Democratic and Popular Republic of Algeria
Ministry of Higher Education and Scientific Research

UNIVERSITY of SAAD DAHLEB BLIDA 1

Faculty of Sciences
Department of Physics

Master Thesis

Theoretical Physics

Theme :

Beyond the Standard Model: The Z' Stueckelberg extension of the Standard Model.

Prepared by : **FICHOUH Nihal**

Defended on july 16, 2023

Before the jury composed of:

Pr. SI LAKHAL Bahia	Prof.	U.S.T.H.B	President
Dr. BOUCHACHIA Karim	M.C.B	U.Y.F.MEDEA	Examiner
Dr. YANALLAH Abdelkader	M.C.B	U.S.D-BLIDA 1	Supervisor
Dr. BOUAYED Noureddine	M.C.A	U.S.D-BLIDA 1	Co-Supervisor

Blida, July 2023

Abstract

The new Z' gauge boson is one of the consequences of some physics Beyond the Standard Model. Such a particle can emerge from a gauge extension of the Standard Model by an additional $U(1)$ symmetry. We are interested in a $U(1)_X$ gauge extension, attached with a Stueckelberg way of attaining the mass of the new Z' boson. The Stueckelberg mechanism is characterized by the scalar σ, which obeys specific gauge transformations, ensuring the gauge invariance. It can be used as an alternative to the spontaneous symmetry breaking mechanism by extending the Higgs sector. This model is grounded in the choice taken for eliminating the unphysical bilinear terms, as well as the choice of the gauge fixing term. The impact of this Stueckelberg Z' is investigated in the context of "$b\bar{b} \to \nu\bar{\nu}$" scattering process.

Key words : BSM, $U(1)$ extension of the SM, Stueckelberg mechanism, Z' gauge boson.

Résumé

Le nouveau boson de jauge Z' est l'une des conséquences de quelques aspects du physique au-delà du Modèle Standard. Une telle particule peut émerger d'une extension de jauge du Modèle Standard par une symétrie supplémentaire $U(1)$. Notre intérêt se porte spécifiquement sur l'extension de jauge $U(1)_X$, associée à une méthode de Stueckelberg permettant d'obtenir la masse du nouveau boson Z'. Le mécanisme de Stueckelberg est caractérisé par le scalaire σ, qui obéit à des transformations de jauge spécifiques, assurant l'invariance de jauge. Il peut être utilisé comme une alternative au mécanisme de brisure spontanée de symétrie en étendant le secteur de Higgs. Le modèle repose sur le choix effectué pour éliminer les termes bilinéaires non physiques, ainsi que sur le choix du terme de fixation de jauge. L'impact de ce Z' de Stueckelberg est étudié dans le contexte du processus de diffusion "$b\bar{b} \to \nu\bar{\nu}$".

Mots clés : la physique au-delà du Modèle Standard, une extension de jauge $U(1)$ du Modèle Standard, le mechanisme de Stueckelberg, le nouveau boson de jauge Z'.

-

Acknowledgment

I would like to express my sincere gratitude to my supervisor, Dr. YANALLAH Abdelkader, and my co-supervisor Dr. BOUAYED Noureddine of the faculty of sciences at the USDB-1. I am so grateful for their invaluable guidance, and support throughout my master's program, I managed to acquire new insights, as well as having a rich environment to enlarge my knowledge packages. This endeavor would not have been possible without their expertise, time and their constructive feedback. I can't thank them enough.

My most profound appreciation will reside in the insights, and deep informative discussions that we have embarked on, as well as the analytical skills that I have accumulated throughout my university years, without forgetting the encouragement, and the opportunities that I received, which thoroughly impacted my development.

I would like also to thank the contributors in process of validating my thesis work: **Pr. SI LAKHAL Bahia** as the president of the jury, and **Dr. BOUCHACHIA Karim** as the examiner of the work. Thank you for accepting to incorporate your expertise, and to direct this work, and my efforts towards a better place.

Dedication

I would like to transmit my feelings of gratitude for my parents, thank you for creating the right atmosphere to enable my development, especially on the academic side. Both of you has made it possible for me to seek the fulfillment of one of human's curiosities attached to the vast reality surrounding us.

A special appreciation, and thanks to my best friend Z. Nabil, his endless support and care has been a strong pillar of encouragement, thank you for always being by my side.

I will always be deeply grateful, and in dept to all of you.

Contents

List of tables

List of figures

Introduction **1**

1 The Standard Model and Beyond **3**

1.1 The Standard Model of particle physics . . . 3

1.1.1 The Standard Model Lagrangian . . . 4

1.1.2 The limitations of the Standard Model . . . 10

1.2 Physics Beyond The Standard Model . . . 13

1.2.1 Grand Unified Theories and Supersymmetry . . . 13

1.2.2 Gauge extensions of the Standard model . . . 17

1.3 Some Samples of Z' models . . . 19

1.3.1 $U(1)_{B-L}$ model . . . 19

1.3.2 Z' in LRSM and GUTs . . . 20

2 A Stueckelberg Extension of the Standard Model **23**

2.1 $U(1)_X$ Extension of The Standard Model . . . 23

2.1.1 Kinetic mixing . . . 25

2.1.2 Neutral Gauge Bosons Masses . . . 28

2.2 Stueckelberg Extension . . . 29

2.2.1 Stueckelberg Lagrangian . . . 31

2.2.2 Stueckelberg Mechanism and the Mass matrix . . . 32

2.3 Generalized Stueckelberg Effect . . . 34

2.3.1 Generalized Stueckelberg Lagrangian . . . 34

2.3.2 Stueckelberg masses . . . 36

3 Identification of the new Z' gauge boson **40**

3.1 Z' Phenomenology . . . 40

3.1.1 Search for the Z' at e^+e^- Colliders . . . 42

3.1.2 Identification of the Z' using b and t quarks . . . 46

3.2 $b\bar{b} \to \nu\bar{\nu}$ Scattering process . . . 51

3.2.1 The square amplitude $|\mathscr{M}|^2$ of the scattering process . . . 56

3.2.2 The differential cross section . . . 61

3.3 Discussion . . . 66

Conclusion **69**

Perspectives **70**

A Fundamental Concepts **71**
A.1 Basics of Group Theory 71
A.1.1 Gauge group SU(2) 72
A.1.2 Gauge group SU(3) 72
A.2 Spontaneous Symmetry Breaking 72

B Standard Model's parameters **74**
B.1 Values of T_3, Y, and Q in the SM 74
B.2 The free parameters of the SM 74
B.3 Some of the properties of SM particles 75

C Particle Physics terminology **77**

D Feynman Rules **81**

E Casimir's trick and the square amplitude $|\mathscr{M}|^2$ **83**

List of Tables

1.1 Representations of various SM fields taken from [29]. 9
1.2 Various observed data from different experiments indicating the ratio of the solar neutrino events with respect to the number of events predicted by the Solar Standard Model [33]. 12
1.3 Two different SSB patterns in G(211) models. 18

3.1 Signal cross sections, taken from [80], for different Z' masses and couplings to first and second generation quarks. The values in this table are calculated with$g_{Z'_{b/t,\bar{b},\bar{t}}} = 1$. 49
3.2 The values of the vector and axial couplings c'_{v_f}, c'_{A_f} for the vertex $Z'\bar{f}f$. . 56
3.3 The values of the vector and axial couplings c_{v_f}, c_{A_f} for the vertex $Z\bar{f}f$. . . 56

B.1 Values of T_3, Y, and Q of various SM's fields 74
B.2 Values of some parameters of the SM. 75
B.3 The particles of the Standard Model and some of their properties. 76

C.1 some of the values of the luminosity for different particle accelerators with their specified type of interactions. 79

D.1 The Feynman rules for the external lines in Feynman diagrams. 81

List of Figures

1.1 Possible symmetry breaking patterns to the G_{SM}. This figure is taken from [29]. 15
1.2 A simulation showing the peak of Z'_{B-L} boson, through the e^+e^- decay channel in the mass $M_{Z'_{B-L}}$ region comprised between 800 and 1000. The data was taken from [56]. 20
1.3 An illustration of some possible intermediate steps throughout the breaking of the E_6 symmetry [60]. 22
1.4 Total decay width $\Gamma_{tot}(Z'(\theta))$ for certain values of $M_{Z'(\theta)}$ and θ. The width Γ is used for $\Gamma(Z'(\theta)).\Gamma(Z'(\theta))$, and $M_{Z'(\theta)}$ are in GeV. 22

2.1 A plot of the discovery limits of Z' in StSM with the discovery limit defined by $5\sqrt{N_{SM}}$, or by 10 events, whichever larger. The inflections in the plots are precisely the points of transitions between the two criteria. Regions to the left and above each curve can be probed by the LHC at a given luminosity. The top point on each curve corresponds to $\varepsilon = .061$. The analysis is done for the ATLAS detector but similar results hold for the CMS detector. 39

3.1 Some e^+e^- observables showing their standard model values, and their values for a Z' as a function of M'_Z for $\sqrt{s} = 500GeV$. The solid line is the standard model value, the dashed line is for Z_χ (from GUTs), the dotted line for Z_η, the dot-dashed line for Z_LR(Left-Right SM), and the dot-dot-dash line for Z_{ALR}(The alternative Left-Right Symmetric Model). The error bars are based on the statistical error assuming an integrated luminosity of $50fb^{-1}$. 44
3.2 A Feynman diagram for the production of a Z' boson via gluon splitting and a pair of b-quarks, in addition to its decay to a pair of b-quarks. 47
3.3 A Feynman diagram for the production of a Z' boson through the fusion of a top quark pair, exhibiting a potential to decay to a pair of bottom quarks, and the two spectator top quarks decay semi-leptonically. 49
3.4 Representative Feynman diagrams for Z' production via $t-\bar{t}$ fusion (to the left) and $t-\bar{t}$ associated production (to the right), taken from [83]. 50
3.5 Feynman Diagrams for the process $b\bar{b} \to \nu\bar{\nu}$ 57
3.6 An illustration of a scattering process in the center of mass frame. 61
3.7 Variation of the axial and vector couplings of the vertex $Z'ff$ for both b quarks, and neutrinos in function of the parameter ϱ. 64
3.8 A plot showing the variation of the differential cross section in function of the parameter ϱ, and through 4 different values for the Z' mass. 65
3.9 A plot showing the variation of the differential cross section in function of $\sqrt{s}$, with 3 different values for the Z' mass. And for the fixed values $\theta = \pi$, $\varrho = 1$. 65

3.10 A plot showing the variation of the differential cross section in function of θ, with 4 different values for the Z' mass. And for the fixed values E= 1 TeV, $\varrho = 1$. 65
3.11 A plot showing the variation of the differential cross section in function of $M_{Z'}$, with the values: E=1 TeV, $\theta = \pi$, $\varrho = 1$. The mass of the Z' is taken with GeV. 66

A.1 An illustration of the concept of spontaneous symmetry breaking in the case of bending a rod. 73

C.1 A geometrical interpretation of the concept of the cross section. 77
C.2 A visualization of the conception of decay width. 78
C.3 An example illustrating a Drell-Yan process, where a quark and anti-quark annihilate, forming a photon decaying into a pair of leptons. 79

Introduction

Our desire to demystify the deep underlying structure of our observable universe contributed to shape the very well-known theory in the realm of particle physics, namely "The Standard Model of particle physics". Despite the theoretical consistency of the model, and its capacity to survive many rigid low-energy experimental tests, the model left so many unanswered questions. Ranging from fundamental issues such as incorporating gravity, the hierarchy problem [1], neutrino oscillations [1], to cosmological phenomena; such as explaining dark matter, dark energy, and comic inflation. Furthermore, experimental inconsistencies such as the anomalous magnetic dipole moment of the "muon $g-2$" [2], B meson decay, and the mass of the W boson [3], has made it clear that we need another source to supply the answers. Therefore, our drive for a new physics is legitimized, such a source is believed to reside within the realm of physics Beyond The Standard Model, where various attempts have been made to extend the Standard Model in order to account for these phenomena, and address these experimental issues. For instance, Grand Unified Theories, and Supersymmetry have been very appealing theoretical routes to consider, even extra dimensions has been an interesting paradigm for expanding the Standard Model, while a less compacted solution has been considered in the form of extensions of the gauge sector of the SM.
However, one particular way of extending the SM has been widely studied, mainly extending the gauge internal structure by a $U(1)$ gauge symmetry. Many models were proposed based on such an extension, and they showed great capacity to take into consideration many phenomena. Such an extension was capable of accommodating cosmological inflation scenario [4], it can offer a suitable candidate for dark matter particles [5], it can account for the light neutrino masses by the seesaw mechanism [6], even accounting for the observed baryon asymmetry of the Universe via leptogenesis [7], and many more. Such an extension is always accompanied with the existence of an additional neutral gauge boson, the so-called Z' gauge boson, leading to a more rich phenomenology, given how such a particle seems to hold a potential for some answers, which make it very appealing to investigate. The Z' bosons have been present in so many locations in the literature [8], and such particles would have profound implications for particle physics, depending on the models from where they arise, given the different possible ranges for their masses and couplings.

It is, therefore, interesting to scrutinize a simple bottom up extension of the SM, where the gauge structure $SU(3)_C \times SU(2)_L \times U(1)_Y$ can be extended by an extra $U(1)_X$ gauge symmetry, leading to the new structure $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ [8]. Models with an additional abelian symmetries also allow for kinetic mixing between the fields, as well as potential $Z - Z'$ mixings, which are intriguing implications of such an extension [9]. On the other hand, providing the new resulting gauge boson with a mass cannot be achieved via the familiar route with the symmetry-breaking mechanism of the extended SM with a single scalar doublet, due to the fact that the extra $U(1)_X$ symmetry will remain unbroken. Thus, one must seek a way to break the symmetry, naturally by extending the scalar sector by an additional doublet, or bi-doublet [10]. Such a possibility is an interesting take of its own, and it is possible to achieve, but it comes with some

reliability, mainly the number of additional parameters, and the issue of fine tuning them. However, such a burden can be avoided via an alternative, the so-called the "Stueckelberg mechanism", where we can attain a massive $Z^{'}$ gauge boson without having to break the additional symmetry, and therefore, having to deal with less parameters.
An economical minimal Stueckelberg extension of the SM can be constructed via adding one field associated with the extra $U(1)$ symmetry, and one additional scalar field σ called "Stueckelberg field", which obeys specific gauge transformations, that ensure the gauge invariance. Thus, it is a viable procedure to generate the gauge bosons masses in the context of an extended SM, in addition to the Higgs mechanism [11].

In this thesis work, we are well-motivated to explore the impact of the $Z^{'}$ gauge boson, which can be revealed via studying some of the properties of this particle, such as its mass, and its couplings to fermions. Such a particle is chosen to be originated from a simple $U(1)$ gauge extension of the SM, and it obtains its mass via Stueckelberg mechanism, in the presence of the Higgs mechanism as well to generate the rest of the gauge bosons masses. The phenomenology of the $Z^{'}$ boson is investigated within the context of its coupling to fermions, both quarks and leptons.
This work is structured as follows; the first chapter is dedicated as an overview of the Standard model, and Beyond the SM physics, where the SM is briefly explored, as well as its deficiencies. Therefore, justifying the need for new physics, which will be explored briefly, then a hasty exploration of some samples of $Z^{'}$ models will be carried out. The second chapter will be marked by a Stueckelberg way of extending the SM, leading the way to Stueckelberg mechanism, and the acquisition of a massive $Z^{'}$ gauge boson. The final chapter will be characterized by some phenomenology of the $Z^{'}$, and its potential discovery at future colliders, precisely studying the $b\bar{b} \to \nu\bar{\nu}$ Scattering process, and finishing with some concluding remarks.
We have added some supplements to the work, four appendices to clarify certain concepts that have been used, as well as some tools such as Feynman rules, and Casimir's trick for calculating the square amplitude.

Chapter 1

The Standard Model and Beyond

Throughout the history of mankind's scientific inquiries, seeking the fundamental structure of our universe has been one major interest. From developing models for atomic structures all the way to constructing a framework based on a quantum field paradigm [12]. Such a step has led to the development of the so called 'the Standard Model of particle physics'(SM). The Standard Model is heavily built on the framework of quantum field theory. It describes reality at a very fundamental level, and its success has made it clear that reductionism might be the path to obtain the final underlying structure behind all the various physical phenomena that we are capable of observing today.
Under such circumstances, we are motivated to explore what such a model has to offer, and what sort of open questions that forced us to look for an alternative. At the end, we will set our preparations to embark on exploring one possible path to obtain such an alternative.

1.1 The Standard Model of particle physics

The Standard Model is a non-abelian gauge theory[1] of the primary constituents of the observable universe, and their interactions. It is a relativistic local field theory describing particles in terms of fields.

Historically, the development of such a model took place in stages. Starting from early development of quantum electrodynamics [13], which gave rise to the conception of gauge theory for abelian groups. It was followed by Yang Chen-Ning, and Robert Mills extending this conception to non-abelian groups, which led to incorporate an explanation for the strong interactions. Another big contribution to the standard model's construction was the work of Sheldon Glashow, where in 1961, he managed to combine the electromagnetic, and weak interactions resulting in the so-called electroweak interaction [14]. After that, Steven Weinberg [15], and Abdus Salam [16] incorporated the Higgs mechanism into Glashow's electroweak interaction.
The modern version of the model didn't take its form until mid 1970, and the solidity of the theory increased as more experimental data confirmed it. Starting from direct evidence of quarks [17], to the discovery of the $W_{\pm}$, and Z_0 bosons in 1983 [18]. Followed by a proof of the top quark 1995 [19], the tau neutrino 2000 [20], and finally at the top of the crown, the Higgs boson in 2012 [21]. Thus, the main ingredients of the model have been set. What's remaining is to establish how these ingredients fit together to give rise to the universe we observe around. While the SM describes the fundamental reality as a collec-

[1] The construction of any particle physics model is based on the gauge principle, which requires the invariance of the Lagrangian under gauge transformations.

tion of quantum fields interacting among each other, it really does distinguish between these quantum fields. Reality according to it is built upon two categories: Fermionic and bosonic fields.

Fermionic fields, also known as matter fields, are responsible for setting up the deep rigid structure. They are described by spinors, accompanied with the property of half integer spin, involving both lepton and quark fields folded in 3 generations. Quarks don't exist individually, a propriety known as "confinement". However, all fermionic fields obey the generic Pauli's exclusion principle. A most common fermionic field is Dirac's field, which is a spin ½ field.

The SM provides us with a new outlook at how interactions fundamentally occur. The entire process got reduced into only exchanging 12 vector bosons, which are the quanta of bosonic fields. The massless photon mediates electromagnetic interactions, the massless gluons strong interactions, while the massive Z_0 and W mediate weak interactions.
At low energies the 3 interactions seem to be distinguishable, but above the electroweak scale, the universe begins to exhibit extraordinary levels of symmetry. Manifesting in the unification of the electromagnetic and weak interactions.
Interestingly enough, the final ingredient is the reason why such a symmetry will break at the electroweak scale due to a process known as "Spontaneous Symmetry Breaking"(SSB)[2].
Within the context of the standard model specifically, such a process can be achieved via the postulation of another quantum field, which permeates space-time, named the Higgs field. Whose quanta are the 0-spin particle called the Higgs boson. The fermions in our universe acquire a mass proportional to the coupling to the Higgs field, leading to the famous "Higgs mechanism" [22].
All SM particles with some of their physical properties are described in the table(B.3) based on the Particle Data Group (PDG) [23].

The qualitative picture of the standard model mentioned above is insufficient to show the depth of the theory. The steps of manufacturing the model on the other hand can reveal how profound such a theory is. And with this regard, the situation is calling for a rigorous description, mainly the mathematical framework underlying the Standard Model of particle physics.

1.1.1 The Standard Model Lagrangian

As we mentioned previously, the standard model is built upon the framework of QFT. Mainly each particle is associated with a dynamical field $\psi_i(x)$, $i = 1......, n$, where $x = (x^0, x^1, x^2, x^3)$. The dynamism of those particles is determined through the Lagrangian density $\mathscr{L}[\psi_i(x), \partial_\mu \psi_i(x)]$(or similarly action S):

$$S = \int dx^4 \mathscr{L}[\psi_i(x), \partial_\mu \psi_i(x)] \tag{1.1}$$

thus, the entire focus will shift to how we can build such a Lagrangian.

The crucial ingredients that will be needed to construct the Lagrangian, and the SM as a whole are:

- Symmetries. Precisely the two classes of symmetries :
  - Space-time symmetries: Poincaré symmetry in 4 dimensions.

[2]For an intuitive understanding of the concept, consult appendix(A.2).

- Internal symmetries.

- Representations of fields.
- A mechanism for Spontaneous Symmetry Breaking.

The model is standing on the pillar of symmetries, such an item has been expressed in the language of group theory. It shows a spectacular role in giving birth to physical particles as well as planting the seed for interactions to take place.
The standard model's gauge symmetry group is defined as :

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{1.2}$$

This compact gauge group will specify the vector field components of the theory, where C is the color charge, L indicates a coupling to only the left-handed fermions , and Y is the weak hypercharge.
On the grand scheme, we can view the SM's gauge group as two sectors, the strong sector described by the $SU(3)_c$ symmetry, and the electroweak sector described by the $SU(2)_L \times U(1)_Y$ symmetry.
The strong sector, described by a theory called Quantum Chromodynamics (QCD), is a non-chiral theory of strong interactions, governed by the $SU(3)$ gauge symmetry, and it primarily tackles the interactions between colored quarks and gluons. The quark fields are denoted by $q_{r\alpha}$, where $\alpha = 1, 2, 3$ represents the color quantum number, and $r = u, d, s, c, b, t$ represents the field's flavors. While the gluon fields are denoted by G^i, there are 8 massless gluons in the theory playing the role of mediators for the strong interaction, with a strong gauge coupling g_s. The representations for both kind of fields are indicated in table 1.1.
The QCD's Lagrangian takes the form [24]:

$$\mathscr{L}_{QCD} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} + \sum_r \bar{q}^\alpha_r i \not{D}^\beta_\alpha q_{r\beta} - \sum_r m_r \bar{q}^\alpha_r q_{r\alpha} + \frac{\theta_{QCD}}{32\pi^2} g_s^2 G^i_{\mu\nu} \tilde{G}^{i\mu\nu} \tag{1.3}$$

$G^i_{\mu\nu}$ is the field strength tensor defined as:

$$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk} G^j_\mu G^k_\nu. \tag{1.4}$$

The covariant derivative:

$$D^{\mu\beta}_\alpha = \partial^\mu \delta^\beta_\alpha + \frac{i g_s}{\sqrt{2}} G^{\mu\beta}_\alpha \tag{1.5}$$

G^α_β are the gluon fields in tensor (matrix) notation. Providing that:

$$G^\beta_\alpha = (G^\alpha_\beta)^\dagger = \sum_{i=1}^{8} \frac{\lambda^i_{\alpha\beta}}{\sqrt{2}} G^i. \tag{1.6}$$

The dual field strength tensor:

$$\tilde{G}^i_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{i\rho\sigma}$$

θ_{QCD} is a dimentionless constant. When it is set to zero, the theory will exhibit additional symmetries called accidental symmetries. The strong interactions seem to be reflection invariant (P), invariant under charge conjugation (C), time reversal (T), and the products (CP), and (CPT). The last term in (1.3) is referred to as the strong CP term[3], when $\theta_{QCD} \neq 0$ we fall in the problem called the strong CP problem, where the theory violates

[3]The term is gauge invariant, and it respects the renormalizability of the theory.

P, T, and CP symmetries.
As we can observe in (1.3), a mass term is included. In QCD, it is permissible to add such a term, and by hand without ruining the gauge invariance, as opposed to the electroweak theory, where adding a mass term will cause further complications as we will see later.

However, the remaining factor $SU(2) \times U(1)$ is governing the underlying of the electroweak theory of the standard model. It is a chiral unifying theory of both electromagnetic and weak nuclear interactions. The $SU(2)$ symmetry yielding 3 gauge boson degrees of freedom $W^i, i = 1, 2, 3$ with gauge coupling g, while the $U(1)$ symmetry yields a gauge boson B with a gauge coupling g'. The fields representations are described in table1.1
The Yang–Mills Lagrangian for the gauge group $SU(2) \times U(1)$is given by [25]:

$$\mathscr{L}_{Yang-Mills} = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R \tag{1.7}$$

where:

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu \tag{1.8}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{1.9}$$

we have incorporated fermionic fields in the Lagrangian. Due to the fact that the theory is chiral, the right and left fermions behave differently under $SU(2)$ group transformations. Given that :

$$\psi_{L,R} = [(1 \mp \gamma_5)/2]\psi. \tag{1.10}$$

As now we can observe in (1.7), there is no sign of a mass term, simply due to the issue coming from adding it by hand. Such a problem has led to seeking other ways by which the particles can gain their masses. It turns out SSB is playing a crucial role in that matter.

This $SU(2) \times U(1)$ will be broken by the Vacuum Expectation Value (VEV) of a scalar field, into a $U(1)_{em}$ at around 100 GeV, through the well-established Brout-Englert-Higgs mechanism.

Historically speaking, what was firstly introduced was the concept of Spontaneous Symmetry Breaking in 1964, separately by R.Brout, and F.Englert [26]. As well as P.Higgs [27]. Through such an idea, we are capable of discerning two phases, the phase of the unbroken symmetry or before SSB, where the physical states are all invariant under the displayed symmetry, and they show up as massless states. And a phase after the SSB, where the symmetry is spontaneously broken, meaning with no external interference, resulting in the physical particles gaining their masses.

The necessity of the mechanism in the SM arose from the fact that a mass term is actually forbidden, if we want to maintain gauge invariance of the theory. Leading to the wonder why the physical entities have the property of mass, while symmetries seem to give the impression that such a thing is not allowed. To allow for such a property again, we postulated the existence of a scalar field ϕ, in such a way that it will be permissible to write mass terms. It turns out, if this scalar field will possess a ground state, that will not share the symmetry of the Lagrangian, that is if the field will acquire a VEV $< \phi >$, then suddenly the mass terms for the particles will be generated.
To see the Higgs mechanism in action, we define the complex iso-doublet scalar field as:

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \tag{1.11}$$

with a potential term:

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \frac{\lambda}{4}(\phi^\dagger\phi)^2. \tag{1.12}$$

Acquiring a VEV $< |\phi| >$ can be easily achieved via seeking the minimum of the potential:

$$(\phi^\dagger\phi)_{min} = \frac{2\mu^2}{\lambda} \tag{1.13}$$

resulting in:

$$< |\phi| >= \sqrt{\frac{2\mu^2}{\lambda}} = \frac{v}{\sqrt{2}} \tag{1.14}$$

the form of the Higgs doublet around the minimum will be:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi_1 + i\chi_2 \\ v + h + i\chi_3 \end{pmatrix}. \tag{1.15}$$

The 3 Nambu-Goldstone bosons, which are contained in $\varphi^+(= \chi_1 + i\chi_2)$ and χ_3 are the result of the breaking of global continuous symmetry (Goldstone's theorem) [28], they are not observed in reality due to the fact that they are absorbed by the physical gauge bosons, and they will contribute into their longitudinal polarization.
The SM gauge group after SSB will be written as :

$$G'_{SM} = SU(3)_c \times U(1)_Q \tag{1.16}$$

where the electric charge becomes:

$$Q = T_3 + \frac{Y}{2} \tag{1.17}$$

Y being the weak hypercharge, and T_3 is the 3^{rd} component of the $SU(2)_L$ generator. Their values for various SM's fields are listed in appendix (B.1).

After SSB, a mixing between the states will lead to the actual physical gauge bosons:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu) \tag{1.18}$$

$$Z^0_\mu = \frac{1}{\sqrt{g'^2 + g^2}}(gW^3_\mu - g'B_\mu) \tag{1.19}$$

$$A_\mu = \frac{1}{\sqrt{g'^2 + g^2}}(g'W^3_\mu + gB_\mu) \tag{1.20}$$

leading to the masses,

$$M_W = \frac{gv}{2} \qquad M_Z = \sqrt{g'^2 + g^2}\frac{v}{2}$$

$$M_\gamma = 0$$

it can be written more elegantly in terms of the so called the Weinberg angle θ_w, which is the angle between W^3_μ and B_μ. It is defined as :

$$Tan\theta_w = \frac{g'}{g} \qquad Cos\theta_w = \frac{M_W}{M_Z} \tag{1.21}$$

resulting in :

$$Z_\mu = Cos\theta_w W^3_\mu - Sin\theta_w B_\mu \tag{1.22}$$
$$A_\mu = Sin\theta_w W^3_\mu + Cos\theta_w B_\mu \tag{1.23}$$

In conclusion, The universe seems to relay on the interaction of bosons and fermions with the scalar background Higgs to generate their masses, and as a result the symmetry must be broken. The validity of the Higgs mechanism has been verified by two experiments : CMS and ATLAS at the LHC [21].

Now that we have taken a closer look at the Higgs mechanism, and established the perspective of the two individual theories residing in the SM, it is only natural to seek the complete comprehension of the theory, which cannot be achieved by only restricting our sight to QCD, and the electroweak theory separately. Such an understanding can be sought via constructing the Lagrangian of the theory. Therefore, we can present the Standard Model Lagrangian in the following form [24]:

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge} + \mathscr{L}_f + \mathscr{L}_\phi + \mathscr{L}_{yuk} + \mathscr{L}_{GF} + \mathscr{L}_{Ghost}. \tag{1.24}$$

The first term $\mathscr{L}_{gauge}$ holds the gauge machinery of our model, it takes the simple form:

$$\mathscr{L}_{gauge} = -\frac{1}{4}G^i_{\mu\nu}G^{\mu\nu i} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \tag{1.25}$$

The second term describes the fermionic sector of the model:

$$\mathscr{L}_f = \sum_{i=1}^{3}(\bar{q}_{iL}i\not{D}q_{iL} + \bar{\ell}_{iL}i\not{D}\ell_{iL} + \bar{u}_{iR}i\not{D}u_{iR} + \bar{d}_{iR}i\not{D}d_{iR} + \bar{e}_{iR}i\not{D}e_{iR}). \tag{1.26}$$

The chiral nature of the SM is pretty explicit at this point, the left-handed fermions are the only ones transforming under $SU(2)_L$, while the right handed fermions transform trivially resulting in them having zero isospin. The left handed particles will be represented by isospin doublets, and the right handed as isospin singlets:

$$\begin{aligned}
&q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L\right) \\
&u_{iR} = (u_R, c_R, t_R), \qquad\qquad d_{iR} = (d_R, s_R, b_R) \\
&\ell_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} = \left(\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \mu \end{pmatrix}_L\right) \\
&e_{iR} = (e_R, \mu_R, \tau_R)
\end{aligned}$$

only the quarks take part in the strong interaction, since they are characterized by having a color charge $\alpha = (r, b, g)$, in this case we have repressed the color index.
In addition, No right handed neutrinos are included in the SM.

The representations for fermionic fields are as well stated in table (1.1). In table (1.1), the numbers within the parentheses indicates the representation of the fields under $SU(3)_C$ and $SU(2)_L$, while the the subscript shows the hypercharge.

The covariant derivatives:

$$\begin{aligned}
D^{\mu}q_{iL} &= \left(\partial^{\mu} + \frac{ig_s}{2}\lambda^{a}G^{\mu}_{a} + \frac{ig}{2}\tau^{i}.W^{\mu}_{i} + \frac{ig'}{6}B^{\mu}\right)q_{iL} \\
D^{\mu}\ell_{iL} &= \left(\partial^{\mu} + \frac{ig}{2}\tau^{i}.W^{\mu}_{i} + \frac{ig'}{2}B^{\mu}\right)\ell_{iL} \\
D^{\mu}e_{iR} &= \left(\partial^{\mu} + \frac{ig}{2}\tau^{i}W^{\mu}_{i} + \frac{ig'}{2}B^{\mu}\right)e_{iR} \\
D^{\mu}u_{iR} &= \left(\partial^{\mu} + \frac{ig_s}{2}\lambda^{a}G^{\mu}_{a} + \frac{2ig'}{3}B^{\mu}\right)u_{iR} \\
D^{\mu}d_{iR} &= \left(\partial^{\mu} + \frac{ig}{2}\tau^{i}W^{\mu}_{i} + \frac{ig'}{3}B^{\mu}\right)d_{iR}
\end{aligned} \tag{1.27}$$

λ_a are the 3 × 3 Gell Mann matrices, and τ_b are the 2 × 2 Pauli matrices.

Table 1.1: Representations of various SM fields taken from [29].

	Field	**Representation**
Vector Bosons	B_μ	$(1,1)_0$
	C_μ	$(1,3)_0$
	G_μ	$(8,1)_0$
Fermions	ℓ_{iL}	$(1,2)_{-\frac{1}{2}}$
	e_{iR}	$(1,1)_{-1}$
	q_{iL}	$(3,2)_{\frac{1}{6}}$
	u_{iR}	$(3,1)_{\frac{2}{3}}$
	d_{iR}	$(3,1)_{-\frac{1}{3}}$
Scalar Bosons	ϕ	$(1,2)_{\frac{1}{2}}$

Coming to the shiny section of the model, the Higgs sector which is the mathematical background for the Higgs mechanism described above, represented by $\mathscr{L}_\phi$:

$$\mathscr{L}_\phi = (D_\mu\phi)^{\dagger}D^{\mu}\phi - \mu^2\phi^{\dagger}\phi - \frac{\lambda}{4}(\phi^{\dagger}\phi)^2 \tag{1.28}$$

ϕ is the complex scalar Higgs field, where :

$$D^{\mu}\phi = \left(\partial^{\mu} + \frac{ig}{2}\tau^{i}W^{\mu}_{i} - \frac{ig'}{2}B^{\mu}\right)\phi$$

The next term on the list is called Yukawa term, which represents Yukawa couplings between the Higgs and the fermions, that describes the mass gained by fermions due to this interaction. We have:

$$\mathscr{L}_{Yuk} = -\sum_{i,n=1}^{3}(\Gamma^{u}_{in}\bar{q}_{iL}\tilde{\phi}u_{nR} + \Gamma^{d}_{in}\bar{q}_{iL}\phi d_{nR} + \Gamma^{e}_{in}\bar{\ell}\phi e_{nR}) + h.c. \tag{1.29}$$

Of course, to fix the redundancy, we will have to "fix the gauge", and to gain the capacity to define the propagators, therefore the requirement for the gauge fixing term. It can be written as:

$$\mathscr{L}_{GF} = -\frac{1}{2\xi_G}F_G^2 - \frac{1}{2\xi_A}F_A^2 - \frac{1}{2\xi_Z}F_Z^2 - \frac{1}{\xi_W}F_-F_+ \tag{1.30}$$

where

$$F_G^a = \partial^\mu G_\mu^a \qquad F_A = \partial^\mu A_\mu \qquad F_Z = \partial^\mu Z_\mu + \eta\eta_Z\xi_Z M_Z\varphi_Z$$
$$F_+ = \partial^\mu W_\mu^+ + i\eta\xi_W M_W\varphi^+ \quad F_- = \partial^\mu W_\mu^- - i\eta\xi_W M_W\varphi^- .$$

The final term to be added is known as ghosts term, ghost fields are an indispensable feature of gauge theories. And for our case, it if given by Fadeev-Popov ghosts in the following fashion:

$$\mathscr{L}_{Ghost} = \eta_G \sum_{i=1}^{4} (\bar{c}_+ \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_- \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_z \frac{\partial(\delta F_z)}{\partial\alpha^i} + \bar{c}_A \frac{\partial(\delta F_A)}{\partial\alpha^i})c_i + \eta_G \sum_{a,b=1}^{8} \bar{\omega}^a \frac{\partial(\delta F_G^a)}{\partial\beta^b}\omega^b \tag{1.31}$$

where we adopted the convention that $c_\mp, c_A, c_Z$ are the electroweak ghosts, while ω^a are the strong ghosts. and $\eta_G = \mp 1$ is an optional sign convention with no physical implications.

In conclusion, the overall Lagrangian density of the standard model is given by putting all the pieces together :

$$\begin{aligned}
\mathscr{L}_{SM} = &-\frac{1}{4}G^i_{\mu\nu}G^{\mu\nu i} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+ \sum_{i=1}^{3}(\bar{q}_{iL}i\not{D}q_{iL} + \bar{\ell}_{iL}i\not{D}\ell_{iL} + \bar{u}_{iR}i\not{D}u_{iR} + \bar{d}_{iR}i\not{D}d_{iR} + \bar{\ell}_{iR}i\not{D}\ell_{iR}) \\
&- \sum_{i,n=1}^{3}(\Gamma^u_{in}\bar{q}_{iL}\tilde{\phi}u_{nR} + \Gamma^d_{in}\bar{q}_{iL}\phi d_{nR} + \Gamma^e_{in}\bar{\ell}\phi e_{nR} + \Gamma^\nu_{in}\bar{\ell_{iL}}\tilde{\phi}\nu_{nR}) + h.c \\
&+ (D^\mu\phi)^\dagger D_\mu\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\
&- \frac{1}{2\xi}F_G^2 - \frac{1}{2\xi}F_A^2 - \frac{1}{2\xi}F_Z^2 - \frac{1}{\xi}F_-F_+ \\
&\sum_{i=1}^{4}(\bar{c}_+ \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_- \frac{\partial(\delta F_+)}{\partial\alpha^i} + \bar{c}_z \frac{\partial(\delta F_z)}{\partial\alpha^i} + \bar{c}_A \frac{\partial(\delta F_A)}{\partial\alpha^i})c_i + \sum_{a,b=1}^{8}\bar{\omega}^a \frac{\partial(\delta F_G^a)}{\partial\beta^b}\omega^b
\end{aligned} \tag{1.32}$$

All of the theoretical ingredients for the Standard Model of particle physics can be summarized by (1.32). The success of this theory was astonishing, it survived rigorous experimental tests [30], and it proved its capacities to explain most phenomena at low energy scale. It wasn't long before the theory started being challenged by issues that can't be overlooked. Some of these issues will be mentioned in the following subsection.

1.1.2 The limitations of the Standard Model

The standard model received a lot of admiration, and acceptance among physicists. Mainly due to its capacities to provide valid explanations for various physical phenomena. It wasn't long before various data started pushing us to consider the borders of this theory. Thus, the SM might just be a low-energy approximation of another hidden profound theory.

The standard model fails to account for gravitational effects. Only 3 out of 4 fundamental interactions are part of the model. Gravity as it is manifested in our observable universe is accounted for by Einstein's theory of general relativity, but there are no experimental sings for its compatibility with the standard model. Which means we are in a lack of a theory that can take into consideration the fundamental interactions altogether. There certainly has been attempts to solve this issue from the desperate search for a theory of

quantum gravity to reducing the focus on the prospect of unification, and just aim at the properties of the fabric of space-time [31, 32].
In addition to the failure of the standard model to account for gravity, and its quantization like the remaining forces, it also fails to explain other phenomena such as the existence of only 3 generations or families of fermions. It seems that our observable universe relies heavily on the first generation to build itself, so it leaves us with an empty hole of whether the fundamental structure is constructed out of 3 generations or there are more, and if there is more then the model doesn't supply us with any predictions of such a thing.
The model does not incorporate a valid explanation for dark matter and dark energy, which are so far important ingredients for the observable universe, and we can't speak of a fundamental theory of our universe which does not take these two elements into consideration. Frankly, this is not the only phenomenon that the model does not account for, it as well does not have an explanation for cosmic inflation, matter anti-matter asymmetry, as well as the reason why baryon asymmetry exist in our universe, which showcase how phenomenologically limited the theory is [24, 29].
The model equally does not provide an explanation for charge quantization, and why the electroweak section is a chiral theory unlike QCD.

In fact, what would be unsatisfying about the standard model to many physicists is the number of free parameters within the model. The theory involves 19 parameters[4], 18 of these parameters include: the 6 quark masses, the 3 charged leptons masses, 3 mixing angles, the CP violating phase, 3 coupling constants, the vacuum expectation value, and θ_{QCD}. Leaving us with the final parameter which is the Higgs boson mass measured by the LHC to be around 125.25 GeV [23]. In addition to the large number of free parameters, it is unpleasant that theory depends on experimentally determined parameters, which means their values must be measured independently of the theory, due to the inability of model to bring forth an explanation, or any form of predictions[5]. This issue for sure is an indicator for some that the standard model is the not complete.

An interesting puzzling aspect about this model is the so called the hierarchy problem of the standard model [24]. It is, in a nutshell, the discrepancy between the light mass of the Higgs boson, and Planck scale. As we know the measured mass of the Higgs boson is around 125 GeV, while the plank scale is at around 10^{19} GeV, the issue of why the mass of the Higgs is so small compared to the scale of new physics is not answered by the theory. In addition, according to the model the mass of the Higgs is highly sensitive to any quantum corrections which can lead to an increase in its mass considerably (quadratic divergences) [33].
Another issue that falls in the same category is the question of why neutrino masses are too small in comparison to the quark top mass [33], while the neutrino mass is of order of few eV the mass of the quark top is around 174 GeV, leading to the wonder of why would nature host such big differences in particles masses.

Another unknown within the leptonic sector is the mysteries of neutrinos, they are subatomic particles that don't participate in the strong, and electromagnetic interactions which make them difficult to detect. First deficiency signal started with the so called the Solar Neutrino Problem [33], which refers to the inconsistency between the expected, and observed number of neutrinos from the Sun. Such a discrepancy can be seen through the table (1.2).

[4]This number is considered when the SM is with no right handed neutrinos, and when neutrinos are considered massless.

[5]Some of the free parameters are listed in appendix(B.2).

Table 1.2: Various observed data from different experiments indicating the ratio of the solar neutrino events with respect to the number of events predicted by the Solar Standard Model [33].

Experiment	Data /SSM
^{37}Cl	0.310 ∓ 0.030
Super K	0.413 ∓ 0.015
^{71}Ga	0.540 ∓ 0.039
SNO^{Scatt}	0.540 ∓ 0.039

Such results led to the postulation of a phenomenon known as neutrino oscillations, where neutrinos undergo flavor transformations as they propagate through space. Furthermore, such a phenomenon can take place if there is a physical property distinguishing between the various flavors of neutrinos, and given that they have similar gauge couplings, we are left to assume that what's different among them is the property of mass. Herein, leading to the issue at hand, the Standard Model is unable to provide a theoretical description for neutrino masses, and neutrino oscillations.
However, the nature of neutrinos is still a mystery as well, we don't know if they are Dirac or a Majorana particles [34], and not only that the model is unable to fill all these loopholes, but also it only includes left-handed neutrinos with no signs for their right-handed counterparts.

The list of issues doesn't end here, and what would be critically problematic is the experimental misfit of the theoretical predictions, mainly the muon $g-2$ anomaly, and the W boson mass anomaly.
The capacity of predicting theoretically the magnetic moment of charged leptons was hugely influential, such an achievement was based on defining the anomalous magnetic moment as follows:

$$\alpha = \frac{g - g_{Dirac}}{2} = \frac{g-2}{2} \tag{1.33}$$

But recent experimental evidences from the Muon $g-2$ collaboration[2] showed a conspicuous discrepancy between the measured value and the SM predicted value, a discrepancy of 4.2σ:

$$\Delta\alpha_\mu = \alpha_{\mu,Exp} - \alpha_{\mu,SM} = (251 \pm 59) \times 10^{-11}. \tag{1.34}$$

Such an issue was clearly enough to ignite motives to start seeking answers Beyond The SM.
Similarly, another experimental results reported by CDF II experiment showed a deviation of 7σ of the W boson mass from the theoretically predicted value [3]:

$$m_W^{CDFII} = 80.4335 \pm 0.0094 Gev \tag{1.35}$$

$$m_W^{SM} = 80.357 \pm 0.006 Gev. \tag{1.36}$$

The newly discovered value of the W boson mass is believed to be one other sign of physics BSM [35], if it is confirmed by future experiments the anomaly of the W boson mass certainly cannot be ignored, and it will only require some new layer of physics to make sense of the empirical results.

In conclusion, the standard model is a ravishing theory, but despite its enormous success and its deepest insights, it is still not the final answer. However, in order to extend our understating of such a universe, we are in a dire need to understand deeply the standard model, since such an understanding will constitute the basis to construct new physics, a physics beyond the standard model.

1.2 Physics Beyond The Standard Model

The limitations of the standard model were only an opening to a new era in physics, such limitations are but mere indications of a much deeper layer awaiting to be discovered. So, it is only natural to take a closer look at what such new physics looks like, thus a general outlook on some models beyond the Standard Model is taken and scrutinized.
We start by looking at some aspects of Physics Beyond the Standard Model, all based on extending the previous bounded model under the purpose of providing solutions to its limitations. From exploring some of GUTs, to taking a look at what SUSY has to offer, and reaching out to specific gauge extensions of the SM. All under the motivation of inspecting what such a new physics would look like, and it is noteworthy to mention that we will talk briefly about some models, without the rigorous mathematical details.

1.2.1 Grand Unified Theories and Supersymmetry

The very first compelling route would be unification, which is the major motivation behind Grand Unified Theories. Such theories are an interesting extension of the Standard Model, as they aim to unify the 3 fundamental interactions into one, mainly via making the 3 gauge groups $SU(3) \times SU(2) \times U(1)$ subgroups of one simple[6] group. A very much needed constraint on these theories is the requirement of complex representations, which will help us extract the chiral structure of the SM once the symmetry is broken. In addition, the SM's group is rank 4, therefore the groups G that we are trying to embed in must be at least rank $N - 1 = 4$.

One popular proposition is embedding all the gauge groups of the SM in the special unitary $SU(5)$ group [36], it was the first the grand unified theory, which was put forward first by H. Georgi and S. Glashow in 1974. Such a choice describes the unified force as something mediated by the resulting gauge bosons, there will be 24 generators for SU(5), associated with the vector V_μ^a, a = 1, ..., 24. As a result, we will have 12 new gauge bosons, denoted in the literature X and Y, carrying a color charge and a fractional electric charge. Fermions on the other hand are described differently, leptons and quarks in this model are combined into one irreducible representation, leading to possible violation of baryon number[7] [37].
However, the process of SSB of the $SU(5)$ into $SU(3)_C \times SU(2)_L \times U(1)_Y$ can occur in a similar way as in the SM, it only requires a scalar field to acquire a VEV, leading the symmetry to break down at the GUTs scale[8] to the SM gauge symmetries. Similarly, such a process induces the generation of mass for the gauge bosons including the X and Y gauge bosons, which mediate processes that violate baryon and lepton numbers, an example of such a process is proton decay, but all the SM bosons will remain massless, and they will obtain their mass by the electroweak symmetry breaking.
SU(5) model has many physical implications, it provided an explanation for the hypercharge quantization which the SM couldn't, directing the light to early signs of a functional

[6] A simple group in group theory is a group that has no nontrivial proper normal subgroups.

[7] Such a step can allow for interactions between leptons and quarks that do not conserve baryon number, but still conserve B-L number.

[8] The energy scale at which the couplings unify is defined as M_{GUT}, where $M \sim 10^{15}$ GEV

unified theory. As well as predicting finely the value of the Weinberg angle [38]:

$$Sin^2(\theta_w) = \frac{g'^2}{g^2 + g'^2} \approx 0.21. \tag{1.37}$$

Maybe the most fascinating prediction is the phenomenon of proton decay, where the proton can decay into a positron and a neutral pion[9]: $p \to e^+\pi^0$, with a lifetime:

$$\tau_p \approx \frac{1}{\alpha^2_{su(5)}} \frac{M_X^4}{m_p^5} \tag{1.38}$$

but with the estimated lifetime the model is running into a contradiction with experimental data given that we didn't spot any signs of proton decay. In addition to this issue, $SU(5)$ model fails the B-test [39], which might indicate that the $SU(5)$ model still needs further extensions.
It is worthy to mention that the above description is related to a minimal $SU(5)$ model, and some of the issues such as the lifetime of the proton, and the B-test can solved via adding more particles into the model.

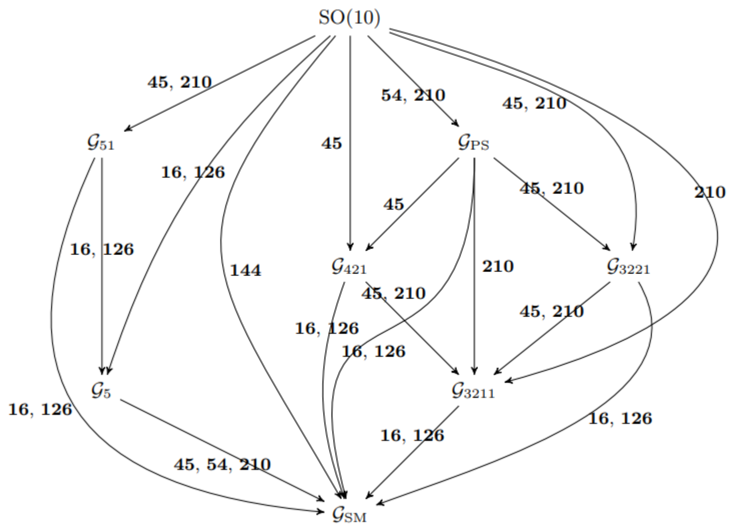
Another choice for a unification group is the $SO(10)$ group, it was first worked upon by Fritzsch and Minkowski [40]. Such a choice is based on a larger Lie group, the special orthogonal group $SO(10)$ with rank 5, and with 45 generators, so there are 45 gauge bosons. And the model combine the fermions of one family into a 16 dimensional spinor representation, thus including the right handed neutrinos.
Furthermore, there are a lot of possible ways to break the $SO(10)$ symmetry to G_{SM}, some of these paths are shown in figure (1.1)[10]. And as we know when the symmetry is spontaneously broken, only the gauge bosons corresponding to the generators that are left unbroken remain massless.
The model offers some impressive phenomenological predictions [29], one of the predictions of the model as we stated before is the is the right-handed neutrino, and thus neutrino masses can be explained by Seesaw models. In addition, axions are naturally embedded in the model, and can provide a solution to Dark Matter. It contains the necessary ingredients to allow for the baryon asymmetry to be generated through the mechanism of leptogenesis. More consequences can appear from the symmetry breaking chains, since various configurations can be formed by the scalar fields, resulting in phenomena such as: domain walls, cosmic strings, or magnetic monopoles.
$SO(10)$ models are very interesting candidate for GUTs, and it constitutes viable models for Physics Beyond The SM.

To summarize, GUTs aim to find a unified layer from where we can extract all the phenomenological verified data of the SM. The search incorporated large Lie groups that might have the SM gauge group as a subgroup, such as $SU(5)$, flipped $SU(5)$, $SO(10)$, and E_6 simple groups models, Pati-Salam Model with the symmetry SU(4) × SU(2) × SU(2), and even semi-simple groups such as $SU(3) \times SU(3) \times SU(3)$ model. These are but few GUTs examples, the literature is still rich, each model being distinguished by the gauge symmetries imposed, fermions representations, mechanisms for SSB, and most importantly a layer of predictions, and physical implications that will play a major role in

[9]There are more possible decays, such as to other mesons and with a muon instead of an electron, but the electron/pion final state is the one with largest branching ratio for SU(5).

[10]The representations next to each path are $SO(10)$ rep. And the notations mean: $G_{51} = SU(5) \times U(1)$, $G_5 = SU(5)$,$G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$, $G_{421} = SU(4)_c \times SU(2)_L \times U(1)_{B-L}$, $G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, $G_{3211} = SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$.

Figure 1.1: Possible symmetry breaking patterns to the G_{SM}. This figure is taken from [29].



establishing the validity of the model as an acceptable, physically viable unified theory.

The second attractive conception to explore would be Supersymmetry (SUSY) [41], it is an extension of the SM by a new symmetry, but unlike GUTs, this symmetry is imposed between fermions and bosons themselves, such that for each fermion there corresponds its bosonic superpartner, and vice versa.
If, let's say, $\mathbf{Q}$ is a SUSY generator of Grassmann nature, then we have:

$$\mathbf{Q}|fermion> = |boson> \qquad\qquad \bar{\mathbf{Q}}|boson> = |fermion> \tag{1.39}$$

In such a theoretical framework, fermions and bosons are arranged in the so called supermultiplets, they typically contain an equal number of bosonic and fermionic degrees of freedom, with the same quantum numbers, except for their spin since it should differ by 1/2.

Supersymmetric theories aim to unify all 4 fundamental interactions, including gravity, and that is why it is predominant in theories such as supergravity, and superstring theories. This might be the most challenging aspect when it comes to this framework, but certainly it is not the only motivation guiding it. Some other motivations include:

- Solution to the Hierarchy problem.
- A base for candidate models to account for dark matter.
- Gauge couplings unification.
- Neutrinos masses.

In SUSY algebra [41], we deal with different representations, among these variations we can construct a representation on a multiplet of fields, these fields will act as components of the so called "superfield ", which lives in the so called superspace. These two notions are pretty useful to the description of particles in supersymmetric field theories.
The two most basic superfields are the chiral and vector ones, and any renormalizable supersymmetric Lagrangian pretty much can be constructed primarily out of these two.

A chiral superfield can contain spin 0 and 1/2 fields, while a vector superfield can contain spin 1/2 and 1 fields. As an illustration, we can represent quarks and leptons as an embodiment in chiral superfields, while gauge bosons in vector superfields.

For further illustration of the utility of Supersymmetric theories, we take a closer look at the Supersymmetric Standard Model(SSM) [41]. As the name indicates, SSM is an extension of the SM that incorporates the formalism of SUSY, which means it is only natural to seek the new version of the SM ingredients within the formalism. As a starter, the Minimal Supersymmetric Standard Model(MSSM) is an attractive route, where we maintain a minimal amount of new particles and interactions necessary. In such a model, to regard the chiral nature of the SM, we will have to represent our fermions as members of the chiral superfield, which will result in a Weyl fermion, and a complex scalar as physical degrees of freedom. For our gauge bosons, they will be part of the vector superfield. As a result, fermions, and bosons will acquire superpartners: for quarks q, they will have their spartners "squarks" $\tilde{q}$, leptons ℓ with "sleptons"$\tilde{\ell}$, and for gauge bosons: gluinos $\tilde{g}$, winos $\tilde{\omega}^{\mp}, \tilde{\omega}^3$, and bino $\tilde{b}$. As well as, a spartner for the Higgs, the higgsinos $\tilde{H}_1, \tilde{H}_2$.
In addition, an important phenomenological aspect that must be mentioned is the fact that there is an imposed symmetry on the MSSM model for the sake of conserving the baryon and lepton number, called the R-parity. Such a constraint prohibits any mixing between particles and their superpartners [42].
Of course, there is one missing ingredient: a mechanism for SSB. The mechanism would break the Supersymmetry [42], that is when an auxiliary field acquire a VEV causing SUSY to break, resulting in a goldstino field.
These are but soft key features about MSSM, its physical implications are what one would judge to be captivating. Not only that, such a model is capable of solving the hierarchy problem [42], since the additional particles counterbalance the quantum corrections to the Higgs mass, but also it provides signatures in leptonic colliders with the diverse channels that can help probe the sparticles [41]. The existence of those sparticles can play a major role in our measurements as well, potentially leading to solving issues, such as the anomalous magnetic moment of the muonμ, and increase precision in the values we already have. Additionally, it constitutes a candidate for dark matter, especially that the model involves the so called "the lightest supersymmetric particle" (LSP), which is already a weakly interacting massive particle (WIMP), making it a good start for explaining dark matter.

The MSSM is but one direction, there has been other attempts to incorporate SUSY such as: Next to Minimal Supersymmetric Standard Model (NMSSM), Supersymmetric E6 Grand Unified Theory (SE6-GUT), $U'(1)$ extended Supersymmetric Standard Model (U(1)' SSM), Gauge Mediated Supersymmetry Breaking (GMSB) Models, and many more. Each one motivated by the will to cure the Standard model, and even in their partial failure they still form a base of insights for further constructions.
Interestingly enough, problems such as proton stability, and the μ problem can be solved through replacing R-parity with a non-anomalous U(1)' gauge symmetry, which already shed the light at the importance of $U'(1)$ extensions of the Standard Model.

Another layer of physics BSM can be constructed via focusing only the gauge sector of the SM, and aim to extend it by one or more gauge symmetries. It turns out that such a process is very interesting to take a look at, and its importance can be seen through the fact that some of these extensions appear in SUSY, and GUTs symmetry breaking patterns.

1.2.2 Gauge extensions of the Standard model

In this subsection, we will dedicate some efforts to see how the gauge sector of the Standard Model can be extended by further gauge symmetry groups. Mainly navigating through 2 choices, in order to get a general outline of what this side of BSM formulations is looking like . Such a step is motivated by the importance of gauge extensions in building a full fledged model, as well as being a substructure of a fundamental theory of our universe. Examples of such theories have been described previously. The exploration also will help preparing the ground specifically for a $U(1)'$ extension of the SM, since it is our primary interest for an analytical study of a massive Z' boson.

Extending the gauge sector of SM can have many forms, among the various existing ones in the literature are: Abelian gauge extensions, and Non-Abelian gauge extensions of the SM. As the name indicates, Abelian gauge extension models are primarily focused on enhancing the SM structure by adding abelian gauge groups or symmetries, while Non-Abelian gauge extensions are tilted towards adding non-abelian gauge groups to the SM gauge sector. Each form has its own strength points and weaknesses, and they range from the simplest case to the most complicated, all depending on what is added, and what is omitted in the model considered.

A general abelian extension is widely studied, and the literature is full of various modifications and imposed constraints. Such models are built on adding the abelian group $U(1)$ while playing on various parameters, such as adding exotic fields, additional mechanisms such as the Seesaw Mechanism, Radiative Mechanism, adding new quantum numbers, or simply considering a general case without further specifications. Among such models we have the U_{B-L} models [43], which consider the difference between the baryon number B, and lepton number L a new quantum number for the various particles. Further elaborations on such models will take place in the following section.
In addition, we can extend the SM based on a very interesting idea, namely that we can gauge global symmetries generated by family-lepton numbers L_e, L_μ, and L_τ [44]. We can have these linear combinations for an anomaly-free global symmetry :

$$L_1 = L_e - L_\mu \qquad L_2 = L_e - L_\tau \qquad L_3 = L_\mu - L_\tau \tag{1.40}$$

resulting in these possible extensions:

$$G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{L_1} \tag{1.41}$$
$$G_2 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{L_2} \tag{1.42}$$
$$G_3 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{L_3} \tag{1.43}$$

of course, one impressive physical implication is the resulting new gauge bosons Z_1', Z_2', and Z_3' in each of the these models. The first two can couple to electrons, which already can tell us a lot about some experimental powers that these two models hold, while the third one couples to τ, ν_τ. Each model of course will vary from the mass and the coupling constants of the new gauge bosons. Such an abelian gauge extension can be further studied to incorporate a mechanism for neutrino masses [45], for dark matter [46], and even provide a solution for the magnetic moment of the muon [47].
Another possible model is a right-handed gauge symmetry $U(1)_R$ extension, which is a simple abelian gauge extension under which only the right-handed fermions are charged. It constitutes one of the promising models capable of incorporating the three right-handed neutrinos needed. Such a model has been explored in [48].
Not only that, but a possibility for a left-handed gauge symmetry $U(1)_L$ extension has

also been explored, under which we find charged left-handed fermions. And even a combination of both is possible, and the mathematical setup for such a model has been studied in [49].
There are many more examples, Abelian gauge extension with four generations, Abelian gauge extension with vector-like fermions[11] Froggatt Nielsen extension of the SM[12] and Stuckelberg extension of the SM [50], which will be tackled in details in the next chapter. In these previous models, the preserved idea was mainly one additional abelian symmetry, but in reality there are models incorporating more than one, which makes their field of studies rich of options, and only experimental evidences can filter the viable models from the non-viable ones.

Nevertheless, there is also a possibility for Non-Abelian gauge extensions, which open the door for more varieties. These extensions are based on extending the SM by a non-abelian symmetries. Given our fixed goals, we will only mention two possible models among many, namely the so called $G(221)$, and $G(331)$ models.
The $G(221)$ models are based on the symmetry $SU(2)_1 \times SU(2)_2 \times U(1)_X$, including examples such as [51]: Left-Right Models(LRM), Lepto-Phobic Models (LPM), Hadron-Phobic Models(HPM), and Fermio-Phobic Models (FPM). They all share different phenomenology despite the gauge group in common. $G(221)$ models can be classified by SSB patterns [51], organized in the following table:

Table 1.3: Two different SSB patterns in G(211) models.

pattern	The gauge structure before SSB	First stage	Second stage
I	$SU(2)_L \times SU(2)_2 \times U(1)_X$	$SU(2)_2 \times U(1)_X \to U(1)_Y$	$SU(2)_L \times U(1)_Y \to U(1)_{em}$
II	$SU(2)_1 \times SU(2)_2 \times U(1)_Y$	$SU(2)_1 \times SU(2)_2 \to SU(2)_L$	$SU(2)_L \times U(1)_Y \to U(1)_{em}$

The second breaking pattern can be present mostly in the Un-Unified Models (UUM), and Non-Universal Models (NUM).
Naturally, a complete model will involve the fermions charge assignments, the state of the Higgs sector, and of course Yukawa couplings. In addition to the most interesting physical implications of $G(211)$ viz. the existence of new heavy W', and Z' bosons.

However, the G(331) models are based on a symmetry of the form $SU(3)_C \times SU(3)_L \times U(1)_X$, such a model has the capacity to explain why there is 3 families of quarks and leptons, since the anomalies cancel only when the number of generations is divisible by three[13] [52]. And by that it is considered a model that holds the answer for one of the puzzling questions we had previously. In these type of models, the symmetry breaking pattern can occur via the VEV of three scalar triplets, as follows :

$$SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y \to U(1)_{em} \tag{1.44}$$

the hypercharge, and the electric charge within the model are:

$$Y = \beta T_8 + IX \qquad Q = \frac{Y + T_3}{2} \tag{1.45}$$

[11] Vector-like fermions are particles whose left and right-handed components transform in a similar way under the gauge group transformations of the model.
[12] Froggatt Nielsen mechanism was used to explain the masses and mixing of fermions within the model.
[13] In the SM, the gauge anomalies cancel for each of the 3 generations of quarks, and leptons. This model brought back the necessity of the 3 families together to cancel out the anomalies.

where, T_3, T_8 are the Gell-Mann matrices of the $SU(3)_L$, and β, I are parameters of the model.
The G(331) models offer physical predictions, such as charged spin-1 bosons, bileptons, which can be seen in e-e scattering, or even p-p scattering at the LHC [53, 54].

To summarize, gauge extensions of the Standard Model are of a great importance to the search of new physics, since they can be viewed as low energy effective theories of a high layer of new physics, such as SUSY, and GUTS. Not only that, but they are phenomenologically rich, and they offer answers to many questions we have about the observable universe. Procedures of gauge extending the SM can vary from just a general Abelian / Non-Abelian extension, to specific models with additional exotic ingredients, which makes the experimental endeavor already one step easier.
As we can see, along with some of the theories and models characterizing the new era of physics, there is one prominent physical implication, namely the existence of new gauge boson Z', which makes the curiosity around it even more persistent. It is only natural to seek deep knowledge about other locations for such a particle, and further justify our interest in it.

1.3 Some Samples of Z' models

As previously described, we have the need for new physics, and among many established models, $U'(1)$ models are prominent, and studied on a large scale. Such models are based on a $U'(1)$ gauge extension of the Standard Model, and in many cases it yields viable models for treating many of the SM deficiencies. We will start by taking a look at some of the interesting $U'(1)$ models, while focusing on those models due to the presence of a new massive Z' boson. The physics of this new gauge boson is exquisite and prosperous, we will discuss it briefly as well some methods used for probing its existence experimentally, and why we have this particular interest in it.

1.3.1 $U(1)_{B-L}$ model

Starting with the $U(1)_{B-L}$ model, it is based on $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ symmetry, with a new quantum number $B-L$ (the baryon number B - the lepton number L), that is conserved, and that also plays the role of a new charge. Under the new U_{B-L} symmetry fields transform as :

$$\psi_{L/R} \to e^{iY_{B-L}\theta(x)}\psi_{L/R} \tag{1.46}$$

via introducing a new gauge field C_μ accounting for the new symmetry extension, it will physically predict the existence of a new gauge boson, namely the new massive Z' [55], with a new coupling constant g'', and a charge Y_{B-L}. Moreover, the existence of this extra gauge boson is of a great experimental interest. For the sake of uniformity, we will label it: Z_{B-L}, indicating that it is the result of this B-L model. In the future examples, we will see that each model is distinguished by the values of the mass and coupling constants ranges, as well as the mechanisms for SSB, and of course its physical predictions.

After the SSB, the particle acquires a mass [56]:

$$M^2_{Z'_{B-L}} = 4g''^2\nu'^2 \tag{1.47}$$

ν' being the B-L symmetry breaking scale, acquired by the singlet complex scalar field χ leading to breaking the symmetry:

$$< \chi >= \frac{\nu'}{\sqrt{2}}. \tag{1.48}$$

The discovery of such a particle can take place at the LHC, mainly via di-lipton channels $Z'_{B-L} \rightarrow \ell^+\ell^-$, they seem to be more efficient compare to $Z'_{B-L} \rightarrow q\bar{q}$. The Z'_{B-L} can be discovered via the e^+e^- decay channel, one might expect :

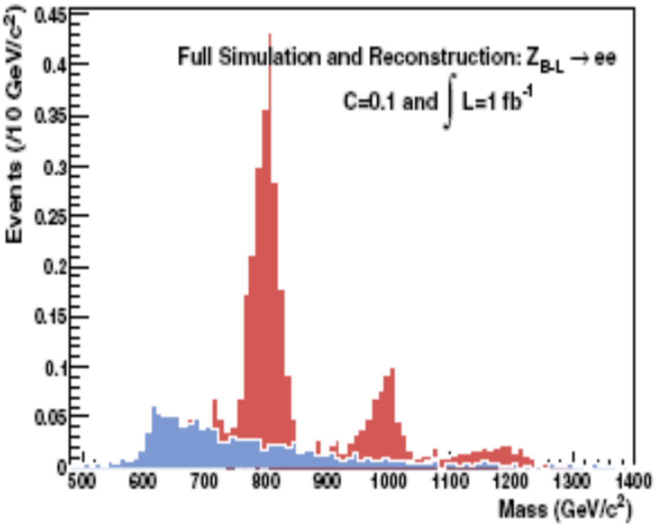


Figure 1.2: A simulation showing the peak of Z'_{B-L} boson, through the e^+e^- decay channel in the mass $M_{Z'_{B-L}}$ region comprised between 800 and 1000. The data was taken from [56].

This model has a great phenomenological potential [56], since it accounts for the experimental data of neutrinos light masses, given that their masses are produced via the seesaw mechanism. Add to that its ability to account for the right handed neutrinos, that is through the existence of 3 SM singlet fermions predicted by the anomaly cancellation conditions. Labeled the 3 right-handed neutrinos, and it accounts for their mixings, through a matrix called the neutrino mass matrix. Interestingly, the lightest particle among these right handed neutrinos fulfill the conditions for a dark matter particle, which shows that the model has the potential to account for dark matter. The model also provides an explanation for the observed baryon asymmetry of the Universe, and that is via the process of leptogenesis.

1.3.2 Z' in LRSM and GUTs

Next on the list of Z' models, we have the so called Left-Right Symmetric Model [55], it introduces us to a faint touch of a GUT scenario, and it is built upon the gauge group:

$$G_{LRSM} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}}. \tag{1.49}$$

As it shows, it introduces an extra $SU(2)_R$ leading to right handed singlets to be grouped together forming right handed doublets under the symmetry, which also means that the model treats left and right handed fields equally. The model also exhibits a higher degree

of symmetry compared to the SM, since LRSM is symmetric under parity transformations, such an additional symmetry is broken when the model is reduced to the SM.
The model [57] offers an explanation for the 3 right-handed neutrinos masses, also via the See-Saw mechanism, but it required an additional term in the Lagrangian called the Majorana mass term, in order to adjust the mass of neutrinos to the observed data. It incorporates charge quantization as a mean to have a renormalizable, and anomaly-free theory. Not only that, but it solves the CP problem due to mirror-symmetry.

In LRSM, the unphysical states $\vec{W_{L\mu}}, \vec{W_{R\mu}}, B_\mu$, with the associated coupling constants g_L, g_R, g' respectively, will yield after the mixing, the 3 SM physical gauge bosons $W_L^{\mp}, Z, \gamma$, in addition to 3 new massive gauge bosons $W_R^{\mp}, Z'_{LRSM}$. Naturally, these gauge bosons will need a mechanism to acquire the property of a mass. Potentially, the model [58] allow for two symmetry breaking paths. In the minimal version of the LRSM, there are 3 scalar fields Φ_1, Φ_2, and Φ_3, the two first are triplets under $SU(2)_R$, while the last one is a bi-doublet under the symmetry $SU(2)_L \times SU(2)_R$. Needless to say, the model requires a scalar sector to break (1.49) into U_{em}. Therefore, using these scalar fields, we can break the symmetry using the old fashion, we break (1.49) to the SM gauge group, followed up by the usual SSB leading to U_{em}. Such a path is constrained by the fact that the VEV of the left triplets need to be taken as zero. Another path would be to break the symmetry in the following manner [58]:

$$SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}} \rightarrow SU(2)_L \times U(1)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em} \tag{1.50}$$

The intermediate extra step is just the SM with a $U(1)$ extension. Of course, these steps take place thanks to 2 complex scalar triplets and one bi-doublet.

After the process of SSB, the particles gain masses(in the approximation $v_2 >> v'$ [58]:

$$M_Z^2 \approx v'^2(g^2 + g''^2) \tag{1.51}$$

$$M^2_{Z'_{LRSM}} \approx \frac{2g^2 v_2^2}{g^2 - g''^2} + v'^2(g^2 - g''^2) \tag{1.52}$$

$$M^2_{W_L} = g^2 v'^2 \tag{1.53}$$

$$M^2_{W_R} = g^2(v_1^2 + v_2^2 + v'^2) \tag{1.54}$$

where $v' = \frac{1}{2}v_H$, and v_1, v_2, are the VEV of Φ_1, Φ_2, where v_3, v_4 are the VEV of Φ_3, with $v'^2 = v_3^2 + v_4^2$.

As we are capable of inferring so far, there are many locations where the Z' neutral gauge boson arises, being it within a GUT scenario or outside. Another possible location for a Z' would be models coming from a E_6 grand unification, where the E_6 symmetry breaking can take the following from:

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi. \tag{1.55}$$

And as we mentioned in section 1.2.1, the $SU(5)$ symmetry will break into 1.2.
In such a case, the resulting neutral gauge boson can be a linear combination of: Z_ψ resulting from $U(1)_\psi$, and Z_χ coming from $U(1)_\chi$, such as [59]:

$$Z_6 = Z_\psi cos(\theta) + Z_\chi sin(\theta). \tag{1.56}$$

Other possible intermediate steps can be illustrated in figure (1.3).

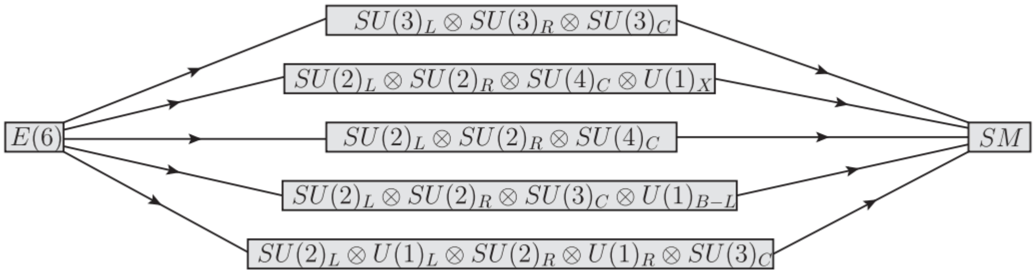


Figure 1.3: An illustration of some possible intermediate steps throughout the breaking of the E_6 symmetry [60].

This extra gauge boson can be discovered experimentally at the LHC. Hence, experimental quantities such as decay widths are of a great importance. The figure (1.4) shows the total decay width $\Gamma_{tot}(Z'(\theta))$[14] for certain different values of $M_{Z'(\theta)}$ and θ.

$M_{Z(\theta)}$	500	600	700	800	900	1000
$\Gamma_{tot}(\theta = 0)$	4.4217	5.3067	6.1905	7.0752	7.9581	8.8464
$\Gamma_{tot}(\theta = 37.8)$	5.7825	6.9387	8.0961	9.2520	10.4088	11.5653
$\Gamma_{tot}(\theta = 90)$	9.6768	11.6112	13.5465	15.4824	17.4462	19.3524
$\Gamma_{tot}(\theta = 127.8)$	8.7723	10.5312	12.2805	14.0349	15.7896	17.5443

Figure 1.4: Total decay width $\Gamma_{tot}(Z'(\theta))$ for certain values of $M_{Z'(\theta)}$ and θ. The width Γ is used for $\Gamma(Z'(\theta)).\Gamma(Z'(\theta))$, and $M_{Z'(\theta)}$ are in GeV.

It is notable that E_6 models can be studies in both nonsupersymmetric and supersymmetric lens. In the supersymmetric case, the $U(1)'$ symmetry can be broken by the VEV of the scalar partners of the S and s the conjugate of the right-handed neutrino ν^c(breaking R-parity as well).

In conclusion, Z' physics is not limited to those few examples mentioned above, there are more and more theoretical attempts to study this neutral gauge boson, given that it is expected to be discovered at the LHC in the future. What also can be observed is the importance of a simple $U(1)'$ extension of the Standard model, given how diverse the locations in which we can encounter such an additional symmetry. And for that particular reason, we will direct our attention, in the next chapter, to how we can extend our SM Lagrangian in grand details. Discussing the implications of such an extension, and finishing by putting a strong emphasis on one among many ways of obtaining the mass term, namely "the Stueckelberg Mechanism". This is motivated by how simple the model is for a Z' starter hunt, in addition to the elegance of the procedure, and of course the prediction of the Z' neutral gauge boson's properties.

[14] The $Z'(\theta)$ here is the same as gauge boson Z_6 mentioned in (1.56).

Chapter 2

A Stueckelberg Extension of the Standard Model

An additional $U'(1)$ symmetry for the SM has proven itself worthy of our consideration. It is a simple process by which we can put our initial footsteps into the realm of physics Beyond the Standard Model. For this small endeavor, we will draw our path from a $U'(1)$ extension of the SM, how such a process influences the structure of the SM, the various changes accompanied by adding such a symmetry, then leading our way to the potential physically viable consequences.

Throughout this journey, various physical implications are discussed, primarily the additional Z' neutral gauge boson. We will attempt to generate the mass term, and show how an alternative way to the Higgs effect, namely " Stueckelberg mechanism ", can help generate the mass for this new Z' without necessarily breaking the new symmetry added. Such a path shows how seeking economical, simple ways of curing the SM pays off, and they are good candidate for preparing the ground for more extensive theoretical work.

2.1 $U(1)_X$ Extension of The Standard Model

As mentioned before, our main aim is to extend the SM by an abelian $U(1)'$ gauge symmetry. A general $U(1)$ gauge extension can be performed via simple steps, the framework will involve:

- Introduce the symmetry.
- Introduce the vector content associated with symmetry, and construct the field strength tensor.
- Introduce a new coupling constant.
- Construct the new covariant derivative.
- Study the various changes on the SM structure, via setting up the mathematical framework of the model.
- Infer the various physical predictions, such as the existence of a new gauge boson Z'.
- Infer its properties (mass, charge, coupling constant..etc), and its impact on the various sectors of the SM.
- Clarify the constraint imposed on the new values of the new parameters, so that they fit with the already established SM values.

And Like any theoretical work, it is necessary to draw the edges on what we will be dealing with in this section, and this will be accomplished via stating the various assumptions taken. Such assumptions are summarized as follows:

- Only one additional $U(1)$ element is added to the gauge group.
- We are considering minimal particle content.
- Automatic anomaly cancellation, meaning no need for anomaly cancellation mechanisms[1].
- The $U(1)_X$ charge is not related to the charge operator[2].

It is recommended to mention that the $U(1)_X$ model is considered as a generalization, since the same structure can be found in models such as the minimal $B-L$ model [62], where the extra abelian symmetry will be assimilated to the $B-L$ gauge symmetry. Same thing can be said for $B+L$ gauge symmetry [63], and $L_\mu - L_\tau$ gauge symmetry. Which gives the advantage of only being restricted to a general case. Furthermore, with minimal particle content, the model offers phenomenological consequences that are of a great interest to us, since it can accommodate cosmological inflation layouts [64], as well as candidate for DM [65], and even add 3 generations of right-handed neutrinos and explain the origin of their tiny masses [66]. Hence, removing the obscurity around fundamental mysteries of the SM.

A final point to mention, the distinction of the various $U'(1)$ models existing in the literature can be seen in:

(a) The coupling constants.

(b) The symmetry $U(1)'$ breaking scale.

(c) The type of scalar responsible for the $U(1)'$ breaking.

(d) The charges of the SM fermions and Higgs doublet.

(e) The necessity of additional exotic fields for anomaly cancellation.

After establishing such a clarification, we are ready to build gradually the process of the extension. Adding the new abelian gauge symmetry will force some changes [11], starting with the new form of the gauge group:

$$G_{SM_X} = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \tag{2.1}$$

the extra symmetry will have a gauge field associated with it, labeled C_μ.
For the sake of simplicity, we will work with the SM Lagrangian in this from:

$$\mathscr{L}_{SMX} = \mathscr{L}_{f_X} + \mathscr{L}_{gauge_X} + \mathscr{L}_{\phi_X} \tag{2.2}$$

focusing on $\mathscr{L}_{gauge}$, extending 1.2 by an extra $U(1)_X$ will imply :

$$\mathscr{L}_{gauge_X} = -\frac{1}{4}G^i_{\mu\nu}G^{\mu\nu i} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} \tag{2.3}$$

where $C_{\mu\nu}$ is the associated strength tensor:

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu. \tag{2.4}$$

[1] Such as the Green-Schwarz mechanism.

[2] There is a possibility in which such a relatedness can be considered. Such as in [61], where the charge operator took the form $Q = T_3 + Y + X$.

The new form for the gauge transformation will be:

$$\phi(x) \to \phi'(x) = e^{-igT_a\theta_a - \frac{i}{2}g'_Y Y_\phi \theta'_Y \mathbb{I} - \frac{i}{2}g_X X_\phi \theta'_X \mathbb{I}} \phi(x) \tag{2.5}$$

where, g'_Y, g'_X denotes the new associated coupling constants, and X_ϕ being the new weak hypercharge of the scalar doublet ϕ.

The new covariant derivative :

$$\bar{D}_\mu = \partial_\mu + \frac{ig_s}{2}\vec{\lambda}.\vec{G}_\mu - \frac{iq_W g}{2}\vec{\tau}.\vec{W}_\mu - \frac{iq_B g'_Y}{2}B_\mu - \frac{iq_C g_X}{2}C_\mu \tag{2.6}$$

$q_{W,B,C}$ are the charges which correspond to the fields on which $\bar{D}_\mu$ is applied to.

The modifications on the fermionic sector will be displayed as follows :

$$\mathscr{L}_{f_X} = \sum_{i=1}^{3}(\bar{q}_{iL} i\bar{\not{D}} q_{iL} + \bar{\ell}_{iL} i\bar{\not{D}} \ell_{iL} + \bar{u}_{iR} i\bar{\not{D}} u_{iR} + \bar{d}_{iR} i\bar{\not{D}} d_{iR} + \bar{\ell}_{iR} i\bar{\not{D}} \ell_{iR}) \tag{2.7}$$

given that :

$$\bar{D}_\mu = \partial_\mu - \frac{ig}{2}\sum_{a=1}^{3}\tau^a W^a_\mu - ig'_Y \frac{Q_{f_Y}}{2}B_\mu - ig_X \frac{Q_{f_X}}{2}C_\mu \tag{2.8}$$

$Q_{f_{Y,X}}$ are the fermions charges under $U(1)_Y$, and $U(1)_X$.

The Higgs part :

$$\mathscr{L}_{\phi_X} = (\bar{D}_\mu \phi)^\dagger \bar{D}^\mu \phi + V(\phi) \tag{2.9}$$

where:

$$\bar{D}_\mu \phi = (\partial_\mu - \frac{ig}{2}\tau_a W^a_\mu - \frac{ig'_Y Y_\phi}{2}B_\mu - \frac{ig_X X_\phi}{2}C_\mu)\phi \tag{2.10}$$

Furthermore, considering the most general case implies that the fermion and scalar contents are not restricted to be the same as in the SM. For example, fermions may include right-handed neutrinos ν_R, while the scalar sector may include as well more than one Higgs doublet, or even singlets.

Such an internal structure allow for multiple consequences, among these consequences: a kinetic mixing between the abelian fields B_μ, C_μ, and an additional gauge boson, labeled Z' gauge boson. In addition to a possibility for a mass mixing between the SM Z boson, and the new Z'.

2.1.1 Kinetic mixing

Models with more than one U(1) gauge symmetry always permit the existing of elements that mix the kinetic terms of the Abelian gauge fields, given it does not violate any known symmetries [9]. A mixing can only occur if there is more than one field strength tensor, and they are each neutral under the gauge symmetry considered, which is substantially why we don't see mixed terms in the context of the SM. And since we are in the context of an abelian extension of the SM by a $U(1)_X$ symmetry, then a mixing between the the two $U(1)'$s is allowed. The kinetic term in such a case can take the general form :

$$\mathscr{L}_{Kinetic} = \alpha B_{\mu\nu}B^{\mu\nu} + \beta C_{\mu\nu}C^{\mu\nu} + \gamma B_{\mu\nu}C^{\mu\nu} \tag{2.11}$$

after scaling the gauge fields:

$$\mathscr{L}_{Kinetic} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{2\rho}{4}B_{\mu\nu}C^{\mu\nu}. \tag{2.12}$$

The last term is the so-called the gauge kinetic mixing term that is Lorentz and gauge invariant.
And the requirement that $|\rho| < 1$[3], where ρ is some coefficient that parametrizes the strength of this so-called kinetic mixing operator.

Interestingly enough, even if the gauge kinetic mixing is set to zero at some high energy scale, for example the GUT scale, it can always be radiatively generated at lower energy scales via quantum corrections [67]. There are ways to actually get rid of the kinetic mixing term. One way is to eliminate it by an appropriate rotation in the space of the abelian gauge fields [9].
In order to see how such a term can be eliminated, we consider an example based the following non-unitary transformations:

$$\begin{aligned} \tilde{B}_\mu &= B_\mu + \rho C_\mu \\ \tilde{C}_\mu &= \sqrt{1-\rho^2}C_\mu \end{aligned} \tag{2.13}$$

as a result:

$$\begin{aligned} B_{\mu\nu} &= \tilde{B}_{\mu\nu} - \frac{\rho}{\sqrt{1-\rho^2}}\tilde{C}_{\mu\nu} \\ C_{\mu\nu} &= \frac{1}{\sqrt{1-\rho^2}}\tilde{C}_{\mu\nu} \end{aligned} \tag{2.14}$$

replacing in (2.12):

$$\begin{aligned} \mathscr{L}_{Kinetic} = &-\frac{1}{4}\left[\tilde{B}_{\mu\nu} - \frac{\rho}{\sqrt{1-\rho^2}}\tilde{C}_{\mu\nu}\right]\left[\tilde{B}^{\mu\nu} - \frac{\rho}{\sqrt{1-\rho^2}}\tilde{C}^{\mu\nu}\right] \\ &- \frac{1}{4}\frac{1}{\sqrt{1-\rho^2}}\tilde{C}_{\mu\nu}\tilde{C}^{\mu\nu} - \frac{2\rho}{4}\left[\tilde{B}_{\mu\nu} - \frac{\rho}{\sqrt{1-\rho^2}}\tilde{C}_{\mu\nu}\right]\frac{\tilde{C}^{\mu\nu}}{\sqrt{1-\alpha^2}} \end{aligned} \tag{2.15}$$

implying :

$$\mathscr{L}_{Kinetic} = -\frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{C}_{\mu\nu}\tilde{C}^{\mu\nu}. \tag{2.16}$$

Thus, the kinetic mixing term was successfully eliminated. One reason why we would seek such an elimination process is the fact that there are no observable signs of its existence in the experimental playground. Therefore, building a kinetic-mixing-free model would be more preferable given the current data that we own.

However, it is always appealing to mention what are major implications that can arise from these kinetic mixing terms, one implication is the mixing between the Z and Z' coming from the symmetry breaking, with a certain angle that can be determined, and such a mixing is ρ-dependent [9]. To see clearly how it arises, let's first see the modifications imposed on the covariant derivative due to (2.14):

$$\begin{aligned} D_\mu &= \partial_\mu - \frac{ig}{2}\tau_a W^a_\mu - \frac{ig'Y}{2}B_\mu - \frac{ig_X X}{2}C_\mu \\ \Longrightarrow D_\mu &= \partial_\mu - \frac{ig}{2}\tau_a W^a_\mu - \frac{ig'Y}{2}\tilde{B}_\mu - \frac{i}{2}\left[\frac{g_X X}{\sqrt{1-\rho^2}} - \frac{\rho g' Y}{\sqrt{1-\rho^2}}\right]\tilde{C}_\mu \end{aligned} \tag{2.17}$$

[3]Positive kinetic energy.

we can notice that $\tilde{B_\mu}$ couples to the same generator as B_μ.
Furthermore, the gauge bosons mass matrix would then be written in terms of the orthogonal fields A_0, Z_0, Z'_0, and it takes the general form:

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M^2_{Z_0} & \delta \\ 0 & \delta & M^2_{Z'} \end{pmatrix} \tag{2.18}$$

The phtoton is clearly the massless eigenstate. And as we can see a non-zero δ implies the $Z - Z'$ mixing, and when ρ does not coincide with 0, the fields A_0, Z_0, Z'_0 are not orthogonal. Of course, they can be related to the orthogonal fields A, Z, Z' via some non-unitary transformation:

$$\begin{pmatrix} A_0 \\ Z_0 \\ Z'_0 \end{pmatrix} = S \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} \tag{2.19}$$

in such a case, the mass matrix can be written in terms of the orthogonal fields:

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a & b \\ 0 & b & c \end{pmatrix} \tag{2.20}$$

where the parameters [9]:

$$\begin{aligned} a &= M^2_{Z_0} \\ b &= \frac{-\rho sin\theta}{\sqrt{1-\rho^2}} M^2_{Z_0} + \frac{\delta}{\sqrt{1-\rho^2}} \\ c &= \frac{M^2_{Z'}}{1-\rho^2} + \frac{-2\rho sin\theta}{1-\rho^2}\delta + \frac{\rho^2 sin^2\theta}{1-\rho^2} M^2_{Z_0} \end{aligned} \tag{2.21}$$

clearly, we can observe that for the mixing to vanish, we require the condition that $b = 0$, which implies:

$$\rho = \frac{\delta}{M^2_{Z_0} sin\theta} \tag{2.22}$$

which indicates straightforwardly that the $Z - Z'$ mixing is ρ-dependent. It is also apparent that due to the mixing there will be some modifications in the properties of the SM Z boson.

It is worthwhile to mention that a $Z - Z'$ mixing can be eliminated via the existence of discrete symmetries. The importance of this resides, first and foremost, in the fact that there is no experimental data validating such a mixing, which will necessarily requires a way to remove it, and it turns out discrete symmetries are a natural way to forbid such a mixing. It can be easily verified. So let's consider the discrete symmetry of the following gauge bosons associated with the two $U(1)'$s in the model :

$$B_1 \leftrightarrow C_1 \tag{2.23}$$

the kinetic Lagrangian that respects the previous symmetry have the form:

$$\mathscr{L}_{Kinetic} = -\frac{1}{4} B_{1\mu\nu} B_1^{\mu\nu} - \frac{1}{4} C_{1\mu\nu} C_1^{\mu\nu} - \frac{2\rho}{4} B_{1\mu\nu} C_1^{\mu\nu}. \tag{2.24}$$

At first sight, one must say that the last term on the right can be identified as a mixing term, but a simple linear combination of B_1, and C_1 can prove otherwise. Considering :

$$B = \frac{B_1 + C_1}{\sqrt{2}} \qquad\qquad C = \frac{B_1 - C_1}{\sqrt{2}} \tag{2.25}$$

$$\implies B_{1\mu\nu} = \frac{1}{\sqrt{2}}[B_{\mu\nu} + C_{\mu\nu}] \qquad\qquad C_{1\mu\nu} = \frac{1}{\sqrt{2}}[B_{\mu\nu} - C_{\mu\nu}] \tag{2.26}$$

with such a step, (2.24) will be re-arranged to showcase that there is no kinetic mixing term. Therefore, by mere existence of discrete symmetries within the model we can ensure that no $Z - Z'$ mixing is allowed.

As a final note, in the non-abelian case, the field strengths $B_i^{\mu\nu}$ and $C_i^{\mu\nu}$ will not be considered gauge invariant quantities, and thus any terms proportional to them will be forbidden.
Now that we have briefly talked about the potential kinetic mixing that can arise due to the abelian extension we considered, it is safe to assume from now on that we won't be considering it in future development for the sake of simplicity, and to keep the model experimentally consistent. In the following subsection, we would direct our efforts towards the mass term, and how can the neutral gauge bosons within our model gain their masses.

2.1.2 Neutral Gauge Bosons Masses

Naturally, to obtain the mass term, we will employ the well-established Higgs mechanism. The Higgs doublet, before SBB and after, famously takes the form:

$$\phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\chi_3) \end{pmatrix} \rightarrow <\phi> = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h) \end{pmatrix} \tag{2.27}$$

Given that we are interested in obtaining the mass for the Z, Z' and γ, it is useful to focus on the relevant factors only:

$$\mathscr{L}_M \rightarrow (\bar{D}_\mu <\phi>)^\dagger (\bar{D}_\mu <\phi>) = |\bar{D}_\mu <\phi>|^2 \tag{2.28}$$

and $\bar{D}_\mu$ is given by 2.10.
Making use of :

$$\frac{1}{2}\tau_a W_\mu^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{W_\mu^3}{\sqrt{2}} & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{\sqrt{2}} \end{pmatrix} \tag{2.29}$$

W_μ^- , W_μ^+ are given in 1.18.
We find :

$$|\overline{D}_\mu <\phi>|^2 \rightarrow |\frac{-ig(v+h)}{2} W_\mu^+|^2 + |\frac{ig(v+h)}{2\sqrt{2}} W_\mu^3 - \frac{i(v+h)}{2\sqrt{2}} g'_Y Y_\phi B_\mu - \frac{i(v+h)}{2\sqrt{2}} g_X X_\phi C_\mu|^2 \tag{2.30}$$

given our interest, we can take only the relevant terms from the neutral current. Leading to :

$$\begin{aligned} &\rightarrow \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v^2}{8} W_\mu^3 W^{3\mu} + \frac{g_Y'^2 Y_\phi^2 v^2}{8} B_\mu B^\mu + \frac{g_X^2 X_\phi^2 v^2}{8} C_\mu C^\mu \\ &- \frac{g g'_Y Y_\phi v^2}{8} W_\mu^3 B^\mu - \frac{g g_X X_\phi v^2}{8} W_\mu^3 C^\mu - \frac{g'_Y g Y_\phi v^2}{8} B_\mu W^{3\mu} \\ &+ \frac{g'_Y g_X Y_\phi X_\phi v^2}{8} B_\mu C^\mu - \frac{g g_X X_\phi v^2}{8} C_\mu W^{3\mu} + \frac{g'_Y g_X Y_\phi X_\phi v^2}{8} C_\mu B^\mu . \end{aligned} \tag{2.31}$$

Such an expression can be put in the following form :

$$\mathscr{L}_M \to \frac{g^2v^2}{4}W_\mu^+W^{\mu-} + \begin{pmatrix} W^{3\mu} & B^\mu & C^\mu \end{pmatrix} M \begin{pmatrix} W_\mu^3 \\ B_\mu \\ C_\mu \end{pmatrix} \tag{2.32}$$

where the matrix M can be easily deduced :

$$M = \begin{pmatrix} \frac{g^2v^2}{8} & -\frac{gv^2}{8}g_Y'Y_\phi & -\frac{gv^2}{8}g_XX_\phi \\ -\frac{gv^2}{8}g_Y'Y_\phi & \frac{v^2}{8}(g_Y'Y_\phi)^2 & \frac{v^2}{8}g_Y'g_XY_\phi X_\phi \\ -\frac{gv^2}{8}g_XX_\phi & \frac{v^2}{8}g_Y'g_XY_\phi X_\phi & \frac{v^2}{8}(g_XX_\phi)^2 \end{pmatrix} \tag{2.33}$$

after diagonalization:

$$M_{Diag} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & \frac{v^2}{8}(g^2 + g_X^2X_\phi^2 + g_Y'^2Y_\phi^2) \end{pmatrix} \tag{2.34}$$

we can identify the mass of the Z boson : $M_Z = \frac{v^2}{8}(g^2 + g_X^2X_\phi^2 + g_Y'^2Y_\phi^2$, while the remaining terms can be identified with the mass of the massless photon and the Z' boson: $M_\gamma = M_Z' = 0$

Clearly, we have an issue. The Z' boson is massless, and this is not phenomenologically possible, otherwise it would have been observed already. The origin of the issue will be tracked down to the additional symmetry U_X we added, we conclude that the symmetry wasn't broken by the Higgs doublet. Resulting in this symmetry breaking pattern at the electroweak scale:

$$SU(2)_L \times U(1)_Y \times U(1)_X \to U(1)_{em} \times U(1)_Z' \tag{2.35}$$

Therefore, we need to give the Z' its mass. This can be achieved via multiple methods, one of them is via the Higgs mechanism, where we will introduce a new singlet complex scalar field χ with a certain VEV v_χ to break the extra $U(1)_X$. The singlet's VEV should be about an order of magnitude larger than the typical Higgs doublet's VEV, since a Z' has not yet been observed. Of course, such a procedure will have additional phenomenological implications, that can be studied within the frame of a particular model. More details on the approach can be consulted in [10].

Another possibility, which turns out to be our main interest in this section is an approach where the Z' can obtain its mass without necessarily breaking the extra symmetry we added. Such a mechanism is the well-known "Stueckelberg mechanism". Through such a mechanism, we will preserve the symmetry, thus keeping the model at a minimum number of parameters, and the neutral gauge boson will acquire its mass via a coupling to a new scalar field σ that we will introduce. More details on the approach will be discussed in the upcoming section.

2.2 Stueckelberg Extension

One of the most interesting paths to investigate when trying to extend the SM is a Stueckelberg way of extending the SM, in which the extra gauge boson resulting from the additional $U(1)_X$ symmetry can couple to an axionic scalar field[4], namely Stueckelberg field σ. In

[4] We refer to σ as an axionic scalar because of the pseudo-scalar nature that it possesses.

this following section, we will attempt to take a look at the minimal [5]Stueckelberg contribution to the extended SM, followed up by how the new Z' boson can obtain its mass, in a relatively new manner, without breaking the symmetry, and via a coupling to the field σ.

Originally, in 1938 Stueckelberg [68] proposed to reformulate the Proca action for the massive vector field by introducing a new scalar field $B(x)$ of positive definite metric accompanied with the four-vector components A_μ, resulting in five fields, to describe the 3 polarizations of a massive vector field, while maintaining Lorentz covariance, and gauge invariance. Thus, Stueckelberg's field was a brilliant idea to recover the gauge invariance broken by adding a mass term. He postulated that $B(x)$ obeys the same equation as A_μ and has the same mass:

$$(\partial^2 + m)B(x) = 0 \tag{2.36}$$

A historical review about Steuckelberg method can be found in reference [68].

The general scheme followed in order to make use of Stueckelberg's ideas starts with identifying the gauge invariant, and the non-gauge invariant elements in the Lagrangian. Taking as an example massive vector fields, the gauge invariant element would be the Maxwell Lagrangian, while the non-invariant part would simply be the mass term. So, after making such an identification, Stueckelberg field will be included as a parameter of the gauge transformation of the invariant part, and its own gauge symmetry is defined as the composition of original gauge transformations, which will result in a compensation by the gauge transformation of Stueckelberg field to the change of the non-invariant part of the Lagrangian.

Let's see how the above description fits with a simple case, mainly for a massive vector A_μ. In such a scenario, Stueckelberg field will be denoted $\sigma(x)$. Stueckelberg Lagrangian will then be [69]:

$$\mathscr{L}_{Stueck} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(mA_\mu + \partial_\mu\sigma\right)\left(mA^\mu + \partial^\mu\sigma\right) \tag{2.37}$$

with an obvious mass term. And the gauge transformations will be [6]:

$$\begin{aligned} A_\mu &\to A_\mu + \partial_\mu\epsilon \\ \sigma &\to \sigma - m\epsilon \end{aligned} \tag{2.38}$$

the gauge invariance through such transformations of the Lagrangian can be easily verified. The total Lagrangian takes then the from, after fixing the gauge:

$$\mathscr{L}_{tot} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\xi m^2}{2}\sigma^2 \tag{2.39}$$

where the gauge fixing term added was:

$$\mathscr{L}_{GF} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi m\sigma\right)^2 \tag{2.40}$$

where it can be seen that the mixing terms between A^μ and σ has been canceled out, we say that the fields have been decoupled.

We can conclude that the general scheme includes two parts. One is the extension of the standard mass term through a mixing term that involves a differential of the scalar

[5]Minimal in the context of only adding one σ field, and one abelian vector field C_μ.

[6]Through such a split, σ takes the role of the longitudinal component of the massive vector

field added. The second part is the choice of a special gauge fixing term to cancel out the mixing between the scalar and the gauge field. And we managed to acquire a massive vector field without spoiling gauge invariance.

We are now motivated to use Stueckelberg's ideas in the context of the extended SM. To see specifically how the previous idea can be used in the context of the SM, [70] has a detailed work on how a Stueckelberg massive $U(1)$ can be combined with the SM, and what are the phenomenological consequences of such a process. In our case, we want to take Stueckelberg idea, and apply it to an extended version of the SM.

2.2.1 Stueckelberg Lagrangian

As indicated earlier, we are interested in a Stueckelberg's contribution to the SM, more precisely a minimal Stueckelberg extension of the SM, while making sure that the gauge invariance is not spoiled. In the following context, we will explore how the SM Lagrangian will be extended by a simple abelian vector, in addition to a Stuecklberg's kind of coupling through a Stueckelberg field σ. For the the sake of simplicity, we won't bother taking the entire SM Lagrangian, we will focus only on the relevant parts for the extension:

$$\mathscr{L}_{SM} \supset -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - D_\mu\phi^\dagger D^\mu\phi - V(\phi^\dagger\phi) \tag{2.41}$$

where like usual, $W_{\mu\nu}, B_{\mu\nu}$ are the $SU(2)_L, U(1)_Y$ field strengths respectively, and $\phi$is the Higgs doublets with a potential $V(\phi^\dagger\phi)$ attaining its minimum at $\frac{v}{2}$.

For a minimal Stuckelberg extension [71], we only add minimal fields content to the previous Lagrangian, mainly we add one more abelian vector field C_μ associated to a $U(1)_X$ with a field strength $C_{\mu\nu}$, and one axionic scalar σ. Leading to the effective Lagrangian:

$$\begin{aligned} \mathscr{L}_{StSM} &= \mathscr{L}_{SM} + \mathscr{L}_{St} \\ \mathscr{L}_{St} &= -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{1}{2}\left(\partial_\mu\sigma + M_1 C_\mu + M_2 B_\mu\right)^2 \end{aligned} \tag{2.42}$$

$M_{1,2}$ are mass parameters related to the Stueckelberg extension.

The gauge invariance for $U(1)_Y$:

$$\begin{aligned} \delta_Y B_\mu &= \partial_\mu \epsilon_Y \\ \delta_Y \sigma &= -M_2 \epsilon_Y \end{aligned} \tag{2.43}$$

and for $U(1)_X$:

$$\begin{aligned} \delta_X C_\mu &= \partial_\mu \epsilon_X \\ \delta_X \sigma &= -M_1 \epsilon_X \end{aligned} \tag{2.44}$$

It is important to mention that in the context of our extension, the σ field has Stueckelberg coupling to all the abelian gauge bosons, i.e. B_μ, C_μ, while we assume that the charged gauge bosons are neutral under the $U(1)_X$ symmetry.
In order to observe the effects of a minimal Stueckelberg extension, it is required to generate the modified version of the mass matrix, which is what will attempt to do at the point of the development.

2.2.2 Stueckelberg Mechanism and the Mass matrix

The expected question that would arise at this level is how can we possibly generate the mass of the gauge bosons, and naturally we would lean towards familiarity, that is to say, a symmetry breaking process with the Higgs mechanism. Such an initial guess would take us from the usual electro-weak symmetry breaking via the Higgs doublet to breaking the additional $U(1)_X$, such a procedure is similar to that of the SM.
We can safely expect a symmetry breaking pattern to take the following form :

$$SU(3)_C \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Higgs } \phi} \times U(1)_X. \tag{2.45}$$

And in order to give the new Z' a mass we would break the remaining symmetry using, let's say, the scalar field χ:

$$SU(3)_C \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Higgs } \phi} \times \underbrace{U(1)_X}_{\chi} \rightarrow SU(3)_C \times U(1)_{em}. \tag{2.46}$$

Such a thinking process is completely valid, the mass of this boson will be generated in several ways, for instance via spontaneous symmetry breaking of a scalar, which is singlet, or doublet under the $SU(2)$ symmetry group. But it is here where Stueckelberg's idea can play a role. Due to his thought process, we can explore an alternative to the previous mechanism, and obtain a massive Z' boson without having to break the additional symmetry, especially that breaking it will cost us the price of additional parameters that have to be finely tuned. This is what "Stueckelberg mechanism" is capable of offering to our current subject of study.

It is noteworthy to mention that the SM's Higgs mechanism cannot be replaced by Stuckelberg mechanism, since extending such a mechanism[7] to non-abelian gauge theories will violate unitarity[8]. Therefore, in the context of the SM, we will risk losing the renormalizability of the theory. Which justifies the fact that we are only interested in the mechanism insofar of an abelian context, that to say, only to the $U(1)'$s symmetries.
It is equally important to also have a clear distinction between the utility of both the Higgs, and Stuckelberg mechanisms. One primary aspect is the physical consequences of both, while the Higgs effect is associated with the existence of the Higgs boson, Stueckelberg's way of attaining mass has no visible physical signs, since all the degrees of freedom will be absorbed by the vector boson, with none left to observe.

We are finally ready to use the above extension to obtain the neutral gauge bosons masses, since we can't replace the Higgs mechanism in the SM, we will consider both processes, with the scalar sector being neutral under the additional symmetry. That is to say:

$$SU(3)_C \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Higgs } \phi} \times U(1)_X. \tag{2.47}$$

$$SU(3)_C \times SU(2)_L \times \underbrace{U(1)_Y \times U(1)_X}_{\text{Stueckelberg } \sigma}. \tag{2.48}$$

thus with the electro-weak symmetry breaking, the mass terms of the neutral gauge bosons will align with the following form:

$$-\frac{1}{2} V_\mu^T M_{Stueck}^2 V^\mu \tag{2.49}$$

[7]Stueckelberg mechanism is only compatible with abelian gauge symmetries.
[8]Because the longitudinal components of the vector fields cannot be decoupled from the physical Hilbert space.

where:

$$V_\mu = \begin{pmatrix} W_\mu^3 \\ B_\mu \\ C_\mu \end{pmatrix} \tag{2.50}$$

while the mass matrix will end up taking the following shape:

$$M^2_{Stueck} = \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{v^2}{4} g'^2 & -\frac{v^2}{4} g g' \\ 0 & -\frac{v^2}{4} g g' & \frac{v^2}{4} g^2 \end{pmatrix} \tag{2.51}$$

it is easily verified that $Det(M^2_{Stueck}) = 0$, which means we should expect a one eigenvalue to be equal to 0.

After diagonalizaton :

$$M^2_{Stueck_{Diag}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_-^2 & 0 \\ 0 & 0 & M_+^2 \end{pmatrix} \tag{2.52}$$

where:

$$\begin{aligned} M_\pm^2 = \frac{1}{8} \Big[& 4M_1^2 + 4M_2^2 + g^2 v^2 + g'^2 v^2 \pm \\ & \sqrt{\left(4M_1^2 + 4M_2^2 + g^2 v^2 + g'^2 v^2\right)^2 - 16 \left(M_1^2 g^2 v^2 + M_2^2 g^2 v^2 + M_1^2 v^2 g'^2\right)} \Big] \end{aligned} \tag{2.53}$$

we can rewrite this in a more presentable manner:

$$\begin{aligned} M_\pm^2 = \frac{1}{2} \Bigg[& \frac{v^2}{4}(g^2 + g'^2) + M_1^2 \left(1 + \frac{M_2^2}{M_1^2}\right) \pm \\ & \sqrt{\left(\frac{v^2}{4}(g^2 + g'^2) + M_1^2 \left(1 + \frac{M_2^2}{M_1^2}\right)\right)^2 - 4M_1^2 \left(\frac{v^2}{4}(g^2 + g'^2) + \frac{M_W^2 M_2^2}{M_1^2}\right)} \Bigg] \end{aligned} \tag{2.54}$$

making use of Stueckelberg parameters, where:

$$\begin{aligned} \delta &= \frac{M_2}{M_1} \\ M_0^2 &= \frac{v^2}{4}(g^2 + g'^2) \end{aligned} \tag{2.55}$$

resulting in:

$$\begin{aligned} M_\pm^2 = \frac{1}{2} \Big[& M_0^2 + M_1^2 \left(1 + \delta^2\right) \pm \\ & \sqrt{\left(M_0^2 + M_1^2 \left(1 + \delta^2\right)\right)^2 - 4M_1^2 \left(M_0^2 + M_W^2 \delta^2\right)} \Big]. \end{aligned} \tag{2.56}$$

Of course, the zero-eigen mode is associated with the masseless photon, and the heavy mass M_+^2 is identified as the mass of the Z' boson M_Z', while the lighter mass M_-^2 as the mass of the Z boson M_Z^2. The total degrees of freedom indicates that the two gauge bosons out of the 3 will get massive by absorbing the two scalars.
In the limit, $M_2 <<< M_1$, we notice that the Stueckelberg sector decouples from the SM.

2.3 Generalized Stueckelberg Effect

As our theoretical development progressed so far, we have seen that a neutral Z' gauge boson can be obtained via a simple bottom-up extension of the SM, it was achieved via the addition of a $U(1)_X$ symmetry. And in the previous section, we have investigated a Stueckelberg technique that allow the resulting Z' to obtain a mass. But throughout the process, we assumed that the scalar sector is left unaffected, and that is neutral under the gauge symmetry, which led us to wonder what if such an assumption is discarded ? It turns out we can approach the process from a different angle, this time it will be through building an invariant construct from the Stueckelberg field, to be injected later on in the mass term [11]. In this section, we will explore such a pathway, which we referred to it, in this thesis work, as the "Generalized Stueckelberg Effect". We won't exclude C_μ from the scalar covariant derivative, and we will attempt to derive the new modified mass matrix, to finally write the mass of the Z'.

2.3.1 Generalized Stueckelberg Lagrangian

We start off by tracing the idea of Stueckelberg again, while keeping in mind that the purpose is to put together ingredients to add to the Lagrangian, which may result in a mass term without having to lose gauge invariance.

The Stueckelber's field $\sigma(x)$, which obeys the following gauge transformation :

$$\sigma(x) \to \sigma'(x) = \sigma(x) - M\theta(x) \tag{2.57}$$

such a field can enable the construction of the invariant quantity:

$$\Gamma_\mu = A_\mu + \frac{1}{M}\partial_\mu \sigma \tag{2.58}$$

with:

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \theta(x). \tag{2.59}$$

The first attempt was to build the following construct :

$$\mathscr{L}_M = \frac{1}{2}M^2 \Gamma_\mu \Gamma^\mu \tag{2.60}$$

such a procedure is considered a failure given that it doesn't generate a mass for the scalar field $\sigma(x)$, and it yields extra bilinear terms which are hard to justify physically [11]. Nevertheless, the issue can be repaired if we consider a different form for the invariant construct :

$$\Delta = \sigma - \frac{1}{M}\partial_\mu A^\mu \tag{2.61}$$

under certain restrictions, the mass term can be written as :

$$\mathscr{L}_M = -\frac{1}{2}M^2 \Delta^2. \tag{2.62}$$

Historically, the fundamental issue with Stueckelberg theory was the issue of mass degeneracy of the scalar $\sigma(x)$, resulting in it not gaining much interest at that time [11]. However, the elegance of the idea should not be dismissed, and for that reason, we can attempt a different construct.

Making use of the two quantities:

$$\Gamma_\mu = \partial_\mu \sigma + M_Y B_\mu + M_X C_\mu \tag{2.63}$$

and:

$$\Delta = \sigma - \frac{1}{M_Y}\partial_\mu B^\mu - \frac{1}{M_X}\partial_\mu C^\mu \tag{2.64}$$

under the restriction:

$$(\Box + M_Y^2)\theta'_Y = (\Box + M_X^2)\theta'_X = 0 \tag{2.65}$$

where M_Y^2, and M_X^2 are parameters of a mass dimension. They can also be referred to as Stueckelberg parameters, since their presence is characterized solely to a Stueckelberg extension.

The new transformation for the Stueckelberg's scalar :

$$\sigma \to \sigma' = \sigma - M_Y\theta'_Y - M_X\theta'_X. \tag{2.66}$$

Making use of these ingredients, we are now in a position to construct the Stueckelberg Lagrangian[9], which initially takes the form:

$$\mathscr{L}_S = \frac{1}{2M^2}(M^2 + \lambda_1\phi^\dagger\phi)\Gamma_\mu\Gamma^\mu - \frac{1}{2}(M^2 + \lambda_2\phi^\dagger\phi)\Delta^2 \tag{2.67}$$

where λ_1, and λ_2 are new coupling constants. And M is the parameter of mass.
In such a construction, we permit the Higgs field to couple with σ, allowing the latter to secure its mass.

The previous Lagrangian is valid as long as we are not considering SSB, otherwise considering a SSB in the Higgs sector implies replacing the factor $\phi^\dagger\phi$ with $\frac{v^2}{2}$, leading to mass, and bilinear terms. In such a phase, the Stueckelberg Lagrangian will be written as :

$$\mathscr{L}_S = \mathscr{L}_\sigma + \mathscr{L}_{bil} + \mathscr{L}_{gf} \tag{2.68}$$

the first term is the free scalar Lagrangian. When expanding 2.67, and isolating the scalar, we find:

$$\mathscr{L}_\sigma = \frac{1}{2}(1 + \frac{\lambda_1 v^2}{2M^2})\partial_\mu\sigma_1\partial^\mu\sigma_1 - \frac{1}{2}M^2(1 + \frac{\lambda_2 v^2}{2M^2})\sigma_1^2 \tag{2.69}$$

we can rewrite 2.69 in a clean manner :

$$\mathscr{L}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M_\sigma^2\sigma^2 \tag{2.70}$$

where,

$$\sigma = \sqrt{1 + \frac{\lambda_1 v^2}{2M^2}}\sigma_1 = \sqrt{Z_\sigma}\sigma_1 \tag{2.71}$$

and the physical mass of Stueckelberg scalar can then be written as:

$$M_\sigma = M\sqrt{(1 + \frac{\lambda_2 v^2}{2M^2})(1 + \frac{\lambda_1 v^2}{2M^2})^{-1}} \tag{2.72}$$

the bilinear terms representing mixed terms between σ, B_μ and C_μ, will show up after the expansion as :

[9]Stuecklberg Lagrangian is a kinetic gauge invariant term, valid for abelian massive vector fields.

$$\begin{aligned}\mathscr{L}_{bil} =&(1+\frac{\lambda_1 v^2}{2M^2})M_Y\partial_\mu\sigma B^\mu + (1+\frac{\lambda_2 v^2}{2M^2})\frac{M^2}{M_Y}\sigma\partial_\mu B^\mu \\ &+(1+\frac{\lambda_1 v^2}{2M^2})M_X\partial_\mu\sigma C^\mu + (1+\frac{\lambda_2 v^2}{2M^2})\frac{M^2}{M_X}\sigma\partial_\mu C^\mu .\end{aligned} \tag{2.73}$$

we can consider a simple outcome via canceling the bilinear terms, and that is achieved by imposing the constraint $M_X = -M_Y$, or since it has no physical relevance we can just take the constraint to be $M_X = M_Y$.

Finally, the last piece of the Lagrangian would be the gauge fixing terms, taking the form:

$$\mathscr{L}_{gf} = -\frac{M^2}{2M_Y^2}\left(1+\frac{\lambda_2 v^2}{2M^2}\right)\left((\partial_\mu B^\mu)^2 + (\partial_\mu C^\mu)^2 + 2\partial_\mu B^\mu \partial_\nu C^\nu\right) \tag{2.74}$$

such a term can be rewritten in a concise manner :

$$\mathscr{L}_{gf} = -\frac{1}{2\omega_B}\left(\partial_\mu \tilde{B}^\mu\right)^2 \tag{2.75}$$

where the following change was imposed:

$$\begin{aligned}B_\mu =& \frac{1}{\sqrt{2}}\tilde{B}_\mu + \frac{1}{\sqrt{2}}\tilde{C}_\mu \\ C_\mu =& \frac{1}{\sqrt{2}}\tilde{B}_\mu - \frac{1}{\sqrt{2}}\tilde{C}_\mu\end{aligned} \tag{2.76}$$

and :

$$\omega_B = \frac{M_Y^2}{2M^2}\left(1+\frac{\lambda_2 v^2}{2M^2}\right). \tag{2.77}$$

And by this, the final Stueckelberg Lagrangian is all set up :

$$\mathscr{L}_S = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M_\sigma^2\sigma^2 - \frac{1}{2\omega_B}\left(\partial_\mu \tilde{B}^\mu\right)^2 \tag{2.78}$$

Followed up by this construction, we will try next to see how (2.76), and (2.67) can help us get the new modified mass matrix, and thus the mass of the new Z'.

2.3.2 Stueckelberg masses

Now we are ready to see the neutral gauge bosons masses, starting from the changes imposed on the covariant derivatives from 2.76:

$$\overline{D}_\mu < \phi >= \left(\partial_\mu - \frac{ig}{2}\tau_a W_\mu^a - \left(\frac{ig'_Y Y_\phi}{2\sqrt{2}} + \frac{ig_X X_\phi}{2\sqrt{2}}\right)\tilde{B}_\mu - \left(\frac{ig'_Y Y_\phi}{2\sqrt{2}} - \frac{ig_X X_\phi}{2\sqrt{2}}\right)\tilde{C}_\mu\right) < \phi > \tag{2.79}$$

Focusing on the relevant parts of $|\overline{D}_\mu < \phi > |^2$, amounting to :

$$
\begin{aligned}
&\to \frac{g^2v^2}{4}W_\mu^+W^{-\mu} + \frac{g^2v^2}{8}W_\mu^3W^{3\mu} + \frac{v^2(g_Y^{'}Y_\phi + g_XX_\phi)^2}{16}\tilde{B}_\mu\tilde{B}^\mu + \frac{v^2(g_Y^{'}Y_\phi - g_XX_\phi)^2}{16}\tilde{C}_\mu\tilde{C}^\mu \\
&- \frac{gv^2(g_Y^{'}Y_\phi + g_XX_\phi)}{8\sqrt{2}}W_\mu^3\tilde{B}^\mu - \frac{gv^2(g_Y^{'}Y_\phi - g_XX_\phi)}{8\sqrt{2}}W_\mu^3\tilde{C}^\mu - \frac{gv^2(g_Y^{'}Y_\phi + g_XX_\phi)}{8\sqrt{2}}\tilde{B}_\mu W^{3\mu} \\
&+ \frac{v^2(g_Y^{'}Y_\phi + g_XX_\phi)(g_Y^{'}Y_\phi - g_XX_\phi)}{16}\tilde{B}_\mu\tilde{C}^\mu - \frac{gv^2(g_Y^{'}Y_\phi - g_XX_\phi)}{8\sqrt{2}}\tilde{C}_\mu W^{3\mu} \\
&+ \frac{v^2(g_Y^{'}Y_\phi + g_XX_\phi)(g_Y^{'}Y_\phi - g_XX_\phi)}{16}\tilde{C}_\mu\tilde{B}^\mu.
\end{aligned}
\tag{2.80}
$$

Now, to derive the contribution from the Stueckelberg Lagrangian to the mass term, we focus also on the relevant parts in (2.67). Implying:

$$
\mathscr{L}_S \to \frac{1}{2M^2}\left(M^2 + \frac{v^2\lambda_1}{2}\right)\Gamma_\mu\Gamma^\mu \tag{2.81}
$$

it expands to,

$$
\begin{aligned}
&\to \frac{1}{2M^2}\left(M^2 + \frac{v^2\lambda_1}{2}\right)\left[\left(\frac{M_X^2}{2} + \frac{M_Y^2}{2} + M_XM_Y\right)\tilde{B}_\mu\tilde{B}^\mu\right. \\
&\left. + \left(\frac{M_X^2}{2} + \frac{M_Y^2}{2} - M_XM_Y\right)\tilde{C}_\mu\tilde{C}^\mu + \left(M_X^2 - M_Y^2\right)\tilde{B}_\mu\tilde{C}^\mu\right]
\end{aligned}
\tag{2.82}
$$

by incorporating the constraint $M_X = M_Y$, we notice clearly that the two last factors to the right vanish. So that the Stueckelberg mass contribution will simply be:

$$
\to \frac{M_X^2}{M^2}\left(M^2 + \frac{v^2\lambda_1}{2}\right)\tilde{B}_\mu\tilde{B}^\mu. \tag{2.83}
$$

From (2.73), and with the constraint imposed, we know that the coefficient must satisfy :

$$
\left(1 + \frac{v^2\lambda_1}{2M^2}\right)M_X = \left(1 + \frac{v^2\lambda_2}{2M^2}\right)\frac{M^2}{M_X} \tag{2.84}
$$

which enables us to rewrite (2.83) as:

$$
\to \left(1 + \frac{v^2\lambda_2}{M^2}\right)M^2\tilde{B}_\mu\tilde{B}^\mu \tag{2.85}
$$

finally, the set up is ready to obtain the modified mass matrix. For the sake of simplicity, we use :

$$
M_W^2 = \frac{g^2v^2}{4} \qquad \eta_\phi = \frac{g_Y^{'}Y_\phi + g_XX_\phi}{g} \tag{2.86}
$$

$$
\varepsilon_\phi = \frac{g_Y^{'}Y_\phi - g_XX_\phi}{g} \tag{2.87}
$$

We combine the mass contribution, and making use of the above notation, we obtain :

$$\mathscr{L}_M \to M_W^2 W_\mu^+ W^{-\mu} + \frac{M_W^2}{2} W_\mu^3 W^{3\mu} + \left[\frac{M_W^2 \eta_\phi^2}{4} + \left(1 + \frac{v^2\lambda_2}{M^2}\right) M^2\right] \tilde{B}_\mu \tilde{B}^\mu + \frac{M_W^2 \varepsilon_\phi^2}{4} \tilde{C}_\mu \tilde{C}^\mu$$
$$- \frac{M_W^2 \eta_\phi}{2\sqrt{2}} W_\mu^3 \tilde{B}^\mu - \frac{M_W^2 \varepsilon_\phi}{2\sqrt{2}} W_\mu^3 \tilde{C}^\mu - \frac{M_W^2 \eta_\phi}{2\sqrt{2}} \tilde{B}_\mu W^{3\mu}$$
$$+ \frac{M_W^2 \eta_\phi \varepsilon_\phi}{4} \tilde{B}_\mu \tilde{C}^\mu - \frac{M_W^2 \varepsilon_\phi}{2\sqrt{2}} \tilde{C}_\mu W^{3\mu} + \frac{M_W^2 \eta_\phi \varepsilon_\phi}{4} \tilde{C}_\mu \tilde{B}^\mu. \tag{2.88}$$

rewritten in the from:

$$\mathscr{L}_M \to M_W^2 W_\mu^+ W^{\mu-} + \begin{pmatrix} W^{3\mu} & \tilde{B}^\mu & \tilde{C}^\mu \end{pmatrix} M \begin{pmatrix} W_\mu^3 \\ \tilde{B}_\mu \\ \tilde{C}_\mu \end{pmatrix} \tag{2.89}$$

where the mass matrix M is now:

$$M' = \begin{pmatrix} \frac{M_W^2}{2} & -\frac{M_W^2}{2\sqrt{2}}\eta_\phi & -\frac{M_W^2}{2\sqrt{2}}\varepsilon_\phi \\ -\frac{M_W^2}{2\sqrt{2}}\eta_\phi & \frac{M_W^2}{4}\left(\eta_\phi^2 + 4\mu^2\right) & \frac{M_W^2}{4}\eta_\phi\varepsilon_\phi \\ -\frac{M_W^2}{2\sqrt{2}}\varepsilon_\phi & \frac{M_W^2}{4}\eta_\phi\varepsilon_\phi & \frac{M_W^2}{4}\varepsilon_\phi^2 \end{pmatrix} \tag{2.90}$$

and we have labeled a new coefficient :

$$\mu^2 = \frac{M^2}{M_W^2}\left(1 + \frac{v^2\lambda_2}{2M^2}\right) = Z_\sigma \frac{M_\sigma^2}{M_W^2}. \tag{2.91}$$

A diagonalization will lead to :

$$M'_{Diag} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_-^2 & 0 \\ 0 & 0 & M_+^2 \end{pmatrix} \tag{2.92}$$

where:

$$M_\pm^2 = \frac{M_W^2}{8}\left[2 + 4\mu^2 + \eta_\phi^2 + \varepsilon_\phi^2 \pm \sqrt{4(-8\mu^2 - 4\mu^2\varepsilon_\phi) + (2 + 4\mu^2 + \varepsilon_\phi^2 + \eta_\phi^2)^2}\right] \tag{2.93}$$

the lighter mass will be identified as the Z boson mass $M_-^2 = M_Z^2$. Constraining it with the well established SM value:

$$M_Z = \frac{M_W}{cos\theta_W} \to \begin{cases} \eta_\phi = 0 \\ \varepsilon_\phi^2 = 2tan^2\theta_W \end{cases} \tag{2.94}$$

which will retain the mass matrix to have the new form:

$$M' = \frac{M_W^2}{4}\begin{pmatrix} 2 & 0 & -2tan\theta_W \\ 0 & 4\mu^2 & 0 \\ -2tan\theta_W & 0 & 2tan^2\theta_W \end{pmatrix} \tag{2.95}$$

finally, the mass of the gauge boson Z' :

$$M'_Z = M_W\mu = M\sqrt{1 + \frac{v^2\lambda_2}{2M^2}}. \tag{2.96}$$

The mass of the new gauge boson was generated wihout having to break the new symmetry added, and with a very economical number of parameters. The model contains only two additional elements, the field σ with a mass M_σ, and a gauge boson Z' with a mass M'_Z, both of these quantities depends on 3 other unknown parameters M, and $\lambda_{1,2}$.

Finally, the Stueckelberg mechanism has proven itself useful when it comes to the generation of the mass for an abelian gauge boson, without having to break the additional symmetry, and while maintaining gauge invariance. We have examined such a procedure in the case of a gauged $U(1)_X$ extension of the SM. However, to an experimentalist the theoretical procedures are half cooked, and such a gauge boson has to be sought at the heart of particle accelerators. An exploration of the Stueckelberg Z' has been analyzed in different references [72, 73], which shows promising prospects for its discovery. Specifically, Stueckelberg's extensions allow for narrow resonances, in our case associated with the Z'. Such a narrow resonance can potentially be discovered using the dilepton production in the Drell-Yan process. More details have been carried out in [72], where in such an analysis it was indicated that a narrow Z' resonance of a Stueckelberg origin can be explored up to about 2 TeV with 100 fb^{-1} integrated luminosity[10], and even up to 2.5 TeV with 300 fb^{-1} integrated luminosity, and an exploration can even be carried out to 3 Tev with 1000 fb^{-1} of integrated luminosity. For a better illustration of these results, we have taken partial data from [72], shown in figure (2.1).

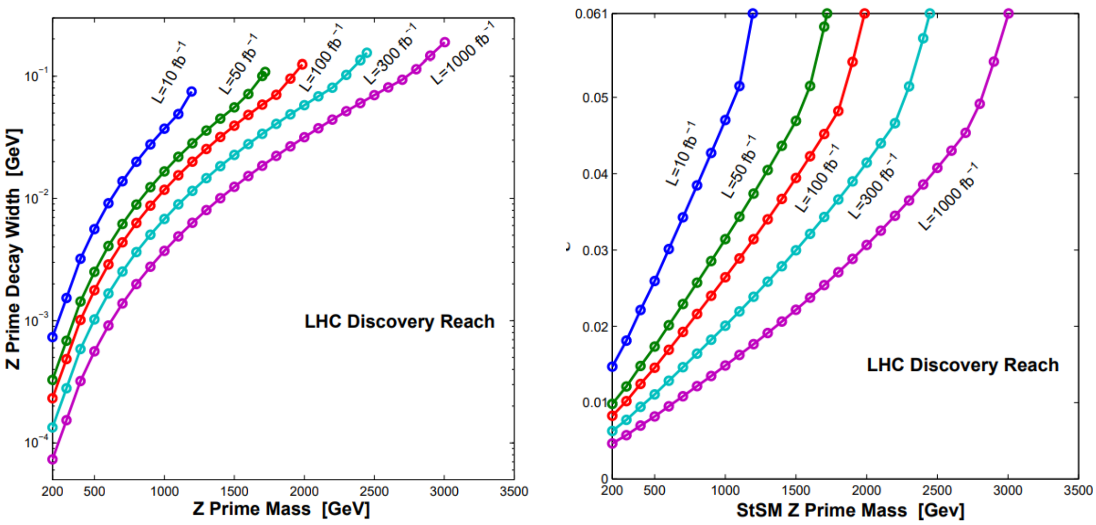


Figure 2.1: A plot of the discovery limits of Z' in StSM with the discovery limit defined by $5\sqrt{N_{SM}}$, or by 10 events, whichever larger. The inflections in the plots are precisely the points of transitions between the two criteria. Regions to the left and above each curve can be probed by the LHC at a given luminosity. The top point on each curve corresponds to $\varepsilon = .061$. The analysis is done for the ATLAS detector but similar results hold for the CMS detector.

Furthermore, the experimental search of this new gauge boson is of a great interest to our work, regardless of its origins. For this purpose, we will generalize our focus in the upcoming chapter to some phenomenological consequences associated with a general Z' gauge boson. Such an exploration will be compacted, since the main aim is to give general feature on Z' searches, that will help us interpret our final results regarding the impact of the Z'.

[10] For more details about the experimental terminology being used, check Appendix (C)

Chapter 3

Identification of the new Z' gauge boson

The Z' neutral gauge boson has been predicted by many models of physics BSM, and we have seen it can be a result of a $U(1)_X$ extension of the SM. As any theoretical prediction, experimental talk needs to take place. Given that there are many locations from where this new Z' can originate, it is crucial to answer questions such as : is it possible to observe such a boson at the LHC?. If so then, by which means we can detect its existence ?, and how can we identify which model is actually valid given the measurements we have ?, and many more. For that purpose, we will attempt to explore some of these questions, starting from general procedures to identify such a particle, to narrowing our interest to a specific scattering process, namely the $b\bar{b} \to \nu\bar{\nu}$ scattering process. Where the Stuckelberg Z' boson is exchanged, so that we can infer what might be the effect of such an entity, if it exists. Finally, a discussion around the variations of the differential cross section is planned, leading to concluding remarks about the model under scrutiny.

3.1 Z' Phenomenology

After the hard work of seeking theoretical models that can supply us with insights about the new coating of physics BSM, it is heavily important to validate our theoretical work with some experimental data, or at least shed some light on which of our many models is the closest to describe our actual physical world. Being motivated by such a purpose, it is necessary to take a look at what available tools we have to probe the existence of the new neutral Z'. It is expected that strenuous searches for the signature of this new Z' will be performed at present, and future high energy colliders. And given that the Z's can be massive enough to fall outside the scope of CERN LHC's discovery reach, in which case indirect signatures of Z' exchanges are more likely to be beneficial, and more likely to take place at future colliders, which justifies our interest.

The entire experimental process can be summarized as follows:

- Probe and identify the existence of the new Z' gauge boson.
- Measure the properties of this new Z' boson (characterization of its spin and couplings ..etc.).
- Compare the experimental measurements with the theoretical predictions, and identify the origin the of the Z'.

It is known that the Z' is a neutral gauge boson with spin-1[1], and without a color charge. In addition to the assumption that it is similar to the SM Z boson, in the sense that it is expected to be a very short–lived particle. Such a particle has very high chances to be observed at the next generation of colliders, and therefore, the study of the Z' phenomenology is a crucial part of the scientific program of every future collider. We can take glimpses at its existence through its decay products, or its indirect effects. And we can observe it in very high energy processes, where the energy of the particles collided must be high enough to produce it. We can also infer its existence in precision experiments, where the experimental errors with the errors due to the theoretical predictions must be smaller than the derivations caused by the Z' [74, 75].
If such a boson exists, and is accessible at the TeV scale, it is possible that it may first be directly produced via the Drell-Yan process at the LHC, provided the Z' couples to both quarks and leptons, since the majority of the models assume that. But if the Z' does not couple to leptons, then it cannot be produced at the NLC. In this case, searches at the LHC would also be difficult. If the Z' couples only to leptons, then the NLC give a unique mean to study it. It is possible that a Z' even if it exists, it may not be seen at the LHC due to its mass, and the nature of its couplings to fermions. However, in such a scenario, the NLC might be the solution [76], since it can extend the search reach for some models, and it may be able to determine both the mass and couplings of the Z'. In the case where the mass of the Z' is less than the NLC center-of-mass energy, we can aim to determine all of the Z' couplings to fermions by sitting on-resonance and repeating the LEP/SLC experimental program. Another possibility is $M_Z' > \sqrt{s}$, in such a case the deviations in the cross section, and the asymmetries for various flavors can be used to look for indirect Z' effects, this indirect reach can be as large as $M_Z' = 10\sqrt{s}$. This methodology can be applied the same way we carried out the observation of the Z-SM at PEP, PETRA, and TRISTAN below the resonance.

Before we attempt to talk about ways to probe the existence of the Z', it is advisable to set forth some global ideas about the properties of the Z' particle, while ignoring for now the constraints imposed by our specific choice of the model. Starting with the Z' couplings to fermions [74], generally we can expect that the particle to have 24 distinct couplings, to the 12 different fermions with different chiralities (L,R). In such a case, the coupling is said to be Non-universal [2], which can allow for flavor changing neutral currents[3] (FCNC) in low-energy processes. In such a case, the Z' mass must be of order ~ 100 TeV or more, which makes it outside the scope of the LHC.
However, if there will be a mechanism to repress the FCNC[4], then we can expect that the Z' can be light enough to be observed at the LHC. In such a case, the number of couplings can be reduced to 8 couplings, and they are said to be generation-independent couplings. The couplings of a Z' boson to the first-generation fermions can be written in the following manner:

$$\begin{aligned} Z'_\mu \Big(& g_u^L u_L \gamma^\mu \bar{u}_L + g_d^L d_L \gamma^\mu \bar{d}_L + g_u^R u_R \gamma^\mu \bar{u}_R + g_d^R d_R \gamma^\mu \bar{d}_R + g_\nu^L \nu_L \gamma^\mu \bar{\nu}_L + g_e^L e_L \gamma^\mu \bar{e}_L \\ & + g_e^R e_R \gamma^\mu \bar{e}_R \Big) \end{aligned} \tag{3.1}$$

where u, d, ν, e are the quarks and lepton fields in the mass eigenstate basis, and the coef-

[1] It is important to mention that the spin-1 Z' here is the result of a gauge extension of the SM, while there are other theoretical interpretations, such as: a spin-0 in R-parity violating SUSY, a spin-2 Kaluza-Klein(KK) excitation of the graviton as in the Randall-Sundrum(RS) model, or even a spin-1 KK excitation from some extra dimensional model [74].

[2] Family or generation-dependent.

[3] They are interactions that allow for a change in the flavor of a fermion without altering its electric charge.

[4] Examples of such mechanisms is the Glashow–Iliopoulos–Maiani GIM mechanism.

ficients $g_u^L, g_d^L, g_u^R, g_d^R, g_\nu^L, g_e^L, g_e^R$ are real dimensionless parameters.

And as we said previously, If the Z' couplings to quarks and leptons are family-universal, then these seven parameters simply describe the Z' couplings to all the SM fermions. However, if the Z' couplings to fermions are non-universal, then Eq. (3.1) will be written with family indices $i, j = 1, 2, 3$ labeling the quark and lepton fields.
Not only the coupling to fermions is a parameter of interest, there are other quantities that are relevant to the new gauge boson Z' [75]. To summarize:

- Coupling strength.
- Vector, and axial coupling to fermions.
- The $Z - Z'$ mixing angle.
- The mass and width of the mass-eigenstate.

All of the above parameters are important for the process of Z' searches. Such searches can be categorized into two categories: Direct and Indirect searches [74]. Indirect searches simply focus on looking for deviations from the SM already verified data that can be associated as a cause for the presence of the Z' particle. This can involve for example, precision electroweak measurements at above(or below) the Z-pole, the cross section and forward backward asymmetry ..etc. Indirect searches for Z' bosons are carried out at electron-positron colliders, since these type of colliders are characterized by high-precision measurements of the properties of the SM-Z gauge boson. Some of the available constraints come from the $Z - Z'$ mixing, and they are model-dependent. The current limits are from the CERN LEP collider, which puts lower bound limits on the Z' mass, it is constrained to be heavier than a few hundred GeV.
While in direct searches, we mostly rely on the Drell-Yan process [5], these searches are carried out at hadron colliders, since they allow for better access to the highest energies available. The process is based on a search for high-mass dilepton resonances, where the Z' boson would be produced by quark-antiquark annihilation, and decays to an electron-positron pair or a pair of opposite-charged muons. The most strict current limits come from the Fermilab Tevatron, and depend on the couplings of the Z' boson. [6].

To our own interest, we will just take a brief look at what the identification of the new Z' gauge boson would look like, considering first possible signatures at e^+e^- Colliders, then its identification using the process $pp(p\bar{p}) \to \ell^+\ell^- + X$. Lastly, using b and t quarks in the context of restricting the Z' couplings to only third generation fermions.

3.1.1 Search for the Z' at e^+e^- Colliders

As we established previously, theoretically the Z' is a neutral particle, colorless, and it's its own anti-particle[7]. Its origins are also diverse, since it appears in so many locations within the theoretical literature of physics BSM, as we have seen in section (1.3). Experimentally, the Z' is a resonance, it is expected to be heavy than the Z_{SM} boson, and it can be observed at the LHC.

[5] It is a process proposed by Sid Drell and Tung-Mow Yan to describe high-mass lepton-pair production in hadron-hadron collisions.

[6] which also control the production cross section.

[7] Self adjoint.

Many processes can be sensitive to a Z' [75], among these we can use fermion pair production, or Bhabha and Møller scatterings, since they have larger event rates compared to fermion pair production. On the other hand, W pair production is extremely sensitive to ZZ' mixing, and this sensitivity can be enhanced at very high energies. In addition to e^-e^+, and $\mu^-\mu^+$ collisions, which have the capacity to yield different reactions that can probe some properties of an extra Z' boson. Although we will explicitly mention mostly e^-e^+ collisions, the ideas are also applicable to $\mu^-\mu^+$ o $\mu^-\mu^-$ collisions.

At e^-e^+ colliders, the discovery limits are indirect, they will be mostly from deviations from the SM predictions for various cross sections, and from asymmetries due to interference between the Z' propagator, and the γ, Z propagators [75], such an effect can be assimilated to the scenario where PEP/PETRA saw the SM-Z as deviations from QED predictions.
Considering the simple reaction $e^+e^- \to f\bar{f}$, where $f = (e, \mu, \tau, u, d, c, s, b, t)$. The cross section in this case takes the from:

$$\frac{d\sigma_L}{dcos\theta} = \frac{\pi\alpha^2}{4s}\left[|C_{LL}|^2(1+cos\theta)^2 + |C_{LR}|^2(1-cos\theta)^2\right] \tag{3.2}$$

where the coupling parameters C_{ij} are written in terms of the SM-Z couplings, and the Z' couplings.
Once we have (3.2), we can compute the rest of the quantities of interest such as the total differential cross section :

$$\frac{d\sigma}{dcos\theta} = \frac{1}{2}\left[\frac{d\sigma_L}{dcos\theta} + \frac{d\sigma_R}{dcos\theta}\right] \tag{3.3}$$

Specifically, we can use this reaction, and look at the Z' through its effects in some observables, among these we mention [77]:

- The cross section to some specific fermionic final state σ^f.
- The ratio of the hadronic to the QED point cross section $R^{had} = \frac{\sigma^{had}}{\sigma_0}$.
- The forward-backward asymmetry to a certain fermion f in the final state A^f_{FB}.
- The left-right asymmetry with a fermion in the final state A^f_{LR}.
- The left-right asymmetry with hadrons in the final state A^{had}_{LR}.
- The tau polarization P_τ.

The form of the amplitude for $e^+e^- \to (\gamma, Z, Z') \to f\bar{f}$ can be written as :

$$\mathscr{M} = \sum_n \frac{g_n^2}{s - m_n^2}\bar{v}(e)\gamma_\beta[v_e(n) - \gamma_5 a_e(n)]u(e)\bar{u}(f)\gamma^\beta[v_f(n) - \gamma_5 a_f(n)]v(f) \tag{3.4}$$

the summation runs over the exchanged gauge bosons. And a_e, a_f are the couplings.

In addition, to get the so-called discovery limits for new physics, which in this case implies evidence for extra neutral gauge bosons, we look for statistically significant deviations from the SM expectations. Figure (3.1) plays the role of an illustrative example, as in how a number of observables behave with their standard model values, and for various Z' from different models, in function of the Z' mass [77].

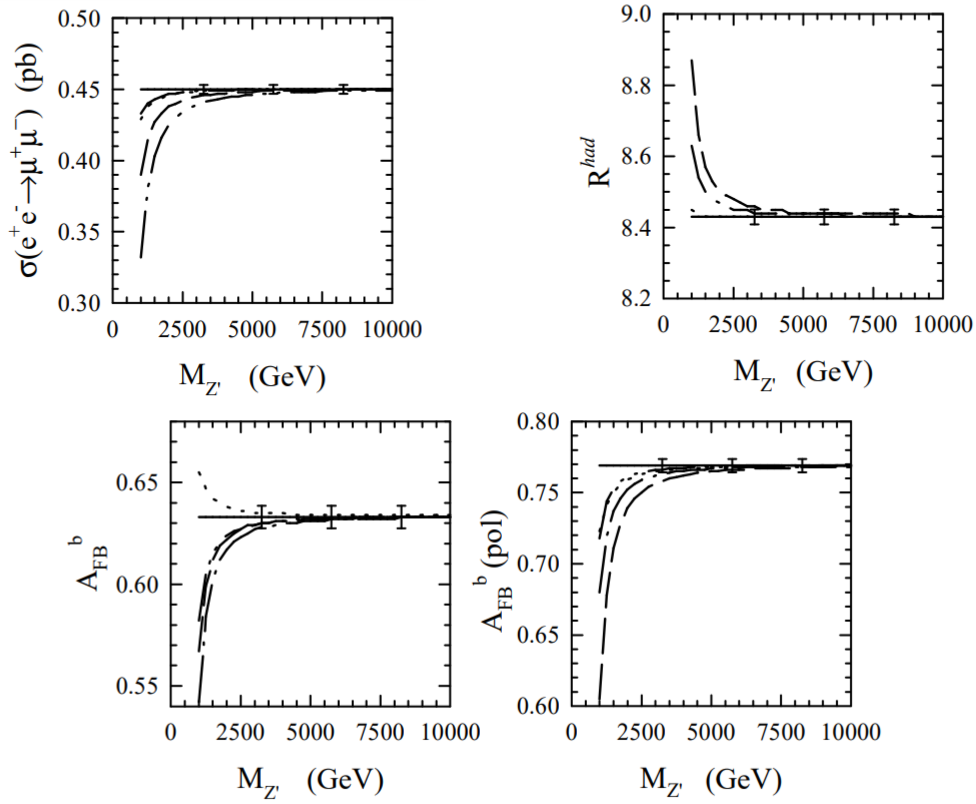


Figure 3.1: Some e^+e^- observables showing their standard model values, and their values for a Z' as a function of M_Z' for $\sqrt{s} = 500GeV$. The solid line is the standard model value, the dashed line is for Z_χ (from GUTs), the dotted line for Z_η, the dot-dashed line for Z_LR(Left-Right SM), and the dot-dot-dash line for Z_{ALR}(The alternative Left-Right Symmetric Model). The error bars are based on the statistical error assuming an integrated luminosity of $50fb^{-1}$.

It is note worthy that we mentioned only few observables, the full data is mentioned in reference [77]. In addition to the fact that the data above didn't include b-quark tagging efficiencies in the statistical errors, therefore some of the error bars should be bigger than what is shown.

We can also probe the Z' using Bhabha and Møller scatterings, but this would be probing the Z' couplings to electrons only. Bhabha events are considered as additional observables in e^+e^- collisions, while Møller scattering requires the e^-e^- option of a linear collider. In Bhabha scattering [75], electrons and positrons appear in the final state, and the angular distribution of such a scattering is peaked in the forward direction. In such a case, the neutral gauge bosons are exchanged in the s and t channels. While on the other hand, in Møller scattering, only electrons appear in the final state, and the scattering has a symmetrical angular distribution. In addition to the neutral gauge bosons being exchanged this time in the t and u channels. Such processes are insensitive to $Z - Z'$ mixing, and therefore neglecting the mixing is generally assumed.

For illustrative purposes, the Born cross section of Bhabha scattering including the Z' exchange is [75]:

$$\frac{d\sigma}{dc} = \frac{\pi\alpha^2}{2s}\left(f_0 + (\lambda_+ - \lambda_-)f_1 + \lambda_+\lambda_- f_2\right) \tag{3.5}$$

where the f_k derive from the squared sum of matrix elements M_u with s- and t-channel exchange of a photon, Z-boson, or Z'-boson.
Among the various observables, with Bhabha scattering one can use unpolarized beams to measure contributions proportional to f_0, while with polarized electrons one can attempt to measure the left-right asymmetry A_{LR}, given that it is sensitive to contributions proportional to f_1. And with two polarized beams, we are allowed to access a measurement of the asymmetryA_{2L}, which is a quantity sensitive to contributions proportional to f_2. And it is worth mentioning that such process gives us the advantage to probe new information, since fermion pair production, and W pair production for example don't supply further data if we are to use two polarized beams [75].
Furthermore, with Møller scattering we can have both electron beams as highly polarized beams, which would enable one to measure several angular distributions, such as:

$$\frac{1}{\sigma}\frac{d\sigma}{dc}, \frac{1}{\sigma^{LL}}\frac{d\sigma^{LL}}{dc}, \frac{1}{\sigma^{RR}}\frac{d\sigma^{RR}}{dc}, \frac{1}{\sigma^{LR}}\frac{d\sigma^{LR}}{dc} \tag{3.6}$$

As we can see, indirect searches can be pretty helpful as way to obtain more data about the potential Z' boson. In addition to Bhabha and Møller scatterings, one can even make use of the process $e^+e^- \to W^+W^-$ to infer some Z' properties, but such a reaction is sensitive to a Z' particle only in the case of a non-zero $Z - Z'$ mixing, more details can be found in [75].

Moreover, since we have tackled briefly how the Z' searches are looking like in e^-e^+ collisions, it is also worth mentioning that promising prospects also are present in hadronic colliders, and the Z' signal can also be observed from direct production at $pp/p\bar{p}$ collisions, with the constraint that the Z' mass is smaller than the center-mass-energy of the colliding protons. The Z' can be detected through its decay products, since it would show up as resonances in the invariant mass distribution of the decay products, which can be challenging in hadronic colliders, in the case of the Z', its decay products must be separated from the SM background, otherwise it will be unlikely to spot its existence. We can distinguish many possible processes that can be judged to be useful in hadron collisions, and have the potential to gather further information about the Z'. Among these processes, we mention the possibility of the Z' decaying to a pair of fermions $Z' \to f\bar{f}$, which will be of our interset to investigate in the upcoming subsection. To a W pairs, but this is only possible if there is $Z - Z'$ mixing. In addition to high order processes such as: a rare decay $Z' \to f_1\bar{f}_2V$, where $V = W, Z$, and f_1, f_2 are just ordinary fermions. And an associated Z' production with other gauge bosons, $pp \to Z'V$, where $V = Z, W, \gamma$ [75].

Our interest would be biased toward the search of Z' in the so-called Drell-Yan (DY) process, given that these processes have been studied for over 40 years, which makes us more familiarized with such a thing, and in addition to the fact that the discoveries of the SM W, and Z resonances at the Super Proton Synchrotron (SPS) were thanks to these channels. Furthermore, the signals of the Drell-Yan channels can be considered as one of the cleanest especially with the difficult environment of a high-energy hadronic collider as we mentioned prevsiously. And since these processes involve the production of lepton-anti-lepton pairs with high invariant mass, then when it comes to the final state one woud try to identify a purely leptonic frame, and since there would be no color charge in the final state then we will be dealing with fewer diagrams, making the entire procedure a simple process to study. On the other hand, the cross-section is also lower compared to other non-leptonic processes, with Less backgroud, therefore, easier identification. Which overall ensure a high efficiency even with the reduced event-rate.

Taking for example the reaction:

$$pp(p\bar{p}) \to (\gamma, Z, Z')X \to f\bar{f}X \tag{3.7}$$

This reaction is much less sensitive to $Z - Z'$ mixing, so it is permissible to take any $Z - Z'$ mixing effects as neglegeable. In such a case the Born cross section of a produced Z', and that will decay to a fermion pair is [75] :

$$\sigma_A^f = \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \sigma_A(sx_1x_2; q\bar{q} \to f\bar{f}) G_A^q(x_1, x_2, M_{Z'}^2)\theta(x_1x_2s - M_{\sum}^2) \tag{3.8}$$

after we used the notation:

$$\sigma_A \left(pp(p\bar{p}) \to (\gamma, Z, Z')X \to f\bar{f}X\right) \equiv \sigma_A^f \tag{3.9}$$

where: $\sqrt{s}$ is the center of mass energy, $M_{\sum}$ is the sum of the masses of the particles in the final state, σ_A, $A = T, FB$ are the total, Forward-Backward Born cross sections, and $G_A^q(x_1, x_2, M_{Z'}^2)$ are the structure functions for quarks, $A = T, FB$. They can be given by the expressions given in reference [75].

We can re-write 3.8 in terms of the rapidity "y", since:

$$x_{1,2} = \sqrt{\frac{Q^2}{s}} e^{\pm y} \tag{3.10}$$

resulting in :

$$\sigma_A^f = \sum_q \frac{M}{s} \int_{M_{\sum}}^{s} dQ^2 \int_{-y^{max}}^{y^{max}} dy \sigma_A(sx_1x_2; q\bar{q} \to f\bar{f}) G_A^q(x_1, x_2, M_{Z'}^2)\theta(x_1x_2s - M_{\sum}^2) \tag{3.11}$$

The fermions coming from Z' decay have an invariant mass, but also other fermion pairs with the same invariant mass can result from other places such as being produced by gluons, photon or Z boson.
We can neglect the interference with the SM contribution to $f\bar{f}$ production, which is a good approximation for a narrow Z' resonance, such as a Stueckelberg Z' as we mentioned in the previous chapter. In such a case, Z' production, in the s-channel, would have a cross section of the form [78]:

$$\sigma(pp \to Z'X \to f\bar{f}) \approx \frac{\pi}{48s} \sum_q c_q^f w_q(s, M_Z'^2) \tag{3.12}$$

The functions w_q carry the information about all the parton distributions and QCD corrections, while the coefficients c_q^f carry all the dependence on the Z' couplings, and they are defined as :

$$c_q^f = [(g_q^L)^2 + (g_q^R)^2] B(Z' \to f\bar{f}) \tag{3.13}$$

3.1.2 Identification of the Z' using b and t quarks

Given the possibility that the neutral gauge boson Z' can be discovered at the Large Hadron Collider program, it is worthy to walk through a brief description of how a Z' decaying to b- and t-quark final states can act as an effective way for not only identifying for the particle, but also for measuring its properties and have a way to discriminate between the various theoretical models predicting its existence.
One important feature when it comes to the new gauge boson is its coupling to fermions,

its coupling to leptons for example can be measured using 3 ingredients : the cross section to leptons, the forward backward asymmetry A_{FB}, and the width $\Gamma_{Z'}$. While when it comes to quarks, the quark couplings can be measured through processes where we can identify the b and t quarks in the final states, since it can be an extremely powerful to measure those couplings and use them to draw the distinction between the available models.

Some research have pointed out the possibility of using the 3^{rd} generation quarks to identify any additional gauge bosons [78], and it proved itself useful to actually provide a tool to distinguish between the models, while using the 3^{rd} generation t and b quarks showed more promising aspects, specially when it comes to differentiate between models of extra neutral gauge boson, since identifying heavy quark flavors as final states render the task of measuring individual quark couplings musch easier compared to light quarks.

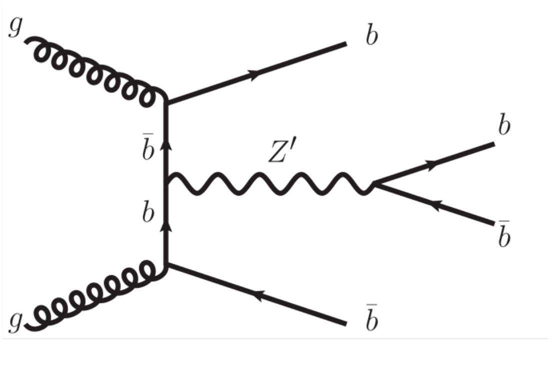
The two quantities of inetrest would be to compute the cross sections $\sigma \to Z' \to b\bar{b}$, $\sigma \to Z' \to t\bar{t}$, as described by the Drell-Yan cross section with the extra Z'. According to [78], measuring the latter at the LHC would be accompanied by two challenging aspects. First is the issue with achieving sufficiently high b- and t-quark identification efficiencies, so that the resulting data can be statistically sufficient to discern between models. While the other challenge resides in having the capacity to distinguish the Z' signal from the large SM QCD backgrounds.

Not only that, but most of the work done in the literature was executed under the assumption that these Z' bosons interact universally with the fermions of all families, but when considering otherwise, it was shown for example that a neutral gauge boson interacting differently with the heaviest fermions naturally leads to a realistic mass hierarchy and Kobayashi-Maskawa mixing matrix. Other theoretical work showed that excluding the coupling of the Z' boson to only the third generation has many implications worth considering, such an assumption has been explored by many so far [79, 80]. In our case, we work through a brief overview of how using b and t quarks can either facilitate or makes the Z' difficult to achieve.

An additional Z' coupling to only the third generation fermions can be expected to contribute to processes where τ, b and t quarks are produced, given that the quarks of the third family are significantly heavier than those of the other two families, so one may speculate that such a boson may be involved in the formation of this difference, and thereby interact directly with these quarks.

Examples can be shown involving b quarks, where the Z' gauge particle can be produced via gluon splitting in the LHC experiments, and its decay into also a pair of b-quarks [81], as the figure (3.2)shows. Other possible production modes at ($pp/p\bar{p}$) colliders can be

Figure 3.2: A Feynman diagram for the production of a Z' boson via gluon splitting and a pair of b-quarks, in addition to its decay to a pair of b-quarks.



quark annihilation $b\bar{b}$, which means via the annihilation of a b quark from one proton (or anti-proton) and a$\bar{b}$ from the other. In such a case, The matrix element is proportional to [79]:

$$\mathscr{M} = g_{Z'}^2 \left[\bar{v}(p_2)\gamma^{\mu}(v_b' - a_b'\gamma_5)u(p_1)\right] \frac{-g_{\mu\nu + \frac{k_\nu k_\mu}{M_Z'}}}{k^2 - M_Z'^2 + iM_Z'\Gamma_Z'} \left[\bar{u}(q)\gamma^{\nu}(v_q' - a_q'\gamma_5)v(\bar{q})\right] \quad (3.14)$$

where $p_{1,2}$ are the momenta of the incident proton, and antiproton, and $q/\bar{q}$ are the final-state quark and antiquark momenta. With $k = p_1 + p_2$.
Furthermore, For the decay of the Z' to a certain fermion ant-ifermion pair, we consider the following fermionic partial decay width at the tree level:

$$\Gamma_f = \frac{g_Z'^2 M_Z'}{12\pi}\sqrt{1 - \left(\frac{2m_f}{M_Z'}\right)^2}\left[v_f'^2\left(1 + \frac{2m_f^2}{M_{Z'}^2}\right) + a_f'^2\left(1 - \frac{4m_f^2}{M_Z'^2}\right)\right] \quad (3.15)$$

where: M_Z' denotes the mass of the Z', m_f is the mass of the fermions, and v_f', a_f' represents the vector and axial couplings respectively.
In such a case the total fermionic width takes the form:

$$\Gamma_Z' = \Gamma_b + \Gamma_t + \Gamma_\tau + \Gamma_{\nu\tau} \quad (3.16)$$

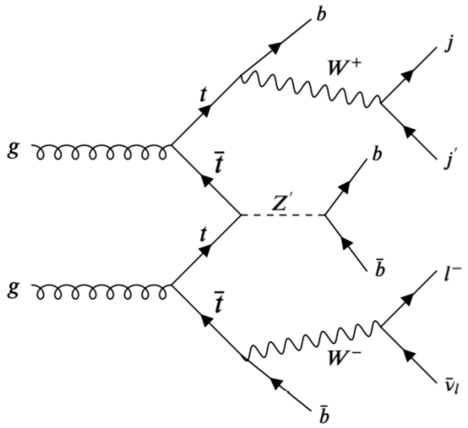
At the LHC the sea-quark and gluon distributions are much less suppressed, so it is only natural to wonder if the Z' production can compete with the QCD background. Such an analysis was carried out in reference [79], where it was noticeable that it would be extremely hard to lay down the hand at the discovery such vector bosons in hadronic collisions.

However, searches for new resonance using the other elements in the third generation are also interesting to consider. The top quark (t) is the most massive known fundamental particle in the SM. It has a Yukawa coupling to the Higgs field that is near unity, and it is also a factor contributing to the hierarchy problem, where the largest corrections to the Higgs mass arise from top quark loops. Thus, it is natural to question the possibility of this particle to host the answer to the issues we are facing with the SM.

An interesting place to look at a vector-like Z' boson was proposed in [80], where an analytical study was carried out about the production of a Z' at the CERN's Large Hadron Collider (LHC), using $p-p$ collisions at $\sqrt{s} = 13TeV$, and $14TeV$. Specifically, the work was done under the assumption that the new particle has couplings only to the third generation of fermions, referred to as "tritogenophilic". Such an assumption was motivated by the fact that many searches conducted so far at the LHC are built upon on the assumption that additional new particles have similar couplings to all generations of fermions[8], thus this might be one leading factor for why extensive searches have found no firm indication of new phenomena so far. The Z' is produced through the fusion of a $t\bar{t}$ pair, and can decay to a pair of b quarks ($Z' \to b\bar{b}$), as the figure (3.3) indicates.
In addition, table (3.1) indicates the production cross sections for different Z' masses, considering pp collisions at $\sqrt{s} = 13TeV$, and 14 TeV. While also considering two possible scenarios, one where $g_q = 0$ which implies that the couplings of the Z' to light quarks are suppressed. And the case $g_q = 1$, which allows for couplings to light quarks, and thus it opens the possibility for other $t\bar{t}Z'$ production processes to contribute.
The couplings of the Z' to the first and second generation SM quarks is defined as $g_{Z'_{q\bar{q}}} =$

[8]including couplings to the partons inside the proton.

Figure 3.3: A Feynman diagram for the production of a Z' boson through the fusion of a top quark pair, exhibiting a potential to decay to a pair of bottom quarks, and the two spectator top quarks decay semi-leptonically.



$g_q \times g_{Z_{q\bar{q}}}$, where $g_{Z_{q\bar{q}}}$ is the SM Z boson coupling to first and second generation quarks, and g_q is the "modifier" for the coupling. Furthermore, the couplings of the Z' to the third generation SM quarks is defined as $g_{Z'_{b/t,\bar{b},\bar{t}}} \times g_{Z_{b/t,\bar{b},\bar{t}}}$, where the modifier in this case is taken as the modifier to the Sequential Standard Model couplings, and it is referred to as $g_{Z'_{b/t,\bar{b},\bar{t}}}$.

Table 3.1: Signal cross sections, taken from [80], for different Z' masses and couplings to first and second generation quarks. The values in this table are calculated with$g_{Z'_{b/t,\bar{b},\bar{t}}} = 1$.

	13 TeV		14 TeV	
Z' mass (GeV)	$\sigma_{g_q=0}$	$\sigma_{g_q=1}$	$\sigma_{g_q=0}$	$\sigma_{g_q=1}$
250	51.34	72.87	64.32	90.21
300	32.61	47.73	41.22	58.94
325	26.40	39.47	33.48	48.33
350	21.58	32.84	27.14	40.53
375	17.71	27.48	22.50	33.77
400	14.58	23.14	18.97	28.62
500	7.379	12.53	9.557	15.67
750	1.700	3.546	2.315	4.516
1000	0.502	1.285	0.703	1.681
2000	0.011	0.066	0.017	0.093

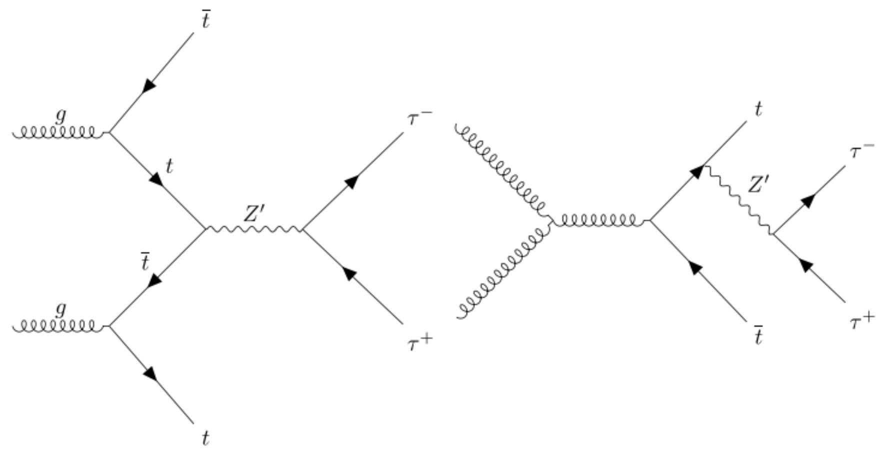
The data presented in the table is interesting to look at given how various coupling scenarios for the Z', being it suppressed couplings to light flavour quarks ($g_q = 0$), enhanced couplings to third generation fermions, and preferential couplings to top and bottom quarks ($g_{Z'_{b/t,\bar{b},\bar{t}}} = 1$), have the capability to alter the discovery potential of the particle. The work done in [80] shines the light on an important issue that must be looked at in the future, as the LHC continues to run with pp collisions at the highest energy, it became an important matter to ponder on why certain searches for new physics have not provided strong evidence for and consnew physics, and thus considering some unexplored possibilities. Among these possibilities a Z' boson that favors higher-generation fermions, or the so-called "An anogenophilic Z'", and in particular coupling to third generation fermions

" A tritogenophilic Z'. The scenario of an anogenophilic Z' has been explored for various cases, we mention a case in which the new boson is produced Z' boson is produced by top quarks and decaying to tau leptons, as the figure (3.4) indicates. Or a case in which the "Top-philic Z'" is produced [82], at the LHC, in association with two top quarks at the tree level, or with a jet at the one-loop level, and can decay to two top quarks, since it was mentioned that the 3 dominant decay modes of the Z' boson are:

$$\begin{aligned}
\Gamma(Z' \to \mu^- \mu^+) &= \frac{c_L^2 c_E^2}{24\pi} M_Z' \\
\Gamma(Z' \to \nu\bar{\nu}) &= \frac{c_L^2}{24\pi} M_Z' \\
\Gamma(Z' \to t\bar{t}) &= \frac{c_{t_R}}{8\pi}\left(1 - \frac{m_t^2}{M_Z'^2}\right)\sqrt{1 - 4\frac{m_t^2}{M_Z'^2}} M_Z'
\end{aligned} \tag{3.17}$$

The coupling to the top quarks is defined as $c_t = \sqrt{c_L^2 + c_R^2}$, where c_L, c_R are components that couple only to left-handed and right-handed top-quarks respectively.

Figure 3.4: Representative Feynman diagrams for Z' production via $t - \bar{t}$ fusion (to the left) and $t - \bar{t}$ associated production (to the right), taken from [83].



In conclusion, final states containing heavy flavor quarks such as the top quark have yet to be explored to their fullest potential, and with the increase interest in the Z' potential discovery, this new frontier is waiting to be explored, and considered in future Z' searches at the LHC, by both the ATLAS and the CMS collaboration.

In this section, we have shed some beams of light upon the vast Z' phenomenology, what we mentioned is still just a brief overview of the potential Z' searches, many references offer more. To our particular goal, we will carry the previous package of data, and we will attempt to compute the amplitude of a simple reaction $b\bar{b} \to (Z, Z')\nu\bar{\nu}$, and try to interpret the resulting data with the amount of knowledge package we currently carry.

3.2 $b\bar{b} \to \nu\bar{\nu}$ Scattering process

The Z' influence is of great interest to investigate, and as we saw earlier, its coupling to fermions might be a strong route to probe its existence. Being motivated by such arguments, we try to consider a simple process including the Z' contributions. Such an attempt will be worked on in the context of the model we developed in chapter (2), and throughout the details of the work, we will mention the assumption adopted, as well as inferring some as a result of some choices we made.
The reaction under consideration is :

$$b\bar{b} \to (Z, Z') \to \nu\bar{\nu} \tag{3.18}$$

as it can be seen, the photon γ is not a factor participating in the reaction, due to the simple reason that neutrinos don't interact with photons. This will be of some importance later on to set up some constraints.
Given our interest, we will focus on the gauge fermions interactions, and this will be given by :

$$\begin{aligned}\mathscr{L}_f = \sum_{i=k}^{3} \Big[& i\bar{q}_{kL}\gamma^\mu D_\mu^{(q)_L} q_{kL} + i\bar{\ell}_{kL}\gamma^\mu D_\mu^{(\ell)_L}\ell_{kL} + i\bar{u}_{kR}\gamma^\mu D_\mu^{(u)_R} u_{kR} + i\bar{d}_{kR}\gamma^\mu D_\mu^{(d)_R} d_{kR} \\ & + i\bar{e}_{kR}\gamma^\mu D_\mu^{(e)_R} e_{kR}\Big]\end{aligned} \tag{3.19}$$

where, we distinguish :

$$\begin{aligned} D_\mu^{(f)_L} &= \left[\partial_\mu \mathbb{I} - \frac{i}{2} g \tau_a W_\mu^a - \frac{i}{2\sqrt{2}}(g'_Y Y_f + g_X X_f)\tilde{B}_\mu \mathbb{I} - \frac{i}{2\sqrt{2}}(g'_Y Y_f - g_X X_f)\tilde{C}_\mu \mathbb{I}\right] \\ D_\mu^{(f)_R} &= \left[\partial_\mu - \frac{i}{2\sqrt{2}}(g'_Y Y_f + g_X X_f)\tilde{B}_\mu - \frac{i}{2\sqrt{2}}(g'_Y Y_f - g_X X_f)\tilde{C}_\mu\right]\end{aligned} \tag{3.20}$$

$\tilde{B}_\mu, \tilde{C}_\mu$ are given by (2.76). And using a similar notation to the one we adopted here (2.86):

$$\begin{aligned} D_\mu^{(f)_L} &= \left[\partial_\mu \mathbb{I} - \frac{i}{2} g \tau_a W_\mu^a - \frac{ig}{2\sqrt{2}}\eta_f \tilde{B}_\mu \mathbb{I} - \frac{ig}{2\sqrt{2}}\varepsilon_f \tilde{C}_\mu \mathbb{I}\right] \\ D_\mu^{(f)_R} &= \left[\partial_\mu - \frac{ig}{2\sqrt{2}}\eta_f \tilde{B}_\mu - \frac{ig}{2\sqrt{2}}\varepsilon_f \tilde{C}_\mu\right]\end{aligned} \tag{3.21}$$

the relevant terms for a neutral current would then be:

$$i\bar{f}\gamma^\mu D_\mu^{(f)} f \to \left[i\bar{q}_{3L}\gamma^\mu D_\mu^{(q)_L} q_{3L} + i\bar{\ell}_{kL}\gamma^\mu D_\mu^{(\ell)_L}\ell_{kL} + i\bar{d}_{3R}\gamma^\mu D_\mu^{(d)_R} d_{3R}\right] \tag{3.22}$$

where the fermionic fields are explicitly written in (1.27). This will lead to :

$$\begin{aligned} i\bar{f}\gamma^\mu D_\mu^{(f)} f \to\ & i\begin{pmatrix}\bar{t} & \bar{b}\end{pmatrix}_L \begin{pmatrix}\gamma^\mu\partial_\mu & 0 \\ 0 & \gamma^\mu\partial_\mu\end{pmatrix}\begin{pmatrix} t \\ b\end{pmatrix}_L + \frac{g}{2\sqrt{2}}\begin{pmatrix}\bar{t} & \bar{b}\end{pmatrix}_L \gamma^\mu \begin{pmatrix}\frac{W_\mu^3}{\sqrt{2}} & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{\sqrt{2}}\end{pmatrix}\begin{pmatrix} t \\ b\end{pmatrix}_L \\ & + \frac{g}{2\sqrt{2}}\eta_q \begin{pmatrix}\bar{t} & \bar{b}\end{pmatrix}_L \begin{pmatrix}\gamma^\mu\tilde{B}_\mu & 0 \\ 0 & \gamma^\mu\tilde{B}_\mu\end{pmatrix}\begin{pmatrix} t \\ b\end{pmatrix}_L + \frac{g}{2\sqrt{2}}\varepsilon_q \begin{pmatrix}\bar{t} & \bar{b}\end{pmatrix}_L \begin{pmatrix}\gamma^\mu\tilde{C}_\mu & 0 \\ 0 & \gamma^\mu\tilde{C}_\mu\end{pmatrix}\begin{pmatrix} t \\ b\end{pmatrix}_L \\ & + i\begin{pmatrix}\bar{\nu}_{kL} & \bar{e}_{kL}\end{pmatrix}\begin{pmatrix}\gamma^\mu\partial_\mu & 0 \\ 0 & \gamma^\mu\partial_\mu\end{pmatrix}\begin{pmatrix}\nu_{kL} \\ e_{kL}\end{pmatrix} + \frac{g}{2\sqrt{2}}\begin{pmatrix}\bar{\nu}_{kL} & \bar{e}_{kL}\end{pmatrix}\gamma^\mu\begin{pmatrix}\frac{W_\mu^3}{\sqrt{2}} & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\nu_{kL} \\ e_{kL}\end{pmatrix} \\ & + \frac{g}{2\sqrt{2}}\eta_\ell \begin{pmatrix}\bar{\nu}_{kL} & \bar{e}_{kL}\end{pmatrix}\begin{pmatrix}\gamma^\mu\tilde{B}_\mu & 0 \\ 0 & \gamma^\mu\tilde{B}_\mu\end{pmatrix}\begin{pmatrix}\nu_{kL} \\ e_{kL}\end{pmatrix} + \frac{g}{2\sqrt{2}}\varepsilon_\ell \begin{pmatrix}\bar{\nu}_{kL} & \bar{e}_{kL}\end{pmatrix}\begin{pmatrix}\gamma^\mu\tilde{C}_\mu & 0 \\ 0 & \gamma^\mu\tilde{C}_\mu\end{pmatrix}\begin{pmatrix}\nu_{kL} \\ e_{kL}\end{pmatrix} \\ & + i\bar{b}_R\gamma^\mu\partial_\mu b_R + \frac{g}{2\sqrt{2}}\eta_d \bar{b}_R\gamma^\mu\tilde{B}_\mu b_R + \frac{g}{2\sqrt{2}}\varepsilon_d \bar{b}_R\gamma^\mu\tilde{C}_\mu b_R\end{aligned} \tag{3.23}$$

Expanding (3.23), and selecting once again the relevant terms for a neutral current, resulting in :

$$\begin{aligned}
&\rightarrow -\frac{g}{4}\bar{b}_L\gamma^\mu W^3_\mu b_L + \frac{g}{2\sqrt{2}}\eta_{b_L}\bar{b}_L\gamma^\mu\tilde{B}_\mu b_L + \frac{g}{2\sqrt{2}}\varepsilon_{b_L}\bar{b}_L\gamma^\mu\tilde{C}_\mu b_L \\
&+\frac{g}{4}\bar{\nu}_{kL}\gamma^\mu W^3_\mu \nu_{kL} + \frac{g}{2\sqrt{2}}\eta_{\nu_L}\bar{\nu}_{kL}\gamma^\mu\tilde{B}_\mu\nu_{kL} + \frac{g}{2\sqrt{2}}\varepsilon_{\nu_L}\bar{\nu}_{kL}\gamma^\mu\tilde{C}_\mu\nu_{kL} \\
&+\frac{g}{2\sqrt{2}}\eta_{b_R}\bar{b}_R\gamma^\mu\tilde{B}_\mu b_R + \frac{g}{2\sqrt{2}}\varepsilon_{b_R}\bar{b}_R\gamma^\mu\tilde{C}_\mu b_R
\end{aligned} \tag{3.24}$$

After reaching such a stage, it is necessary to switch to the physical state Z_μ, Z'_μ, γ. To achieve such a purpose, one might use the general parametrization:

$$O = \begin{pmatrix} c_\psi c_\phi - s_\theta s_\phi s_\psi & s_\psi c_\phi + s_\theta s_\phi c_\psi & -c_\theta s_\phi \\ c_\psi s_\phi + s_\theta c_\phi s_\psi & s_\psi s_\phi - s_\theta c_\phi c_\psi & c_\theta c_\phi \\ -c_\theta s_\psi & c_\theta c_\psi & s_\theta \end{pmatrix} \tag{3.25}$$

where: c, s represent the Cos, Sin of the respective angles (θ, ψ, ϕ).
While such a choice is valid, but our narrow focus will be leaning towards maintaining familiarity; in such a situation, our choice will be made to assimilate the parametrization to the of the SM. We choose :

$$\begin{aligned}
Z_\mu &= cos\theta_w W^3_\mu - sin\theta_w\tilde{C}_\mu &&= cos\theta_w W^3_\mu - sin\theta_w\left(\frac{B_\mu}{\sqrt{2}} - \frac{C_\mu}{\sqrt{2}}\right) \\
A_\mu &= sin\theta_w W^3_\mu + cos\theta_w\tilde{C}_\mu &&= sin\theta_w W^3_\mu + cos\theta_w\left(\frac{B_\mu}{\sqrt{2}} - \frac{C_\mu}{\sqrt{2}}\right) \\
Z'_\mu &= \tilde{B}_\mu
\end{aligned} \tag{3.26}$$

It is note worthy that such a choice will automatically make the gauge fixing in (2.75) relevant to the Z' field.
Leading to :

$$\begin{pmatrix} W^3_\mu \\ \tilde{C}_\mu \\ \tilde{B}_\mu \end{pmatrix} = \begin{pmatrix} cos\theta_w & sin\theta_w & 0 \\ -sin\theta_w & cos\theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \\ Z'_\mu \end{pmatrix} \tag{3.27}$$

replacing in (3.24) will yield:

$$\begin{aligned}
&\rightarrow -\frac{g}{4}\bar{b}_L\gamma^\mu[cos\theta_w Z_\mu + sin\theta_w A_\mu]b_L + \frac{g}{2\sqrt{2}}\eta_{b_L}\bar{b}_L\gamma^\mu Z'_\mu b_L + \frac{g}{2\sqrt{2}}\varepsilon_{b_L}\bar{b}_L\gamma^\mu[-sin\theta_w Z_\mu \\
&+ cos\theta_w A_\mu]b_L \\
&+\frac{g}{4}\bar{\nu}_{kL}\gamma^\mu[cos\theta_w Z_\mu + sin\theta_w A_\mu]\nu_{kL} + \frac{g}{2\sqrt{2}}\eta_{\nu_L}\bar{\nu}_{kL}\gamma^\mu Z'_\mu\nu_{kL} + \frac{g}{2\sqrt{2}}\varepsilon_{\nu_L}\bar{\nu}_{kL}\gamma^\mu[-sin\theta_w Z_\mu \\
&+ cos\theta_w A_\mu]\nu_{kL} \\
&+\frac{g}{2\sqrt{2}}\eta_{b_R}\bar{b}_R\gamma^\mu Z'_\mu b_R + \frac{g}{2\sqrt{2}}\varepsilon_{b_R}\bar{b}_R\gamma^\mu[-sin\theta_w Z_\mu + cos\theta_w A_\mu]b_R.
\end{aligned} \tag{3.28}$$

Finally, a rearrangement will lead to :

$$\begin{aligned}
&\rightarrow \left[\frac{g}{2\sqrt{2}}\eta_{b_L}\right]\bar{b}_L\gamma^\mu Z'_\mu b_L + \left[-\frac{g}{4}cos\theta_w - \frac{g}{2\sqrt{2}}\varepsilon_{b_L}sin\theta_w\right]\bar{b}_L\gamma^\mu Z_\mu b_L \\
&+\left[-\frac{g}{4}sin\theta_w + \frac{g}{2\sqrt{2}}\varepsilon_{b_L}cos\theta_w\right]\bar{b}_L\gamma^\mu A_\mu b_L \\
&+\left[\frac{g}{2\sqrt{2}}\eta_{\nu_L}\right]\bar{\nu}_{kL}\gamma^\mu Z'_\mu\nu_{kL} + \left[\frac{g}{4}cos\theta_w - \frac{g}{2\sqrt{2}}\varepsilon_{\nu_L}sin\theta_w\right]\bar{\nu}_{kL}\gamma^\mu Z_\mu\nu_{kL} \\
&+\left[\frac{g}{2\sqrt{2}}\eta_{b_R}\right]\bar{b}_R\gamma^\mu Z'_\mu b_R + \left[-\frac{g}{2\sqrt{2}}\varepsilon_{b_R}sin\theta_w\right]\bar{b}_R\gamma^\mu Z_\mu b_R + \left[\frac{g}{2\sqrt{2}}\varepsilon_{b_R}cos\theta_w\right]\bar{b}_R\gamma^\mu A_\mu b_R
\end{aligned} \tag{3.29}$$

with the constraint :

$$\left[\frac{g}{4}sin\theta_w + \frac{g}{2\sqrt{2}}\varepsilon_{\nu_{kL}}cos\theta_w\right] = 0 \tag{3.30}$$

At this level, we have successfully wrote the relevant terms to our computations, and it is easier to proceed from here and deduce the relevant Feynman diagrams, thus embarking on calculating the amplitude for the reaction at hand. But, what would be preferable to do is to rewrite the vertices in a more concise manner, and to find $(\eta/\varepsilon)_{b_L,b_R,\nu}$ in terms of the known parameters of the SM, for the purpose of easing the comparison later on.

Starting with:

$$\begin{aligned} b_L &= (\frac{1-\gamma_5}{2})b \qquad \bar{b}_L = \bar{b}(\frac{1+\gamma_5}{2}) \\ b_R &= (\frac{1+\gamma_5}{2})b \qquad \bar{b}_R = \bar{b}(\frac{1-\gamma_5}{2}) \\ \nu_{kL} &= (\frac{1-\gamma_5}{2})\nu_k \end{aligned} \tag{3.31}$$

as (1.10) indicates[9].
As a result of making use of (3.31), we obtain:

$$\begin{aligned} &i\bar{f}\gamma^\mu D_\mu^{(f)} f \rightarrow \frac{g}{4\sqrt{2}}\left[(\eta_{b_L}+\eta_{b_R}) + (\eta_{b_R}-\eta_{b_L})\gamma_5\right]\bar{b}\gamma^\mu Z'_\mu b \\ &+\left[\left(-\frac{g}{8}cos\theta_w - \frac{g}{4\sqrt{2}}sin\theta_w(\varepsilon_{b_L}+\varepsilon_{b_R})\right) + \left(\frac{g}{8}cos\theta_w - \frac{g}{4\sqrt{2}}sin\theta_w(\varepsilon_{b_R}-\varepsilon_{b_L})\right)\gamma_5\right]\bar{b}\gamma^\mu Z_\mu b \\ &+\left[\left(-\frac{g}{8}sin\theta_w + \frac{g}{4\sqrt{2}}cos\theta_w(\varepsilon_{b_L}+\varepsilon_{b_R})\right) + \left(\frac{g}{8}sin\theta_w - \frac{g}{4\sqrt{2}}cos\theta_w(\varepsilon_{b_R}-\varepsilon_{b_L})\right)\gamma_5\right]\bar{b}\gamma^\mu A_\mu b \\ &+\frac{g}{4\sqrt{2}}\left[(\eta_{\nu_L}) - (\eta_{\nu_L})\gamma_5\right]\bar{\nu}_k\gamma^\mu Z'_\mu\nu_k \\ &+\left[\left(\frac{g}{8}cos\theta_w - \frac{g}{4\sqrt{2}}\varepsilon_{\nu_L}sin\theta_w\right) - \left(\frac{g}{8}cos\theta_w - \frac{g}{4\sqrt{2}}\varepsilon_{\nu_L}sin\theta_w\right)\gamma_5\right]\bar{\nu}_k\gamma^\mu Z_\mu\nu_k \end{aligned} \tag{3.32}$$

All what's remaining is the parameters $\eta_{(b_L,b_R,\nu)}, \varepsilon_{(b_L,b_R,\nu)}$. ε_ν can be easily determined from (3.30):

$$\varepsilon_\nu = -\sqrt{2}tan\theta_w \tag{3.33}$$

While in order to determine the remaining factors, we will rewrite the covariant derivative in (3.21) in terms of the physical states. In the upcoming procedure, we will drop the chirality distinctions, for the sole reason that they won't make any difference, except making the procedure a strenuous work. We will bring it back when we acquire the final results.

Starting from:

$$D_\mu^{(f)} = \left[\partial_\mu\mathbb{I} - ig(T_+W_\mu^+ + T_-W_\mu^-) - igT_3W_\mu^3 - \frac{ig}{2\sqrt{2}}\eta_f\tilde{B}_\mu\mathbb{I} - \frac{ig}{2\sqrt{2}}\varepsilon_f\tilde{C}_\mu\mathbb{I}\right] \tag{3.34}$$

where: $T_\pm = T_1 \pm iT_2$, and $W_\mu^\pm$ are given by (1.18).
Using (3.27), resulting in :

$$\begin{aligned} D_\mu^{(f)} = &\left[\partial_\mu\mathbb{I} - ig(T_+W_\mu^+ + T_-W_\mu^-) - igcos\theta_w\left(T_3 - \frac{\varepsilon_f tan\theta_w}{2\sqrt{2}}\mathbb{I}\right)Z_\mu - ig\left(\frac{\eta_f}{2\sqrt{2}}\mathbb{I}\right)Z'_\mu\right. \\ &\left.-igsin\theta_w\left(T_3 + \frac{\varepsilon_f}{2\sqrt{2}tan\theta_w}\mathbb{I}\right)A_\mu\right] \end{aligned} \tag{3.35}$$

[9]Knowing that : $\gamma_5^\dagger = \gamma_5$, $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$, and $\gamma_5^2 = 1$.

labeling the generators as:

$$
\begin{aligned}
Q_Z &= T_3 - \frac{\varepsilon_f tan\theta_w}{2\sqrt{2}}\mathbb{I} \\
Q_Z' &= \frac{\eta_f}{2\sqrt{2}}\mathbb{I} \\
Q_A &= T_3 + \frac{\varepsilon_f}{2\sqrt{2}tan\theta_w}\mathbb{I}
\end{aligned}
\tag{3.36}
$$

After such a step, the methodology will consist of identifying the factor in terms of something known, in this case Y_{SM}. So, we will attempt to write η_f, ε_f in terms of Y_{SM}.
The parameter ε_f can be easily deduced via identifying the charge operator of that of the SM:

$$
\begin{cases}
Q_{SM} = T_3 + \dfrac{Y_{SM}}{2}\mathbb{I} \\
Q_A = T_3 + \dfrac{\varepsilon_f}{2\sqrt{2}tan\theta_w}\mathbb{I}
\end{cases}
\Rightarrow \varepsilon_f = \sqrt{2}tan\theta_w Y_{SM} \Longleftrightarrow \varepsilon_f = \frac{\sqrt{2}g'}{g}Y_{SM}
\tag{3.37}
$$

Now, to obtain η_f, we make use of:

$$
\begin{aligned}
\eta_\phi &= 0 \rightarrow g_Y' Y_\phi = -g_X' X_\phi \\
\varepsilon_\phi &= \sqrt{2}tan\theta_w \rightarrow \frac{2g_Y' Y_\phi}{g} = \sqrt{2}\frac{g'}{g}
\end{aligned}
\tag{3.38}
$$

which will lead to:

$$
\begin{aligned}
Y_\phi &= \frac{g'}{\sqrt{2}g_Y'} \Longleftrightarrow g_Y' = \frac{g'}{\sqrt{2}Y_\phi} \\
X_\phi &= -\frac{g'}{\sqrt{2}g_X'} \Longleftrightarrow g_X' = -\frac{g'}{\sqrt{2}X_\phi}
\end{aligned}
\tag{3.39}
$$

combining (3.39) with:

$$
\begin{aligned}
\eta_f &= \frac{g_Y' Y_f + g_X' X_f}{g} \\
\varepsilon_f &= \frac{g_Y' Y_f - g_X' X_f}{g}
\end{aligned}
\tag{3.40}
$$

will yield:

$$
\begin{aligned}
\eta_f &= \frac{g'}{\sqrt{2}g}\left[\frac{Y_f}{Y_\phi} - \frac{X_f}{X_\phi}\right] \\
\varepsilon_f &= \frac{g'}{\sqrt{2}g}\left[\frac{Y_f}{Y_\phi} + \frac{X_f}{X_\phi}\right]
\end{aligned}
\tag{3.41}
$$

making use of (3.37):

$$
\begin{aligned}
\frac{g'}{\sqrt{2}g}\left[\frac{Y_f}{Y_\phi} + \frac{X_f}{X_\phi}\right] &= \frac{\sqrt{2}g'}{g}Y_{SM} \\
\Longleftrightarrow \left[\frac{Y_f}{Y_\phi} + \frac{X_f}{X_\phi}\right] &= 2Y_{SM}
\end{aligned}
\tag{3.42}
$$

and replacing in the expression of η_f, and pulling X_f, such that :

$$
X_f = \frac{g_Y'}{g_X'}Y_f - \frac{\sqrt{2}g'}{g_X'}Y_{SM}
\tag{3.43}
$$

which will enable us to write the expression of η_f:

$$\eta_f = \frac{2g_Y^{'}}{g} Y_f - \frac{\sqrt{2}g^{'}}{g} Y_{SM} \tag{3.44}$$

So that all what's remaining is Y_f. The determination of the factor can be easily achieved if we impose the conditions of anomaly cancellation, thus ensuring that our model is anomaly free, and at the same time obtaining the final expressions. The conditions are :

$$\begin{aligned} Tr[\mathbb{Y}] = Tr[\mathbb{X}] = 0 \\ Tr[\mathbb{Y}^3] = Tr[\mathbb{Y}^2\mathbb{X}] = Tr[\mathbb{Y}\mathbb{X}^2] = Tr[\mathbb{X}^3] = 0 \end{aligned} \tag{3.45}$$

and since we know that the Y_{SM} fulfills the conditions $Tr[\mathbb{Y}_{SM}] = Tr[\mathbb{Y}_{SM}^3] = 0$, then the easiest choice available is to choose $Y_f \propto Y_{SM}$. Then we can write :

$$Y_f = \kappa Y_{SM} \tag{3.46}$$

leading to :

$$\eta_f = \left[\frac{2g_Y^{'}}{g}\kappa - \frac{\sqrt{2}g^{'}}{g} \right] Y_{SM} = \varrho Y_{SM} \tag{3.47}$$

As we can see, ϱ is family-universal, and that is exactly the reason why at the beginning we dropped the chirality distinctions. By bringing it back we can finally write:

$$\begin{aligned} \eta_{f_L} &= \varrho Y_{SM}^{f_L} & \eta_{f_R} &= \varrho Y_{SM}^{f_R} \\ \varepsilon_{f_L} &= \frac{\sqrt{2}g^{'}}{g} Y_{SM}^{f_L} & \varepsilon_{f_R} &= \frac{\sqrt{2}g^{'}}{g} Y_{SM}^{f_R} \end{aligned} \tag{3.48}$$

And finally, we have obtained the final form of the vertices, we write:

$$\begin{aligned} i\bar{f}\gamma^\mu D_\mu^{(f)} f \rightarrow & \frac{g\varrho}{4\sqrt{2}} \left[(Y_{SM}^{b_L} + Y_{SM}^{b_R}) + (Y_{SM}^{b_R} - Y_{SM}^{b_L})\gamma_5 \right] \bar{b}\gamma^\mu Z_\mu^{'} b \\ &+ \left[\left(-\frac{g}{8}cos\theta_w - \frac{g^{'}}{4}sin\theta_w (Y_{SM}^{b_L} + Y_{SM}^{b_R}) \right) + \left(\frac{g}{8}cos\theta_w - \frac{g^{'}}{4}sin\theta_w (Y_{SM}^{b_R} - Y_{SM}^{b_L}) \right) \gamma_5 \right] \bar{b}\gamma^\mu Z_\mu b \\ &+ \left[\left(-\frac{g}{8}sin\theta_w + \frac{g^{'}}{4}cos\theta_w (Y_{SM}^{b_L} + Y_{SM}^{b_R}) \right) + \left(\frac{g}{8}sin\theta_w - \frac{g^{'}}{4}cos\theta_w (Y_{SM}^{b_R} - Y_{SM}^{b_L}) \right) \gamma_5 \right] \bar{b}\gamma^\mu A_\mu b \\ &+ \frac{g\varrho}{4\sqrt{2}} \left[(Y_{SM}^{\nu_L}) - (Y_{SM}^{\nu_L})\gamma_5 \right] \bar{\nu}_k \gamma^\mu Z_\mu^{'} \nu_k \\ &+ \left[\left(\frac{g}{8}cos\theta_w - \frac{g^{'}}{4}sin\theta_w Y_{SM}^{\nu_L} \right) - \left(\frac{g}{8}cos\theta_w - \frac{g^{'}}{4}sin\theta_w Y_{SM}^{\nu_L} \right) \gamma_5 \right] \bar{\nu}_k \gamma^\mu Z_\mu \nu_k \end{aligned} \tag{3.49}$$

we can make use of the fact that $e = gsin\theta_w = g^{'}cos\theta_w$, and further useful simplifications will lead to:

$$\begin{aligned} i\bar{f}\gamma^\mu D_\mu^{(f)} f \rightarrow & \frac{g\varrho}{4\sqrt{2}} \left[(Y_{SM}^{b_L} + Y_{SM}^{b_R}) + (Y_{SM}^{b_R} - Y_{SM}^{b_L})\gamma_5 \right] \bar{b}\gamma^\mu Z_\mu^{'} b \\ &+ \frac{g}{4cos\theta_w} \left[\left(\frac{-1 + sin^2\theta_w}{2} - sin^2\theta_w (Y_{SM}^{b_L} + Y_{SM}^{b_R}) \right) + \left(\frac{1 - sin^2\theta_w}{2} - sin^2\theta_w (Y_{SM}^{b_R} - Y_{SM}^{b_L}) \right) \gamma_5 \right] \bar{b}\gamma^\mu Z_\mu b \\ &+ \frac{e}{4} \left[\left(-\frac{1}{2} + (Y_{SM}^{b_L} + Y_{SM}^{b_R}) \right) + \left(\frac{1}{2} - (Y_{SM}^{b_R} - Y_{SM}^{b_L}) \right) \gamma_5 \right] \bar{b}\gamma^\mu A_\mu b \\ &+ \frac{g\varrho}{4\sqrt{2}} \left[(Y_{SM}^{\nu_L}) - (Y_{SM}^{\nu_L})\gamma_5 \right] \bar{\nu}_k \gamma^\mu Z_\mu^{'} \nu_k \\ &+ \frac{g}{4cos\theta_w} \left[\left(\frac{1 - sin^2\theta_w}{2} - sin^2\theta_w Y_{SM}^{\nu_L} \right) - \left(\frac{1 - sin^2\theta_w}{2} - sin^2\theta_w Y_{SM}^{\nu_L} \right) \gamma_5 \right] \bar{\nu}_k \gamma^\mu Z_\mu \nu_k \end{aligned} \tag{3.50}$$

As we can see, the vertices are of the form:

$$\begin{aligned}
\mathscr{L}_{Z'\bar{f}f} &= \sum_f \bar{\psi}_f \gamma^\mu (c'_{v_f} + c'_{A_f}\gamma_5)\psi_f \\
\mathscr{L}_{Z\bar{f}f} &= \sum_f \bar{\psi}_f \gamma^\mu (c_{v_f} + c_{A_f}\gamma_5)\psi_f \\
\mathscr{L}_{A\bar{f}f} &= \sum_f \bar{\psi}_f \gamma^\mu (c''_{v_f} + c''_{A_f}\gamma_5)\psi_f
\end{aligned} \tag{3.51}$$

where $c_{v_f}, c'_{v_f}, c''_{v_f}$, $c_{A_f}, c'_{A_f}, c''_{A_f}$ and are called vector and axial couplings, respectively.

Furthermore, we have obtained the relevant vertices to our computations, as a final step we will calculate the vector and axial couplings for the two vertices :$Z'\bar{f}f, Z\bar{f}f$. The results are indicated in the tables (3.2, 3.3).

Table 3.2: The values of the vector and axial couplings c'_{v_f}, c'_{A_f} for the vertex $Z'\bar{f}f$.

f	c'_{v_f}	c'_{A_f}
b	$-\frac{g\varrho}{12\sqrt{2}}$	$-\frac{g\varrho}{4\sqrt{2}}$
ν_k	$-\frac{g\varrho}{4\sqrt{2}}$	$\frac{g\varrho}{4\sqrt{2}}$

Table 3.3: The values of the vector and axial couplings c_{v_f}, c_{A_f} for the vertex $Z\bar{f}f$.

f	c_{v_f}	c_{A_f}
b	$\left(\frac{g}{4cos\theta_w}\right)\left(\frac{-3+5sin^2\theta_w}{6}\right)$	$\left(\frac{g}{4cos\theta_w}\right)\left(\frac{1+sin^2\theta_w}{2}\right)$
ν_k	$\left(\frac{g}{4cos\theta_w}\right)\left(\frac{1+sin^2\theta_w}{2}\right)$	$\left(\frac{g}{4cos\theta_w}\right)\left(\frac{-1-sin^2\theta_w}{2}\right)$

The values of Y^f_{SM} can be found in appendix (B).

In conclusion, we are ready to draw the Feynman diagrams associated with the interaction Lagrangian we have obtained, and embark on the procedure of calculating the Feynman amplitude for the transition, and its square value, since we need it to evaluate the differential cross section later on.

3.2.1 The square amplitude $|\mathscr{M}|^2$ of the scattering process

The scattering process under evaluation is :

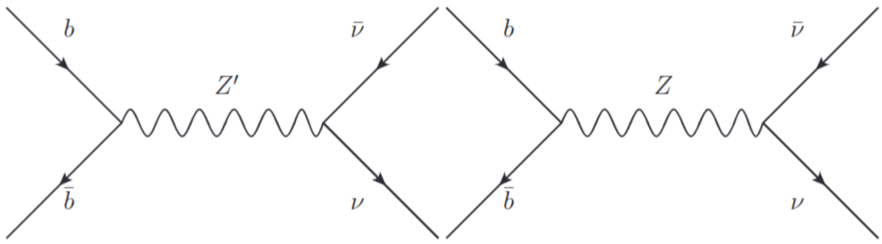
$$b(p_1) + \bar{b}(p_2) \rightarrow \nu_k(p_3) + \bar{\nu}_k(p_4) \tag{3.52}$$

Given the expression (3.49), we are in a position to infer and draw the associating Feynman diagrams. Thus, we obtain the diagrams illustrated in figure(3.5).

Making use of these diagrams, we will attempt to use the Feynman rules[10], and derive

[10] The rules are mentioned in appendix (D).

Figure 3.5: Feynman Diagrams for the process $b\bar{b} \to \nu\bar{\nu}$



the amplitude associated with each diagram, so we can compute the square amplitude of the entire process. Such that :

$$|\mathscr{M}|^2 = |\mathscr{M}_1 + \mathscr{M}_2|^2 \tag{3.53}$$

Starting off with the first diagram:

$$i\mathscr{M}_1 = \left[\bar{v}^{s_2}(p_2)i\gamma^\alpha(c'_{v_b} + c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\frac{-ig_{\alpha\beta}}{k^2 - M_{Z'}^2 + i\epsilon}\right]\left[\bar{u}^{s_3}(p_3)i\gamma^\beta(c'_{v_{\nu_k}} + c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right] \tag{3.54}$$

leading to :

$$\mathscr{M}_1 = \frac{-1}{k^2 - M_{Z'}^2 + i\epsilon}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b} + c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}} + c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right] \tag{3.55}$$

similarly for the second diagram:

$$i\mathscr{M}_2 = \left[\bar{v}^{s_2}(p_2)i\gamma^\alpha(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\frac{-ig_{\alpha\beta}}{k^2 - M_Z^2 + i\epsilon}\right]\left[\bar{u}^{s_3}(p_3)i\gamma^\beta(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right] \tag{3.56}$$

which leads to :

$$\mathscr{M}_2 = \frac{-1}{k^2 - M_Z^2 + i\epsilon}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right] \tag{3.57}$$

where $k^2 = (p_1 + p_2)^2$, and the vector / axial couplings are given by the tables (3.2, 3.3). The square amplitude then can be calculated, the starting expression would then be:

$$|\mathscr{M}|^2 = (\mathscr{M}_1 + \mathscr{M}_2)(\mathscr{M}_1 + \mathscr{M}_2)^\dagger \tag{3.58}$$

which can be explicitly written as:

$$\begin{aligned}
|\mathscr{M}|^2 = &\left(\frac{-1}{k^2 - M_{Z'}^2 + i\epsilon}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b} + c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}} + c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]\right. \\
&\left.-\frac{1}{k^2 - M_Z^2 + i\epsilon}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]\right) \times \\
&\left(\frac{-1}{k^2 - M_{Z'}^2 - i\epsilon}\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c'_{v_{\nu_k}} + c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c'_{v_b} + c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\right. \\
&\left.-\frac{1}{k^2 - M_Z^2 - i\epsilon}\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\right)
\end{aligned} \tag{3.59}$$

which leads to:

$$|\mathscr{M}|^2 = \Bigg(\frac{1}{(k^2-M_{Z'}^2)^2+\epsilon^2}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]$$
$$\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\Bigg)$$
$$+\Bigg(\frac{1}{(k^2-M_Z^2)^2+\epsilon^2}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c_{v_b}+c_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c_{v_{\nu_k}}+c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]$$
$$\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c_{v_{\nu_k}}+c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c_{v_b}+c_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\Bigg)$$
$$+\Bigg(\frac{1}{(k^2-M_{Z'}^2)(k^2-M_Z^2)+i\epsilon M_{Z'}^2-i\epsilon M_Z^2+\epsilon^2}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]$$
$$\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c_{v_{\nu_k}}+c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger$$
$$\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c_{v_b}+c_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\Bigg)$$
$$+\Bigg(\frac{1}{(k^2-M_{Z'}^2)(k^2-M_Z^2)-i\epsilon M_{Z'}^2+i\epsilon M_Z^2+\epsilon^2}\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c_{v_b}+c_{A_b}\gamma_5)u^{s_1}(p_1)\right]$$
$$\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c_{v_{\nu_k}}+c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger$$
$$\left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger\Bigg) \tag{3.60}$$

In order to compute this expression, we will make use of the so-called Casimir's trick for calculating the square amplitude [11]. And we will calculate each term separately, starting with :

$$\begin{aligned}|\mathscr{M}_1|^2 \sim &\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]\\ &\left[\bar{v}^{s_4}(p_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha u^{s_3}(p_3)\right]\left[\bar{u}^{s_1}(p_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha v^{s_2}(p_2)\right]\end{aligned} \tag{3.61}$$

we sum over the spins gradually, and using the completeness relations:

$$\begin{aligned}\sum_{s_i=1,2} u_i^{s_i}\bar{u}_i^{s_i} &= (\not{p}_i+m_i)\\ \sum_{s_i=1,2} v_i^{s_i}\bar{v}_i^{s_i} &= (\not{p}_i-m_i)\end{aligned} \tag{3.62}$$

we obtain:

$$\begin{aligned}\sum_{s_4}|\mathscr{M}_1|^2 \sim &\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)u^{s_1}(p_1)\right]\\ &Tr\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha u^{s_3}(p_3)\right]\\ &\left[\bar{u}^{s_1}(p_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha v^{s_2}(p_2)\right]\end{aligned} \tag{3.63}$$

similarly,

$$\begin{aligned}\sum_{s_1,s_4}|\mathscr{M}_1|^2 \sim\ &Tr\left[\bar{v}^{s_2}(p_2)\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)(\not{p}_1+m_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha v^{s_2}(p_2)\right]\\ &Tr\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha u^{s_3}(p_3)\right]\end{aligned} \tag{3.64}$$

[11] More details can be found in appendix (E.)

using the cyclic symmetry property of the traces, we find:

$$\sum_{s_1,s_2,s_3,s_4} |\mathscr{M}_1|^2 \sim Tr\left[\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)(\not{p}_1+m_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha(\not{p}_2-m_2)\right]$$
$$Tr\left[\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha(\not{p}_3+m_3)\right] \tag{3.65}$$

via averaging over the initial spins, we get:

$$< |\mathscr{M}_1|^2 > \sim \frac{1}{4}Tr\left[\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)(\not{p}_1+m_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha(\not{p}_2-m_2)\right]$$
$$Tr\left[\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha(\not{p}_3+m_3)\right] \tag{3.66}$$

working on each trace separately will yield:

$$\begin{aligned}
&Tr\left[\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)(\not{p}_1+m_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha(\not{p}_2-m_2)\right]=\\
&\quad c'^2_{v_b}Tr[\gamma_\beta\not{p}_1\gamma^0\gamma_0\gamma_\alpha\not{p}_2]-m_2c'^2_{v_b}Tr[\gamma_\beta\not{p}_1\gamma^0\gamma_0\gamma_\alpha]+c'_{v_b}c'_{A_b}Tr[\gamma_\beta\not{p}_1\gamma^0\gamma_5\gamma_0\gamma_\alpha\not{p}_2]\\
&\quad -m_2c'_{v_b}c'_{A_b}Tr[\gamma_\beta\not{p}_1\gamma^0\gamma_5\gamma_0\gamma_\alpha]+m_1c'^2_{v_b}Tr[\gamma_\beta\gamma^0\gamma_0\gamma_\alpha\not{p}_2]-m_1m_2c'^2_{v_b}Tr[\gamma_\beta\gamma^0\gamma_0\gamma_\alpha]\\
&\quad +m_1c'_{v_b}c'_{A_b}Tr[\gamma_\beta\gamma^0\gamma_5\gamma_0\gamma_\alpha\not{p}_2]-m_1m_2c'_{v_b}c'_{A_b}Tr[\gamma_\beta\gamma^0\gamma_5\gamma_0\gamma_\alpha]+c'_{v_b}c'_{A_b}Tr[\gamma_\beta\gamma_5\not{p}_1\gamma^0\gamma_5\gamma_0\gamma_\alpha\not{p}_2]\\
&\quad -m_2c'_{v_b}c'_{A_b}Tr[\gamma_\beta\gamma_5\not{p}_1\gamma^0\gamma_0\gamma_\alpha]+c'^2_{A_b}Tr[\gamma_\beta\gamma_5\not{p}_1\gamma^0\gamma_5\gamma_0\gamma_\alpha\not{p}_2]-m_2c'^2_{A_b}Tr[\gamma_\beta\gamma_5\not{p}_1\gamma^0\gamma_5\gamma_0\gamma_\alpha]\\
&\quad +m_1c'_{A_b}c'_{v_b}Tr[\gamma_\beta\gamma_5\gamma^0\gamma_0\gamma_\alpha\not{p}_2]-m_1m_2c'_{A_b}c'_{v_b}Tr[\gamma_\beta\gamma_5\gamma^0\gamma_0\gamma_\alpha]+m_1c'^2_{A_b}Tr[\gamma_\beta\gamma_5\gamma^0\gamma_5\gamma_0\gamma_\alpha\not{p}_2]\\
&\quad -m_1m_2c'^2_{A_b}Tr[\gamma_\beta\gamma_5\gamma^0\gamma_5\gamma_0\gamma_\alpha].
\end{aligned} \tag{3.67}$$

and,

$$\begin{aligned}
&Tr\left[\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha(\not{p}_3+m_3)\right]=\\
&\quad c'^2_{v_{\nu_k}}Tr[\gamma^\beta\not{p}_4\gamma^0\gamma^0\gamma^\alpha\not{p}_3]-m_3c'^2_{v_{\nu_k}}Tr[\gamma^\beta\not{p}_4\gamma^0\gamma^0\gamma^\alpha]+c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\not{p}_4\gamma^0\gamma_5\gamma^0\gamma^\alpha\not{p}_3]\\
&\quad +m_3c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\not{p}_4\gamma^0\gamma_5\gamma^0\gamma_\alpha]-m_4c'^2_{v_{\nu_k}}Tr[\gamma^\beta\gamma^0\gamma^0\gamma^\alpha\not{p}_3]-m_4m_3c'^2_{v_{\nu_k}}Tr[\gamma^\beta\gamma^0\gamma^0\gamma^\alpha]\\
&\quad -m_4c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\gamma^0\gamma_5\gamma^0\gamma^\alpha\not{p}_3]-m_4m_3c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\gamma^0\gamma_5\gamma^0\gamma^\alpha]+c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\not{p}_4\gamma^0\gamma^0\gamma^\alpha\not{p}_3]\\
&\quad +m_3c'_{v_{\nu_k}}c'_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\not{p}_4\gamma^0\gamma^0\gamma^\alpha]+c'^2_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\not{p}_4\gamma^0\gamma_5\gamma^0\gamma^\alpha\not{p}_3]+m_3c'^2_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\not{p}_4\gamma^0\gamma_5\gamma^0\gamma^\alpha]\\
&\quad -m_4c'_{A_{\nu_k}}c'_{v_{\nu_k}}Tr[\gamma^\beta\gamma_5\gamma^0\gamma^0\gamma^\alpha\not{p}_3]-m_4m_3c'_{A_{\nu_k}}c'_{v_{\nu_k}}Tr[\gamma^\beta\gamma_5\gamma^0\gamma^0\gamma^\alpha]-m_4c'^2_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\gamma^0\gamma_5\gamma^0\gamma^\alpha\not{p}_3]\\
&\quad -m_4m_3c'^2_{A_{\nu_k}}Tr[\gamma^\beta\gamma_5\gamma^0\gamma_5\gamma^0\gamma^\alpha].
\end{aligned} \tag{3.68}$$

working out the traces [12], and a simplification will result in:

$$\begin{aligned}
&Tr\left[\gamma_\beta(c'_{v_b}+c'_{A_b}\gamma_5)(\not{p}_1+m_1)\gamma^0(c'_{v_b}+c'_{A_b}\gamma_5)\gamma_0\gamma_\alpha(\not{p}_2-m_2)\right]=4m_1m_2(c'^2_{A_b}-c'^2_{v_b})g_{\beta\alpha}\\
&\quad +(c'^2_{A_b}+c'^2_{v_b})\left[4p_{2\beta}p_{1\alpha}-4(p_2p_1)g_{\beta\alpha}+4p_{1\alpha}p_{2\beta}\right]
\end{aligned} \tag{3.69}$$

and,

$$\begin{aligned}
&Tr\left[\gamma^\beta(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)(\not{p}_4-m_4)\gamma^0(c'_{v_{\nu_k}}+c'_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha(\not{p}_3+m_3)\right]=4m_4m_3(c'^2_{A_{\nu_k}}-c'^2_{v_\nu})g^{\beta\alpha}\\
&\quad +(c'^2_{A_{\nu_k}}+c'^2_{v_{\nu_k}})\left[4p_3^\beta p_4^\alpha-4(p_3p_4)g^{\beta\alpha}+4p_3^\alpha p_4^\beta\right]
\end{aligned} \tag{3.70}$$

[12]Some details about the calculations are carried out in appendix (E).

resulting in,

$$
\begin{aligned}
&< |\mathscr{M}_1|^2 >\sim 4m_1m_2m_3m_4(c'^2_{A_{\nu_k}} - c'^2_{v_{\nu_k}})(c'^2_{A_b} - c'^2_{v_b}) \\
&+ 8(c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})[(p_2.p_3)(p_1.p_4) + (p_2.p_4)(p_1.p_3) - (p_3.p_4)(p_2.p_1) + (p_2.p_1)(p_3.p_4)] \\
&- 8m_1m_2(p_3p_4)(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}}) - 8m_3m_4(p_2.p_1)(c'^2_{v_b} + c'^2_{A_b})(c'^2_{A_{\nu_k}} - c'^2_{v_{\nu_k}})
\end{aligned} \tag{3.71}
$$

finally replacing $m_{1,2} = m_b, m_{3,4} = m_\nu = 0$, we obtain:

$$
\begin{aligned}
< |\mathscr{M}_1|^2 > = \frac{8}{(k^2 - M^2_{Z'})^2} \Big[& m_b^2(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})(p_3.p_4) \\
&+ (c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})[(p_2.p_3)(p_1.p_4) + (p_1.p_3)(p_2.p_4)] \Big]
\end{aligned} \tag{3.72}
$$

Similarly for the second term, which is just $|\mathscr{M}_2|^2$:

$$
\begin{aligned}
|\mathscr{M}_2|^2 \sim & \left[\bar{v}^{s_2}(p_2)\gamma_\beta(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]\left[\bar{u}^{s_3}(p_3)\gamma^\beta(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right] \\
& \left[\bar{u}^{s_3}(p_3)\gamma^\alpha(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)v^{s_4}(p_4)\right]^\dagger \left[\bar{v}^{s_2}(p_2)\gamma_\alpha(c_{v_b} + c_{A_b}\gamma_5)u^{s_1}(p_1)\right]^\dagger
\end{aligned} \tag{3.73}
$$

a similar treatment will lead to the following expression;

$$
\begin{aligned}
< |\mathscr{M}_2|^2 > \sim & \frac{1}{4} Tr\left[\gamma_\beta(c_{v_b} + c_{A_b}\gamma_5)(\not{p}_1 + m_1)\gamma^0(c_{v_b} + c_{A_b}\gamma_5)\gamma_0\gamma_\alpha(\not{p}_2 - m_2)\right] \\
& Tr\left[\gamma^\beta(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)(\not{p}_4 - m_4)\gamma^0(c_{v_{\nu_k}} + c_{A_{\nu_k}}\gamma_5)\gamma^0\gamma^\alpha(\not{p}_3 + m_3)\right]
\end{aligned} \tag{3.74}
$$

while following the same previous procedure will yield the following result :

$$
\begin{aligned}
&< |\mathscr{M}_2|^2 >\sim 4m_1m_2m_3m_4(c^2_{A_{\nu_k}} - c^2_{v_{\nu_k}})(c^2_{A_b} - c^2_{v_b}) \\
&+ 8(c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})[(p_2.p_3)(p_1.p_4) + (p_2.p_4)(p_1.p_3) - (p_3.p_4)(p_2.p_1) + (p_2.p_1)(p_3.p_4)] \\
&- 8m_1m_2(p_3p_4)(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}}) - 8m_3m_4(p_2.p_1)(c^2_{v_b} + c^2_{A_b})(c^2_{A_{\nu_k}} - c^2_{v_{\nu_k}})
\end{aligned} \tag{3.75}
$$

resulting in,

$$
\begin{aligned}
< |\mathscr{M}_2|^2 > = \frac{8}{(k^2 - M^2_{Z})^2} \Big[& m_b^2(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})(p_3.p_4) \\
&+ (c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})[(p_2.p_3)(p_1.p_4) + (p_1.p_3)(p_2.p_4)] \Big]
\end{aligned} \tag{3.76}
$$

The mixed terms will also be treated in a similar fashion, resulting in:

$$
\begin{aligned}
2 < \mathscr{M}_1\mathscr{M}_2^\dagger > = \frac{16}{(k^2 - M^2_{Z'})(k^2 - M^2_Z)} \Big[& m_b^2(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})(p_3.p_4) \\
&+ (c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})[(p_2.p_3)(p_1.p_4) + (p_1.p_3)(p_2.p_4)] \Big]
\end{aligned} \tag{3.77}
$$

Which enables us to finally write the expression for the square amplitude of our process:

$$
\begin{aligned}
<|\mathscr{M}|^2> = 8m_b^2 & \left[\frac{(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2} \right. \\
& \left. +2 \frac{(c'_{A_b} c_{A_b} + c'_{v_b} c_{v_b})(c'_{A_{\nu_k}} c_{A_{\nu_k}} - c'_{v_{\nu_k}} c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})} \right] (p_3.p_4) \\
& + 8 \left[\frac{(c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}} c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2} \right. \\
& \left. +2 \frac{(c'_{A_b} c_{A_b} + c'_{v_b} c_{v_b})(c'_{A_{\nu_k}} c_{A_{\nu_k}} - c'_{v_{\nu_k}} c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})} \right] [(p_2.p_3)(p_1.p_4) + (p_1.p_3)(p_2.p_4)]
\end{aligned}
\tag{3.78}
$$

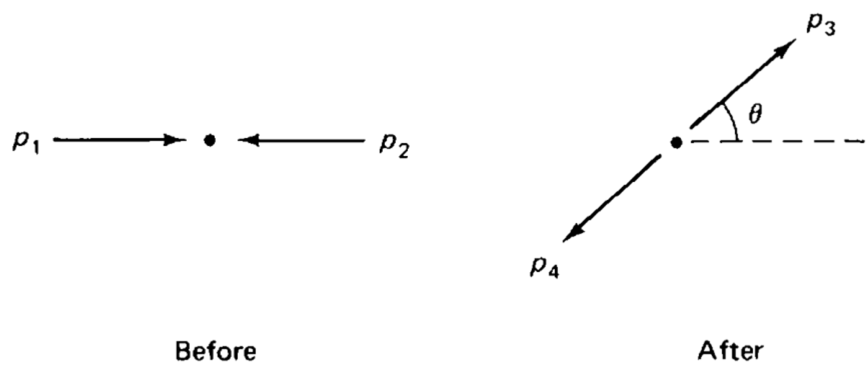
The expression (3.78) can be simplified further by making use of the values of the axial and vector couplings mentioned in tables (3.2, 3.3). In our case, we maintain the compacted form, since it is easier to handle for our next step of calculating the differential cross section "$\frac{d\sigma}{d\Omega}$" associated with the process under study.

3.2.2 The differential cross section

We have taken formerly a close look at the procedure of calculating the Feynman amplitude, and its square value associated with our scattering process. What hasn't been shown yet is the value of the differential cross section, which is the purpose of this subsection.

We are interested in calculating $\frac{d\sigma}{d\Omega}$ in the center of mass frame, in the case of unpolarized particles, where the scattering will look something similar to what is shown in figure (3.6).

Figure 3.6: An illustration of a scattering process in the center of mass frame.



The differential cross section will take the form:

$$
\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{|\vec{p}_f|}{|\vec{p}_i|} \frac{<|\mathscr{M}|^2>}{(E_1 + E_2)^2} \tag{3.79}
$$

we consider natural units $\hbar = c = 1$, thus:

$$
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{|\vec{p}_3|}{|\vec{p}_1|} \frac{<|\mathscr{M}|^2>}{(E_1 + E_2)^2}. \tag{3.80}
$$

The 4-momenta associated with the incoming, and outgoing particles can be written as:

$$p_1 = (E_1, \vec{p}_1) \qquad p_2 = (E_2, \vec{p}_2) \tag{3.81}$$

$$p_3 = (E_3, \vec{p}_3) \qquad p_4 = (E_4, \vec{p}_4) \tag{3.82}$$

we consider the case where $E_1 = E_2 = E_3 = E_4 = E$, and we make use of the formula :

$$E_i = \sqrt{m_i^2 + \vec{p}_i^{\,2}} \tag{3.83}$$

combined with the fact that $\vec{p}_1 + \vec{p}_2 = 0$ in the center of mass frame, and $m_1 = m_2 = m_b$, $m_3 = m_4 = 0$, we obtain[13]:

$$\begin{aligned} |\vec{p}_1| &= \sqrt{E^2 - m_b^2} \\ |\vec{p}_3| &= E \end{aligned} \tag{3.84}$$

leading to :

$$p_1 = \left(E, 0, 0, \sqrt{E^2 - m_b^2}\right) \qquad p_2 = \left(E, 0, 0, -\sqrt{E^2 - m_b^2}\right) \tag{3.85}$$

$$p_3 = (E, 0, Esin\theta, Ecos\theta) \qquad p_4 = (E, 0, -Esin\theta, -Ecos\theta) \tag{3.86}$$

where we have chosen a particular frame.

We can now work out the dot products :

$$\begin{aligned} p_1.p_3 &= E^2 - \vec{p}_1\vec{p}_3 = E^2 - (|\vec{p}_1||\vec{p}_3|cos(\pi - \theta)) = E^2 + E\sqrt{E^2 - m_b^2}cos\theta \\ p_1.p_4 &= E^2 - E\sqrt{E^2 - m_b^2}cos\theta \\ p_2.p_4 &= E^2 + E\sqrt{E^2 - m_b^2}cos\theta \\ p_2.p_3 &= E^2 - E\sqrt{E^2 - m_b^2}cos\theta \\ p_3.p_4 &= 2E^2 \end{aligned} \tag{3.87}$$

leading to :

$$\begin{aligned} (p_2.p_3)(p_1.p_4) + (p_1.p_3)(p_2.p_4) &= E^4 \left[\left(1 - \sqrt{1 - \frac{m_b^2}{E^2}}cos\theta\right)^2 + \left(1 + \sqrt{1 - \frac{m_b^2}{E^2}}cos\theta\right)^2\right] \\ &= 2E^4 \left[1 + \left(1 - \frac{m_b^2}{E^2}\right) cos^2\theta\right] \end{aligned} \tag{3.88}$$

enabling us to write $< |\mathscr{M}|^2 >$ in function of E, and θ:

$$\begin{aligned} < |\mathscr{M}|^2 > = 16m_b^2E^2 &\left[\frac{(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right. \\ &\left. +2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right] \\ + 16E^4 &\left[\frac{(c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}}c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right. \\ &\left. +2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right] \left[1 + \left(1 - \frac{m_b^2}{E^2}\right) cos^2\theta\right] \end{aligned} \tag{3.89}$$

[13] $|\vec{p}_1| = |\vec{p}_2|$, and $|\vec{p}_3| = |\vec{p}_4|$.

we can clearly see that, in the case where $m_b <<< E$, we are allowed to work with the approximation :

$$\left[1 + \left(1 - \frac{m_b^2}{E^2}\right) cos^2\theta\right] \sim \left[1 + cos^2\theta\right] \tag{3.90}$$

$$\begin{aligned}
< |\mathscr{M}|^2 > = 16m_b^2E^2 &\left[\frac{(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right.\\
&\left.+2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right]\\
&+ 16E^4\left[1 + cos^2\theta\right]\left[\frac{(c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right.\\
&\left.+2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right].
\end{aligned} \tag{3.91}$$

Finally, the unpolarized differential cross section in the case of $m_b <<< E$:

$$\begin{aligned}
\frac{d\sigma}{d\Omega} = \frac{m_b^2}{16\pi^2}&\left[\frac{(c'^2_{A_b} - c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} - c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right.\\
&\left.+2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right]\\
&+ \frac{E^2(1 + cos^2\theta)}{16\pi^2}\left[\frac{(c'^2_{A_b} + c'^2_{v_b})(c'^2_{A_{\nu_k}} + c'^2_{v_{\nu_k}})}{(k^2 - M^2_{Z'})^2} + \frac{(c^2_{A_b} + c^2_{v_b})(c^2_{A_{\nu_k}} + c^2_{v_{\nu_k}})}{(k^2 - M^2_Z)^2}\right.\\
&\left.+2\frac{(c'_{A_b}c_{A_b} + c'_{v_b}c_{v_b})(c'_{A_{\nu_k}}c_{A_{\nu_k}} - c'_{v_{\nu_k}}c_{v_{\nu_k}})}{(k^2 - M^2_Z)(k^2 - M^2_{Z'})}\right]
\end{aligned} \tag{3.92}$$

it implies that the form of the differential section can be written as[14]:

$$\frac{d\sigma}{d\Omega} = A(E, M_{Z'}, \varrho) + B(E, M_{Z'}, \varrho)cos^2\theta \tag{3.93}$$

where :

$$\begin{aligned}
A(E, M_{Z'}, \varrho) = &\frac{aE^2}{(4E^2 - M^2_{Z'})^2}\varrho^4 + \frac{b}{(4E^2 - M^2_{Z'})^2}\varrho^4 - \frac{cE^2}{(4E^2 - M^2_{Z'})(4E^2 - M^2_Z)}\varrho^2\\
&- \frac{d}{(4E^2 - M^2_{Z'})(4E^2 - M^2_Z)}\varrho^2 + \frac{fE^2}{(4E^2 - M^2_{Z'})(4E^2 - M^2_Z)}\varrho + \frac{g}{(4E^2 - M^2_{Z'})(4E^2 - M^2_Z)}\varrho\\
&+ \frac{hE^2}{(4E^2 - M^2_Z)^2} + \frac{k}{(4E^2 - M^2_Z)^2}
\end{aligned} \tag{3.94}$$

[14] It is note worthy that a QCD correction must take place in order to acquire the correct value of the cross section. Given that the quarks in the initial states have an internal degree of freedom, that is the color charge (r,b,g), the correct value for the cross section would just be 3σ.

and,

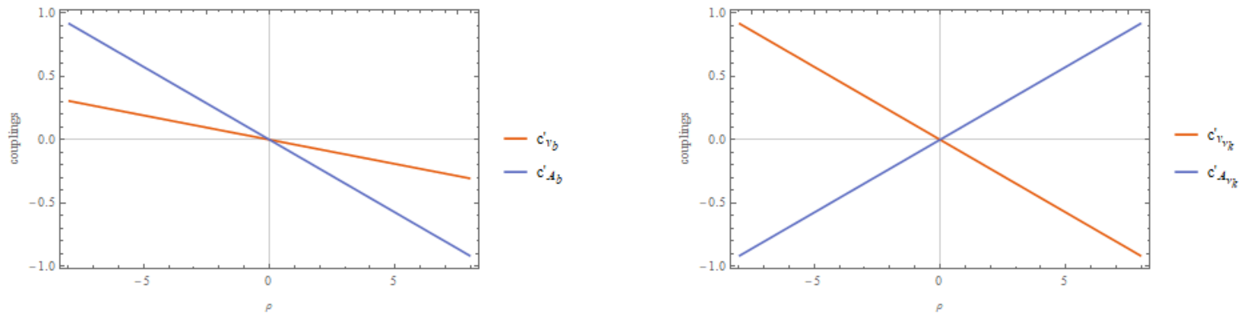
$$B(E, M_{Z'}, \varrho) = \frac{a'E^2}{(4E^2 - M_{Z'}^2)^2}\varrho^4 - \frac{b'E^2}{(4E^2 - M_{Z'}^2)(4E^2 - M_Z^2)}\varrho^2 + \frac{c'E^2}{(4E^2 - M_{Z'}^2)(4E^2 - M_Z^2)}\varrho + \frac{d'E^2}{(4E^2 - M_Z^2)^2} \tag{3.95}$$

$a, b, c, d, f, g, h, a', b', c', d'$ are real numbers. which can be obtained using the following numerical substitutions:

$$g = 0.6483 \qquad m_b = 4.18\ GeV \qquad M_Z = 91.1876\ GeV \tag{3.96}$$

$$cos\theta_w = 0.88153 \qquad sin^2\theta_w = 0.23152. \tag{3.97}$$

What can be observed is that the differential cross section depends on 4 crucial variables:"$E, \varrho, \theta, M_{Z'}$". Therefore, in order to infer how this quantity is subjected to those variables, we make use of MATHEMATICA, and we pass to a numerical evaluation of the differential cross section, with certain choice of fixed values. Among the values that should be fixed is the range of the variable ϱ, this can be inferred by drawing the variation of the couplings in function of ϱ. The results are shown in (3.7).



(a) Variation of the axial and vector couplings of the b quark in function of the parameter ϱ.

(b) Variation of the axial and vector couplings of the neutrino ν_k in function of the parameter ϱ.

Figure 3.7: Variation of the axial and vector couplings of the vertex $Z'ff$ for both b quarks, and neutrinos in function of the parameter ϱ.

As can be observed, the couplings diverge for bigger values of ϱ, thus to be confined to the perturbative range, a choice of $-2 < \varrho < 2$ can be made.
Taking such a choice, and choosing: $E = 1$ TeV, $\theta = \pi$, we can compute the variation of the differential cross section in function of the variable ϱ, with 4 different values for the mass of the Z'. The results are depicted in (3.8).

Taking $\varrho = 1$, $\theta = \pi$, and different Z' mass values, we can also illustrate the variation of the differential cross section in function of the energy. In such a situation, it is important to pay attention to the poles originating from the propagators in (3.92), such that $2E = M_{Z'}$. For such a case, we make use the energy of center of The mass $\sqrt{s} = 2E$. The results are shown in (3.9).

An evaluation of the variation of the differential cross section in function of the angle θ can also be drawn, as depicted in(3.10).

Furthermore, the variation of the differential cross section can be seen as well in function of the mass of the Z', while fixing the values: E=1 Tev, $\theta = \pi$, $\varrho = 1$. The results

Figure 3.8: A plot showing the variation of the differential cross section in function of the parameter ϱ, and through 4 different values for the Z' mass.

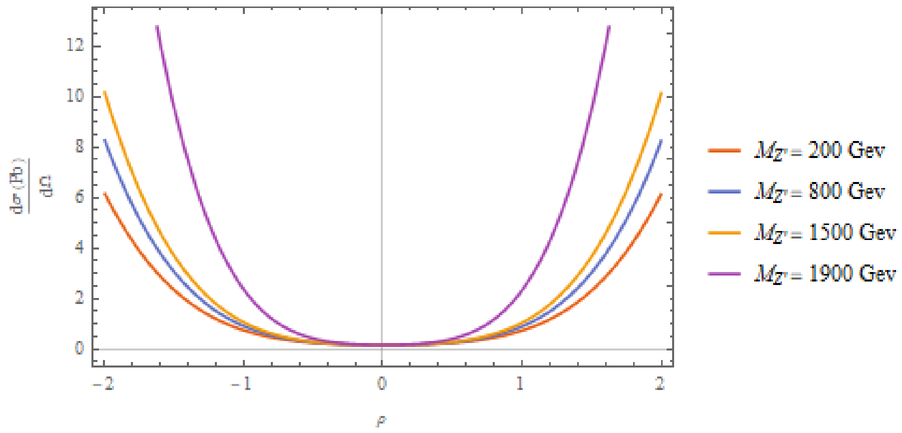


Figure 3.9: A plot showing the variation of the differential cross section in function of $\sqrt{s}$, with 3 different values for the Z' mass. And for the fixed values $\theta = \pi$, $\varrho = 1$.

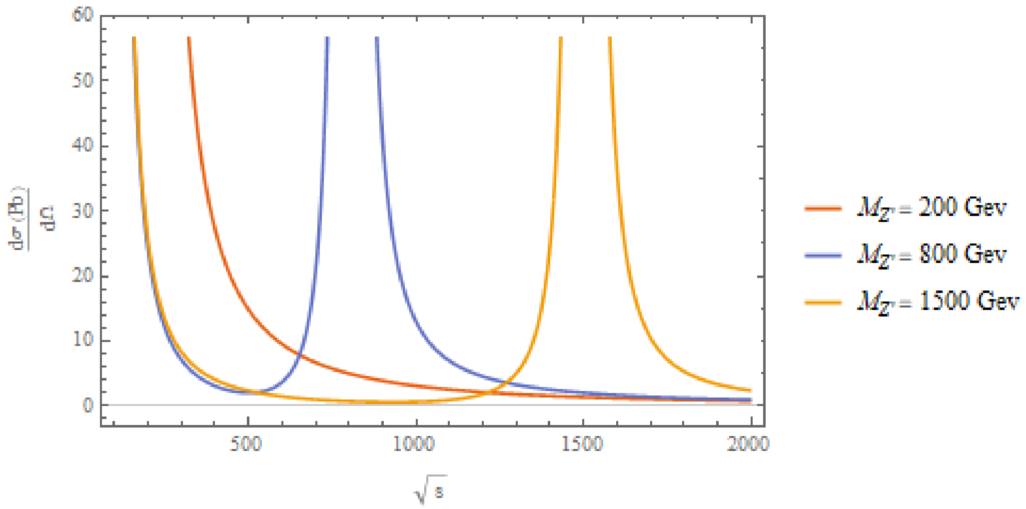
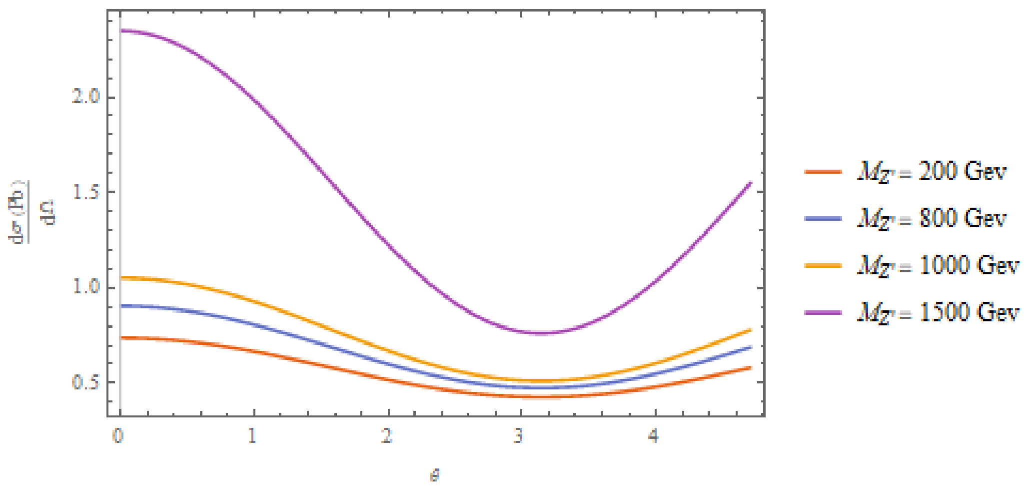
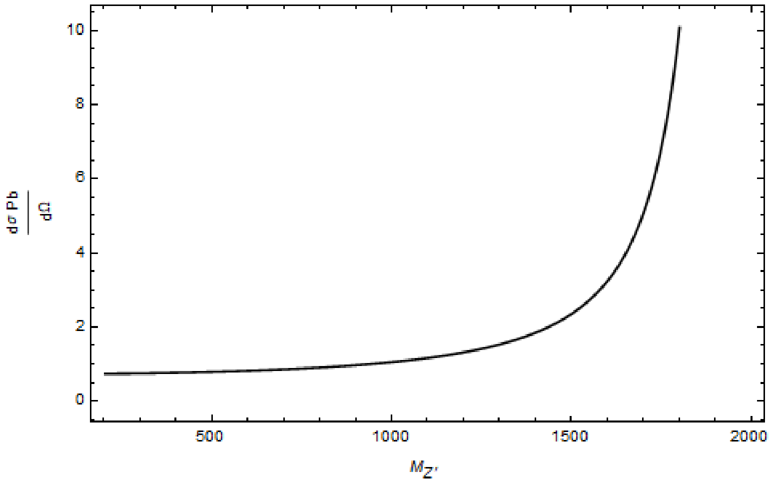


Figure 3.10: A plot showing the variation of the differential cross section in function of θ, with 4 different values for the Z' mass. And for the fixed values E= 1 TeV, $\varrho = 1$.



are shown in (3.11).

Figure 3.11: A plot showing the variation of the differential cross section in function of $M_{Z'}$, with the values: E=1 TeV, $\theta = \pi$, $\varrho = 1$. The mass of the Z' is taken with GeV.



3.3 Discussion

The examined Stueckelberg Z' gained its mass via the generalized Stueckelberg mechanism, where its mass is dependent on two free parameters: M, λ_2. The Stuckelberg contribution to the mass term was controlled by two major elements: the bilinear term cancellation constraint $M_X = M_Y$, and the gauge fixing choice of the fields $\tilde{B}_\mu, \tilde{C}_\mu$. The model got further cornered by our third choice, where the parametrization was picked to assimilate the SM's parametrization, while selecting the Z'_μ field as a pure state of $\tilde{B}_\mu$. This contributed to shape very strictly the parameter of mass, as well as the couplings to fermions.

The first crucial result of our work was the dependence of the Z' couplings to both b quarks, and neutrinos ν_k on the parameter ϱ, which due to our specific choice that all the hypercharges Y_f are proportional to those of the SM, the parameter ended up being universal to all the fermions. Such a result enabled us to plot the variation of the couplings in terms of the parameter ϱ, which shows that the Z' couplings are relatively stronger compared to the Z_{SM} couplings. In addition to the observation that a decoupling will occur at the limit $\varrho \to 0$. Such results contributed to impose a specific range for the ϱ values, mainly to ensure that the couplings remain within the perturbative range, that to say $-2 < \varrho < 2$.
It is note worthy that the results obtained were influenced by the choice made for the physical field Z'_μ, since it has been chosen as a pure state of $\tilde{B}_\mu$.

Through the calculations, we managed to obtain the unpolarized differential cross section, which can be written in the form (3.93), after assuming that $m_b << E$, which can be interpreted as us dealing with high energy scales. Such an expression shows its dependence on the 4 parameters: $M_{Z'}, E, \varrho, \theta$. A variation of the differential cross section in function of the center of mass energy $\sqrt{s}$ is depicted in (3.9). It displays two resonances, at the poles : $\sqrt{s} = M'_Z = 800, 1500$ GeV. The two resonances are the particle being spotted. If by experimental measurements in the future we obtain values for the differential cross section, then through such values we can both confirm the existence of the particle, as well as fixing its mass.

In addition, a variation of the differential cross section in terms of the universal parameter ϱ can also be studied. The results shown in (3.8) reveal the influence of such a parameter, which puts a heavy emphasis on the role played by the couplings on the Z' detection. At first sight, we can see that the effect of the mass is relatively similar among the masses $M_{Z'} = 200, 800, 1000$ GeV, while noticeable difference can be spotted in the plot associated with $M_{Z'} = 1500$ GeV. Furthermore, we observe a vanishing differential cross section for all of the plots at the limit $\varrho \to 0$. It can be interpreted as a difficulty to observe the Z' particle when it doesn't couple to fermions.

A plot of the variation of $\frac{d\sigma}{d\Omega}$ in function of the angle θ, while fixing the remaining parameters E= 1 TeV, $\varrho = 1$, is shown in (3.10). We can see the 3 plots with $M'_Z = 200, 800, 1000$ GeV behaving relatively the same, with the mass $M'_Z = 1000$ Gev having the highest value for the differential cross section compared to the other two. An apparent difference is seen with the mass $M'_Z = 1500$ GeV. It seems that once again the role of the Z' mass can be discerned.

Moreover, the influence of the mass of the Z' gauge boson on the differential cross section can be seen clearly in the plot (3.11). At energy 1 TeV, the differential cross section seems to increase as we go to high mass ranges. At the beginning, the increase appears at a slow pace for masses less than $M'_Z < 1000$ GeV. Then, as we are moving to masses of the scale $M'_Z = 1500$ GeV and above, the increase in its value escalates considerably. Once again, showcasing that indeed the parameter of mass has an important role to play.

Conclusion

Throughout this modest scientific inquiry, we attempted to formulate a presentable frame of the impact of the Z' gauge boson, by means of one of the BSM formulations, namely a simple $U(1)_X$ extension of the Standard Model with minimal field content. Such an extension was carried out by making use of the Stueckelberg mechanism, under the purpose of obtaining the mass of the extra gauge boson Z', as an economical alternative to the spontaneous symmetry breaking mechanism by extending the Higgs sector.

In chapter 1, we initiated the path by an overview of the Standard Model of particle physics, through a brief construction of its Lagrangian. We attached a justified list of the Standard Model's limitations to vindicate the necessity of physics BSM. Then, a brief expedition of such a realm has taken place to assess the importance of one particularity, namely Z' physics.

In chapter 2, the strategy was marked by a choice of one of the BSM formulations, called a minimal Stueckelberg extension of the SM. It is based on enlarging the gauge structure of the SM by a $U(1)_X$ gauge symmetry, and adding a Stueckelberg scalar σ. Throughout the analysis, a brief discussion about the possibility of kinetic, and mass mixing has taken place, mainly to reach the conclusion that, even though theoretically mixing abelian fields is possible, but due to the absence of any experimental signs, it is preferable to build a kinetic mixing free model. In addition to how a mass mixing can be eliminated as well. Then, the necessity for a mechanism to acquire the mass of the new gauge boson Z' has been established by showing that relying only on the Higgs sector of the SM will yield a massless Z', which is phenomenologically impossible. Thus, instead of seeking another extension of the Higgs sector, the mass of the Z' gauge boson can be attained via the Stueckelberg mechanism without breaking the additional gauge symmetry. The mass generation comes from a coupling to the Stueckelberg scalar σ, which has coupling to all of the gauge bosons, and which is characterized by its special gauge transformation with the two mass-dimension parameters M_X, M_Y.
The study is heavily based the step of the bilinear terms cancellation in the Lagrangian by imposing a constraint, combined with a specific choice of the unphysical fields $\tilde{B}_\mu, \tilde{C}_\mu$ providing the gauge fixing term. As a result, the mass matrix was rendered simple to handle, and with very few associated free parameters.

Chapter 3 was an important element of our research inquiry, since it was marked by the attempt to pin point the impact of the Z', in order to help us assimilate the potential of its discovery. A short phenomenological approach was executed, firstly by pinning what might be relevant to the Z' empirical search, such as its coupling to fermions. Secondly, its potential discovery in e^+e^-, and $p\bar{p}$ colliders was implemented, as well as using third generation quarks to identify it. It is worth mentioning that the data stated weren't attached to a specific origin for the Z' particle.
Furthermore, given the importance of the Z''s coupling to fermions, an application to infer its effect in the scattering process $b\bar{b} \to \nu\bar{\nu}$ was further developed. The scattering

process includes both contributions from the Z, and Z', while the Z' gauge boson is originating from a generalized Stueckelberg extension of the SM. Its couplings to the b quark, and neutrinos has been constructed so that they are proportional to the SM hypercharge, mainly via choosing the simplest option for anomaly cancellation. Resulting in the appearance of an important parameter ϱ, that was shown to be universal. A nice feature of the analysis is the acquisition of the unpolarized differential cross section in function of only 4 free parameters: $M_{Z'}, E, \varrho, \theta$, while considering the case of very high energies, in the case $m_b <<< E$. Such a feature enabled us to discuss the influence of the Z' gauge boson through two of its properties. Thus, inferring that the Z' mass plays an important role, alongside with its couplings to fermions being characterized by that one proportionality factor.

In conclusion, the model under scrutiny has the power of yielding few free parameters to fix, which makes the procedure of fine tuning less of labor to handle. But it didn't fail to account for a fraction of the Z' impact in physics, and how such an impact can manifest via only two of the Z' properties. Further development of the work, and further phenomenological examination are required before making any definitive claim.

Perspectives

Along the journey of this modest research procedure, we had the capacity to draw a frame on the model studied, given the large range of possibilities. Thus, starting from this localized work, we can suggest further directions that can be taken in the future to extend this scientific inquiry.

1. Study the scattering process proposed at higher orders, under the assumption that the new particle has couplings only to the third generation of fermions, the so-called the tritogenophilic Z'.

2. Pick a different choice of parametrization, such as the general case, and discuss the various coupling patterns.

3. Investigate different choices for the bilinear terms cancellation for the construction of the Stueckelberg Lagrangian, and other possibilities for anomaly cancellation conditions, or mechanisms.

4. Investigate the observability of the Stueckelberg Z', such as the question of which process can be considered the best to probe its existence.

Appendix A

Fundamental Concepts

This section is meant to provide the reader with some background knowledge of some key-concepts that are useful to deepen the understanding of the Standard Model.

A.1 Basics of Group Theory

A very important concept that has been excessively used is the conception of symmetries, or a symmetry transformation. A symmetry transformation is a transformation that can act on the states of a theory, and the operators, while it preserves all the physics. We can write :

$$|\psi> = U|\psi> \qquad\qquad \mathscr{O} = U\mathscr{O}U^* \tag{A.1}$$

An internal symmetry is a symmetry in which a local operator $\mathscr{O}$ is transformed into a local operator at the same point. Such symmetries can be considered separately from spacetime symmetries, such as translations, rotations, and boosts.

Furthermore, a symmetry group is an algebraic structure that plays a central role in the construction of the SM. Interestingly enough, it is not the symmetry groups themselves that are of primary significance, but instead the so-called representations of these symmetry groups. A representation R is defined as a homomorphism between the group G, and the set of linear operators, acting on a vector field :

$$R : G \to GL(n, F) \tag{A.2}$$

in simple terms, if we have a set of matrices associated with elements of a group, and which satisfy the condition of respecting the group law, then these matrices are called a representation of the group. In any representation, the identity element of the group must be mapped into the identity matrix$\mathbb{I}$.
A representation can be classified into two categories: reducible rep[1], or irreducible rep.

The groups that are of our interest are namely Lie groups, such groups are characterized by the so-called generators T_a, where $T_a = -\partial_a g|_e$. Those generators satisfy commutation relations:

$$[T_a, T_b] = i f_{abc} T_c \tag{A.3}$$

the f_{abc} are called the structure constants of the group.

[1] A reducible rep has a matrix form with block diagonal matrices.

Every group has a representation, called the singlet or trivial representation, where the element gets mapped to the identity matrix. In addition, every group also contains a representation called the adjoint representation, set up by $n \times n$ real matrices. If we consider an abelian group for example, the structure constant vanishes, and thus the adjoint representation is the same as the singlet representation. While for the group of rotations $SU(2)$, the adjoint rep is actually the spin-one representation.

The 3 main gauge groups used in the SM are $U(1)$, $SU(2)$, and $SU(3)$. The group $U(1)$ for example is the familiar group of phase rotations, parameterized by θ, and with the famous element $e^{i\theta}$. More information about the remaining groups are mentioned below.

A.1.1 Gauge group SU(2)

The $SU(2)$ is the simplest non-abelian group, it is a 3-dimensional group, its group elements are defined as:

$$g(X) = e^{iX^a J^a} \tag{A.4}$$

these group elements are unitary, and have determinant $+1$, with the requirements that:

$$(J^a) = (J^a)^\dagger \qquad\qquad Tr(J^a) = 0 \tag{A.5}$$

the Lie algebra of the group is defined as well:

$$[J_a, J_b] = i\epsilon_{abc} J_c \tag{A.6}$$

defining $J_a = \frac{\sigma_a}{2}$, where σ_a are Pauli matrices[2].
Such a definition provides a simple representation, which corresponds to 2×2 unitary matrices.

A.1.2 Gauge group SU(3)

The group SU(3) is the exact equivalent of SU(2), but in 3 complex dimensions rather than two. Generally, a $SU(n)$ matrix has $n^2 - 1$ free parameters, thus for $n = 3$, we have 8 independent degrees of freedom, meaning 8 generators of the group. As an analogy to Pauli matrices, there are Gell-Mann matrices λ_a.
For the Lie algebra :

$$[\frac{\lambda_k}{2}, \frac{\lambda_l}{2}] = i \sum_m f_{klm} \frac{\lambda_m}{2} \tag{A.7}$$

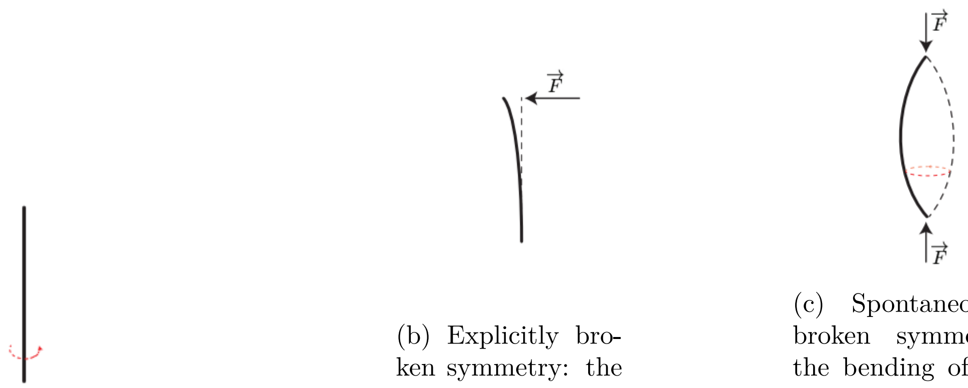
where f_{klm} are the structure constants of the group.
In addition to the fact that the renormalization condition requires :

$$Tr(\lambda_i, \lambda_j) = 2\delta_{ij} \tag{A.8}$$

A.2 Spontaneous Symmetry Breaking

The gauge principle is crucial to the construction of the Standard Model, not only that but in general it can serve as a dynamical principle to help guide us constructing theories. As a matter of fact, global gauge invariance combined with Noether's theorem implies the existence of a conserved current, while local gauge invariance necessitates the existence of massless vector gauge bosons. And it was at this level that we faced an issue, this

[2] In our work, we have adopted the notation τ_a for Pauli matrices.



(a) The unbroken symmetry of the rod: rotational invariance.

(b) Explicitly broken symmetry: the bending of the rod due to an external force, thus losing the rotational invariance.

(c) Spontaneously broken symmetry: the bending of the rod in an arbitrary direction, the ground state breaks the rotational invariance.

Figure A.1: An illustration of the concept of spontaneous symmetry breaking in the case of bending a rod.

gauge principle has led to theories in which all the interactions are mediated by massless vector bosons. To our knowledge, only the photon and the gluons are massless, but the vector bosons mediating the weak interactions, the so-called the W and Z bosons, are in reality massive. Such a problem has been solved via a phenomenon called "Spontaneous Symmetry Breaking", where the Lagrangian of the theory still obeys the symmetries, and is still invariant under the symmetries of the theory, but the physical vacuum does not conserve the symmetry.
For the sake of exemplification, let's consider the rotational symmetry of a rod around its axis as it is depicted in figure (A.1a). We can break the symmetry by applying an external force to only one end of the rod, leading to the rod bending, and thus losing its rotational symmetry, as we can see in figure(A.1b). This case in particular is what we refer to as an explicit symmetry breaking, such a breaking is caused by an external force that is involved in the equations of motion. On the other hand, we can obtain the case of a spontaneous symmetry breaking if we apply the force in the longitudinal direction of the rod, as the figure (A.1c) indicates. At first sight, we notice that the rod will now also bend, and is no longer rotationally invariant, but it can be seen otherwise by focusing on the properties of spontaneously broken state. Starting with the direction of the bending, such a direction is completely arbitrary, which shows that there are multiple ground states, implying the degeneracy of the ground states. In addition to the fact that these directions are related to the rotational symmetry, which demonstrates that the symmetry did actually not vanish, but it is 'hidden'. This implies that the equations of motion still respect the rotational symmetry, but the ground states do not.

Appendix B

Standard Model's parameters

We present here a summary of the values for the various parameters of the standard model as has been determined so far, and which were relevant to our work.

B.1 Values of T_3, Y, and Q in the SM

Recall:

$$Q_{SM} = T_3 + \frac{Y_{SM}}{2} \tag{B.1}$$

thus,

Table B.1: Values of T_3, Y, and Q of various SM's fields

Field	ℓ_L	ℓ_R	ν_L	u_L	d_L	u_R	d_R	φ^+	φ^0
T_3	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Y	- 1	- 2	-1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	1	1
Q	-1	-1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	1	0

B.2 The free parameters of the SM

The Standard Model contains at least 26 free parameters :

- ⇒ 3 gauge couplings(g_s, g, g').
- ⇒ 6 quarks masses.
- ⇒ 6 lepton masses.
- ⇒ The 4 independent values in the Kobayashi-Maskawa matrix.
- ⇒ The 4 independent values in the Pontecorvo-Maki-Nakagawa-Sakata matrix[1].
- ⇒ Higgs mass.
- ⇒ VEV of the Higgs field

[1] If we consider the Standard Model with its original formulation with massless neutrinos, the 4 parameters of the PMNS matrix should not be listed, and therefore we count only obtaining 19 parameters. While if we consider the SM with massive neutrinos, we count the 25 free parameters

$\Rightarrow$ 1 CP- violating angle.

We present the values of some parameters that has been set up by experiment in table (B.2).

Table B.2: Values of some parameters of the SM.

Parameter	Description	Value (dimensionless)
θ_{12}	CKM 12−mixing angle	13.1°
θ_{23}	CKM 23−mixing angle	2.4°
θ_{13}	CKM 13−mixing angle	0.2°
δ	CKM CP−violating phase	0.995
$g^{'}$	$U(1)$ gauge coupling	0.357
g	$SU(2)$ gauge coupling	0.652
g_s	$SU(3)$ gauge coupling	1.221
θ_{QCD}	QCD vacuum angle	~ 0
V	the Higgs field VEV	246 GeV

Other values such as the masses of the various quarks, and fermions are listed in table(B.3).

B.3 Some of the properties of SM particles

The following table indicates some of the properties of SM particles based on the PDG [23].

Table B.3: The particles of the Standard Model and some of their properties.

Particles		Name (Symbol)	Spin	Electric charge (e)	Mass (Unit)	Color charge
Fermions	Leptons	Electron (e^-)	$\frac{1}{2}$	-1	$548.57990906 \pm 0.000000016 (10^{-6} u)$	No
		Muon (μ^-)	$\frac{1}{2}$	-1	$0.113428925 \pm 0.0000000025 (u)$	No
		Tau (τ^-)	$\frac{1}{2}$	-1	1776.86 ± 0.12	No
		Electron neutrino (ν_e)	$\frac{1}{2}$	0	< 0.8(Ev)	No
		Muon neutrino (ν_μ)	$\frac{1}{2}$	0	< 0.19(MeV)	No
		Tau neutrino (ν_τ)	$\frac{1}{2}$	0	< 18.2(MeV)	No
	Quarks	Up (u)	$\frac{1}{2}$	$\frac{2}{3}$	$2.16^{+0.49}_{-0.26}$ (MeV)	RGB
		Down (d)	$\frac{1}{2}$	$-\frac{1}{3}$	$4.67^{+0.48}_{-0.17}$ (MeV)	RGB
		Charm (c)	$\frac{1}{2}$	$\frac{2}{3}$	1.27 ± 0.02(GeV)	RGB
		Strange (s)	$\frac{1}{2}$	$-\frac{1}{3}$	$93.4^{+8.6}_{-3.4}$ (MeV)	RGB
		Top (t)	$\frac{1}{2}$	$\frac{2}{3}$	172.69 ± 0.30(GeV)	RGB
		Bottom (b)	$\frac{1}{2}$	$-\frac{1}{3}$	$4.18^{+0.03}_{-0.02}$(GeV)	RGB
Particles		Name (Symbol)	Spin	Electric charge	Mass (GeV)	Color charge
Vector Bosons		Photon(γ)	1	0	0	No
		W boson ($W^\mp$)	1	∓ 1	80.377 ± 0.012	No
		Z boson (Z^0)	1	0	91.1876 ± 0.0021	No
		Gluon (g)	1	0	0	Color(RBG)+ Anti-color($\bar{R}\bar{B}\bar{G}$)
		Higgs boson (H)	0	0	125.25 ± 0.17	No

Appendix C

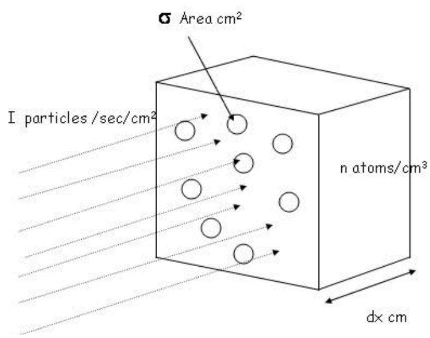
Particle Physics terminology

This appendix serves as a guideline to understand the various concepts that has been used in this thesis work, mainly focusing on the field of particle physics and its experimental most-often used concepts.

- **Cross section** : it is a measure of the probability that a specific process or reaction will occur. It concerns the interactions of elementary particles, nuclei, or atoms with each other. As an illustration, given a particle approaching another particle, the cross section for this process is simply the probability that the two particles interact with each other. Geometrically, the cross-section can be thought of as the area within which a reaction will occur as the figure (C.1) indicates, henceforth it has the units of an area.

  The strength of an interaction can be described by the cross section, and it is

  Figure C.1: A geometrical interpretation of the concept of the cross section.

  measured in barn $= 10^{-28} m^2$.

- **Resonance**: A resonance ideally appears as a peak in the total cross section, found around a certain energy of scattering experiments. Sometimes it is used to describe particles with very short lifetimes; for the purpose of illustration, consider the following process: $A + B \to O \to C + D$. The cross section in this case is given by the Breit-Wigner formula:

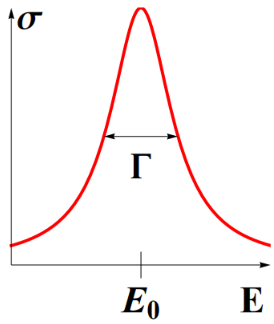
$$\sigma_{i \to O \to f} = \frac{\pi}{k^2} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \tag{C.1}$$

  where: Γ_i is the is the partial width of the resonance to decay to the initial state A + B, Γ_f is the partial width of the resonance to decay to the final state C + D, Γ is the full width of the resonance, k is the wave-number of the incoming projectile, E is the centre-of-mass energy of the system, and E_0 is the rest mass energy of the resonance.

As can be seen, σ is non-zero at any energy, but has a sharp peak at energies $E \sim E_0$. The sharp peak is known as **resonance**. Longer lived intermediate particles have smaller Γ, and hence they exhibit sharper peaks.

- **Decay width**: it is a measure of the probability of a specific decay process to occur within a given amount of time in the parent particle's rest frame.
When measurements are made of the rest mass energy of an unstable particle with no instrument errors a graph is obtained as in figure (C.2).The width of such a distribution is called the decay width Γ, and is measured in units of energy.
Long-lived particles have narrow widths and well-defined energies. Short-lived

Figure C.2: A visualization of the conception of decay width.



particles have large widths and less well defined energies. When the state is so short-lived that its width Γ is similar to its mass, then the decay is so rapid that it is no longer useful to think of it as a particle.
It is related to the particle lifetime as : $\Gamma = \frac{1}{\tau}$, and it's related tothe uncertainty in energy as follows: $\Gamma = 2\Delta E = \hbar\lambda = \frac{\hbar}{\tau}$. And if a particle has multiple decay modes, then one can associate a decay rate for each mode, and the total rate, will be the sum of the partial decay widths rates: $\Gamma_{tot} = \sum_i^n \Gamma_i$.
From such a quantity, we can derive another quantity of interest called the branching fractions, which refer to the probabilities of the decay by individual modes. The branching fraction of a certain mode i:

$$B_i = \frac{\Gamma_i}{\Gamma_{tot}} \tag{C.2}$$

- **Tree level / leading order**: terms used to refer to the simplest diagrams for any process with the smallest number of g factors, where they contain no closed loops.

- **Center of mass-energy**: also known as the CM energy or E_{CM} is the total energy of a collision in the center-of-mass frame of reference [1], measured in units of electron volts (eV), giga-electron volts (GeV), or tera-electron volts (TeV). It is given by :

$$E_{CM} = \sqrt{s} \tag{C.3}$$

where s is the square of the total energy of the collision in the laboratory frame.

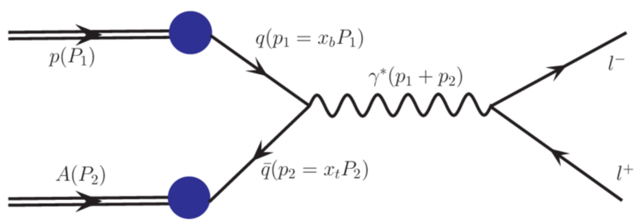
- **Vector / axial couplings**: the couplings of a particle can be expressed in terms of two quantities: the vector coupling V_μ, which is the quantity that get to be presented

[1]The center-of-mass frame is a frame in which the total momentum of the two colliding particles is zero.

as $\propto V_\mu \bar{\psi}\gamma^\mu\psi$, and a second quantity labeled the axial coupling, which is presented as $\propto V_\mu \bar{\psi}\gamma^\mu\gamma^5\psi$.

- **Drell-Yan process**: it is a process first proposed by Sid Drell, and Tung-Mow Yan as an attempt to describe high-mass lepton-pair production in hadron-hadron collisions, in such a process a quark of momentum fraction p_1' originating from one hadron of momentum P_1 annihilates with an anti-quark of momentum fraction p_2' coming from another hadron of some momentum P_2. Resulting in the formation of a virtual particle with a certain momentum q, and a large invariant mass, which will decay into a pair of leptons. An example is shown in figure (C.3) shows.

Figure C.3: An example illustrating a Drell-Yan process, where a quark and anti-quark annihilate, forming a photon decaying into a pair of leptons.



- **Luminosity, and integrated luminosity** : these two quantities are important parameters for particle accelerators, since they provide data about their performance. The luminosity is defined as:

$$L = \frac{1}{\sigma}\frac{dN}{dt} \tag{C.4}$$

which just the ratio of the number of events detected in frame of time to the cross section. While the integrated luminosity is defined as :

$$L_{int} = \int L dt \tag{C.5}$$

Each particle accelerator seek the maximization of such quantities, since they allow for high performance, and thus more accessible data.
Table (C.1) serves as an illustration of some of the values of the luminosity for different particle accelerators.

Table C.1: some of the values of the luminosity for different particle accelerators with their specified type of interactions.

Collider	Type of interaction	$L(cm^{-2}.s^{-1})$
LHC	p + p	2.1×10^{34}
LHC	pb + pb	8.5×10^{29}
PEP	$e^- + e^+$	3×10^{33}
LEP	$e^- + e^+$	1×10^{32}
Tevatron	$p + \bar{p}$	4×10^{32}

- **Standard deviation $\boldsymbol{\sigma}$**: it is a measure of how incompatible a set of data is, given a certain hypothesis. The higher the number of sigma, the more incompatible the data are with the hypothesis.

- **Event rate**: it is defined as: luminosity $L \times$ cross section σ.

- **Background**: the background is considered the set of results expected to be seen. If an experiment shows more instances, which means excess of a certain type of event than what is expected theoretically, then it is to be seen as part of the background. It is usually used to search for signs of new physics.

- **Invariant mass**: when using the term mass, we usually refer to the rest mass "m", which is proportional to the inertia of the particle when it is at rest. Furthermore, when a particle decays, its mass before the decay can be calculated from the energies and momenta of the decay products. The mass value being inferred is called "invariant", due to the fact that it is independent of the reference frame in which the energies and momenta are measured. Such a concept can be generalized and used in different processes rather than just decay processes.

- **Rapidity y**: it is a dimensionless variable describing the rate at which the particle is moving with respect to a specific frame of reference. It is defined as :

$$y = tanh^{-1}h\beta = \frac{1}{2}Ln\frac{1+\beta}{1-\beta} \tag{C.6}$$

where $\beta = \frac{v}{c}$. While, if one wishes to have the expression in function of E, P, then a simple replacement $\beta = \frac{P}{E}$ will do the job.

- **Born cross section**: the term is used to refer to the term to the calculated cross section at lowest order in perturbation theory.

- **Forward-Backward Asymmetry** : it is a physical observable, dominantly used when the mass distribution exhibits a form of asymmetry. it is defined as

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \tag{C.7}$$

where σ_F is the cross section for fermion production in the forward hemisphere $[0^\circ, 90^\circ]$, and σ_B is the cross section for fermion production in the backward hemisphere $[90^\circ, 180^\circ]$.

Appendix D

Feynman Rules

This part is dedicated as a supplement for the purpose of clarifying the central rules used to calculate scattering amplitudes and cross sections, such a conception is called Feynman rules.

The following items are the recipe and the ingredients necessary to calculate ($-i\mathscr{M}_{fi}$) for a given Feynman diagram for tree-level processes.
To obtain $i\mathscr{M}$, we construct the Feynman rules in the following way, and of course based on a certain diagram:

1. **External lines**: they are are represented by the appropriate polarization vector or spinor, as table (D.1) indicates.

Table D.1: The Feynman rules for the external lines in Feynman diagrams.

Particle	Feynman rule
Ingoing fermion	u
Outgoing fermion	$\bar{u}$
Ingoing anti-fermion	$\bar{v}$
Outgoing anti-fermion	v
Ingoing photon	ϵ^{μ}
Outgoing photon	$\epsilon^{\mu *}$
Ingoing scalar	1
Outgoing scalar	1

2. **The propagator**:the propagator gives us information about the contribution to the amplitude from a particle traveling through space and time. it is defined as "i" times the inverse of the kinetic operator in the momentum space[1].

$$\begin{aligned}
&\text{Spin } 0: \quad \frac{i}{q^2-(mc)^2}\\
&\text{Spin}\frac{1}{2}: \quad \frac{i(\not{q}+mc)}{q^2-(mc)^2}\\
&\text{Spin } 1: \begin{cases} \text{Massless}: & \dfrac{-ig_{\mu\nu}}{q^2}\\[2ex] \text{Massive:} & \dfrac{-i[g_{\mu\nu}-q_\mu q_\nu/(mc)^2]}{q^2-(mc)^2}\end{cases}
\end{aligned} \tag{D.1}$$

[1]Sometimes we make use of the substitution $\partial^{\mu} \rightarrow -ip^{\mu}$.

The general form for a propagator is : $\frac{i}{p^2-m^2+i\epsilon}$.

3. **vertices**: vertices graphically are points where lines join together. they represent positions where particles are created or annihilated. They originate from the interaction term in the Lagrangian in the momentum space. They are represented by "i" times factors of the coupling constants.

The process of implementation will take place as follows:

1. Draw all topologically distinct diagrams.
2. Associate for each incoming/ outgoing particle, incoming/ outgoing antiparticle a factor, like the table (D.1) indicates.
3. For each internal line associate the convenient propagator.
4. Associate the relevant vertex factor for each vertex on the diagram.
5. A factor "-1" between diagrams that differ by exchange of fermionic lines.

while the steps for calculating the unpolarized scattering cross-section:

(i) Write $-i\mathscr{M}$ using the Feynman rules.

(ii) Squaring $\mathscr{M}$, and using the Casimir trick to obtain traces.

(iii) Evaluating the traces.

(iv) Applying the appropriate kinematics of the chosen frame.

(v) Integrating over the phase space.

Some remarks to add:

(A) Internal lines are integrated over all time and space.

(B) Extra rules are needed for diagrams containing loops.

(C) There is a symmetry factor to take into consideration if there are identical particles in the final state.

(D) In higher orders it is necessary to correct the external lines with some factors.

Appendix E

Casimir's trick and the square amplitude $|\mathscr{M}|^2$

This appendix is meant to incorporate details about the so-called Casimir's trick, that we made use of to calculate the square amplitude in the subsection(3.2.1).
In our case, our amplitude was of the from:

$$\mathscr{M} \sim \bar{u}^{s_1}\Gamma u_2^{s_2} \rightarrow |\mathscr{M}|^2 \sim [\bar{u}_1^{s_1}\Gamma u_2^{s_2}][\bar{u}_2^{s_2}\bar{\Gamma} u_2^{s_1}] \tag{E.1}$$

which can be proven:

$$\begin{aligned} |\mathscr{M}|^2 &\sim [\bar{u}_1\Gamma u_2][\bar{u}_1\Gamma u_2]^\dagger \\ &\sim [\bar{u}_1\Gamma u_2][u_1^\dagger\gamma^0\Gamma u_2]^\dagger \\ &\sim [\bar{u}_1\Gamma u_2][u_2^\dagger\Gamma^\dagger\gamma^0 u_1]^\dagger \\ &\sim [\bar{u}_1\Gamma u_2][u_2^\dagger\gamma^0\gamma^0\Gamma^\dagger\gamma^0 u_1]^\dagger \\ &\sim [\bar{u}_1\Gamma u_2][\bar{u}_2\bar{\Gamma} u_1]^\dagger \end{aligned} \tag{E.2}$$

and as we saw, a sum over all the spins, as well as averaging over the initial spins results in:

$$\begin{aligned} \sum_{s_2}|\mathscr{M}|^2 &\sim Tr[\bar{u}_1\Gamma(\not{p}_2+m_2)\bar{\Gamma}u_1] \\ \sum_{s_2,s_1}|\mathscr{M}|^2 &\sim Tr[\Gamma(\not{p}_2+m_2)\bar{\Gamma}(\not{p}_1+m_1)] \\ <|\mathscr{M}|^2> &\sim \frac{1}{2}Tr[\Gamma(\not{p}_2+m_2)\bar{\Gamma}(\not{p}_1+m_1)] \end{aligned} \tag{E.3}$$

The factor $\frac{1}{2}$ came from the action of averaging over the initial spins, for each spin coming from an initial state we multiply by $\frac{1}{2}$. If neither of the spins is in the initial state, then the factor is 1. However, the previous procedure justifies what we have obtained in the expressions (3.66, 3.74).
Furthermore, in order to show how we worked out the traces, we just take an example of (3.67, 3.68). The rest has been worked out exactly in a similar fashion.

$$\begin{aligned} c_{v_b}'^2 Tr[\gamma_\beta\not{p}_1\gamma^0\gamma_0\gamma_\alpha\not{p}_2] = c_{v_b}'^2 Tr[\not{p}_2\gamma_\beta\not{p}_1\gamma^0\gamma_0\gamma_\alpha] = c_{v_b}'^2 p_2^\mu p_1^\nu Tr[\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\alpha] \\ = c_{v_b}'^2 p_2^\mu p_1^\nu Tr[\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\alpha] = c_{v_b}'^2[4p_{2\beta}p_{1\alpha} + 4p_{2\alpha}p_{1\beta} - 4(p_2p_1)g_{\beta\alpha}] \end{aligned} \tag{E.4}$$

where : $\gamma^0\gamma_0 = (\gamma^0)^2 = 1$, $\not{p}_2 = \gamma_\mu p_2^\mu$, and $\not{p}_1 = \gamma_\nu p_1^\nu$. And the property:

$$Tr[\gamma^\kappa\gamma^\lambda\gamma^\mu\gamma^\nu] = 4g^{\kappa\lambda}g^{\mu\nu} + 4g^{\kappa\nu}g^{\lambda\mu} - 4g^{\kappa\mu}g^{\lambda\nu} \tag{E.5}$$

The second term:

$$-m_2 c_{v_b}^{\prime 2} Tr[\gamma_\beta \not{p}_1 \gamma^0 \gamma_0 \gamma_\alpha] = -m_2 c_{v_b}^{\prime 2} Tr[\gamma_\beta \not{p}_1 \gamma_\alpha] = -m_2 c_{v_b}^{\prime 2} p_1^\nu Tr[\gamma_\beta \gamma_\nu \gamma_\alpha] = 0 \tag{E.6}$$

because the traces of a single γ matrix is zero, and similarly the trace of any odd number of γ matrices.

Also,

$$\begin{aligned} c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \not{p}_1 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha \not{p}_2] &= -c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \not{p}_1 \gamma^0 \gamma_0 \gamma_5 \gamma_\alpha \not{p}_2] = -c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_5 \gamma_\alpha \not{p}_2 \gamma_\beta \not{p}_1] \\ &= -c_{v_b}^{\prime} c_{A_b}^{\prime} p_2^\mu p_1^\nu Tr[\gamma_5 \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_\nu] = 4i c_{v_b}^{\prime} c_{A_b}^{\prime} p_2^\mu p_1^\nu \epsilon_{\alpha\mu\beta\nu} \end{aligned} \tag{E.7}$$

where γ^5 anti-commutes with all γ-matrices, and : $Tr[\gamma^5 \gamma^\kappa \gamma^\lambda \gamma^\mu \gamma^\nu] = -4i\epsilon_{\kappa\lambda\mu\nu}$.

The next term,

$$-m_2 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \not{p}_1 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha] = m_2 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \not{p}_1 \gamma_5 \gamma_\alpha] = m_2 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_5 \gamma_\alpha \gamma_\beta \not{p}_1] = 0 \tag{E.8}$$

because:

$$Tr[\gamma^5] = 0 \qquad Tr[\gamma^5 \gamma^\nu] = 0$$
$$Tr[\gamma^5 \gamma^\mu \gamma^\nu] = 0 \qquad Tr[\gamma^5 \gamma^\lambda \gamma^\mu \gamma^\nu] = 0$$

We can write all the vanishing terms:

$$\begin{aligned} & m_1 c_{v_b}^{\prime 2} Tr[\gamma_\beta \gamma^0 \gamma_0 \gamma_\alpha \not{p}_2] = 0 \\ & m_1 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha \not{p}_2] = 0 \\ & - m_1 m_2 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha] = 0 \\ & - m_2 c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \gamma_5 \not{p}_1 \gamma^0 \gamma_0 \gamma_\alpha] = 0 \\ & - m_2 c_{A_b}^{\prime 2} Tr[\gamma_\beta \gamma_5 \not{p}_1 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha] = 0 \\ & - m_1 m_2 c_{A_b}^{\prime} c_{v_b}^{\prime} Tr[\gamma_\beta \gamma_5 \gamma^0 \gamma_0 \gamma_\alpha] = 0 \\ & + m_1 c_{A_b}^{\prime} c_{v_b}^{\prime} Tr[\gamma_\beta \gamma_5 \gamma^0 \gamma_0 \gamma_\alpha \not{p}_2] = 0 \\ & + m_1 c_{A_b}^{\prime 2} Tr[\gamma_\beta \gamma_5 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha \not{p}_2] = 0 \\ & + c_{v_b}^{\prime} c_{A_b}^{\prime} Tr[\gamma_\beta \gamma_5 \not{p}_1 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha \not{p}_2] = 0 \end{aligned} \tag{E.9}$$

where the remaining terms are:

$$\begin{aligned} & - m_1 m_2 c_{v_b}^{\prime 2} Tr[\gamma_\beta \gamma^0 \gamma_0 \gamma_\alpha] = -4 m_1 m_2 c_{v_b}^{\prime 2} g_{\beta\alpha} \\ & c_{A_b}^{\prime 2} Tr[\gamma_\beta \not{p}_1 \gamma_\alpha \not{p}_2] = c_{A_b}^{\prime 2} [4 p_{2\beta} p_{1\alpha} + 4 p_{2\alpha} p_{1\beta} - 4(p_2 p_1) g_{\beta\alpha}] \\ & - m_1 m_2 c_{A_b}^{\prime 2} Tr[\gamma_\beta \gamma_5 \gamma^0 \gamma_5 \gamma_0 \gamma_\alpha] = 4 m_1 m_2 c_{A_b}^{\prime 2} g_{\beta\alpha} \end{aligned} \tag{E.10}$$

All of the remaining results we have obtained can be replicated following exactly the same previous procedure.

Bibliography

[1] J. Barranco. Some standard model problems and possible solutions. *Journal of Physics: Conference Series*, 761(1):012007, oct 2016.

[2] Measurement of the positive muon anomalous magnetic moment to 0.46 ppm. *Phys. Rev. Lett.*, 126:141801, Apr 2021.

[3] T. Aaltonen et al. High-precision measurement of the W boson mass with the CDF II detector. *Science*, 376(6589):170–176, 2022.

[4] Digesh Raut. *Gauged U(1) extension of the standard model and phenomenology.* PhD thesis, Alabama U., U. Alabama, Tuscaloosa, 2018.

[5] Nobuchika Okada and Satomi Okada. *Physical Review D*, 93(7), apr 2016.

[6] Osamu Sawada and Akio Sugamoto, editors. *Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe: Tsukuba, Japan, February 13-14, 1979*, Tsukuba, Japan, 1979. Natl.Lab.High Energy Phys.

[7] Ujjal Kumar Dey, Tapoja Jha, Ananya Mukherjee, and Nirakar Sahoo. Leptogenesis in an extended seesaw model with $u(1)_{B-L}$ symmetry. *Journal of Physics G: Nuclear and Particle Physics*, 50(1):015004, dec 2022.

[8] Paul Langacker. The physics of heavy $Z^{'}$ gauge bosons. *Rev. Mod. Phys.*, 81:1199–1228, Aug 2009.

[9] Robert Foot and Xiao-Gang He. Comment on Z Z-prime mixing in extended gauge theories. *Phys. Lett. B*, 267:509–512, 1991.

[10] J. S. Alvarado, S. F. Mantilla, R. Martinez, and F. Ochoa. A non-universal $u(1)_X$ extension to the standard model to study the b meson anomaly and muon $g-2$, 2021.

[11] Radhika Vinze and Sreerup Raychaudhuri. A viable u(1) extended standard model with a massive z' invoking the stueckelberg mechanism, 2021.

[12] Matthew D. Schwartz. *Quantum Field Theory and the Standard Model.* Cambridge University Press, 3 2014.

[13] RICHARD P. FEYNMAN. *QED: The Strange Theory of Light and Matter.* Princeton University Press, rev - revised edition, 1985.

[14] S. L. Glashow. Partial Symmetries of Weak Interactions. *Nucl. Phys.*, 22:579–588, 1961.

[15] Steven Weinberg. A model of leptons. *Phys. Rev. Lett.*, 19:1264–1266, Nov 1967.

[16] Abdus Salam. Weak and Electromagnetic Interactions. *Conf. Proc. C*, 680519:367–377, 1968.

[17] E. M. Riordan. The Discovery of quarks. *Science*, 256:1287–1293, 1992.

[18] Luigi Di Lella and Carlo Rubbia. The Discovery of the W and Z Particles. *Adv. Ser. Direct. High Energy Phys.*, 23:137–163, 2015.

[19] Observation of top quark production in $\overline{p}p$ collisions with the collider detector at fermilab. *Phys. Rev. Lett.*, 74:2626–2631, Apr 1995.

[20] Observation of tau neutrino interactions. *Physics Letters B*, 504(3):218–224, apr 2001.

[21] Observation of a new particle in the search for the standard model higgs boson with the ATLAS detector at the LHC. *Physics Letters B*, 716(1):1–29, sep 2012.

[22] Abdelhak Djouadi. The higgs mechanism and the origin of mass. *Fundamental Theories of Physics*, 162:1–23, 01 2011.

[23] R. L. Workman et al. (Particle Data group), Review of Particle Physics. *Progress of Theoretical and Experimental Physics (PTEP)*, 2022:083C01, 2022.

[24] Takhamsib Aliev, Namik Kemal Pak, and Meltem Serin, editors. *The standard model and beyond*. Springer Proceedings in Physics. Springer, Berlin, Germany, 2008 edition, November 2007.

[25] Guido Altarelli. The standard electroweak theory and beyond. *arXiv: High Energy Physics - Phenomenology*, 1998.

[26] F. Englert and R. Brout. Broken symmetry and the mass of gauge vector mesons. *Phys. Rev. Lett.*, 13:321–323, Aug 1964.

[27] Peter W. Higgs. Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.*, 13:508–509, Oct 1964.

[28] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Broken symmetries and the Goldstone theorem. *Adv. Part. Phys.*, 2:567–708, 1968.

[29] Marcus Pernow. *Models of SO(10) Grand Unified Theories : Yukawa Sector and Gauge Coupling Unification*. PhD thesis, KTH, Physics, 2021.

[30] Tomio Kobayashi. Experimental verification of the standard model of particle physics. *Proc. Jpn. Acad. Ser. B Phys. Biol. Sci.*, 97(5):211–235, 2021.

[31] Carlo Rovelli. *Quantum Gravity*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2004.

[32] Carlo Rovelli. Loop quantum gravity. *Living Reviews in Relativity*, 1(1), jan 1998.

[33] J Barranco. Some standard model problems and possible solutions. *J. Phys. Conf. Ser.*, 761:012007, October 2016.

[34] Ben Gripaios. Gauge field theory. *University of Cambridge*, 2016.

[35] Z. Li and S. Zhang. Domain-wall dynamics and spin-wave excitations with spin-transfer torques. *Phys. Rev. Lett.*, 92:207203, May 2004.

[36] Jean Iliopoulos. Physics beyond the standard model. *arXiv preprint arXiv:0807.4841*, July 2008.

[37] Djuna Croon, Tomás E. Gonzalo, Lukas Graf, Nejc Košnik, and Graham White. Gut physics in the era of the lhc. *Frontiers in Physics*, 7, 2019.

[38] Marina von Steinkirch. The gauge group su(5) as a simple gut. *State University of New York*, June 2010.

[39] Thorén, Johan. Grand Unified Theories: SU(5), SO(10) and supersymmetric SU(5). *Lund University*, July 2012. Student Paper, Bachelor Thesis.

[40] Harald Fritzsch and Peter Minkowski. Unified interactions of leptons and hadrons. *Annals of Physics*, 93(1):193–266, 1975.

[41] D. I. Kazakov. Beyond the standard model. *arXiv preprint arXiv:hep-ph/0411064*, Nov 2004.

[42] Csaba Csáki and Philip Tanedo. Beyond the Standard Model. In *2013 European School of High-Energy Physics*, pages 169–268, 2015.

[43] Nicholas Pollard. *B-L and Other U(1) Extensions of the Standard Model.* PhD thesis, UC, Riverside, 2017.

[44] Xiao-Gang He, G. C. Joshi, H. Lew, and R. R. Volkas. Simplest $Z^{'}$ model. *Phys. Rev. D*, 44:2118–2132, Oct 1991.

[45] Seungwon Baek, Hiroshi Okada, and Kei Yagyu. Flavour dependent gauged radiative neutrino mass model. *Journal of High Energy Physics*, 2015(4), apr 2015.

[46] Seungwon Baek and Pyungwon Ko. Phenomenology of $\mathrm{u}(1)_{L_\mu - L_\tau}$ charged dark matter at PAMELA/FERMI and colliders. *Journal of Cosmology and Astroparticle Physics*, 2009(10):011–011, oct 2009.

[47] Julian Heeck and Werner Rodejohann. Gauged $l_\mu - l_\tau$ symmetry at the electroweak scale. *Physical Review D*, 84(7), oct 2011.

[48] Sudip Jana, P. K. Vishnu, and Shaikh Saad. Minimal dirac neutrino mass models from $u(1)_r$ gauge symmetry and left-right asymmetry at colliders. *The European Physical Journal C*, 79(11), nov 2019.

[49] Takaaki Nomura and Hiroshi Okada. Left-handed and right-handed u(1) gauge symmetry. *J. High Energy Phys.*, 2018(1), January 2018.

[50] Boris Kors and Pran Nath. A Stueckelberg extension of the standard model. *Phys. Lett. B*, 586:366–372, 2004.

[51] Ken Hsieh, Kai Schmitz, Jiang-Hao Yu, and C. P. Yuan. Global Analysis of General SU(2) x SU(2) x U(1) Models with Precision Data. *Phys. Rev. D*, 82:035011, 2010.

[52] Qing-Hong Cao, Bin Yan, and Dong-Ming Zhang. Simple non-abelian extensions of the standard model gauge group and the diboson excesses at the LHC. *Physical Review D*, 92(9), nov 2015.

[53] Benjamin W. Lee and Steven Weinberg. Su(3) $\otimes$ u(1) gauge theory of the weak and electromagnetic interactions. *Phys. Rev. Lett.*, 38:1237–1240, May 1977.

[54] E. Ramirez-Barreto, Y. Amaral Coutinho, and J. Sá Borges. Extra neutral gauge boson from two versions of the 3-3-1 model in future linear colliders. *The European Physical Journal C*, 50:909–917, 2007.

[55] Paul Langacker. The physics of heavy $z^{'}$ gauge bosons. *Reviews of Modern Physics*, 81(3):1199–1228, aug 2009.

[56] Shaaban Khalil. Low-scale b–l extension of the standard model. *Journal of Physics G: Nuclear and Particle Physics*, 35(5):055001, mar 2008.

[57] Joydeep Chakrabortty. Road map of left-right symmetric model. *IIT Kanpur*, 2018.

[58] Melissa Harris. Left-right symmetric model : Putting lower bounds on the mass of the heavy, charged wr gauge boson. diva-portal, 2017.

[59] David London and Jonathan L. Rosner. Extra gauge bosons in e_6. *Phys. Rev. D*, 34:1530–1546, Sep 1986.

[60] Joydeep Chakrabortty, Rinku Maji, Sunando Kumar Patra, Tripurari Srivastava, and Subhendra Mohanty. Roadmap of left-right models based on guts. *Phys. Rev. D*, 97:095010, May 2018.

[61] Cao H. Nam. Flipped u(1) extended standard model and majorana dark matter. *The European Physical Journal C*, 80(12), dec 2020.

[62] Rabindra N. Mohapatra and R. E. Marshak. Local B-L Symmetry of Electroweak Interactions, Majorana Neutrinos and Neutron Oscillations. *Phys. Rev. Lett.*, 44:1316–1319, 1980. [Erratum: Phys.Rev.Lett. 44, 1643 (1980)].

[63] Julian Heeck and Werner Rodejohann. Gauged $L_\mu - L_\tau$ Symmetry at the Electroweak Scale. *Phys. Rev. D*, 84:075007, 2011.

[64] Nobuchika Okada and Satomi Okada. z'-portal right-handed neutrino dark matter in the minimal $u(1)_x$ extended standard model. *Physical Review D*, 95(3), feb 2017.

[65] Nobuchika Okada, Satomi Okada, and Digesh Raut. Inflection-point inflation in a hyper-charge oriented $u(1)_x$ model. *Physical Review D*, 95(5), mar 2017.

[66] Arindam Das, Nobuchika Okada, and Digesh Raut. Heavy majorana neutrino pair productions at the LHC in minimal u(1) extended standard model. *The European Physical Journal C*, 78(9), aug 2018.

[67] Hieu Minh Tran. Kinetic mixing in models with an extra abelian gauge symmetry. *Commun. Phys.*, 28(1):41, July 2018.

[68] HENRI RUEGG and MARTÍ RUIZ-ALTABA. THE STUECKELBERG FIELD. *International Journal of Modern Physics A*, 19(20):3265–3347, aug 2004.

[69] Boris Körs and Pran Nath. Aspects of the stueckelberg extension. *Journal of High Energy Physics*, 2005(07):069–069, jul 2005.

[70] S. V. Kuzmin and D. G. C. McKeon. Stueckelberg mass in the Glashow-Weinberg-Salam model. *Mod. Phys. Lett. A*, 16:747–753, 2001.

[71] BORIS KÖRS and PRAN NATH. HOW STUECKELBERG EXTENDS THE STANDARD MODEL AND THE MSSM. In *PASCOS 2004*. World Scientific Publishing Company, aug 2005.

[72] Daniel Feldman, Zuowei Liu, and Pran Nath. The stueckelberg z prime at the LHC: discovery potential, signature spaces and model discrimination. *Journal of High Energy Physics*, 2006(11):007–007, nov 2006.

[73] W S Emam and S A Moussa. Search for stueckelberg z at LHC. *Eur. Phys. J. C Part. Fields*, 60(3):441–447, April 2009.

[74] Thomas G. Rizzo. Z' phenomenology and the LHC. In *Theoretical Advanced Study Institute in Elementary Particle Physics: Exploring New Frontiers Using Colliders and Neutrinos*, pages 537–575, 10 2006.

[75] A. Leike. The phenomenology of extra neutral gauge bosons. *Physics Reports*, 317(3-4):143–250, aug 1999.

[76] Mirjam Cvetic. Comparison of diagnostic Z-prime physics at future p p and e+ e- colliders. In *International Europhysics Conference on High-energy Physics*, 12 1993.

[77] Stephen Godfrey. Search limits for extra neutral gauge bosons at high-energy lepton colliders. *eConf*, C960625:NEW138, 1996.

[78] Marcela Carena, Alejandro Daleo, Bogdan A. Dobrescu, and Tim M. P. Tait. z' gauge bosons at the fermilab tevatron. *Physical Review D*, 70(9), nov 2004.

[79] A. A. Andrianov, P. Osland, A. A. Pankov, N. V. Romanenko, and J. Sirkka. Phenomenology of a z' coupling only to third-family fermions. *Physical Review D*, 58(7), aug 1998.

[80] Diego Barbosa, Felipe Daiz, Liliana Quintero, Andres Flrez, Manuel Sanchez, Alfredo Gurrola, Elijah Sheridan, and Francesco Romeo. Probing a z' with non-universal fermion couplings through top quark fusion, decays to bottom quarks, and machine learning techniques. *The European Physical Journal C*, 83(5), may 2023.

[81] Search for heavy particles in the b-tagged dijet mass distribution with additional b-tagged jets in proton-proton collisions at $\sqrt{s} = 13$ TeV with the atlas experiment. *Phys. Rev. D*, 105:012001, Jan 2022.

[82] Patrick J. Fox, Ian Low, and Yue Zhang. Top-philic z′ forces at the LHC. *Journal of High Energy Physics*, 2018(3), mar 2018.

[83] Mohammad Abdullah, Mykhailo Dalchenko, Teruki Kamon, Denis Rathjens, and Adrian Thompson. A heavy neutral gauge boson near the z boson mass pole via third generation fermions at the LHC. *Physics Letters B*, 803:135326, apr 2020.