

We use these last form of renormalizeation constants in the third section to calculte the fermion mass mixing matrix elements.

Now, we back to renormalize the remains fields and parameteres. In the next sections we will treat the gauge bosons propagators.

2.3.2 Renormalization of the gauge sector

Applying the same procedure used in the fermionic propagator calculation [18], [20], we can compute the W boson propagator [31]. First, we pick only the free gauge lagrangian term that correspond to the W boson propagator:

$$\mathcal{L}_{W,0} = W^{+}_{\mu,0}\left[g^{\mu\nu}\Box - M^2_{W,0}g^{\mu\nu}\right]W^{-}_{\nu,0}. \tag{2.55}$$

Then, using the same substitution as in the fermionic propagator case [32]:

$$\begin{gathered}\overrightarrow{\partial}^{\mu} \longrightarrow -ip^{\mu}, \qquad \overleftarrow{\partial}^{\mu} \longrightarrow ip^{\mu} \\ \Rightarrow \Box = \overleftarrow{\partial}_{\mu}\overrightarrow{\partial}^{\mu} \longrightarrow p^2,\end{gathered} \tag{2.56}$$

we obtain

$$\begin{aligned}\mathcal{L}_{W,0} &= W^{+}_{\mu,0}g^{\mu\nu}(p^2 - M^2_{W,0})W^{-}_{\nu,0} && (2.57)\\ &= W^{+}_{\mu,0}\left[\mathcal{D}(p)\right]W^{-}_{\nu,0} && (2.58)\end{aligned}$$

where

$$\mathcal{D}(p) = g^{\mu\nu}(p^2 - M^2_{W,0}) \tag{2.59}$$

By inverting (2.59), we obtain the W boson propagator

$$D^{\mu\nu}_{W,0} = -ig^{\mu\nu}\frac{1}{p^2 - M^2_V} \tag{2.60}$$

Following the same logic in the fermion sector, the full W boson propagator can be written as a series containinig more and more insertion of the W boson self energy expression:

$$D^{\mu\nu}_{W} = -ig^{\mu\nu}\left(\frac{1}{p^2 - M^2_W} - \frac{1}{p^2 - M^2_W}\widehat{\Sigma}^{W}(p)\frac{1}{p^2 - M^2_W} + \cdots\right), \tag{2.61}$$

where the W boson self energy $\widehat{\Sigma}_W(p)$ is given by [31]

$$\widehat{\Sigma}^{\mu\nu}_{W}(p) = \left(g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^2}\right)\widehat{\Sigma}^{T}_{W}(p) + \frac{p^{\mu}p^{\nu}}{p^2}\widehat{\Sigma}^{L}_{W}(p). \tag{2.62}$$

We have to eliminate the longitudinal part. We need to renormalize the W boson self energy to cancel the divergences in $\widehat{\Sigma}^{T}_{W}(p)$. We use here the previous counter terms method. By substituting (2.19) and (2.13) in (2.57) we get

$$\mathcal{L}_{W,0} = Z_W W^+_\mu g^{\mu\nu}(-p^2 + M_W^2 + \delta M_W^2)W^-_\nu \tag{2.63}$$

$$= (1+\delta Z_W)W^+_\mu g^{\mu\nu}\left[-p^2 + M_W^2 + \delta M_W^2\right] W^-_{\nu,0} \tag{2.64}$$

$$= W^+_\mu\left[-g^{\mu\nu}(p^2 - M_W^2)\right] + W^+_\mu\left[-g^{\mu\nu}\delta Z_W(p^2 - M_W^2) + g^{\mu\nu}\delta M_W^2\right] W^-_\nu\cdot$$

As in the fermion case, the W boson Lagrangian splits into two terms. The second one is the counter term lagrangian. This enables us to write

$$\widehat{\Sigma}^T_{W,R}(p^2) = \widehat{\Sigma}^T_W(p^2) - \delta M_W^2 + \delta Z_W^2(p^2 - M_W^2) \tag{2.65}$$

Then we define the on-shell renormalization canditions as follow [18], [31]

$$Re\widehat{\Sigma}^T_W(p^2)|_{p^2=M_W^2} = 0, \qquad Re\frac{\partial}{\partial p^2}\widehat{\Sigma}^T_W(p^2)|_{p^2=M_W^2} = 0\cdot \tag{2.66}$$

The cases of A, Z bosons are very similar th the W boson case. procedure. We notice that there is a self-energy of transition between the photon and the Z boson:

$$\widehat{\Sigma}_{ZA}(p) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\widehat{\Sigma}^T_{ZA}(p) + \frac{p^\mu p^\nu}{p^2}\widehat{\Sigma}^L_{ZA}(p). \tag{2.67}$$

similarly to (2.65), and after using (2.20) and (2.13), we write

$$\widehat{\Sigma}^T_{ZZ,R}(p^2) = \widehat{\Sigma}^T_{ZZ}(p^2) - \delta M_Z^2 + \delta Z_{ZZ}^2(p^2 - M_Z^2) \tag{2.68}$$

$$\widehat{\Sigma}^T_{AA,R}(p^2) = \widehat{\Sigma}^T_{AA}(p^2) + \delta Z_{AA}^2 p^2 \tag{2.69}$$

$$\widehat{\Sigma}^T_{AZ,R}(p^2) = \widehat{\Sigma}^T_{AZ}(p^2) - \frac{1}{2}\delta Z_{AZ}^2 p^2 + \frac{1}{2}\delta Z_{ZA}^2(p^2 - M_Z^2)\cdot \tag{2.70}$$

The imposed renormalization conditions are as follow [5], [31]

$$Re\widehat{\Sigma}^T_{ZZ}(p^2)|_{p^2=M_Z^2} = 0, \qquad Re\frac{\partial}{\partial p^2}\widehat{\Sigma}^T_{ZZ}(p^2)|_{p^2=M_Z^2} = 0, \tag{2.71}$$

$$\widehat{\Sigma}^T_{AA}(0) = 0, \qquad \frac{\partial}{\partial p^2}\widehat{\Sigma}^T_{AA}(0) = 0, \tag{2.72}$$

$$\widehat{\Sigma}^T_{AZ}(0) = 0, \qquad Re\widehat{\Sigma}^T_{AZ}(p^2)|_{p^2=M_W^2} = 0\cdot \tag{2.73}$$

To summarize, we have eight renormalization condition for 7 renormalization constants: δZ_{ZZ}, δZ_{AZ}, δZ_{ZA}, δZ_{AA}, δM_Z^2, δZ_W and δM_W^2.

To complete the representation we give all the counter terms of the gauge sector

- The mass counter terms:

$$\begin{aligned}
\delta M_W^2 &= Re\widehat{\Sigma}_W^T(M_W^2) && (2.74)\\
\delta M_Z^2 &= Re\widehat{\Sigma}_{ZZ}^T(M_Z^2) && (2.75)
\end{aligned}$$

- The wave function counter terms:

$$\delta Z_{ZZ} = -Re\frac{\partial}{\partial p^2}\widehat{\Sigma}_{ZZ}^T(M_Z^2), \qquad \delta Z_W = -Re\frac{\partial}{\partial p^2}\widehat{\Sigma}_W^T(M_Z^2), \tag{2.76}$$

$$\delta Z_{ZA} = \frac{2}{M_Z^2}\widehat{\Sigma}_{AZ}^T(0), \qquad \delta Z_{AZ} = -\frac{2}{M_Z^2}Re\widehat{\Sigma}_{ZA}^T(M_Z^2), \tag{2.77}$$

$$\delta Z_{AA} = -\frac{\partial}{\partial p^2}\widehat{\Sigma}_{AA}^T(0)\cdot \tag{2.78}$$

We make here a general remark that all renormalization constants are obtained from self energies. The mass counter terms $\delta M_{W,Z}^2$ are related to the fundamental renormalization constants by [5]

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta M_W^2}{M_W^2} = \frac{S_W}{C_W}(3\delta Z_{ZA} - 2\delta Z_{AZ}). \tag{2.79}$$

It is also useful to renormalize the Weinberg angle. As part of on-shell renormalization scheme, it is natural to use its definition from masses of W and Z bosons:

$$\sin^2(\theta_W) = 1 - \frac{M_W^2}{M_Z^2}. \tag{2.80}$$

This definition is independent of a specific process, and, it is valid to all orders of perturbation theory. We then get the following counterterms [5]

$$\begin{aligned}
\delta S_W &= -\frac{1}{2}\frac{C_W^2}{S_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) && (2.81)\\
\frac{\delta C_W}{C_W} &= -\frac{S_W^2}{C_W^2}\frac{\delta S_W}{S_W} && (2.82)
\end{aligned}$$

2.4 Charge renormalization constant δZ_e

We can detrmine the charge renormalization constant through any charged particle vertex with the photon. Let us consider the electromagnetic vertex. At tree level the eeA vertex correspond s to the following amplitude [18], [34], [31]

$$\Gamma_{\mu}^{eeA} = -ieA_{\mu}. \tag{2.83}$$

The one loop correction and the counterterm of this vertex are represented in the above figure. This counter term modifies the relation (2.83) to give a full contribution at one loop level

$$\Gamma_{R\mu}^{eeA} = \Gamma_{\mu}^{eeA} + \delta\Gamma_{\mu}^{eeA} \tag{2.84}$$

where $\delta\Gamma_{\mu}^{eeA}$ contains electric charge counterterms and the renormalization constants of electron and photon wave functions. The amplitude then becomes

$$\Gamma_{R\mu}^{eeA} = \Gamma_{\mu}^{eeA}\left(1 + \frac{\delta Z_e}{e} + \frac{1}{2}\delta Z_{AA} - \frac{1}{2}\frac{S_W}{C_W}\delta Z_{ZA}\right). \tag{2.85}$$

This expression should not depend on the renormalization constants of the electron wave function. It is universaly valid for all charged fermions(with diffrent factor Q_f).
To obtain the charge renormalization constant, we impose a condition where the coupling represented in (2.83) will not change:

$$\Gamma_{R\mu}^{eeA} = -ieA_{\mu} \tag{2.86}$$

$$\Rightarrow \frac{\delta Z_e}{e} + \frac{1}{2}\delta Z_{AA} - \frac{1}{2}\frac{S_W}{C_W}\delta Z_{ZA} = 0 \tag{2.87}$$

leading to the counterm of the electric charge

$$\frac{\delta Z_e}{e} = -\frac{1}{2}\delta Z_{AA} + \frac{1}{2}\frac{S_W}{C_W}\delta Z_{ZA} \tag{2.88}$$

2.5 Renormalization of mass mixing matrix elements

As we mentioned in the first chapter, the mixing of particles is expressed in terms of mixing matrix. Because of the fact that mass eigenstates at the tree-level mix with each other by radiative corrections, the mixing matrices have to be renormalized to obtain ultraviolet (UV) finite amplitudes. In this section we review the one loop on-shell renormalization of the mixing matrices. As introduced in the first chapter, the relation between the fermion mass eigenstates and the interaction eigenstates is given by [5]

$$f_i = U_{i,L,R} f_i^I, \qquad f_i^I = U_{i,L,R}^{\dagger} f_i. \tag{2.89}$$

By radiative corrections, the wave functions of f_i should be renormalized. The relation between the on-shell renormalized fields f_i and unrenormalized fields f_i^0 is defined by (See (2.24)) [5]

$$f_i^0 = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}\right) f_j \tag{2.90}$$

The off-diagonal parts of δZ_{ij} $(i \neq j)$ represent the mixing between f_i an f_j. The relation (2.89) is modified as

$$f_i^I = U_{i,L,R}^{\dagger,0} f_i^0 = U_{i,L,R}^{\dagger,0}\left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}\right) f_j \tag{2.91}$$

The explicit form of δZ_{ij} for $i \neq j$ is given in terms of the fermionic fields (see (2.51)). The factor $1/(m_i^2 - m_j^2)$ in (2.51) is unique for the off-diagonal wave function corrections. δZ_{ij} diverge when the masses m_i, m_j of f_i and f_j , respectively, become close to each other.
For the cancellation of the UV divergences of off-diagonal δZ_{ij}, the mixing matrix U has to be renormalized. Assume that the renormalized U is related to the bare U_0 by

$$U_{i,L,R}^0 = (\delta_{ij} + \delta u_{ij})U_{j,L,R} \tag{2.92}$$

Since both U^0 and U are unitary, the counterterm δu should be anti-hermite [5]. The UV divergent part of δu is determined such as to cancel that of the anti-hermitian part of δZ. In the study of the radiative correction to the CKM matrix, Denner and Sack proposed to cancel the total anti-hermitian part of δZ_{ij} by δu, by choosing

$$\delta u_{ij} = \frac{1}{4}(\delta Z_{ij} - \delta Z_{ij}^\dagger). \tag{2.93}$$

This is usually called the on-shell renormalization of the mixing matrix.

2.5.1 Renormalization of the quark mixing matrix elements

The quark mixing matrix and the corresponding renormalization constant are defined as follows, (The renormalizaion of both of CKM AND PMNS matrices elements are found in ref [5]).

$$V_{0,ij} = (U_1 V U_2^\dagger)_{ij} = V_{ij} + \delta V_{ij}. \tag{2.94}$$

$V_{0,ij}$ and V_{ij} are are both unitary. The lowest order of V_{ij} is giving by [22]

$$V_{0,ij} = U_{i,k}^{u,L} U_{k,j}^{d,L\dagger} \tag{2.95}$$

We define, up to the first order, the renormalized quark mixing matrix as follow

$$\begin{aligned} V_{ij} &= (\delta_{ik} + \frac{1}{2}\delta Z^{u\dagger}_{ik}) U^{uL}_{km} U^{d,L\dagger}_{mn} (\delta_{nj} + \frac{1}{2}\delta Z^{d}_{nj}) \qquad (2.96) \\ &= V_{0,ij} + \frac{1}{2}\delta Z^{u\dagger}_{ik} V_{0,kj} + \frac{1}{2} V_{0,in} \delta Z^{d}_{nj}. \qquad (2.97) \end{aligned}$$

Using (2.94) we obtain:

$$\begin{aligned} \delta V_{ij} &= V_{0,ij} - V_{ij} \qquad (2.98) \\ &= \frac{1}{2}\delta Z^{u,AH\dagger}_{ik} V_{0,kj} + \frac{1}{2} V_{0,in} \delta Z^{d,AH}_{nj} \qquad (2.99) \end{aligned}$$

we have from the referance [5]:

$$\delta Z^{f,AH}_{ij} = \frac{1}{2}(\delta Z^{f,L}_{ij} - \delta Z^{f,L\dagger}_{ij}) \qquad (2.100)$$

$$\delta Z^{f,AH}_{ij} = \delta Z^{f,AH\dagger}_{ij} \qquad (2.101)$$

After substituting (2.100) in (2.96), we get the quark mixing matrix counter term

$$\delta V_{ij} = \frac{1}{4}\left[(\delta Z^{u,L}_{ik} - \delta Z^{u,L\dagger}_{ik}) V_{kj} - V_{ik}(\delta Z^{d,L}_{kj} - \delta Z^{d,L\dagger}_{kj})\right]. \qquad (2.102)$$

By inserting the renormalization constants (2.51) and (2.53) in (2.102) , we obtain the final form of δV_{ij} [5]

$$\begin{aligned} \delta V_{ij} \quad &= \tfrac{1}{2}\Big\{ \tfrac{1}{m^2_{u,i} - m^2_{u,k}} \Big[m^2_{u,k}\Sigma^{u,L}_{ik}(m^2_{u,k}) + m^2_{u,i}\Sigma^{u,L}_{ik}(m^2_{u,i}) + m_{u,i}m_{u,k}\left(\Sigma^{u,R}_{ik}(m^2_{u,k}) + \Sigma^{u,R}_{ik}(m^2_{u,i})\right) \\ &\qquad + (m^2_{u,k} + m^2_{u,i})\left(\Sigma^{u,S}_{ik}(m^2_{u,k}) + \Sigma^{u,S}_{ik}(m^2_{u,i})\right)\Big] V_{kj} \\ &\quad - V_{ik} \tfrac{1}{m^2_{d,k} - m^2_{d,j}} \Big[m^2_{d,j}\Sigma^{d,L}_{kj}(m^2_{d,j}) + m^2_{d,k}\Sigma^{d,L}_{kj}(m^2_{d,k}) + m_{d,k}m_{d,j}\left(\Sigma^{d,R}_{kj}(m^2_{d,j}) + \Sigma^{d,R}_{kj}(m^2_{d,k})\right) \\ &\qquad + (m^2_{d,k} + m^2_{d,j})\left(\Sigma^{d,S}_{kj}(m^2_{d,k}) + \Sigma^{d,S}_{kj}(m^2_{d,j})\right)\Big]\Big\}. \qquad (2.103) \end{aligned}$$

Chapter 3

Mass Mixing effect on Higgs Width at One Loop Order

3.1 Introduction

Now that we have discovered the Higgs boson, we have a chance to study its properties. In our opinion, the most interesting measurable quantities associated with the Higgs boson are partial widths to it various decay modes and its branching ratios. In the SM the Higgs boson width is very precisely predicted once the Higgs boson mass is known. A Higgs boson mass of about 125 Gev allows to explore the Higgs boson couplings to many SM particles.

In this chapter we will study the Higgs boson decay width of the following channels: $H \to b\bar{b}$, $H \to c\bar{c}$, $H \to \tau\bar{\tau}$, $H \to AA$, $H \to ZA$, $H \to gg$, to compare between them, then to calculate the branching ratios of each channel and we end our work by study the effect of the quark mixing matrix element δV_{ij} on the Higgs and the W boson decay channels.

3.2 Higgs decays channels in Standard Model

The Higgs boson can decay in several different ways depending on its mass. In general, the study of Higgs decay focuses on kinematics of two massive particles. Let m_1 and m_2 represent the considered masses, and λ_1 and λ_2 their polarizations. The process is as follow [28]

$$H(p,0) \to A(k_1,\lambda_1) + B(k_2,\lambda_2) \tag{3.1}$$

According to the conservation momentum-energy law, we have in the reference frame of the higgs boson

$$\overrightarrow{0} = \overrightarrow{k}_1 + \overrightarrow{k}_2 \tag{3.2}$$
$$M_H = E_1 + E_2 = \sqrt{m_1^2 + |\overrightarrow{k}_1|^2} + \sqrt{m_2^2 + |\overrightarrow{k}_2|^2} \tag{3.3}$$

It is easy to deduce from these equations

$$E_1 = \frac{M_H^2 + m_1^2 - m_2^2}{2M_H} \tag{3.4}$$
$$E_2 = \frac{M_H^2 + m_2^2 - m_1^2}{2M_H} \tag{3.5}$$
$$|\overrightarrow{k}_1| = |\overrightarrow{k}_2| = \frac{1}{2M_H}\sqrt{M_H^4 + m_1^4 + m_2^4 - 2m_1^2m_2^2 - 2M_H^2(m_1^2 + m_2^2)}. \tag{3.6}$$

In the case $m_1 = m_2 = m$ the last equations reduce to

$$E_1 = E_2 = \frac{M_H}{2} \tag{3.7}$$
$$|\overrightarrow{k}_1| = |\overrightarrow{k}_2| = \frac{M_H}{2}\sqrt{1 - \frac{4m^2}{M_H^2}} \tag{3.8}$$

3.2.1 The higgs boson decay width

First, we need to define the decay width denoted Γ, which reperesents the number of decays per unit of time. According to a given channel (i) of disintegration, the width Γ_i obeys to the golden rule of Fermi [30][1]

Golden Rule for Decays

Suppose particle 1 decays into several other particles $2, 3, \cdots, n$:

$$1 \to 2, 3, \cdots, n \tag{3.9}$$

The decay width is given by the formula:

$$d\Gamma_i(1 \to 2 + 3 + ... + n) = \left(\frac{1}{n!}\right)\frac{1}{2m_1}\left[\left(\frac{d^3p_2}{(2\pi)^3 2E_2}\right)\left(\frac{d^3p_3}{(2\pi)^3 2E_3}\right)...\left(\frac{d^3p_n}{(2\pi)^3 2E_n}\right)\right]|\mathcal{M}|^2 \times (2\pi)^4\delta^4(p_1 - p_2 - p_3 - \cdots - p_n) \tag{3.10}$$

[1] The decay width for a given process is determined by the amplitude and the phase space according to Fermis Golden Rule $\Gamma = \frac{2\pi}{\hbar}|\mathcal{M}|^2(phase\ space)$

where $p_i = (E_i, \overrightarrow{p}_i)$, is the four-momentum for the i-th particle ($i = 2, cdots, n$), and $\mathcal{M}$ is a scalar function which represents the transition amplitude from the initial state formed by the Higgs particle on a mass shell towards a final state formed by $n-1$ particles. The delta function enforces conservation of energy and momentum. It is zero unless $p_1 = p_2 + p_3 + \cdots + p_n$.

Since we are not interested in the individual momenta of the decay products, we integrate over all outgoing momenta to get the total decay width Γ. Further, we consider only the disintegration of the Higgs boson into just two particles. So we get after putting $p_1 = p, p_2 = k_1$ and $p_3 = k_2$:

$$\Gamma = \left(\frac{1}{6}\right)\frac{1}{2M_H}\int\left[\left(\frac{d^3k_1}{(2\pi)^3 2E)_1}\right)\left(\frac{d^3k_2}{(2\pi)^3 2E_2}\right)\right]|\mathcal{M}^2(k_1,k_2)| \times (2\pi)^4\delta^4(p-k_1-k_2) \quad (3.11)$$

In addition, it is possible to write the delta function in the form [28]

$$\begin{aligned}\delta^4(p-k_1-k_2) &= \delta(M_H - E_1 - E_2)\delta^3(-\overrightarrow{k}_1 - \overrightarrow{k}_2) \\ &= \delta\left(M_H - \sqrt{m_1^2 + |\overrightarrow{k}_1|^2} - \sqrt{m_2^2 + |\overrightarrow{k}_2|^2}\right)\delta^3(-\overrightarrow{k}_1 - \overrightarrow{k}_2)\end{aligned} \quad (3.12)$$

Using the properties of the delta function, we integrate the equation (3.11) according to $\overrightarrow{k}_2$

$$\Gamma_i = \frac{1}{6}\frac{1}{16\pi^2 M_H}\int|\mathcal{M}(|\overrightarrow{k}_1|)|^2\frac{\delta\left(M_H - \sqrt{m_1^2 + |\overrightarrow{k}_1|^2} - \sqrt{m_2^2 + |\overrightarrow{k}_1|^2}\right)}{\sqrt{m_1^2 + |\overrightarrow{k}_1|^2}\sqrt{m_2^2 + |\overrightarrow{k}_1|^2}}dk^3 \quad (3.13)$$

Then integrating the angular variables of $dk^3 = k^2 \sin\theta dk d\theta d\varphi$ in the last expression we get:

$$\Gamma = \left(\frac{1}{6}\right)\frac{1}{8\pi M_H}\int|\mathcal{M}(|\overrightarrow{k}_1|)|^2\frac{\delta\left(M_H - \sqrt{m_1^2 + |\overrightarrow{k}_1|^2} - \sqrt{m_2^2 + |\overrightarrow{k}_1|^2}\right)}{\sqrt{m_1^2 + |\overrightarrow{k}_1|^2}\sqrt{m_2^2 + |\overrightarrow{k}_1|^2}}|\overrightarrow{k}_1|^2 d|\overrightarrow{k}_1| \quad (3.14)$$

Then suppose that:

$$E = \sqrt{m_1^2 + |\overrightarrow{k}_1|^2} + \sqrt{m_2^2 + |\overrightarrow{k}_1|^2} \quad (3.15)$$

Its differential

$$dE = \frac{E|\overrightarrow{k}_1|dk_1}{\sqrt{m_1^2 + |\overrightarrow{k}_1|^2}\sqrt{m_2^2 + |\overrightarrow{k}_1|^2}} \quad (3.16)$$

This allows us to transform (3.14) into the form:

$$\Gamma_i = \frac{1}{3!}\frac{1}{8\pi M_H}\int\limits_{m_1+m_2}^{\infty} |\mathcal{M}(|\overrightarrow{k}|)|^2|\overrightarrow{k}|_{(E)}\delta(E-M_H)\frac{dE}{E}$$
$$= \frac{1}{3!}\frac{1}{8\pi M_H^2}|\mathcal{M}(|\overrightarrow{k}|_{E=M_H})|^2|\overrightarrow{k}|_{E=M_H} \tag{3.17}$$

we replace k by (3.6) and we get the Higgs decay width in two particles of different masses:

$$\Gamma_i^{\neq} = \frac{1}{6}\frac{1}{16\pi M_H^2}|\mathcal{M}(|\overrightarrow{k}_1|_{E=M_H})|^2\sqrt{1+\frac{m_1^4+m_2^4-2m_1^2m_2^2}{M_H^4}-\frac{2(m_1^2+m_2^2)}{M_H^2}} \tag{3.18}$$

And for the case $m_1 = m_2 = m$, we get the Higgs decay width in two particles of identical masses:

$$\Gamma_i^{=} = \frac{1}{6}\frac{1}{16\pi M_H^2}|\mathcal{M}(|\overrightarrow{k}_1|_{E=M_H})|^2\sqrt{1-\frac{4m^2}{M_H^2}} \tag{3.19}$$

3.2.2 Higgs decay into fermion antifermion pairs

We consider the decay process of the scalar Higgs boson in pairs: one fermion of mass m_f and spin s_1 and the other an anti-fermion of same mass m_f and spin s_2, These fermion anti-fermion pairs can be leptons : $(e_i, \bar{e}_i), (\nu_i, \bar{\nu}_i)$ or quarks: $(d_i, \bar{d}_i), (u_i, \bar{u}_i); i = 1, 2, 3$. The opening of these channels is governed by the condition: $2m_f = M_H$ [28].

Moreover, note that since the fermion and its anti-fermion have the same mass, we use the formula (3.19) to calculate the decay width $\Gamma(H \longrightarrow \bar{f}f)$. Now it suffices to calculate the expression of the square of the amplitude $|\mathcal{M}(|\overrightarrow{k}_1|_{E=M_H})|^2$ of the process $H \longrightarrow \bar{f}f$. To do all these calculation we use $FeynCalc$ and $FaynArt$ for getting the total decay width and drawing the Feynman diagram of each process.

Higgs decay into lepton antilepton pairs

We start by activating the $FeynCalc$ program inside Mathematica, then we get the Feynman diagram of the $H \longrightarrow l\bar{l}$ process by running:

```
LoadFeynArts = True;
<<FeynCalc`
He = CreateTopologies [0, 1 -> 2 ]
Hef = InsertFields[CreateTopologies[0, 1 -> 2], {S[1]} -> {F[2, {1}], -F
    [2, {1}]},
InsertionLevel -> {Classes}];
Paint[Hef]
```

Here, by creating a 1 by 2 topology and inserting physical fields, we get:

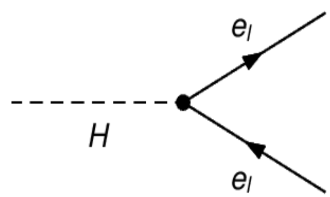


Figure 3.1: Tree level of the process Hee

Now we generate the Feynman amplitude of the diagram:

```
Heamp = FCFAConvert[CreateFeynAmp[Hef], IncomingMomenta -> {pH},
ChangeDimension -> 4,DropSumOver -> True, SMP -> True, Contract -> True,
    UndoChiralSplittings -> True,FinalSubstitutions -> {MLE[1] -> SMP["m_e"
    ]}]
Heeamp = DiracSimplify[Heamp]
```

After fixing the kinematics:

```
SP[p1, p2] = (SMP["m_H"]^2 - 2*SMP["m_e"]^2)/2;
SP[p1, p1] = SMP["m_e"]^2;
SP[p2, p2] = SMP["m_e"]^2;
SP[pH, pH] = SMP["m_H"]^2;
```

we obtain:

$$|\mathcal{M}|^2 = \frac{e^2(m_e^2 M_H^2 - 4m_e^4)}{2m_W^2(\sin(\theta_W))^2} \tag{3.20}$$

To obtain the total decay rate we run the following instructions

```
$Assumptions = {SMP["m_H"] > 0, SMP["m_e"] > 0};
phaseSpacePrefactor[m_] := (1/(16*Pi*SMP["m_H"]))*Sqrt[1 - 4*(m^2/SMP["
  m_H"]^2)];
totalDecayRateHe = Simplify[(#1 /. SMP["e"]^2 -> 4*Pi*SMP["alpha_fs"] &)
  [phaseSpacePrefactor[SMP["m_e"]]*HeeampSq]]
```

which have the final output

$$\Gamma_{H\longrightarrow e\bar{e}} = \frac{\alpha\sqrt{m_H^2 - 4m_e^2}(m_e^2 m_H^2 - 4m_e^4)}{8M_H^2 m_W^2(\sin(\theta_W))^2}, \tag{3.21}$$

where $\alpha = 1/137.035999139(31)$ is the fine structure constant. Similar pattern is followed to get the remain of $H\bar{l}l$ decay width:

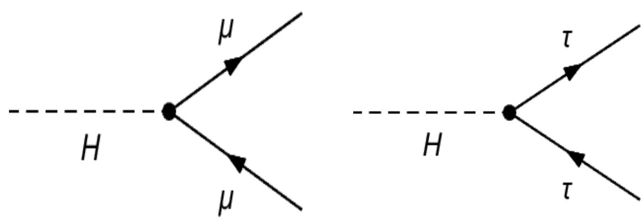


Figure 3.2: Tree level of the process $H\mu\mu$ et $H\tau\tau$

Higgs decay into quark antiquark pairs

We follow the same previous steps as in the case of leptons

```
LoadFeynArts = True;
<<FeynCalc`
Hu = CreateTopologies [0, 1 -> 2 ]
Huf = InsertFields[CreateTopologies[0, 1 -> 2], {S[1]} -> {F[3, {1}], -F
  [3, {1}]},
InsertionLevel -> {Classes}];
Paint[Huf]
```

The $H\bar{u}u$ squared amplitude is given by

$$|\mathcal{M}|^2 = \frac{e^2 C_A m_u^2(M_H^2 - 4m_e^2)}{2m_W^2(\sin(\theta))^2} \tag{3.22}$$

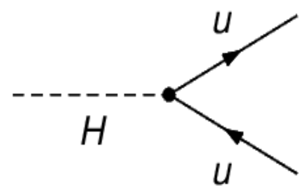


Figure 3.3: Tree level of the process $Hu\bar{u}$

and the associated total decay width is then

$$\Gamma_{H\longrightarrow u\bar{u}} = \frac{\alpha C_A m_u^2 (m_H^2 - 4m_u^2)^{\frac{3}{2}}}{8M_H^2 m_W^2 (\sin(\theta_W))^2} \tag{3.23}$$

3.2.3 Higgs decay into pairs of massive vector bosons:

We consider the decay process of the scalar Higgs boson in two different massive vector bosons pairs:

$$H(p,0) \to W^+(k_1,s_1)W^-(k_2,s_2) \tag{3.24}$$

$$H(p,0) \to Z(k_1,s_1)Z(k_2,s_2) \tag{3.25}$$

The code sequences to these processes is

```
1 LoadFeynArts = True;
2 << FeynCalc`
HZf = InsertFields[
    CreateTopologies[0, 1 -> 2], {S[1]} -> {V[2], V[2]},
    InsertionLevel -> {Classes}];
Hwf = InsertFields[
    CreateTopologies[0, 1 -> 2], {S[1]} -> {-V[3], V[3]},
    InsertionLevel -> {Classes}];
Paint[HWf]
Paint[HZf]
```

and the associated squared amplitudes, after fixing the kinematics, are

$$|\mathcal{M}|^2_{HZZ} = \frac{e^2 m_W^2(-4m_H^2 m_Z^2 + m_H^4 + 12m_Z^4)}{8m_Z^4(\cos(\theta_W))^4(\sin(\theta_W))^2} \tag{3.26}$$

$$|\mathcal{M}|^2_{HWW} = \frac{e^2 m_W^2(-4m_H^2 m_W^2 + m_H^4 + 12m_W^4)}{4m_W^2(\sin(\theta_W))^2} \tag{3.27}$$

Then from the last expressions we get the total decay rates:

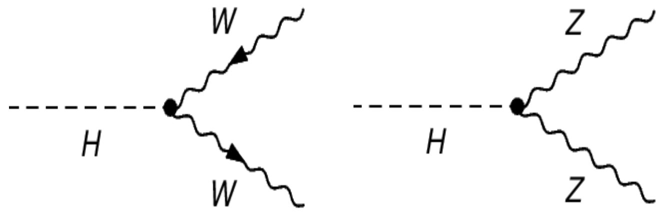


Figure 3.4: Tree level of the process HW^-W^+ et HZZ

$$\Gamma_{H\longrightarrow WW} = \frac{\alpha\sqrt{m_H^2 - 4m_W^2}(-4m_H^2 m_W^2 + m_H^4 + 12m_W^4)}{16M_H^2 m_W^2(\sin(\theta_W))^2} \tag{3.28}$$

$$\Gamma_{H\longrightarrow ZZ} = \frac{\alpha\sqrt{m_H^2 - 4m_Z^2}(-4m_H^2 m_Z^2 + m_H^4 + 12m_Z^4)}{32M_H^2 m_W^2(\sin(\theta_W))^2} \tag{3.29}$$

The processes $H \longrightarrow gg$, $t \longrightarrow AA$, and $H \longrightarrow ZA$ are not possible at tree level and hence their amplitude couldnt be generated by $FeynCalc$. However in the next section it will be possible to achieve these goals with $FormCalc$, which is more efficient for one loop level computation.

3.3 Computing Higgs width through FormCalc

We want to compute the Higgs width at one loop level through FormCalc which had reached big steps towards automating the one loop calculation. It is a package which calculates and simplifies tree level and one loop level Feynman diagrams amplitudes [18]. Here, we use FormCalc, FeynArts and Looptools [14], [13] together to calculate the squared matrix element for a given process (the Higgs decay process at $M_H = 125$ GeV. We start with creating a Mathematica file which contains all codes that must be proceeded by FormCalc to get the total decay width of all the Higgs decay processes. We were inspired to write this Mathematica file from Hahn, where he studied the Higgs decay into bottom antibottom quarks.. At first time we set the Higgs decay width at one value $M_H = 125$ GeV. Then after we change the Higgs mass from 0 Gev to 250 GeV. Starting by analysing the fermion anti-fermion channels,we choose for that the processes $H \longrightarrow b\bar{b}$, $H \longrightarrow c\bar{c}$ and $H \longrightarrow \tau\bar{\tau}$ channels. Afterwards, we also need to analyse massless final suitable states like gluons and photons.

3.3.1 The $H \longrightarrow b\bar{b}$, $H \longrightarrow c\bar{c}$, $H \longrightarrow \tau\bar{\tau}$ channels.

$H \longrightarrow b\bar{b}$ is the isotropic scalar two body decay which represents the dominant Higgs decay mode. We generate the following codes by FormCalc

```
(*
        Hbb-SM.m
                generates the Fortran code for
                H -> \bar b b in the electroweak Standard Model
                this file is part of FormCalc
                last modified March 2021

Note: the QED contributions are not taken into account here.
To plug the QED part back in, remove the ExcludeParticles -> V[1]
from the InsertFields options below.
*)
Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

process = S[1] -> {-F[4, {3}], F[4, {3}]}

name ="/home/zghiche/formcalc/Hbb-SM."

(* 18 May 06: careful, not UV finite if photon is taken out! *)

SetOptions[InsertFields, Model -> "SM"]

SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

 take the comments out if you want the diagrams painted

DoPaint[diags_, file_] := (
  If[ FileType["diagrams"] =!= Directory,
    CreateDirectory["diagrams"] ];
  Paint[diags,
    DisplayFunction -> (Display["diagrams/" <> file <> ".ps", #]&)]

Print["Born"]

tops = CreateTopologies[0, 1 -> 2];
ins = InsertFields[tops, process];
DoPaint[ins, "born"];
born = CalcFeynAmp[CreateFeynAmp[ins]]
```

```
Print["Counter terms"]

tops = CreateCTTopologies[1, 1 -> 2,
  ExcludeTopologies -> {TadpoleCTs, WFCorrectionCTs}];
ins = InsertFields[tops, process];
DoPaint[ins, "counter"];
counter = CreateFeynAmp[ins]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];
vert = CalcFeynAmp[CreateFeynAmp[ins], counter]

amps = {born, vert}

{born, vert} = Abbreviate[amps, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

col = ColourME[All, born]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[born, vert, col, abbr, subexpr, dir]

WriteRenConst[amps, dir]

Print["time used: ", SessionTime[] - time1]
```

After generating this Fortran codes we obtain the drawings of all possible Feynman diagrams for this process (see Annex E) and the numerical values of the $H \longrightarrow b\bar{b}$ decay width. Next, we consider similarly the processes $H \longrightarrow c\bar{c}$ and $H \longrightarrow \tau\bar{\tau}$ by creating their Mathematica file (see Annex B).

3.3.2 The $H \longrightarrow gg$, $H \longrightarrow AA$, $H \longrightarrow ZA$ channels.

The Higgs boson does not couple directly to the photon and gluons, but the decay occurs indirectly. The Higgs boson can emit massive particles and absorb them immediately. These virtual

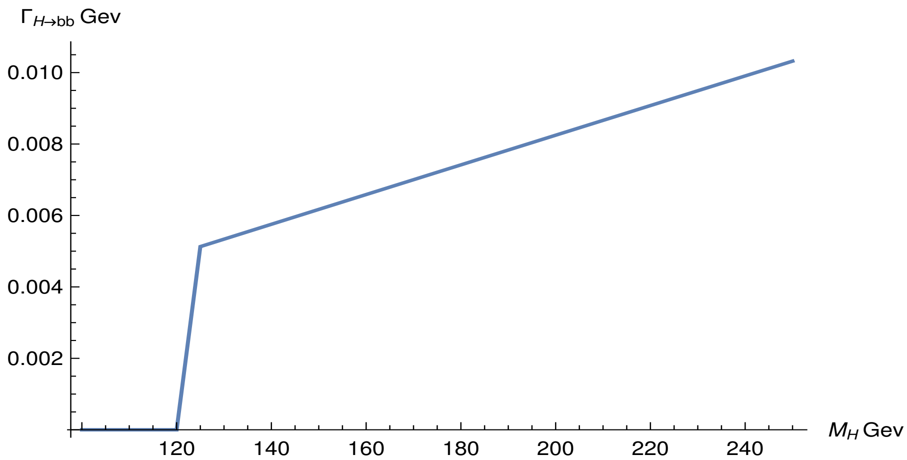


Figure 3.5: The $H \longrightarrow b\bar{b}$ total decay width

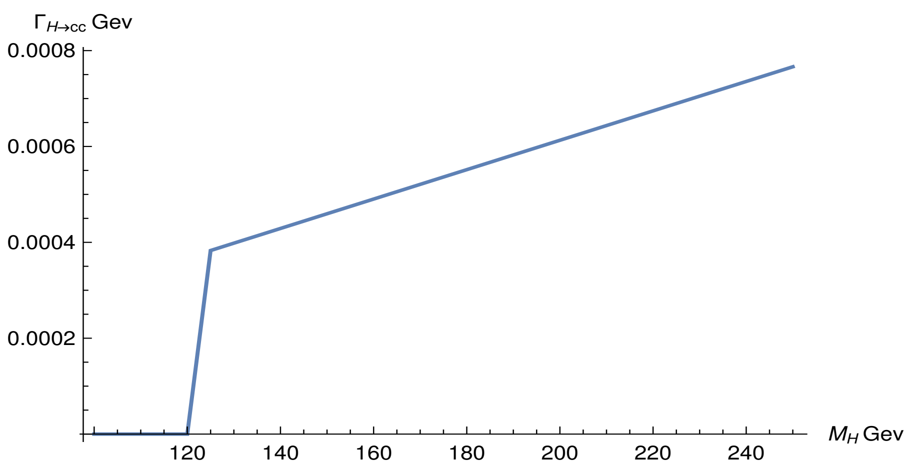


Figure 3.6: The $H \longrightarrow c\bar{c}$ total decay width

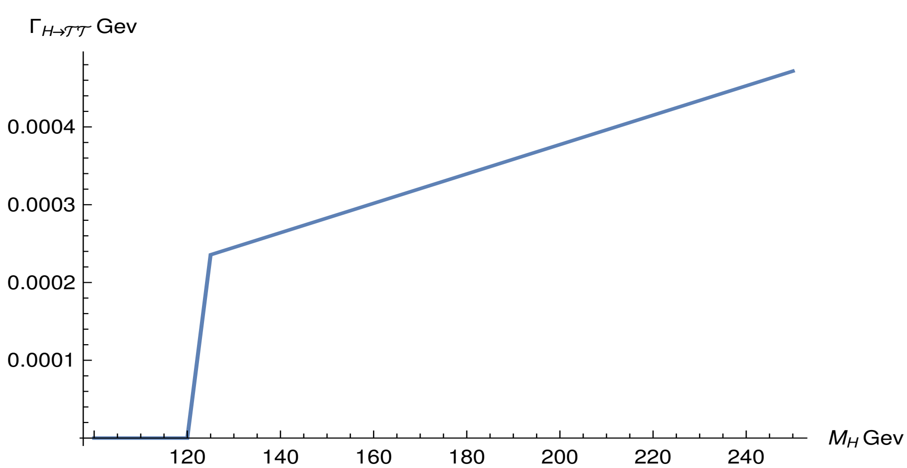


Figure 3.7: The $H \longrightarrow \tau\bar{\tau}$ total decay width

particles can emit photons and/or gluons. The two photon mode is rare, as consequence of the weak intensity of the electromagnetic interaction compared to the strong interaction. Because of the absence of the tree level here, the lowest order amplitude is generated by one loop fermionic and bosonic diagrams. The decay of the neutral Higgs boson to two photons and a photon plus Z boson are mediated by W and heavy fermion loops.

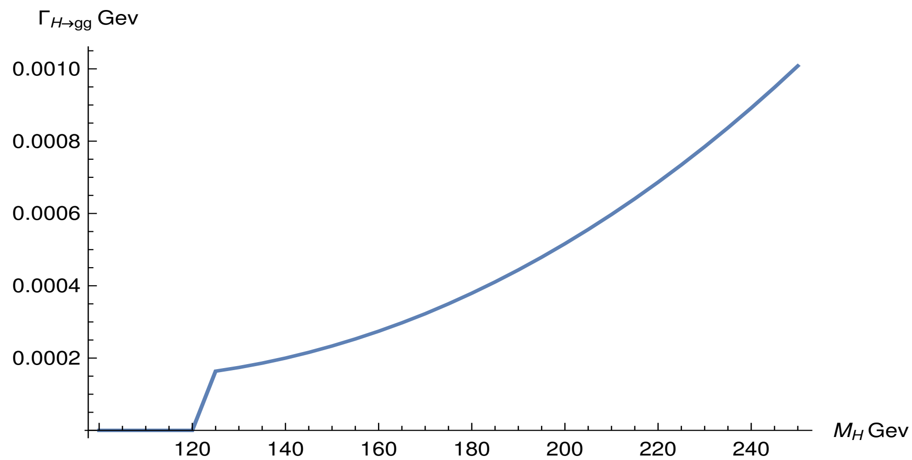


Figure 3.8: The $H \longrightarrow gg$ total decay width

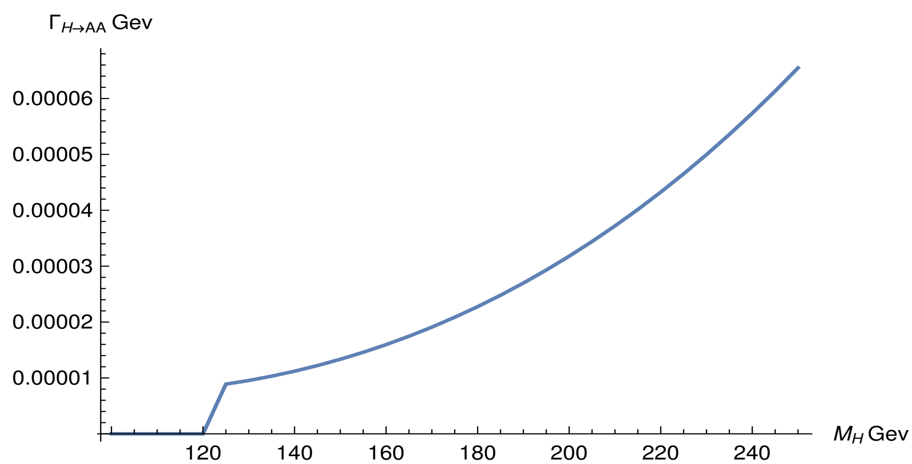


Figure 3.9: The $H \longrightarrow AA$ total decay width

All graphics: Fig 3.5, Fig 3.6, Fig 3.7, Fig 3.8, Fig 3.9, Fig 3.10 are plotted for the fixed values: $M_H = 125$ GeV, $M_b = 4.18$ GeV, $M_C = 1.275$ GeV ,$M_\tau = 1.77682$ GeV, $M_Z = 91.1876$ GeV, $M_{W^\pm} = 80.385$ GeV, $C_W = M_W/M_Z$, $S_W^2 = (1 - C_W)(1 + C_W)$, $\alpha = 1/137.035999074$, and the the CKM parameters in Wolfenstein parameterization are: $\lambda = 0.2257^{+0.0009}_{-0.0010}$, $A = 0.814^{+0.021}_{-0.022}$, $\rho = 0.135^{+0.031}_{-0.016}$, and $\eta = 0.349^{+0.015}_{-0.017}$.

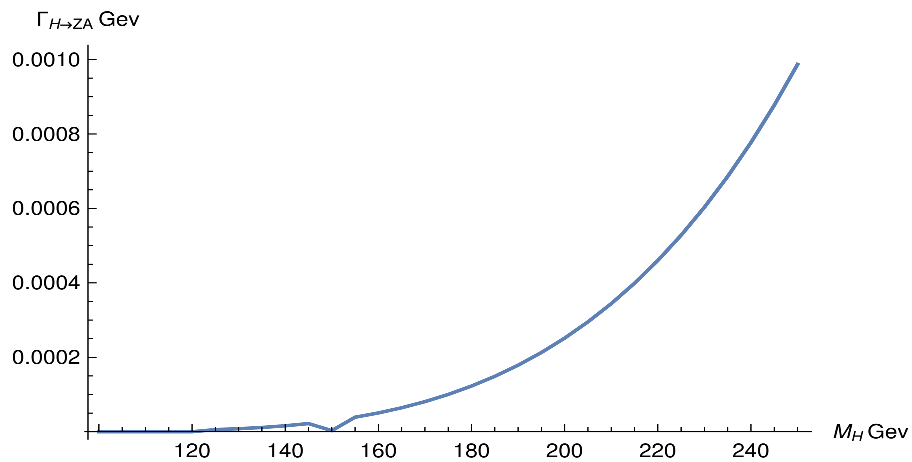


Figure 3.10: The $H \longrightarrow ZA$ total decay width

For an accurate comparison of all the previous curves, we have collected them in one plot as shown below

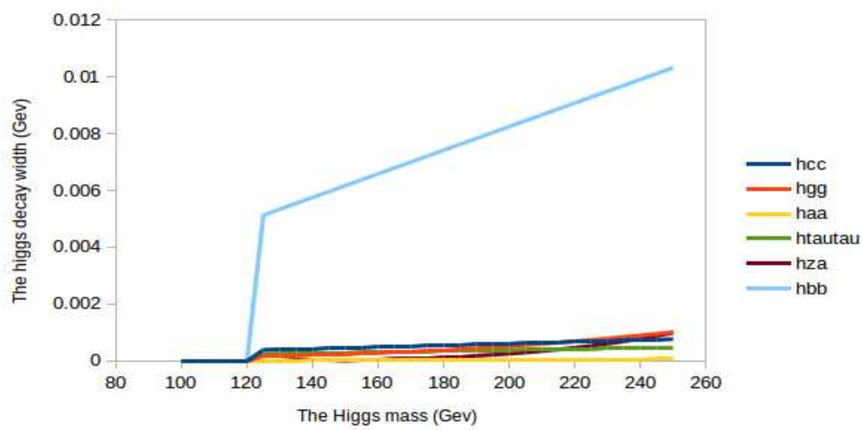


Figure 3.11: The Higgs decay width

We find that at low Higgs mass (of the order of 100 GeV), the Higgs boson preferably disintegrates into a pair of quark b. It is the dominant Higgs decay mode, thus we are going to compare the remaining decay widths without the $H \longrightarrow b\bar{b}$.

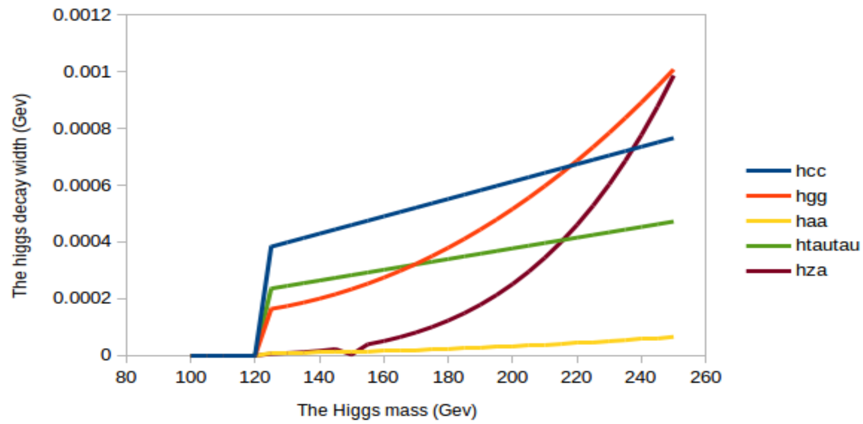


Figure 3.12: The Higgs decay width

In the low mass range ($\Gamma_H < 100$ GeV) the Higgs boson is very narrow, but the width becomes rapidly wider for masses larger than 125 GeV. The main decay mode in the range until 250 GeV is $H \longrightarrow b\bar{b}$ followed by the decay into $c\bar{c}$ and $\tau\bar{\tau}$.

Regarding the $H \longrightarrow AA$, it is a very interesting channel for a low mass of the Higgs boson. We cav argue that $H \longrightarrow AA$ and $H \longrightarrow ZA$ final states are very small compared to all athers.

For large masses ($M_H > 500$ GeV), the Higgs decay width becomes comparable to its mass, the main modes in this range is the decay into the messive gauge bosons $H \longrightarrow ZZ$, $H \longrightarrow WW$ and than the top quark $H \longrightarrow t\bar{t}$.

We conclude that the total decay width of the higgs boson, the inverse of its lifetime, is only a few MeV for a mass close to 100 GeV, but increases considerably to reach values of the order of the Higgs mass. In other spoken words the Higgs is a narrow resonance for low mass and becomes a very wide resonance for very heavy Higgs.

Since we have six decay Higgs channels, we can obtain the Higgs total decay width form through the following relation:

$$\Gamma_T = \sum_{\mathrm{i}} \Gamma_i \tag{3.30}$$

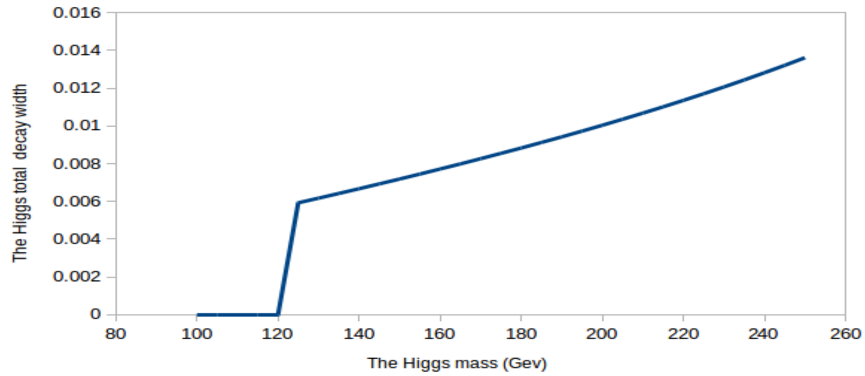


Figure 3.13: The Higgs total decay width

Now we are going to fixe the Higgs mass at 125 Gev and calculate the Higgs decay width after a numerical integration over $\cos\theta$:

	Tree-level	One loop level
$\Gamma(H \to b\bar{b})$	5.1437×10^{-3}	5.1281×10^{-3}
$\Gamma(H \to c\bar{c})$	3.8584×10^{-4}	3.8294×10^{-4}
$\Gamma(H \to \tau\bar{\tau})$	2.4963×10^{-4}	8.3564×10^{-4}
$\Gamma(H \to AA)$	0	5.7594×10^{-6}
$\Gamma(H \to GG)$	0	1.6379×10^{-4}
$\Gamma(H \to ZA)$	0	5.7594×10^{-6}

Table 3.1: The Higgs decay width for $M_H = 125$ GeV

3.4 The branching ratios of the Higgs decay

The branching ratios have been derived from all the partial widths that we calculate before [28]

$$BR_i = \frac{\Gamma_i}{\sum_{\mathrm{i}} \Gamma_i} = \frac{\Gamma_i}{\Gamma_T} \tag{3.31}$$

The b quark is the most important branching ratio followed by the charmed quark c and lepton τ. The gluon comes after and finally the decay into two photons and photon plus Z boson follows.

	Branching ratios
$H \to bb$	8.65483×10^{-1}
$H \to c\bar{c}$	6.4369×10^{-2}
$H \to \tau\bar{\tau}$	3.97707×10^{-2}
$H \to AA$	1.49976×10^{-3}
$H \to gg$	2.76439×10^{-2}
$H \to ZA$	0.972039×10^{-3}

Table 3.2: The Higgs decay branching ratios for $M_H = 125$ GeV

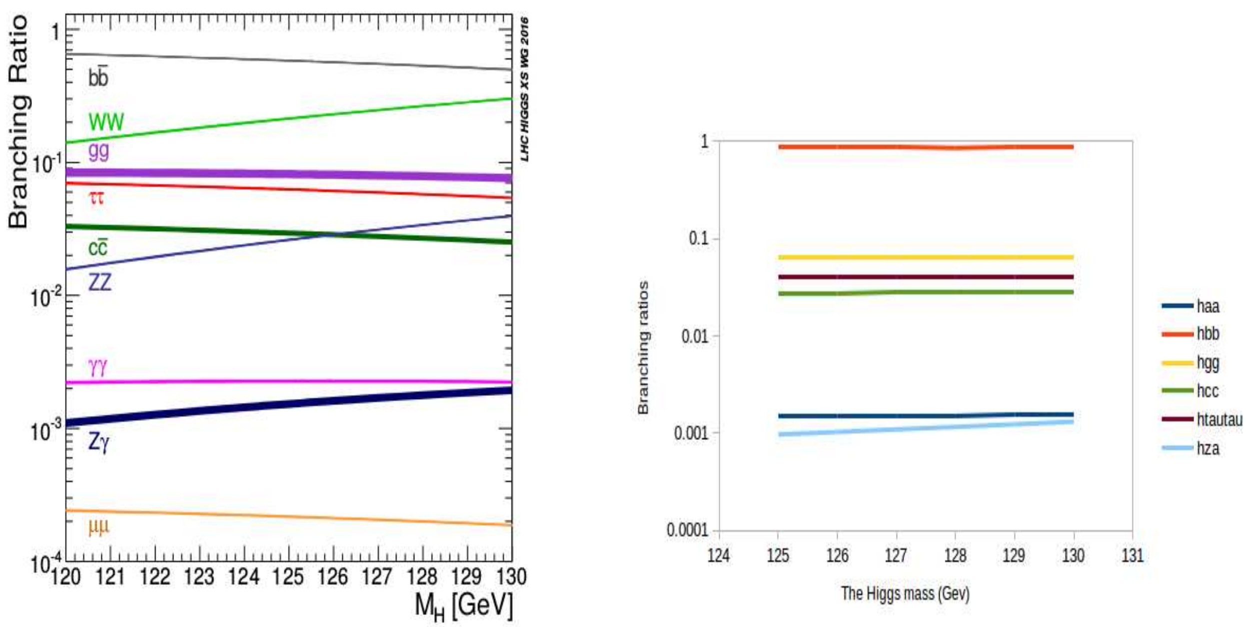


Figure 3.14: Plot of the branching ratios of the higgs decay channels

At $(BR) \sim 10^{-3}$ the processes existed are $H \to AA$, $H \to ZA$. Then we find the processes $H \to c\bar{c}$, $H \to \tau\bar{\tau}$, $H \to gg$ at $10^{-1} < (BR) < 10^{-2}$. The dominate Higgs decay channel is the $H \to b\bar{b}$ process near $(BR) = 1$.

3.5 Effects of mass mixing on Higgs width

In the theory of the SM the only vertices ,containing the mass mixing, are $\phi^{\pm} q\bar{q}$ and $Wq\bar{q}$. The one loop amplitude calculation for the process $Wq\bar{q}$ is in ref [33]. The $\delta V_{q\bar{q}}$ counterterm is necessary to obtain a finit amplitude $\mathcal{M}_{Wq\bar{q}}$. To know the relative effect of the mass mixing on Higgs width, we choose the process of the vertex $Wc\bar{s}$ which has confirmed effect of the mass mixing on its physical states [5](Annexe A equation 16), and comparing the results obtained by FormCalc between the W boson decay and the Higgs decay. We set the W boson mass M_W in the range 70 GeV to 150 GeV and then activate the mixing effect CKM option in the Mathematica file :

```
CKM = IndexDelta -> $CKM = True
```

This make effective the mass mixing in the one loop decay width calculation. We observe that the mass mixing effect appears directly after the W boson mass reaches the 80 GeV (All the partial widths (in GeV) of the W -boson decay channels at the one-loop level are found in ref [9]).

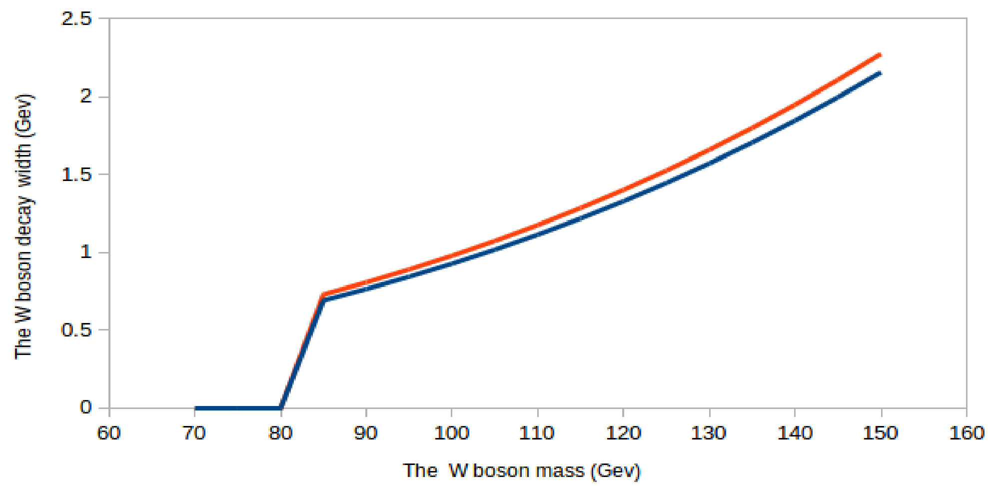


Figure 3.15: The W boson total decay width; the red line is the decay width the mixing effect

What we did next was the same mixing activation for all the Higgs decay channels treated previously. But what we obtained is the same results that we got in the previous section, namely without noticeable change. According to the equation (A.16) in the ref [5], the mass mixing has theoretically no effect on the Higgs fermion interaction. On the other hand, according to the same equation, the fermion-Goldstone interaction has this effect and then needs the counterterm $\delta V_{\phi q\bar{q}}$ for the one loop calculation which is not the case of the Higgs fermion interaction.

Conclusion

In this Master thesis we have been interested in studying the fermion mass mixing and the the Higgs boson decay at one loop level.

We have first began reviewing the Standard Model of particles with all its contents (Lagrangian, particles, intearction and kinetic terms) and through it we have tried to understand the work of the Higgs mechanism in the spontaneous breaking of The electroweak symmetry and how the particles acquires their mass.

Then we have proceeded to study the renormalization at one loop level in chapter 02, where we were able to find the ultraviolet divergences, and then we have got rid of them by adding counter terms. The main objective of all this was to study the various decay channals of the Higgs boson, our study was performed through the FormCalc program that enabled us to calculate the higgs decay width. We have deeply understand how the program works and we were able to enter all the necessary information into it, whether for decaying into two or more particles. then, we translated all the obtained results into graphs to make them easy to read. Finally, we compare the effect of the fermion mass mixing on the decay of the Higgs boson and on thedecay of the W boson.

Appendix A

Kinematics of two and three body decays

A.1 two body decays

We consider the decay of particle “a” into two particles 1 and 2 in the rest frame of “a”. We symbolize this by $a \longrightarrow 1+2$, and we summarize our notation as follow

Particle	a	1	2
Mass	M	m_1	m_2
Energy	$E = M$	E_1	E_2
Momentum	$p = 0$	$p_1 = \sqrt{E_1^2 - m_1^2}$	$p_2 = \sqrt{E_2^2 - m_2^2}$

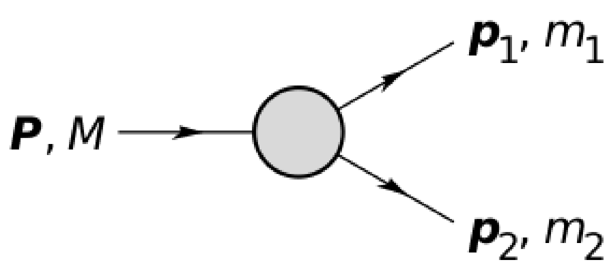


Figure A.1: Definitions of variables for two-body decays.

- Conservation of momentum and energy:

$$p = 0 = p_1 + p_2 \Rightarrow p_1 = -p_2 \tag{A.1}$$

$$M = E_1 + E_2 \tag{A.2}$$

We have from

$$E^2 = p^2 + m^2, \qquad E_1^2 - m_1^2 = E_2^2 - m_2^2 \tag{A.3}$$

Which when Solved for E_1 gives

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}. \tag{A.4}$$

The kinetic energy of particles 1 is then

$$T_1 = E_1 - m_1 C^2. \tag{A.5}$$

We have an expression of the momuntum in the final state

$$p = \mid p_1 \mid = \mid p_2 \mid = \frac{1}{2M_a}\sqrt{[M_a^2 - (m_1+m_2)^2]\,[M_a^2 - (m_1-m_2)^2]}, \tag{A.6}$$

The velocity of the particle is defined by

$$v_i = P_i/E_i \tag{A.7}$$

A.2 Three body decays

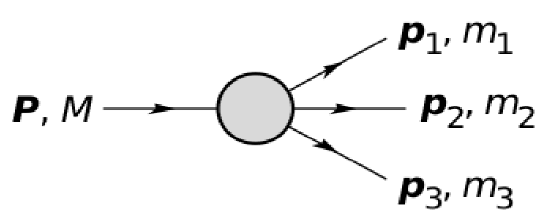


Figure A.2: Definitions of variables for three-body decays.

We define here the quantities $p_{ij} = p_i + p_j, \qquad m_{ij}^2 = p_{ij}^2$, then

$$m_{12}^2 + m_{23}^2 + m_{13}^2 = M^2 + m_1^2 + m_2^2 + m_3^2, \tag{A.8}$$

and

$$m_{12}^2 = (P - p_3)^2 = M^2 + m_3^2 - 2ME_3, \tag{A.9}$$

where E_3 is the energy of particle 3 in the rest frame of M.

The three energies of the three momentum are known, so their relative orientation is fixed. The momenta can be specified in space by givin three Euler angles (α, β, γ) that specify the orientation of the final system relative to the initial particle

$$d\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16M} \mid \mathcal{M} \mid^2 dE_1 dE_1 d\alpha d(\cos\beta) d\gamma. \tag{A.10}$$

Alternatively

$$d\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16M} \mid \mathcal{M} \mid^2 \mid p_1^* \mid p_3 \mid dm_{12} d\Omega_1^* d\Omega_2, \tag{A.11}$$

where $(\mid p_1^* \mid, \Omega_1^*)$ is the momentum of particle 1 in the resy frame of 1 and 2, and Ω_3 is the angle of particle 3 in the rest frame of the decaying particle. $\mid p_1^* \mid$ and $\mid p_3 \mid$ are given by

$$\begin{aligned} \mid p_1^* \mid &= \frac{1}{2m_{12}}\sqrt{[m_{12}^2 - (m_1+m_2)^2(m_{12}^2 - (m_1-m_2)^2]}, && \text{(A.12)} \\ \mid p_3 \mid &= \frac{1}{2M}\sqrt{[M^2 - (m_{12}+m_3)^2(M^2 - (m_{12}-m_3)^2]} && \text{(A.13)} \end{aligned}$$

which have the same form with (A.6).

In a three-body decay the maximum of $\mid p_3 \mid$ (A.14), is acheived when $m_{12} = m_1 + m_2$ i.e particle 1 and 2 have the same vector velocity in the rest frame of the decaying particle. If in addition $m_3 > m_1$ or m_2, then $\mid p_3 \mid_{max} > \mid p_1 \mid_{max}$ or $\mid p_2 \mid_{max}$.

Appendix B

Fortran codes generated by FormCalc

B.1 FormCalc codes for the process $H \longrightarrow c\bar{c}$

```
(*
        Hcc-SM.m
                generates the Fortran code for
                H -> \bar c c in the electroweak Standard Model
                this file is part of FormCalc
                last modified March 2021

Note: the QED contributions are not taken into account here.
To plug the QED part back in, remove the ExcludeParticles -> V[1]
from the InsertFields options below.
*)
Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

process = {S[1]} -> {F[3, {2}], -F[3, {2}]}

name ="/home/zghiche/formcalc/Hcc-SM.m."

(* 18 May 06: careful, not UV finite if photon is taken out! *)

SetOptions[InsertFields, Model -> "SM"]
```

```
SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

take the comments out if you want the diagrams painted
$PaintSE = MkDir[name <> ".diagrams"];
DoPaint[diags_, file_, opt___] := Paint[diags, opt,
  DisplayFunction -> (Export[ToFileName[$PaintSE, file <> ".ps"], #]&)]

Print["Born"]

tops = CreateTopologies[0, 1 -> 2];
ins = InsertFields[tops, process];
DoPaint[ins, "born"];
born = CalcFeynAmp[CreateFeynAmp[ins]]

Print["Counter terms"]

tops = CreateCTTopologies[1, 1 -> 2,
  ExcludeTopologies -> {TadpoleCTs, WFCorrectionCTs}];
ins = InsertFields[tops, process];
DoPaint[ins, "counter"];
counter = CreateFeynAmp[ins]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];
(* vert = CalcFeynAmp[
  CreateFeynAmp[ins],
  Select[counter, DiagramType[#] == 1 &]] *)

vert = CalcFeynAmp[CreateFeynAmp[ins], counter]

amps = {born, vert}

{born, vert} = Abbreviate[amps, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

col = ColourME[All, born]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]
```

```
dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[born, vert, col, abbr, subexpr, dir]

WriteRenConst[amps, dir]

Print["time used: ", SessionTime[] - time1]
```

B.2 FormCalc codes for the process $H \longrightarrow \tau\bar{\tau}$

```
(*
        Htautau-SM.m
                generates the Fortran code for
                H -> tau-bar tau in the electroweak SM
                this file is part of FormCalc
                last modified March 2021

*)

Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

process = {S[1]} -> {F[2, {3}], -F[2, {3}]}

name =" /home/zghiche/formcalc/Htautau-SM4"

SetOptions[InsertFields, Model -> "SM"]

SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

 take the comments out if you want the diagrams painted
$PaintSE = MkDir[name <> ".diagrams"];
DoPaint[diags_, file_, opt___] := Paint[diags, opt,
  DisplayFunction -> (Export[ToFileName[$PaintSE, file <> ".ps"], #]&)]
```

```
Print["Born"]

tops = CreateTopologies[0, 1 -> 2];
ins = InsertFields[tops, process];
DoPaint[ins, "born"];
born = CalcFeynAmp[CreateFeynAmp[ins]]

Print["Counter terms"]

tops = CreateCTTopologies[1, 1 -> 2,
  ExcludeTopologies -> {TadpoleCTs, WFCorrectionCTs}];
ins = InsertFields[tops, process];
DoPaint[ins, "counter"];
counter = CreateFeynAmp[ins]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];
(* vert = CalcFeynAmp[
  CreateFeynAmp[ins],
  Select[counter, DiagramType[#] == 1 &]] *)

vert = CalcFeynAmp[CreateFeynAmp[ins], counter]

amps = {born, vert}

{born, vert} = Abbreviate[amps, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[born, vert, abbr, subexpr, dir]

WriteRenConst[amps, dir]

Print["time used: ", SessionTime[] - time1]
```

B.3 FormCalc codes for the process $H \longrightarrow gg$

```
(*
        HGG-SMQCD.m
                generates the Fortran code for
                H -> G1 G1 in the electroweak SM
                this file is part of FormCalc
                last modified March 2021
*)

Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

process = {S[1]} -> {V[5], V[5]}
name =”/home/zghiche/formcalc/HGG-SMQCD1”

SetOptions[InsertFields, Model -> "SMQCD"]

SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

 take the comments out if you want the diagrams painted
$PaintSE = MkDir[name <> ".diagrams"];
DoPaint[diags_, file_, opt___] := Paint[diags, opt,
  DisplayFunction -> (Export[ToFileName[$PaintSE, file <> ".ps"], #]&)]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];

vert = CalcFeynAmp[CreateFeynAmp[ins]]

vert = Abbreviate[vert, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

col = ColourME[All]
```

```
abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[{}, vert, col, abbr, subexpr, dir]

WriteRenConst[{}, dir]

Print["time used: ", SessionTime[] - time1]
```

B.4 FormCalc codes for the process $H \longrightarrow AA$

```
(*
        HAA-SM.m
                generates the Fortran code for
                H -> gamma gamma in the electroweak SM
                this file is part of FormCalc
                last modified March 2021
*)

Needs["FeynArts`"]

Needs["FormCalc`"]

time1 = SessionTime[]

CKM = IndexDelta

process = {S[1]} -> {V[1], V[1]}
name ="/home/zghiche/formcalc/HAA-SM5"

SetOptions[InsertFields, Model -> "SM"]

 SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]
```

```
 take the comments out if you want the diagrams painted
$PaintSE = MkDir[name <> ".diagrams"];
DoPaint[diags_, file_, opt___] := Paint[diags, opt,
  DisplayFunction -> (Export[ToFileName[$PaintSE, file <> ".ps"], #]&)]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];

vert = CalcFeynAmp[CreateFeynAmp[ins]]

vert = Abbreviate[vert, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

 WriteSquaredME[{}, vert, abbr, subexpr, dir]

WriteRenConst[{}, dir]

Print["time used: ", SessionTime[] - time1]
```

B.5 FormCalc codes for the process $H \longrightarrow ZA$

```
(*
      HZA-SM.m
              generates the Fortran code for
              H -> Z gamma in the electroweak Standard Model
              this file is part of FormCalc
              last modified March 2021

Note: the QED contributions are not taken into account here.
To plug the QED part back in, remove the ExcludeParticles -> V[1]
from the InsertFields options below.

*)

Needs["FeynArts`"]

Needs["FormCalc`"]


time1 = SessionTime[]

CKM = IndexDelta


process = S[1] -> {V[2], V[1]}

name ="/home/zghiche/formcalc/HZA-SM.m."

(* 18 May 06: careful, not UV finite if photon is taken out! *)

SetOptions[InsertFields, Model -> "SM"]

SetOptions[Paint, PaintLevel -> {Classes}, ColumnsXRows -> {4, 5}]

take the comments out if you want the diagrams painted
$PaintSE = MkDir[name <> ".diagrams"];
DoPaint[diags_, file_, opt___] := Paint[diags, opt,
  DisplayFunction -> (Export[ToFileName[$PaintSE, file <> ".ps"], #]&)]
```

```
Print["Counter terms"]

tops = CreateCTTopologies[1, 1 -> 2,
  ExcludeTopologies -> {TadpoleCTs, WFCorrectionCTs}];
ins = InsertFields[tops, process];
DoPaint[ins, "counter"];
counter = CreateFeynAmp[ins]

Print["Vertices"]

tops = CreateTopologies[1, 1 -> 2, TrianglesOnly];
ins = InsertFields[tops, process];
DoPaint[ins, "vert"];
(* vert = CalcFeynAmp[
  CreateFeynAmp[ins],
  Select[counter, DiagramType[#] == 1 &]] *)

vert = CalcFeynAmp[CreateFeynAmp[ins], counter]

vert = Abbreviate[vert, 6,
  Preprocess -> OnSize[100, Simplify, 500, DenCollect]]

abbr = OptimizeAbbr[Abbr[]]

subexpr = OptimizeAbbr[Subexpr[]]

dir = SetupCodeDir[name <> ".fortran", Drivers -> name <> ".drivers"]

WriteSquaredME[{}, vert, abbr, subexpr, dir]

Print["time used: ", SessionTime[] - time1]
```

Appendix C

Dimensional Regularisation

These are steps followed to perform the dimentional regularization of the mass operator $\Sigma(p)$: After evaluation of the integrant numerator:

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{4-D}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{g^{\mu\nu}}{k^2+i\varepsilon}\frac{\gamma_\mu(\not{p}-\not{k}+m_f)\gamma_\nu}{(p-k)^2-m_f^2+i\epsilon} \tag{C.1}$$

The mass operator becomes:

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{4-D}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{(\not{p}-\not{k})(2-D)+m_f D}{[(p-k)^2-m_f^2+i\epsilon][k^2+i\epsilon]} \tag{C.2}$$

- Linearization of denominators via Feynman parametrization:

  By applying the generalized formula of Feynman's parametrization

$$\frac{1}{a^p b^q} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\int_0^1 \mathrm{d}x\frac{x^{p-1}(1-x)^{q-1}}{[ax+b(1-x)]^{p+q}} \tag{C.3}$$

  We have for the denominator

$$\frac{1}{[(p-k)^2-m_f^2+i\epsilon][k^2+i\epsilon]} = \int_0^1 dx\frac{1}{\left[(k-px)^2+p^2x(1-x)-m_f^2x+i\epsilon\right]^2} \tag{C.4}$$

  Then we find:

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{(4-D)}\int_0^1 \mathrm{d}x\left\{\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{(2-D)(\not{p}-\not{k})+Dm_f}{\left[(k-px)^2+p^2x(1-x)-m_f^2x+i\epsilon\right]^2}\right\} \tag{C.5}$$

  To obtain quadratic forms as a function of the integration quadri-momenta in the denominator of the expression (C.5) we apply this change of variable:

$$k = l + px \tag{C.6}$$

$$\begin{aligned} -i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{(4-D)} \int_0^1 \mathrm{d}x\, \{[(2-D)(1-x)p^\mu\gamma_\mu + Dm_f] \\ \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{\left[l^2 - m_f^2 x + p^2 x(1-x) + i\epsilon\right]^2}\right\} \end{aligned} \tag{C.7}$$

Setting that

$$R_x^2 = m_f^2 x - p^2 x(1-x) - i\epsilon \tag{C.8}$$

we get:

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{(4-D)} \int_0^1 \mathrm{d}x \left\{[(2-D)(1-x)p^\mu\gamma_\mu + Dm_f] \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2 - R_x^2]^2}\right\} \tag{C.9}$$

- Calculation of integrals on Euclidean phase space:

  Using:

$$I(r,m) = \int \frac{(l^2)^r}{(l^2 - R_x^2)^m} \frac{\mathrm{d}^D l}{(2\pi)^D} = i\frac{(-1)^{r-m}}{(4\pi)^{\frac{D}{2}}} (R_x^2)^{r+\frac{D}{2}-m} \frac{\Gamma(r+\frac{D}{2})\Gamma(m-r-\frac{D}{2})}{\Gamma(\frac{D}{2})\Gamma(m)}, \tag{C.10}$$

  will give

$$\Rightarrow \int \frac{1}{(l^2 - R_x^2)^2} \frac{\mathrm{d}^D l}{(2\pi)^D} = i\frac{(-1)^2}{(4\pi)^{\frac{D}{2}}} (R_x^2)^{\frac{D}{2}-2} \frac{\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})}{\Gamma(\frac{D}{2})\Gamma(2)} = I(0,2). \tag{C.11}$$

  So we have

$$-i\widehat{\Sigma}(p) = 4\pi\alpha Q_f^2\mu^{(4-D)} \int_0^1 \mathrm{d}x\, \{[(2-D)(1-x)p^\mu\gamma_\mu + Dm_f]\, I(0,2)\} \tag{C.12}$$

  Then using the relation

$$(2-D)(1-x)\not{p} + Dm_f = (2-D)(1-x)(\not{p} - m_f) + m_f[2(1-x) + Dx], \tag{C.13}$$

leads to

$$\widehat{\Sigma}(p) = \frac{\alpha}{4\pi}Q_f^2(4\pi\mu^2)^{(2-\frac{D}{2})}\Gamma(2-\frac{D}{2})\left\{(\not{p}-m_f)\int_0^1 \mathrm{d}x\frac{(2-D)(1-x)}{\left[m_f^2x-x(1-x)p^2-i\epsilon\right]^{2-\frac{D}{2}}}\right.$$
$$\left. + m_f^2\int_0^1 \mathrm{d}x\,\frac{2(1-x)+Dx}{\left[m_f^2x-x(1-x)p^2-i\epsilon\right]^{2-\frac{D}{2}}}\right\} \tag{C.14}$$

It is this expression (C.14) which represents the one loop-order correction of the free fermionic propagator and it intervenes in the denominator of the exact fermionic propagator.

Finally we move to Demonstration of ultraviolet divergence: The expression (C.14) is convergent if $D < 4$ so we can define the positive quantity ε_{uv} by:

$$\varepsilon_{uv} = 2 - \frac{D}{2} \tag{C.15}$$

After few manipulations using the relations

$$\Gamma(\varepsilon_{uv}) = \frac{1}{\varepsilon_{uv}} - \gamma + \mathcal{O}(\varepsilon_{uv}) \tag{C.16}$$

$$a^{\varepsilon_{uv}} = 1 + \varepsilon_{uv}\ln(a) + \mathcal{O}(\varepsilon_{uv}) \tag{C.17}$$

and using (C.15), we get

$$\begin{aligned} \widehat{\Sigma}^{(1)}(p) &= m_f\widehat{\Sigma}_a^{(1)}(p) + (\not{p}-m_f)\widehat{\Sigma}_b^{(1)}(p) \\ &= m_f\widehat{\Sigma}_{m_f}^{(1)}(p) + \not{p}\widehat{\Sigma}_{\not{p}}^{(1)}(p) \end{aligned} \tag{C.18}$$

Where

$$\widehat{\Sigma}_a^{(1)}(p) = \left(\frac{\alpha}{4\pi}Q_f^2\right)\left\{3\left[\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln(\frac{\mu^2}{m_f^2}) - \gamma\right]\right.$$
$$\left. -1-2\int_0^1 \mathrm{d}x(1+x)\;ln\left[x - \frac{p^2}{m_f^2}x(1-x) - i\epsilon\right]\right\}$$

$$\widehat{\Sigma}_b^{(1)}(p) = \left(\frac{\alpha}{4\pi}Q_f^2\right)\left\{-\left[\frac{1}{\varepsilon_{uv}} + ln(4\pi) + ln(\frac{\mu^2}{m_f^2}) - \gamma\right]\right.$$
$$\left. +1+2\int_0^1 \mathrm{d}x(1+x)\;ln\left[x - \frac{p^2}{m_f^2}x(1-x) - i\epsilon\right]\right\}$$

The singular part is

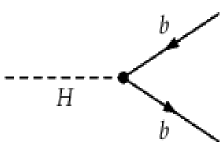
$$\widehat{\Sigma}^{(1)}_{uv}(p) = \left(\frac{\alpha}{4\pi}Q_f^2\right)\left[m_f\frac{4}{\varepsilon_{uv}} - \not{p}\frac{1}{\varepsilon_{uv}}\right] + \mathcal{O}(1) \tag{C.19}$$

Appendix D

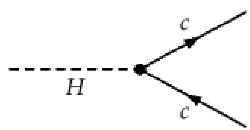
Feynman-digrams-of-the-Higgs-decay-processes

D.1 $H \longrightarrow b\bar{b}$, $H \longrightarrow c\bar{c}$, $H \longrightarrow \tau\bar{\tau}$

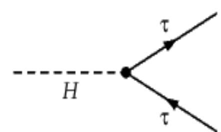
D.1.1 Born



T1 C1 N1

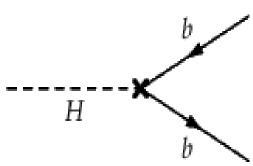


T1 C1 N1

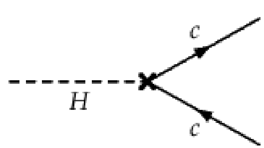


T1 C1 N1

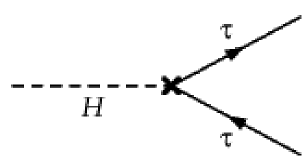
D.1.2 Vertices



T1 C1 N1

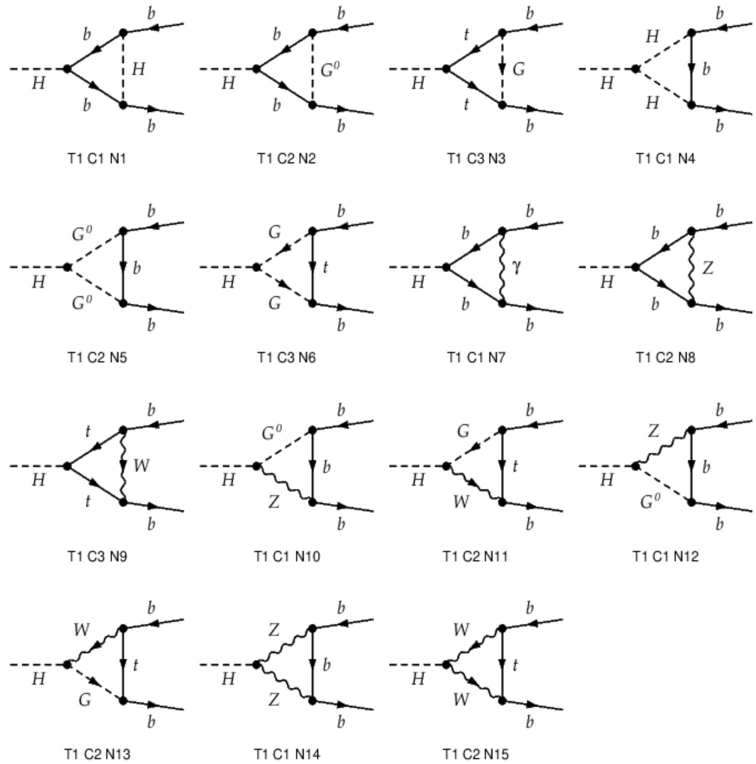


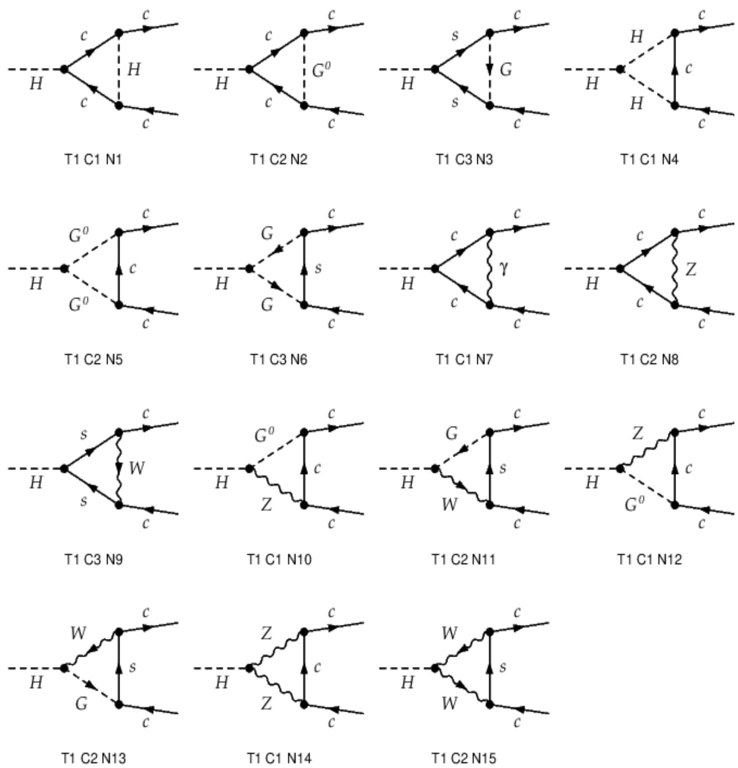
T1 C1 N1

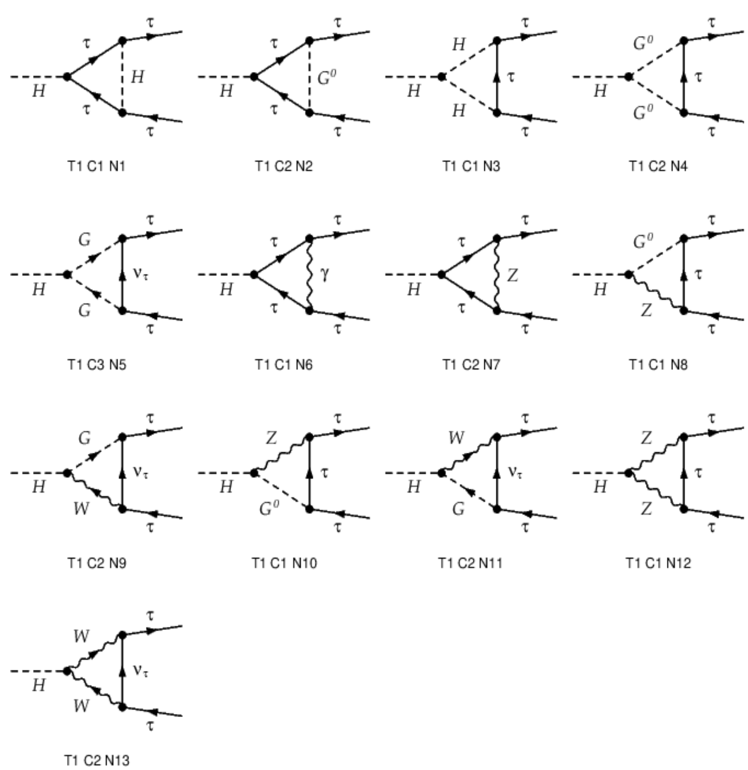


T1 C1 N1

D.1.3 Counterterms

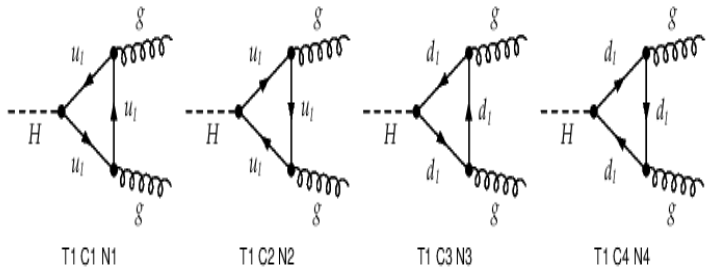


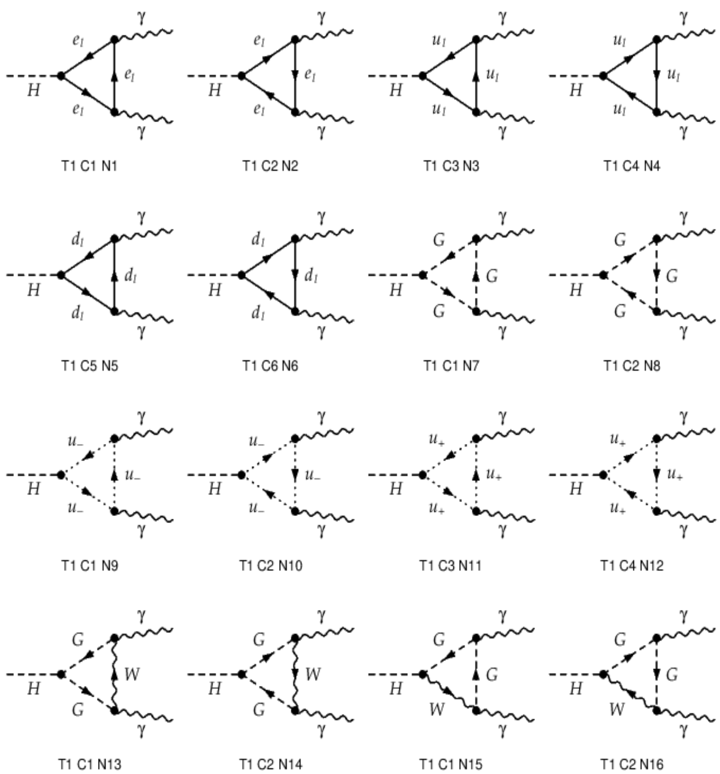

T1 C1 N1
T1 C2 N2
T1 C3 N3
T1 C1 N4
T1 C2 N5
T1 C3 N6
T1 C1 N7
T1 C2 N8
T1 C3 N9
T1 C1 N10
T1 C2 N11
T1 C1 N12
T1 C2 N13
T1 C1 N14
T1 C2 N15

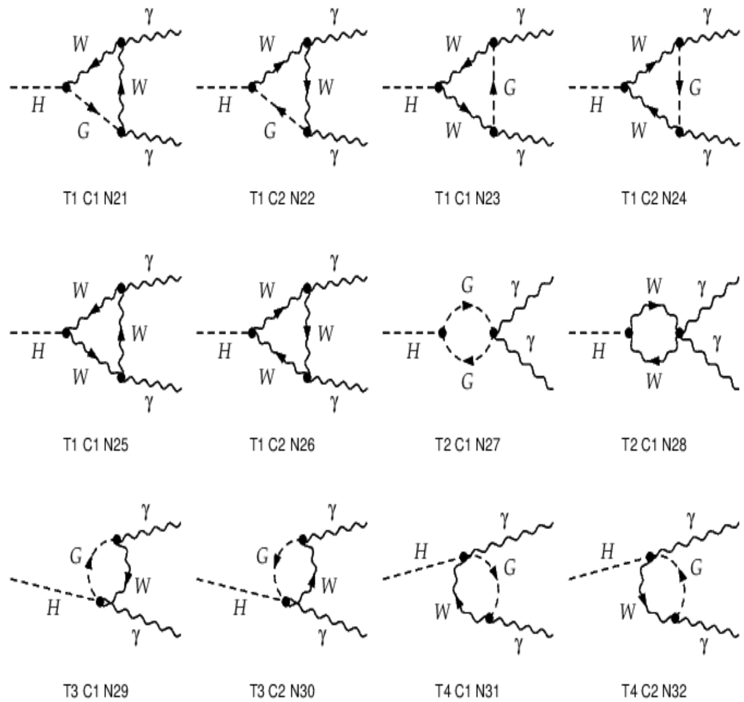

T1 C1 N1
T1 C2 N2
T1 C1 N3
T1 C2 N4
T1 C3 N5
T1 C1 N6
T1 C2 N7
T1 C1 N8
T1 C2 N9
T1 C1 N10
T1 C2 N11
T1 C1 N12
T1 C2 N13

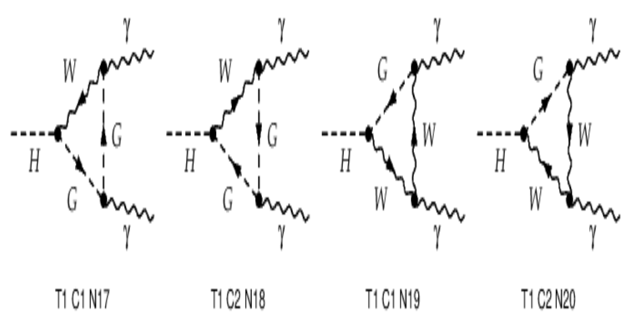
D.2 $H \longrightarrow gg$, $H \longrightarrow AA$, $H \longrightarrow ZA$

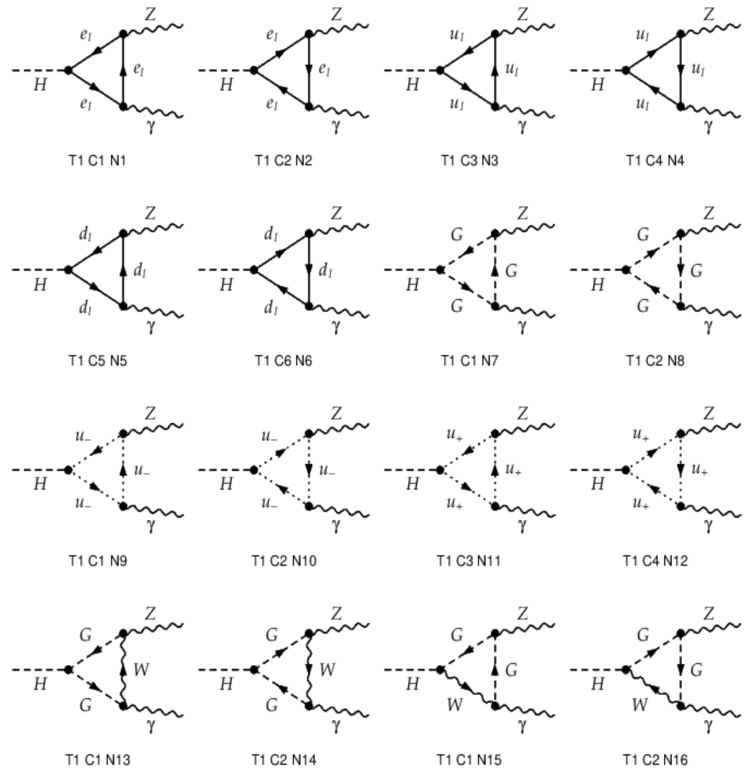
D.2.1 Vertices






T1 C1 N21
T1 C2 N22
T1 C1 N23
T1 C2 N24
T1 C1 N25
T1 C2 N26
T2 C1 N27
T2 C1 N28
T3 C1 N29
T3 C2 N30
T4 C1 N31
T4 C2 N32


T1 C1 N17
T1 C2 N18
T1 C1 N19
T1 C2 N20


T1 C1 N1
T1 C2 N2
T1 C3 N3
T1 C4 N4
T1 C5 N5
T1 C6 N6
T1 C1 N7
T1 C2 N8
T1 C1 N9
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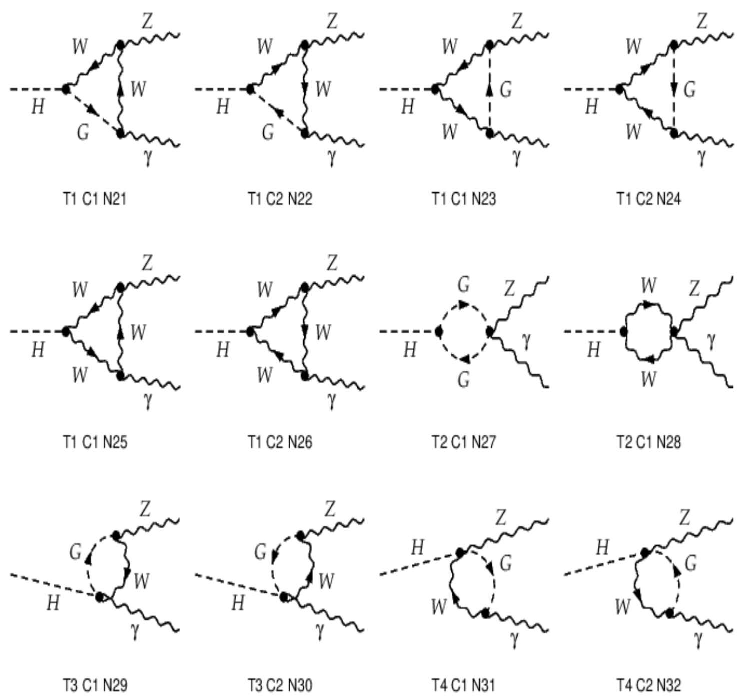
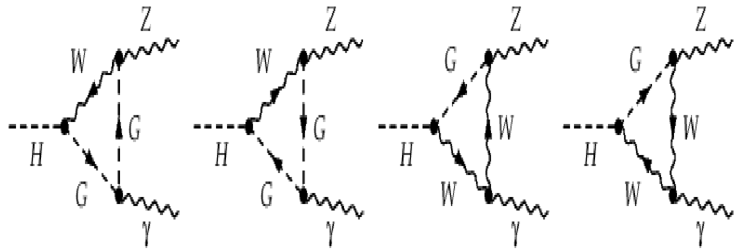
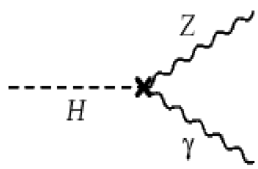
T1 C1 N13
T1 C2 N14
T1 C1 N15
T1 C2 N16


T1 C1 N21
T1 C2 N22
T1 C1 N23
T1 C2 N24
T1 C1 N25
T1 C2 N26
T2 C1 N27
T2 C1 N28
T3 C1 N29
T3 C2 N30
T4 C1 N31
T4 C2 N32



T1 C1 N17 T1 C2 N18 T1 C1 N19 T1 C2 N20

D.2.2 Counterterms



T1 C1 N1

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