PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

University Saàd DAHLAB-blida1

FACULTY OF SCIENCE

Department of Physics

MEMORY

Presented to obtain the diploma of

MASTER

Specialty: Theoretical Physics

By

MISSOUM MOURAD

THEME

Study of open bosonic strings in presence of D-branes

DATE : 15-07-2023

The Jury:

President: Djazi Moussadek MAA U.Blida 1

Examiners: Hadj Moussa M'hamed MCA U.Blida 1

Supervisor: Hamam Douria MCB U.Blida 1

Abstract:

The purpose of this memorandum is the study of the open bosonic string theories in the presence of D-branes.

Essential results are obtained:

- The mass operator is modified leading to a theory with no tachyon.

- Coherence between the development of the partition function and the degeneration of states for each mass level.

- Apart from the ordinary case that we find and which confirms that the presence of parallel D-branes of the same dimensionality does not modify the positive anomaly in the central term of the Virasoro algebra.

Keywords:

D-branes, spectrum, partition fonction, Virasoroalgebra.

Résumé :

L'objet de ce mémoire est l'étude de la théorie d'une corde bosonique en présence de D-brane.

Les résultats obtenus se résument comme suit :

- Modification de l'opérateur de masse impliquant entre autre l'absence de tachyons.

- Cohérence entre le développement de la fonction de partition et la dégénérescence des états pour chaque niveau de masse.

- En dehors du cas ordinaire qu'on retrouve et qui confirme que la présence de D-branes parallèles de même dimensionnalités ne modifie en rien l'anomalie positive dans le terme central de l'algèbre de Virasoro.

Mots clés:

D-branes, spectre, fonction de partition, algèbre de Virasoro.

acknowledgements

First of all, I thank Allah who gave me the strength and the will to be able to complete this work.

I would like to express my gratitude to my supervisor, Mrs Douria Hamam, for her support, and her patience with me,I also thank her for having directed me correctly, for her precision and completeness in every detail, and for her precious advice, I wish her all the best for the future

I address my sincere thanks to all the members of the jury, who accepted to judge this work.

I express my deep gratitude to my family, my dear parents my wife and my kids, who have always been with me, for giving me the love and attention that keeps me going and motivates me to succeed, to my brothers and my two sisters for making me happy and for their continued assistance

Finally, I would like to thank all my colleagues and friends, who were present during this work for their friendship and their encouragement.

Table of figures

List of table

<u>Table of Contents:</u>

Introduction

The study of strings and D-branes has been a topic of interest in physics for several decades. Dirichlet-branes, or D-branes for short, have moved to center stage in string theory In a relatively short time they have gone from being esoteric, and largely ignored, extended objects that existed in certain string theories to playing a key role in the present understanding of non-perturbative dynamics and duality in string theory.

One reason that the importance of D-branes was overlooked is that they are associated with open strings and, at the perturbative level, closed string theory appears much less promising than open string theories, such as the heterotic string theory. With the development of string duality, attention shifted towards non-perturbative issues including solutions, and it was realized that D-branes had precisely the required attributes to be partners with fundamental string states under duality transformations. This realization, along with the fact that D-branes are considerably simpler to describe than other string solutions and often allow explicit calculations in places where only speculations went before, led to an explosion of activity involving D-branes that is still going strong.

The aim of this work is to provide an introduction to open bosonic strings and D-branes. We will begin with a brief overview of the fundamental principles of string theory, including the concept of spacetime and the quantization of strings. We will then introduce the concept of D-branes, which are subspaces of spacetime on which open strings can end. We will discuss the properties of D-branes and their role in string theory. Finally, This paper is intended to provide a foundation for further study of the physics of open bosonic strings and D-branes in the context of D-branes and their role in string theory.

In this paper, we set up the notation needed to describe D-branes, and then we state the appropriate boundary conditions. We let d denote the total number of spatial dimensions in the theory, in the present case, $d= 25$. The total number of space time dimensions is $D=d+1=26$. A Dp-brane with $p < 25$ extends over a p-dimensional subspace of the 25-dimensional space.

The thesis is presented as follows:

The first chapter aims to introduce open string theory and Quantization

In the second chapter we start with openbosonic string on Dp-branes, we will study the spectrum and its degeneracy which will be confronted with the development of the partition function.

In the third chapter, we will focus on the study of open bosonic string between two parallelIDp-branes, in this case, where the open string is between two Dp-branes in parallel we will also develop the calculation of the Virasoro algebra.

Chapter four will examine the open bosonic string between two Dp-, Dq-branes in paralle.

Chapter 1

Open string

1.1.Open string

1.1.1.Equations of motion of the open bosonic string

The bosonic string action is given by:

$$S = \int_{Ti}^{Tf} dt \int_0^{\pi} d\sigma \; L \tag{1.1}$$

when:

$$L = \frac{-1}{2\pi\alpha'}\left[\left(\dot{X}\,X'\right)^2 - \left(\dot{X}^{\,2}X'^2\right)\right]^{\frac{1}{2}} \tag{1.2}$$

From a variation δS (principle of least action $\delta S = 0$) corresponding to a small variation.

$X^{\mu} \rightarrow X^{\mu} + \delta X^{\mu}$ with $\delta X^{\mu}(\sigma, Ti) = \delta X^{\mu}(\sigma, Tf)$and $\delta X^{\mu}(\sigma = 0\,,\,\pi)$arbitrary

The action have to be:

$$\delta S = \int_{Ti}^{Tf} dt \int_0^{\pi} d\sigma \left[\left(\frac{\partial L}{\partial \dot{X}^{\mu}}\right)\frac{\partial}{\partial \sigma}\delta X^{\mu} + \left(\frac{\partial L}{\partial X'^{\mu}}\right)\frac{\partial}{\partial \sigma}\delta X^{\mu}\right]$$

$$\delta S = \int_0^{\pi} d\sigma \left(\frac{\partial L}{\partial \dot{X}^{\mu}}\right)\delta X^{\mu}\,\Big|_{Ti}^{Tf} + \int_{Ti}^{Tf} dt \left(\frac{\partial L}{\partial X'^{\mu}}\right)\delta X^{\mu}\Big|_{\sigma=0}^{\sigma=\tau}$$

$$- \int_{Ti}^{Tf} dt \int_0^{\pi} d\sigma \left[\frac{\partial}{\partial T}\left(\frac{\partial L}{\partial \dot{X}^{\mu}}\right) + \frac{\partial}{\partial \sigma}\left(\frac{\partial L}{\partial X'^{\mu}}\right)\right] \delta X^{\mu} \tag{1.3}$$

The first term vanished when the initial and finale position of the string are taken fixed. The equations of motion are given by:

$$\frac{\partial}{\partial T}\frac{\partial L}{\partial \dot{X}^{\mu}} + \frac{\partial}{\partial \sigma}\frac{\partial L}{\partial X'^{\mu}} = 0 \tag{1.4}$$

With the boundary conditions:

$$\frac{\partial L}{\partial X'^{\mu}}(\sigma = 0\,,\,T) = \frac{\partial L}{\partial X'^{\mu}}(\sigma = \pi\,,\,T) = 0 \tag{1.5}$$

We define:

$$P^{\mu}(\sigma\,,\,T) = -\frac{\partial L}{\partial \dot{X}^{\mu}} = \frac{1}{2\pi\alpha'}\frac{(\dot{X}X')X'\mu - (X')^2\dot{X}\mu}{\sqrt{(\dot{X}X')^2 - (\dot{X})^2(X')^2}} \tag{1.6}$$

Which is the conjugate moment $X^{\mu}(\sigma, T)$ describing the dynamics of the system and verifying the canonical Poisson brackets:

$$\{X^{\mu}(\sigma, T), X^{\upsilon}(\sigma', T)\} = \{P^{\mu}(\sigma, T), P^{\upsilon}(\sigma', T)\} = 0 \tag{1.7}$$

$$\{X^{\mu}(\sigma, T), P^{\upsilon}(\sigma', T)\} = -g^{\mu\upsilon}\delta(\sigma - \sigma') \tag{1.8}$$

$X^{\mu}(\sigma, T)$ and $P^{\mu}(\sigma, T)$are considered as independent dynamic variables.

And:

$$\mathrm{II}^{\mu}(\sigma, T) = -\frac{\partial L}{\partial X'\mu} = -\frac{1}{2\pi\alpha'}\frac{(\dot{\mathrm{X}}X')X^{\cdot}\mu - (\dot{\mathrm{X}})^2 X'\mu}{\sqrt{(\dot{\mathrm{X}}X')^2 - (\dot{\mathrm{X}})^2 (X')^2}} \tag{1.9}$$

(σ denotes the points of the string).

The invariance of the action under the Lorentz transformations ($\delta X\mu = \delta\Lambda\mu\upsilon\, X^{\upsilon}$) with the relations (1.4) and (1.5) leads to the energy and angular momentum of the string. We can demonstrate, in particular, which the flow of energy momentum and moment angle inside the string, which is specified by:

$P^{\mu}(\tau) = \int_0^{\pi} d\sigma\ P^{\mu}(\sigma, \tau)$Total energy moment of the string

$M^{\mu\upsilon}(\tau) = \int_0^{\pi} d\sigma\ (P^{\mu}X^{\upsilon} - P^{\upsilon}X^{\mu})$Total angular momentum of the string

The parameterization symmetry of the action results in the existence of equations of Constraints between $P^{\mu}(\sigma, T)\ et\ X^{\mu}(\sigma, T)$ given by the identities:

$$\begin{cases} P^2 + \dfrac{x'^2}{\pi^2} = 0 \\ P^{\mu} + X'\mu = 0 \end{cases} \tag{1.10}$$

Now the Equations of motion (1.4) are nonlinear, a particular choice of gauge orthogonal makes it possible to reduce them to the following Dalembert equations:

$$\ddot{X}_{\mu}(\sigma, T) - X''\mu(\sigma, T) = 0 \tag{1.11}$$

And the relations (1.10) reduce to the equations

$$\begin{cases} \dot{\mathrm{X}}^2 + X'^2 = 0 \\ \dot{\mathrm{X}}X' = 0 \end{cases} \tag{1.12}$$

Which describe a system of orthonormal coordinates on the world-sheet. So the equations (1.6) and (1.9) become:

$$P^{\mu}(\sigma\,,\,T)=\frac{1}{2\pi\alpha'}\dot{X}^{\,\mu} \tag{1.13}$$

$$\Pi^{\mu}(\sigma\,,\,T)=-\frac{1}{2\pi\alpha'}X'^{\,\mu} \tag{1.14}$$

1.1.2. Solutions to the Equations of Motion

The general solution of the equation of motion (1.10) is given by:

$$X^{\mu}(\sigma\,,\,T)=-\frac{1}{2}\left(f^{\mu}(T+\sigma)+g^{\mu}(T-\sigma)\right) \tag{1.15}$$

In the case of the open string, where the coordinate σ varies between $0\; and\; \pi$, the fields $X^{\,\mu}(\sigma,\,T\;)$ Satisfy boundary conditions. There are two types of ends for open strings:

1- Those which obey the Neumann boundary condition when the two extremities of the string are free

$$\partial_{\sigma}X^{\,\mu}(\,T\,,\,\sigma=0\,,\,\pi\,)=0 \tag{1.16}$$

2- Those which remain fixed in a hyper plane satisfy the boundary condition of Dirichlet when both ends of a moving string are fixed

$$X^{\,\mu}(\,T\,,\,\sigma=0\,,\,\pi\,)=0 \tag{1.17}$$

Note here that the Neumann condition respects the Poincare invariance and therefore the conservation of the energy-momentum tensor. That of Dirichlet breaks this invariance and there is a possibility of momentum loss for directions parallel to the edge of the surface. This is the indication of the presence of an extended object on which the string rests. This is also the origin of the name Dp-brane, the D refers to the Dirichlet conditions. When both ends of the string are free, the Neumann boundary conditions allow equation (1.15) to be written in the following form.

$$X^{\,\mu}(\sigma\,,\,T)=x^{\mu}+2\alpha'\,p^{\mu}T+\sqrt{2\alpha'}\sum_{n=1}^{\infty}\{a_n^{\mu}(0)\exp(-\imath nT)-a_n^{\mu *}(0)\exp(\imath nT)\}\cos n\,\sigma$$

(1.18)

Where $2\alpha'$ is an arbitrary constant with length dimension. X^u and p^u are two integration constants that correspond to the coordinate of the centre of mass and the momentum total length of the string respectively, a_n^u represent the modes of vibration of the string. If we define:

$$a_n^\mu = a_n^\mu(0)\exp(-\iota nT)\ ,\ a_n^{\mu *}(0)\exp(\iota nT) \tag{1.19}$$

$$a_0^\mu = 2\alpha'\, p^\mu \tag{1.20}$$

$$a_n^\mu = \sqrt{2n\alpha' a_n^\mu} \quad , \qquad a_{-n}^\mu = a_n^{\mu *} \quad n \geq 0 \tag{1.21}$$

We can write the equation (1.18) in the form:

$$X^\mu(\sigma, T) = x_0^\mu + \sqrt{2\alpha'\alpha_0^\mu T} + i\sqrt{2a'}\sum_{n=1}^{\infty} -a_n^\mu \exp(-\iota nT)\cos n\sigma \tag{1.22}$$

especially:

$$\dot{X}^\mu(\sigma, T) \pm X'^\mu(\sigma, T) = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} a_n^\mu \exp\big(-\iota n(T \pm \sigma)\big) \tag{1.23}$$

The Virasoro generators are given by:

$$L_n = \frac{1}{2}\pi\alpha' \int_{-\pi}^{\pi} d\sigma\ \ \exp\ \iota n\,(T+\sigma)\left(P^2 + \frac{(X^\mu)^2}{2\pi\alpha'}\right)$$

$$= \frac{1}{4a'}\ \sum_{m=-\infty}^{+\infty} a_{n-m}a_m \tag{1.24}$$

The open string Hamiltonian is given by

$$H = \int_0^\pi d\sigma\big(P_\mu X^{\cdot\mu} - L\big)$$

$$= \int_0^\pi d\sigma\left(P_\mu X^{\cdot\mu} - \frac{1}{4\pi a'}((2\pi a'P)^2 - X'^2)\right)$$

$$= \pi\int_0^\pi d\sigma\left(a'P^2 + \frac{X'^2}{4\pi^2 a'}\right) \tag{1.25}$$

The special case $H = Lo$ and the condition $Lo = 0$ allows the mass of the string to be written as follows

The equation:

$$M^2 = -p^2 = \frac{1}{a'} \sum_{n=1}^{+\infty} a_{n-m}\, a_m \tag{1.26}$$

Which is the relativistic equivalent of the energy expression of a vibrating violin string, and the angular momentum can be written:

$$M^{\mu v} = x^\mu p^v - x^v p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} \left(a_{-n}^\mu \, a_n^v - a_{-n}^v a_n^\mu \right) \tag{1.27}$$

1.2. Quantification

When $X^\mu(\sigma, T)$ and $P^\mu(\sigma, T)$ are treated as independent, the Virasoro constraints are incompatible with Poisson brackets and hence canonical quantization is not more direct, which leads us to use two methods of quantification:

Covariant quantization:

This method treats coordinates and conjugate moments as independent and the constraints (Virasoro conditions) will be considered as initial conditions on the wave functions.

Quantization in the transverse gauge:

Before quantifying the system, we solve the constraint equations within the framework of a specific choice of a parameterization and, this will lead us to keep only the variables effectively independent dynamics.

1.2.1. Covariant gauge

Recall that $X^\mu(\sigma, T)$ is the space-time string coordinate and $P^\mu(\sigma, T)$ is the moment associated with the string.

Commutations relations

We postulate the following commutations relations:

$$[X^{\mu}(\sigma , T), P^{v}(\sigma' , T)] = i\eta^{\mu v}\delta(\sigma - \sigma') \tag{1.28}$$

$$[X^{\mu}(\sigma , T), X^{v}(\sigma' , T)] = [P^{\mu}(\sigma, T), P^{v}(\sigma' , T)] = 0 \tag{1.29}$$

In terms of the dynamic variables x_u ,p_u and $\alpha^{\,n}_{\,u}$ they become :

$$[x^{\mu} , p^{v}] = i\eta^{\mu v} \tag{1.30}$$

$$\left[\alpha^{\,\mu}_{\,n} , \alpha^{\,v}_{\,m}\right] = n\delta_{m+n,o}\eta^{\mu v} \tag{1.31}$$

Where x and p are the center of mass variables and the n can be perceived (analogy to the quantum case of the harmonic oscillator) as creation or annihilation operators for negative or positive n respectively.

Now, it is fundamental to note that the Fock space generated by the application at the ground state of the $a_n^{+\mu}$ is not positive, because the temporal components have a_n^{μ} minus sign, unusual in their commutation relation:

$$[a_n^{0} , a_n^{+0}] = -1 \tag{1.32}$$

And therefore a state of the form $a_n^{+0}\ |0\rangle$ has a negative norm. It is must therefore be taken to eliminate these Ghost states.

You can now match classic Virasoro generators with operators

$$ln = \sum_{m=-\infty}^{\infty} : a^{\mu}_{-n-m} a_{m,\mu}: \tag{1.33}$$

The definition of the operator l_0 poses a problem of order ambiguity, an arbitrary constant could then be added to eliminate this problem: $lo \rightarrow lo - a(0)$the Hamiltonian becomes:

$$H = a'p^2 + \sum_{n=1}^{\infty} a_{-n}a_n - \frac{a(0)}{a'} \tag{1.34}$$

Contrary to the above, the angular momentum remains the same:

$$M^{\mu v} = x^{\mu}p^{v} - x^{v}p^{\mu} - i\sum_{n=1}^{\infty}\frac{1}{n}\left(a_{-n}^{\mu}a_{n}^{v} - a_{-n}^{v}a_{n}^{\mu}\right) \tag{1.35}$$

The annulation condition of l_0 which gave the condition on the mass m of the string written as:

$$\big(lo - a(0)\big)\langle 0\rangle \tag{1.36}$$

And the physical states $|\Psi\rangle_{ph}$ satisfy the following constraints:

$$l_n\,|\Psi\rangle_{ph} = 0\,,\ n \geq 0 \tag{1.37}$$

$$\big(lo - a(0)\big)|\Psi\rangle_{ph} = 0 \tag{1.38}$$

Because the metric $\eta^{\mu v}$is not a positive, we cannot say that there are no ghosts among these physical states. The no-ghost-theorem allows building a Set of so-called transverse operators verifying some fundamental properties which ensure that the transverse operators transform a physical state into other physical states and in particular, when applied to vacum$|0\rangle$ they describe a subspace F of physical states with a positive norm. The theory is then free of ghosts only if $(a(0) \leq 1\,,\, D \leq 25)$ or $(a(0) \leq 1\,,\, D \leq 26) D$ is the space-time dimension. And this is precisely the necessary condition for a consistent string theory.

Virasoro algebra :

Virasoroa algebra provides an extremely powerful frame work for uniting the concepts of Symmetry and locality, where the solution of the equations of motion of an open bosonic string leads the appearance of some constraints which are expressed in terms of an infinite set of Initial condition.

Virasoro operators are defined by:

$$l_n = \frac{1}{2}\sum_{p\in z}\alpha_{n-p}^{\mu}\alpha_{p,\mu} \tag{1.39}$$

And the commutator between two operators is given by [1]

$$[l_n\,, l_m] = (n-m)l_{n+m}\,,\quad m+n \neq 0 \tag{1.40}$$

This commutator is called Virasoro's algebra but with a possible quantum correction in the case where m + n≠ 0. Such a correction has, in the most general case, the form of function of m we therefore obtain:

$$[ln, lm] = (n - m)l_{n+m} + A(m)\,\delta_{n+m} \quad (1.41)$$

The algebra obtained is called the central extension of the Virasoro algebra and $-A\,(m)$ the anomaly of this algebra. We have trivially $A\,(m) = A\left(-\,m\right)$ and $A\,(0) = 0$, and it is therefore we determine $A\,(m)$ for $m > 0$ Using the Jacobi identity

$$\left[l_{k,}[ln, lm]\right] = 0 \quad (1.42)$$

We obtain:

$$[(m-n)A(k) + (n-k)A(m) + (k-m)A(n)]\delta_{n+m+k} = 0$$

Which is equivalent to:

$$(n-m)A(n+m) + (2n+m)A(m) - (2m+n)A(n) = 0$$

For $m = 1$

$$(n-1)A(n+1) + (2+n)A(1) - (2+n)A(n) = 0$$

and so

$$A(n+1) = \frac{(2+n)A(1) - (2+n)A(n)}{(n-1)} \quad (1.43)$$

The general form of the solution of this recurrence relation is

$$A(n) = c_3 n^3 + c_1 n \quad (1.44)$$

The determination of the constants is done by calculating the following mean values $\langle 0|[l_1, l_1]|0\rangle$ and $\left\langle 0\left|\left[l_{2,}\, l_{-2}\right]\right|0\right\rangle$ [1]

Virasoro's algebra then takes the following form:

$$[l_n\,, l_m] = (n-m)l_{n+m} + \frac{D}{12}n(n^2-1)\delta_{n+m,0} \quad (1.45)$$

1.2.2. The transverse gauge

Here, on the other hand, we first solve the constraint equations, by making a specific choice for the parameterizationτ and σ what leads us to the dynamic variables effectively independent $X^i\big(\tau\,;\sigma\,\big)\,,P^j\big(\,\tau;\sigma'\big)\,,x_0^i\;,\;p^i\;,x_0^-,p^+$This is the transverse gauge, and in this gauge, instead of considering directions $x^0, x^1, \dots, x^{D-1}$, we use the directions $x^+\,,x^-\;and\;x^i$of the cone of light where $i = 2, \dots, D-2$ and

$$x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1)$$

$$x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$$

Commutation relations

We postulate the following commutation relations:

$$\big[X^i(\tau,\sigma),\; P^i(\tau,\sigma')\big] = i\eta^{ij}\delta(\sigma - \sigma') \tag{1.46}$$

$$\big[X^i(\tau,\sigma),\; X^i(\tau,\sigma')\big] = \big[P^i(T\,,\,\sigma),\; P^j(T - \sigma')\big] = 0 \tag{1.47}$$

And

$$[x_0^-,\, p^+] = -i \tag{1.48}$$

Other commentators are zero

$$\big[x_0^i\,,\, p^+\big] = 0 \tag{1.49}$$

$$\big[x_0^-\;, X^i(\tau,\sigma)\big] = \;\big[x_0^-\;, P^i(\tau\,,\sigma)\big] = 0 \tag{1.50}$$

$$\big[p^+\;, X^i(\tau,\sigma)\big] = \;\big[p^+\;, P^i(\tau\,,\sigma)\big] = 0 \tag{1.51}$$

The commentator between the modes is given by

$$\big[\alpha\,{}_n^i\,,\;\alpha\,{}_m^j\big] = n\eta^{ij}\delta_{m+n,0} \tag{1.52}$$

Note that the mode α_0^i Commutes with all oscillators, so it is proportional to the moment of the string according to the formula

$$\alpha\,{}_0^i = \sqrt{2\alpha'}p^i$$

Mass operator

The mass operator is defined by :

$$M^2 = 2p^+p^- - p^ip^i \tag{1.54}$$

Where p^- is the canonical conjugate in the direction x^+ and p^+ is the canonical conjugate in the directionx^- . These are connected by the generator l_0 through the relation:

$$\frac{1}{a'}l_0 = 2p^+p^-$$

The operator l_0 can be decomposed in the following form:

$$l_0 = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha^i_{-p}\alpha^i_p$$

$$= \frac{1}{2}\alpha^i_0\alpha^i_0 + \frac{1}{2}\sum_{p=1}^{\infty} \alpha^i_{-p}\alpha^i_p + \frac{1}{2}\sum_{p=1}^{\infty} \alpha^i_p\alpha^i_{-p}$$

And to order the last term we write:

$$\frac{1}{2}\sum_{p=1}^{\infty} \alpha^i_p\alpha^i_{-p} \;\; = \frac{1}{2}\sum_{\mathrm{p}=1}^{\infty}\left(\alpha^i_{-p}\alpha^i_p + \left[\alpha^i_p\,,\,\alpha^i_{-p}\right]\right)$$

$$= \frac{1}{2}\sum_{p=1}^{\infty} \alpha^i_{-p}\,\alpha^i_p + \frac{1}{2}\sum_{p=1}^{\infty}\left(p\eta^{ii}\delta_{p-p,0}\right)$$

$$= \frac{1}{2}\sum_{p=1}^{\infty} \alpha^i_{-p}\,\alpha^i_p + \frac{(D-2)}{2}\sum_{p=1}^{\infty} p$$

The expression of the quantum l_0 becomes:

$$l_0 = \frac{1}{2}\alpha^i_0\alpha^i_0 + \sum_{p=1}^{\infty} \alpha^i_{-p}\,\alpha^i_p + \frac{(D-2)}{2}\sum_{p=1}^{\infty} p \tag{1.55}$$

This expression contains a divergent term, this term plays the role of the additive constant mentioned before and which will be denoted a:

$$a = \frac{(D-2)}{2} \sum_{p=1}^{\infty} p$$

By introducing the zeta $\xi(s)$ function:

$$\xi(s) = \sum_{p=1}^{\infty} \frac{1}{p^s}$$

If s = -1

$$\xi(-1) = 1 + 2 + \cdots$$

$$= -\frac{1}{12}$$

The equation (1.55) becomes:

$$l_0 = \frac{1}{2} \alpha_0^i \alpha_0^i + \sum_{p=1}^{\infty} \alpha_{-p}^i \alpha_p^i - \frac{(D-2)}{24} \tag{1.57}$$

So substituting (1.57) into (1.54) leads to the mass operator formula which we write in terms of operators of creations and annihilations:

$$M^2 = \frac{1}{a'} \left[-\frac{(D-2)}{24} + \sum_{p=1}^{\infty} p \alpha_p^{+i} \alpha_p^i \right] \tag{1.58}$$

Spectrum

Consider Equations (1.58) and determine the mass spectrum of bosonic string states.

Ground state

$$M^2 |p^+ , p_{\vec{T}}\rangle = \frac{-1}{a'} \frac{(D-2)}{24} |p^+ , p_{\vec{T}}\rangle$$

When $|p^+ , p_{\vec{T}}\rangle$ is vacuum state with impulse $p_{\vec{T}}$

$$m^2 = \frac{-1}{a'}\frac{(D-2)}{24}$$

Here, $|p^+ , \vec{p_T}\rangle$ Represent a tachyonic state.

1[St] level excited

After using the relation (1.52) which allows us to write the annihilation operators right of the creation operators we find:

$$M^2\left(a_1^{+j}|p^+ , \vec{p_T}\rangle\right) = \frac{1}{a'}\left[-\frac{(D-2)}{24} + 1\right]\left(a_1^{+j}|p^+ , \vec{p_T}\rangle\right)$$

For the bosonic string D = 26

$$m^2 = 0$$

$a_1^{+j}|p^+ , \vec{p_T}\rangle$ Represents a vector state with spin equal to 1 and mass equal to 0 (photon state)

2[nd]Excited level

The two excited levels are represented by two types of states:

$\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$,$\left(a_2^{+k}|p^+ , \vec{p_T}\rangle\right)$

$$M^2\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right) = \frac{1}{a'}\left[-\frac{(D-2)}{24} + 2\right]\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$$

$$= \frac{1}{a'}\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$$

$$m^2 = \frac{1}{a'}$$

$\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$represents a state tensoriel with mass

$$M^2\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right) = \frac{1}{a'}\left[-\frac{(D-2)}{24} + 2\right]\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$$

$$= \frac{1}{a'}\left(a_1^{+j}a_1^{+k}|p^+ , \vec{p_T}\rangle\right)$$

$\left(a_2^{+k}|p^+ , \vec{p_T}\rangle\right)$Represents a vector state with mass

3rd excited level

The states are defined as follows:

$\left(a_1^{+j} a_1^{+k} a_1^{+l} |p^+ , p_{\vec{T}}\rangle\right), \left(a_2^{+j} a_1^{+k} |p^+ , p_{\vec{T}}\rangle\right), \left(a_3^{+j} |p^+ , p_{\vec{T}}\rangle\right)$

$$M^2\left(a_1^{+j} a_1^{+k} a_1^{+l} |p^+ , p_{\vec{T}}\rangle\right) = \frac{1}{a'}\left(a_1^{+j} a_1^{+k} a_1^{+l} |p^+ , p_{\vec{T}}\rangle\right)$$

$$M^2\left(a_2^{+j} a_1^{+k} |p^+ , p_{\vec{T}}\rangle\right) = \frac{1}{a'}\left(a_2^{+j} a_1^{+k} |p^+ , p_{\vec{T}}\rangle\right)$$

$$M^2\left(a_3^{+j} |p^+ , p_{\vec{T}}\rangle\right) = \frac{1}{a'}\left(a_3^{+j} |p^+ , p_{\vec{T}}\rangle\right)$$

The three states represent states with mass.

Degeneracy

The degeneracy of each type of state regrouped in table below:

Level	Type d'état	Degeneracy
0	$|p^+ , p_{\vec{T}}\rangle$	1
1	$a_1^{+j} |p^+ , p_{\vec{T}}\rangle$	$(D-2)$
2	$a_2^{+j} |p^+ , p_{\vec{T}}\rangle$ $a_1^{+j} a_1^{+k} |p^+ , p_{\vec{T}}\rangle$	$(D-2)$ $(D-2) + \frac{(D-2)(D-3)}{2}$
3	$a_3^{+j} |p^+ , p_{\vec{T}}\rangle$ $a_2^{+j} a_1^{+k} |p^+ , p_{\vec{T}}\rangle$ $a_1^{+j} a_1^{+k} a_1^{+l} |p^+ , p_{\vec{T}}\rangle$	$(D-2)$ $(D-2)^2 (D-2)$ $+ (D-2)(D-3)$ $+ \frac{(D-2)(D-3)(D-4)}{6}$

Table 1.1: Degeneracy of four first levels in open bosonic string in D-branes

Partition function

The study of the spectrum is based on the construction of the generative function of the system. The degeneracy of states for each level of mass increases exponentially, this degeneracy is obtained from a generating function called the partition function, which is given by:

$$f(x) = \sum_{N=0}^{\infty} d(n)\, x^n = Trx^N$$

Where d(n) is the degeneracy at excited level n.

For bosonic strings, the function is defined by:

$$f(x) = \sum_{N=0}^{\infty} d_b(N) x^N$$

With

$d\ (N)$: Number of states for each level

$$N = a'M^2 = \sum_{n=1}^{\infty} n a_{-n}^{i} a_{n}^{i}, \qquad i = 2, (D-2)$$

This leads us to write:

$$f(x) = Trx \sum_{n-1}^{\infty} n a_{-n}^{i} a_{n}^{i}$$

We have:

$$TrA = \sum_{n} \langle n|A|n\rangle$$

It is easily shown that:

$$f(x) = \sum_{n'=0}^{\infty} \left\langle n' \left| x^{\sum_{n=1}^{\infty} \sum_{i=1}^{d-2} n a_{-n}^{i} a_{n}^{i}} \right| n' \right\rangle$$

$$= \prod_{n=1}^{\infty} \prod_{i=1}^{D-2} \sum_{n'=0}^{\infty} x^{nn'}$$

$$= \prod_{n=1}^{\infty} \left(\frac{1}{1 - x^n} \right)^{D-2}$$

The expansion limited to the close proximity of x = 0 gives:

$$f(x) = 1 + (D-2)x + \frac{1}{2}(D-2)(D+1)x^2 + (D-2)\left[2 + (D-2) + (D-3) + \frac{(D-3)(D-4)}{6}\right]x^3 + \cdots$$

The coefficients of this polynomial effectively represent the degeneracy of the states for each mass level that were represented in (Table 1.1)

Chapter 2

Quantizing of open bosonic string on D-branes

The annulations of the variations of the boundary term and the two-dimensional integral term is required by the equations of motion for the universe's surface in order to produce both the equations of motion and the boundary conditions, which specify the points in space-time where the ends of the open strings are connected. These spots collectively make up what is known as a Dirichlet membrane, or D-brane. A D-brane of dimension p, also known as a Dp-brane, propagates in the temporal direction and creates a volume of the universe of dimension $p + 1$.

By extension, we assume that we are dealing with a D25-brane that completely fills space-time when we have Neumann boundary conditions on all of the coordinates.

2.1. Description of Model

The bosonic coordinates verify boundary conditions of the Neumann type for the indices $i = 2, \dots, p$ and Dirichlet type for the d-p transverse coordinates $X^{a=p+1,\dots,d}$. The coordinates of the string $X^{i=2,\dots,p}$. called tangential, satisfy the Neumann conditions at the two free ends of the string (type NN)

$$X'^{i}(\tau,\sigma)|_{\sigma=0} = X'^{i}(\tau,\sigma)|_{\sigma=\pi} = 0 \ , i = 2, \dots, p \quad , \tag{2.1}$$

The coordinates X^a satisfying Dirichlet conditions at the two extremities of the string (known as of type DD) break down as follows

$$X^a(\tau,\sigma) = \bar{x}^a + \sqrt{2a'} \sum_{n \neq 0} \frac{1}{n} \propto_n^a \exp(-in\tau) \sin n\sigma, \tag{2.2}$$

These so-called normal coordinates satisfy the Dirichlet boundary conditions

$$X^a(\tau,\sigma)|_{\sigma=0} = X^a(\tau,\sigma)|_{\sigma=\pi} = \bar{x}^a \ , a = p+1, \dots, d \tag{2.3}$$

Notice that in equation (2.2) the moment has disappeared and therefore the zero modes $\propto_0^a = \sqrt{2 \propto '} p^a$ and the commutation relations are given by

$$\left[X^a(\tau,\sigma), X^b\left(\tau,\sigma^{'}\right)\right] = 2\pi \propto' i\delta^{ab}\delta(\sigma - \sigma'), \tag{2.4}$$

Which will be equivalent to

$$[\propto_m^a, \propto_n^b] = m\delta^{ab}\delta_{m+n,0}, m,n \neq 0 \ , m,n \neq 0 \ , \tag{2.5}$$

2.1. Mass operator:

In the calculation of l_0 we have two directions tangential and normal. Therefore, we have two types of vibration modes.

$$l_0 = \frac{1}{2}\propto_0^i\propto_0^i + \frac{1}{2}\sum_{p=1}^{\infty}\propto_{-p}^i\propto_p^i + \frac{1}{2}\sum_{p=1}^{\infty}\propto_{-p}^a\propto_p^a + \frac{1}{2}\sum_{p=1}^{\infty}\propto_p^i\propto_{-p}^i + \frac{1}{2}\sum_{p=1}^{\infty}\propto_p^a\propto_{-p}^a\ , \qquad (2.6)$$

Let's introduce the expression's usual order. For NN-type modes, we obtain

$$\frac{1}{2}\sum_{p=1}^{\infty}\propto_p^i\propto_{-p}^i = \frac{1}{2}\sum_{p=1}^{\infty}(\propto_{-p}^i\propto_p^i + [\propto_p^i\ ,\propto_{-p}^i])$$

$$= \sum_{p=1}^{\infty}(\propto_{-p}^i\propto_p^i + \frac{1}{2}\sum_{p=1}^{\infty} p\eta^{ii}\ \delta_{p-p,0}$$

$$= \frac{1}{2}\sum_{p=1}^{\infty}\propto_{-p}^i\propto_p^i - \frac{(p-1)}{24}, \qquad (2.7)$$

And for DD type modes

$$\frac{1}{2}\sum_{p=1}^{\infty}\propto_p^a\propto_{-p}^a = \frac{1}{2}\sum_{p=1}^{\infty}\left(\propto_{-p}^a\propto_p^a + [\propto_p^a,\propto_{-p}^a]\right)$$

$$= \frac{1}{2}\sum_{p=1}^{\infty}\propto_{-p}^a\propto_p^a + \frac{1}{2}\sum_{p=1}^{\infty} p\eta^{aa}\ \delta_{p-p,0}$$

$$= \frac{1}{2}\sum_{p=1}^{\infty}\propto_{-p}^a\propto_p^a - \frac{(d-1)}{24}\ , \qquad (2.8)$$

$$l_0 = \frac{1}{2} \propto_0^i \propto_0^i + \sum_{p=1}^{\infty} \propto_{-p}^i \propto_p^i + \sum_{p=1}^{\infty} \propto_{-p}^a \propto_p^a - \frac{(d-1)}{24}, \tag{2.9}$$

Here D indicates the dimension of space where $D = d + 1$ and $D = 26$

So

$$l_0 = \frac{1}{2} \propto_0^i \propto_0^i + \sum_{p=1}^{\infty} \propto_{-p}^i \propto_p^i + \sum_{p=1}^{\infty} \propto_{-p}^a \propto_p^a - 1 \tag{2.10}$$

The mass operator is given by

$$M_1^2 = 2p^+p^- - p^ip^i$$

$$= \frac{1}{\propto'}(l_0 + a) - p^ip^i$$

$$= \frac{1}{\propto'}\left(-1 + \sum_{n=1}^{\infty}\sum_{i=2}^{p} n\, a_n^{+i} a_n^i + \sum_{m=1}^{\infty}\sum_{a=p+1}^{d} m a_m^{+a}\, a_m^a\right) \tag{2.11}$$

$$= \frac{1}{\propto'}(-1 + N_i + N_a)$$

2.2. Spectrum

Ground state

$$M_1^2|p^+,\overrightarrow{p_T}\rangle = \frac{1}{\propto'}\left(-1 + \sum_{n=1}^{\infty}\sum_{i=2}^{p} n\, a_n^{+i} a_n^i + \sum_{m=1}^{\infty}\sum_{a=p+1}^{d} m a_m^{+a}\, a_m^a\right)|p^+,\overrightarrow{p_T}\rangle$$

$$= -\frac{1}{\propto'}|p^+,\overrightarrow{p_T}\rangle \rightarrow tachyonic\ state\ ,$$

1st excited level

This level contains two types of states

$$a_1^{+j}|p^+,\overrightarrow{p_T}\rangle \quad ,j = 2, \dots, p$$

$$a_1^{+b}|p^+,\overrightarrow{p_T}\rangle \quad , \quad b = p+1,\dots,d$$

We have

$$[a^a, a^{j+}] = 0, \qquad [a^i, a^{b+}] = 0 \tag{2.13}$$

So

$$M_1^2\left(a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right) = 0 \tag{2.13}$$

And

$$M_1^2\left(a_1^{b+}|p^+,\overrightarrow{p_T}\rangle\right) = 0, \tag{2.14}$$

Both states are zero mass, except that, $a_1^{+i}|p^+,\overrightarrow{p_T}\rangle$ is a photonic state that will be associated with the Maxwell field on the Brane while, $a_1^{+a}|p^+,\overrightarrow{p_T}\rangle$is a scalar in normal direction.

2nd excited level

States of this level are

$(a_2^{+j}||p^+,\overrightarrow{p_T}\rangle), (a_2^{+b}|p^+,\overrightarrow{p_T}\rangle), \left(a_1^{+b}a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right),$

$\left(a_1^{+b}a_1^{+c}|p^+,\overrightarrow{p_T}\rangle\right), \left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)$

1) $M_1^2\left(a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right) =$
$\frac{1}{\propto'}\left(-1\sum_{n=0}^{\infty}\sum_{i=2}^{p} n a_n^{+i}\, a_n^i + \sum_{m=0}^{\infty}\sum_{a=p-1}^{d} m a_m^{+a}\, a_m^a\right)(a_2^{+j}|p^+,\overrightarrow{p_T}\rangle)$

$$= \frac{1}{\propto'}\left[\left(a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right) + \sum_{i=2}^{p} a_1^{+i}\, a_1^i a_2^{+j}|p^+,\overrightarrow{p_T}\rangle + \sum_{i=2}^{p} 2a_2^{+i}\, a_2^i a_2^{+j}|p^+,\overrightarrow{p_T}\rangle \right.$$
$$\left. + \sum_{p-1}^{d} a_1^{+a} a_1^a a_2^{+j}|p^+,\overrightarrow{p_T}\rangle + \sum_{p-1}^{d} a_2^{+a} a_2^a a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right]$$

$$= \frac{1}{\propto'}\left[\left(-a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right) + \sum_{i=2}^{p} 2a_1^{+i}\left(\eta^{ij} + a_2^{ij} a_2^i\right)|p^+,\overrightarrow{p_T}\rangle\right]$$

$$=\frac{1}{\propto'}\left[\left(-a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right)+\sum_{i=2}^{p}2a_2^{+i}\,\eta^{ij}|p^+,\overrightarrow{p_T}\rangle+\sum_{i=2}^{p}2a_2^{+i}a_2^{+j}\,a_2^{+i}|p^+,\overrightarrow{p_T}\rangle\right]$$

$$=\frac{1}{\propto'}\left[\left(-a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right)+2a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\right]$$

$$=\frac{1}{\propto'}a_2^{+j}|p^+,\overrightarrow{p_T}\rangle\,, \tag{2.15}$$

2) $M_1^2\left(a_2^{+b}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(a_2^{+b}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.16)

3) $M_1^2\left(a_1^{+b}a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(a_1^{+b}a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.17)

4) $M_1^2\left(a_1^{+b}a_1^{+c}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(a_1^{+b}a_1^{+c}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.18)

5) $M^2\left(a_1^{+j},a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(a_1^{+j},a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.19)

3rd excited level

1) $M^2\left(a_3^{+j}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(2a_3^{+j}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.21)

2) $M^2(a_3^{+b}|p^+,\overrightarrow{p_T}\rangle)=\frac{1}{\alpha'}(\,2a_3^{+b}|p^+,\overrightarrow{p_T}\rangle)$ (2.22)

3) $M^2\left(a_2^{+b}a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(2a_2^{+b}a_1^{+j}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.23)

4) $M^2\left(a_2^{+j}\,a_1^{+c}\right)|p^+,\overrightarrow{p_T}\rangle=\frac{1}{\propto'}\left(2a_2^{+j}\,a_1^{+c}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.24)

5) $M^2\left(a_2^{+j}\,a_1^{+k}\right)|p^+,\overrightarrow{p_T}\rangle=\frac{1}{\propto'}\left(2a_2^{+j}\,a_1^{+k}\right)|p^+,\overrightarrow{p_T}\rangle\,,$ (2.25)

6) $M^2\left(a_2^{+b}\,a_1^{+c}\right)|p^+,\overrightarrow{p_T}\rangle=\frac{1}{\propto'}\left(2a_2^{+b}\,a_1^{+c}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.26)

7) $M^2\left(a_1^{+i}\,a_1^{+j}\,a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(2a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right),$ (2.27)

8) $M^2\left(a_1^{+b}\,a_1^{+c}\,a_1^{+d}\right)|p^+,\overrightarrow{p_T}\rangle=\frac{1}{\propto'}\left(2a_1^{+b}\,a_1^{+c}\,a_1^{+d}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.28)

9) $M^2\left(a_1^{+i}\,a_1^{+j}\,a_1^{+b}|p^+,\overrightarrow{p_T}\rangle\right)=\frac{1}{\propto'}\left(2a_1^{+i}\,a_1^{+j}\,a_1^{+b}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.29)

10) $M^2\left(a_1^{+b}\,a_1^{+c}\,a_1^{+i}\right)|p^+,\overrightarrow{p_T}\rangle=\frac{1}{\propto'}\left(2a_1^{+b}\,a_1^{+c}\,a_1^{+i}|p^+,\overrightarrow{p_T}\rangle\right)$ (2.30)

In ordinary case, there are only the vector fields, but here there are scalar fields and vector fields.

2.3. degeneracy

Let us now calculate the degeneration of each level.

Ground state

The fundamental state has no degeneration, so the number of states is equal to 1.

1st excited level

$$a_1^{+\alpha}|p^+,\overrightarrow{p_T}\rangle \rightarrow \propto_S = (d-p),$$
$$a_1^{+i}|p^+,\overrightarrow{p_T}\rangle \rightarrow \propto_V = (p-1),$$

2nd excited level

$$\{a_1^{+a}a_1^{+b}\}|p^+,\overrightarrow{p_T}\rangle \rightarrow \beta_S = (d-p) + \frac{(d-p)(d-p-1)}{2} + (d-p)$$
$$\boldsymbol{\{a_1^{+i}a_1^{+a}\}|p^+,\overrightarrow{p_T}\rangle \rightarrow \beta_V = (p-1) + (d-p)(p-1)},$$

3rd excited level

$$\begin{Bmatrix} a_3^{+a} \\ a_2^{+a}\, a_1^{+b} \\ a_1^{+a}a_1^{+b}a_1^{+c} \end{Bmatrix}|p^+,\overrightarrow{p_T}\rangle \rightarrow \gamma_S = \begin{Bmatrix} (d-p) + (d-p)^2 + (d-p) + (d-p)(d-p-1) \\ + \dfrac{(d-p)(d-p-1)(d-p-2)}{6} \end{Bmatrix},$$

$$\begin{Bmatrix} a_3^{+i} \\ a_2^{+a}a_1^{+i} \\ a_1^{+a}a_2^{+i} \\ a_1^{+i}a_1^{+a}a_1^{+b} \end{Bmatrix}|p^+,\overrightarrow{p_T}\rangle \rightarrow \gamma_V = (p-1) + 3(d-p)(p-1) + (p-1)\frac{(d-p)(d-p-1)}{2},$$

$$\begin{Bmatrix} a_2^{+i}a_1^{+j} \\ a_1^{+i}a_1^{+j}a_1^{+a} \end{Bmatrix}|p^+,\overrightarrow{p_T}\rangle \rightarrow \gamma_{T_2} = (p-1)^2 + (d-p)(p-1) + (d-p)\frac{(p-1)(p-2)}{2},$$

$$a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle \rightarrow \gamma_{T_3} = (p-1) + (p-1)(p-2) + \frac{(p-1)(p-2)(p-3)}{6}$$

For the first three excited levels total degeneration is grouped in the following table

0	1	2	3
1	$\propto_{tot}= d-1$	$\beta_{tot} = \frac{1}{2}d + \frac{1}{2}d^2 - 1$	$\gamma_{tot} = -\frac{7}{6}d + \frac{1}{6}d^3 + d^2$

Table 2.2: Degeneracy of first three excited levels in open bosonic string on a D-branes

With

$$\propto_{tot}=\propto_S+\propto_V$$

$$\beta_{tot} = \beta_S + \beta_V + \beta_{T_2}$$

$$\gamma_{tot} = \gamma_S + \gamma_V + \gamma_{T_2} + \gamma_{T_3}$$

And

S : Scalar , V : Vector , T_2*,*T_3 *: Tensors*

2.4. Partition function

$$f(x) = \sum_{n'_i,n'_a} x^{\langle n'_i,n'_a|N_i+N_a|n'_i,n'_a\rangle}$$

$$= \sum_{n'_i=0}^{\infty}\sum_{,n'_a=0}^{\infty}\left[x^{\langle n'_i,n'_a|\sum_{n=1}^{\infty}\sum_{i=2}^{p} na_n^{+i}a_n^i|n'_i,n'_a\rangle} x^{\langle n'_i,n'_a\left|\sum_{n=1}^{\infty}\sum_{a=p+1}^{p} na_n^{+a}a_n^a\right|n'_i,n'_a\rangle}\right]$$

$$= \left[\prod_{n=1}^{\infty}\prod_{i=2}^{p}\sum_{n'_i}^{\infty} x^{nn'_i}\right]\left[\prod_{n=1}^{\infty}\prod_{a=p+1}^{d}\sum_{n'_a=0}^{\infty} x^{nn'_a}\right]$$

$$= \prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{p-1}\prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{d-p} \tag{2.31}$$

The development of $f(x)$ extended to $x = 0$ given

$$f(x) = 1 + (d-1)x + \left(\frac{1}{2}d + \frac{1}{2}d^2 - 1\right)x^2 + \left(-\frac{7}{6}d + \frac{1}{6}d^3 + d^2\right)x^3 + \cdots, \tag{2.32}$$

The coefficients of this function are identical to those in (Table 2.2), proving the coherence of model.

Chapter 3

Open bosonic between two parallel Dp-branes

3.1. Description of Model

The decomposition of DD type coordinates is written as follows

$$X^a(\tau,\sigma) = \bar{x}_1^a + (\bar{x}_2^a - \bar{x}_1^a)\frac{\sigma}{\pi} + \sqrt{2\propto'}\sum_{n\neq 0}\frac{1}{n}\propto_n^a \exp(in\tau)\sin n\sigma, \qquad (3.1)$$

These coordinates satisfy the Dirichlet boundary conditions

$$X^a(\tau,\sigma)|_{\sigma=0} = \bar{x}_1^a\,,\ \ X^a(\tau,\sigma)|_{\sigma=\pi} = \bar{x}_2^a\,, \qquad a = p+1\ldots d \qquad (3.2)$$

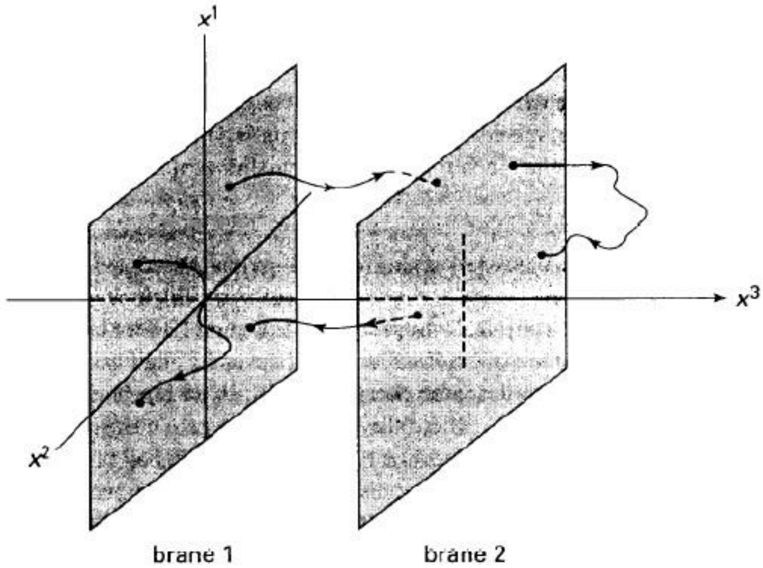

Figure 3.1: String between two parallel Dp-branes.

The constants$\bar{x}_1^a$ and $\bar{x}_2^a$ fix only D-branes and the separation $\bar{x}_2^a - \bar{x}_1^a$ represents the distance between the two ends $\sigma = 0$ and$\sigma = \pi$ [1].

3.2. Mass operator

The mass of a bosonic excited state is given by

$$M_2^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + M_1^2\,, \qquad (3.3)$$

Or $\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2$ is the gap of the mass operator in the positive direction.

3.3. Spectrum

The string state takes the form $|p^+, \overrightarrow{p_T}; [i,j]\rangle$ or $[ij]$ is the sector of the string on the two branes so that $i,j = 1,2$.

Now and for the continuation we consider only the sector [12]

Ground state

$$M_2^2|p^+,\overrightarrow{p_T};[12]\rangle = \left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2 + M_1^2\right]|p^+,\overrightarrow{p_T};[12]\rangle$$

$$= \left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2 - \frac{1}{\propto'}\right]|p^+,\overrightarrow{p_T};[12]\rangle \qquad (3.4)$$

If the separation between the two branes disappears, the usual tachyonic state is obtained. If the two branes are separated by the following particular distance:

$$\frac{|\bar{x}_2^a-\bar{x}_1^a|}{2\pi\propto'} = \frac{1}{\sqrt{\propto'}} \rightarrow |\bar{x}_2^a-\bar{x}_1^a| = 2\pi\sqrt{\propto'} \qquad (3.5)$$

Then, in this case, one obtains a state which represents a scalar field without mass (elimination of the tachyon). For a large separation, the ground state represents a scalar field with mass.

1st level excited

The state $a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle$

$$M_2^2\left(a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right) = \left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2\left(a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.6)$$

The state $\left(a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right)$represents a vector state with mass.

$$M_2^2\left(a_1^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right) = \left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2\left(a_1^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.7)$$

The state $\left(a_1^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right)$represents a scalar with mass.

2nd level excited

We characterize five types of states:

$$M^2\left(a_2^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right) = \left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}\right]\left(a_2^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right), \qquad (3.8)$$

$$M^2\left(a_2^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{1}{\propto'}\right]\left(a_2^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right), \qquad (3.9)$$

$$M^2\left(a_1^{+j}a_1^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{1}{\propto'}\right]\left(a_1^{+j}a_1^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.10)$$

$$M^2\left(a_1^{+b}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{1}{\propto'}\right]\left(a_1^{+b}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.11)$$

$$M^2\left(a_1^{+b}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{1}{\propto'}\right]\left(a_1^{+b}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.12)$$

3[rd.]level excited

$$M^2\left(a_3^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_3^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.13)$$

$$M^2\left(a_3^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_3^{+b}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.14)$$

$$M^2\left(a_2^{+b}a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_2^{+b}a_1^{+j}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.15)$$

$$M^2\left(a_2^{+j}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_2^{+j}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.16)$$

$$M^2\left(a_2^{+j}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_2^{+j}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.17)$$

$$M^2\left(a_2^{+b}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(a_2^{+b}a_1^{+c}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.18)$$

$$M^2\left(a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(2a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.19)$$

$$M^2\left(a_1^{+b}a_1^{+c}a_1^{+d}|p^+,\overrightarrow{p_T};[12]\rangle\right)=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(2a_1^{+b}a_1^{+c}a_1^{+d}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.20)$$

$$M^2\left(a_1^{+i}a_1^{+j}a_1^{+d}|p^+,\overrightarrow{p_T};[12]\rangle\right)$$
$$=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(2a_1^{+i}a_1^{+j}a_1^{+d}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.21)$$
$$M^2\left(a_1^{+b}a_1^{+c}a_1^{+i}|p^+,\overrightarrow{p_T};[12]\rangle\right)$$
$$=\left[\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\propto'}\right)^2+\frac{2}{\propto'}\right]\left(2a_1^{+b}a_1^{+c}a_1^{+i}|p^+,\overrightarrow{p_T};[12]\rangle\right) \qquad (3.22)$$

Noticed :

In this case, the operator N is same as when a string is considered on a Dp-brane (last chapter), this lead to the same partition function and degeneracy, except that here the ground state is defined by $|p^+,\overrightarrow{p_T};[12]\rangle$.

3.4. Virasoro algebra

Let us consider the Virasoro generators decomposed according to the types of coordinates NN, DD

$$L_n^{tot}=L_n^{NN}+L_n^{DD} \qquad (3.23)$$

Remember that Virasoro operators are defined by:

$$L_n^{NN}=\frac{1}{2}\sum_{p\in z}\alpha_{n-p}^{i}\alpha_{p,i} \qquad ,n\neq 0, \qquad (3.24)$$

$$L_n^{DD}=\frac{1}{2}\sum_{p\in z}\alpha_{n-p}^{a}\alpha_{p,a} \qquad ,n\neq 0, \qquad (3.25)$$

The calculation of the Virasoro algebra requires the determination of the following commentators, $[L_n^{NN},L_m^{NN}]$and $[L_n^{DD},L_m^{DD}]$(all mixed switches are zero)

$$[L_n^{DD},L_m^{NN}]=0$$

First, we will calculate the commutation between the Virasoro operators and the modes a_n [1]

$$[l_n\,,\ \alpha^{\,j}_{\,m}]=\frac{1}{2}\sum_{p\in\mathbb{Z}}[\alpha_{n-p}^{i}\alpha_{p,i}\alpha_m^{j}]$$

$$= \frac{1}{2}\sum_{p\in\mathbb{Z}}\left(\alpha^i_{n-p}\left[\alpha_{p,i}\alpha^j_m\right] + \left[\alpha^i_{n-p}\alpha^j_m\ \right]\alpha_{p,i}\right)$$

Using the relation (1.31) we find:

$$\left[ln, \alpha^j_m\ \right] = \frac{1}{2}\sum_{p\in z}\left(p\alpha^i_{n-p}\delta_{p+m,0}\eta^j_i + \alpha_{p,i}(n-p)\delta_{n-p+m,0}\eta^{ij}\right)$$

$$= -m\alpha^j_{n+m}$$

Now, we are going to calculate the commutation between the Virasoro operators.

$$[l_n\ ,\ l_m] = \frac{1}{2}\sum_{p\in\mathbb{Z}}\ \left[l_n, \alpha^i_{m-p}\alpha_{p,i}\right]$$

$$= \frac{1}{2}\sum_{p\in\mathbb{Z}}\left(\alpha^i_{m-p}\left[l_n, \alpha_{p,i}\right] + \left[l_n\ , \alpha^j_{m-p}\ \right]\alpha_{p,i}\right)$$

$$= \frac{1}{2}\sum_{p\in z}\left(-p\alpha^i_{m-p}\alpha_{n+p,i} - (m-p)\alpha^i_{n+m-p}\alpha_{p,i}\right) \qquad (3.26)$$

By replacing (p) by $(p-\ n)$ in the first term, we obtain:

$$[l_n\ , l_m] = (n-m)l_{n+m}\ ,\quad m+n \neq 0 \qquad (3.27)$$

This commentator is called Virasoro's algebra but with a possible quantum correction in the case where m + n≠ 0. Such a correction has, in the most general case, the form of function of m, we therefore obtain:

$$[ln, lm] = (n-m)l_{n+m}\ +\ A(m)\ \delta_{n+m} \qquad (3.28)$$

To determine the constants we calculate the following mean values $\langle 0|[l_1, l_1]|0\rangle$

And $\langle 0|[l_2\ ;\ l_{-2}\ ]|0\rangle$ (see annex B) wefind:

$$[l^{NN}_n, l^{NN}_m] = (n-m)l^{NN}_{n+m}\ + \frac{(p-1)}{12}n(n^2-1)\delta_{n+m,0} \qquad (3.29)$$

By the same way, we obtain:

$$[l_n^{DD}, l_m^{DD}] = (n-m)l_{n+m}^{DD} + \frac{(d-p)}{12}n(n^2-1)\delta_{n+m,0} \qquad (3.30)$$

This lead to the total algebra:

$$[l_n^{tot}, l_m^{tot}] = (n-m)l_{n+m}^{tot} + \frac{(d-1)}{12}n(n^2-1)\delta_{n+m,0} \qquad (3.31)$$

Compared to the usual Virasoro algebra, and by substitution$(d-1) \to (D-2)$

The central term is unchanged and the model rest coherent

Chapter 4

The open bosonic string between two parallel Dp-Dq-branes

4. The open bosonic string between two parallel Dp-Dq-branes

In this part, we examine open bosonic string between two parallel Dp- and Dq-branes where $p$and q are integers satisfying $1 \leq q < p \leq 25$(Figure 2) $[1]$.

In addition of the NN and DD coordinates, we have a third type of coordinates are introduced called Neumann-Dirichlet ND, which we will note

X^r or $r = \overline{q+1, p}$ knowing that the coordinates NN, the index $i = \overline{2, q}$ and for DD coordinates DD the index $a = \overline{p+1, d}$

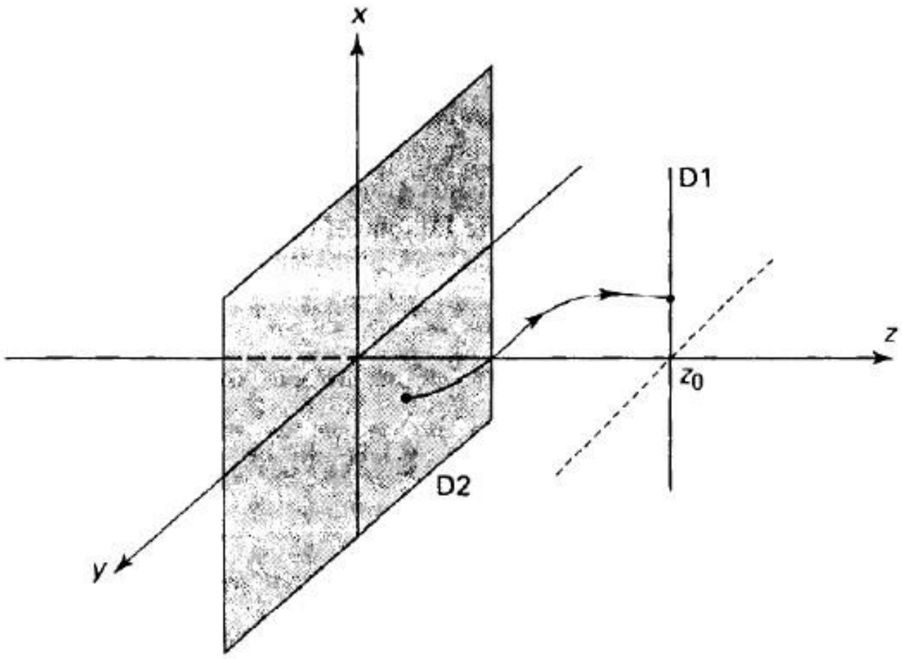

Figure 4.2: String between two parallel Dp-, Dq-branes

The figure above illustrates that the coordinates along the x direction are tangential coordinates of type NN, the coordinates along the y direction are mixed coordinates of type ND (N at $\sigma = 0$ because y is tangent to the D2-brane and D at $\sigma = \pi$ because y and normal to the D1-brane) and the coordinates following the direction z are coordinates normal to bothbranes therefore of type DD: Here, the position of the Dp-brane is specified by the coordinates $\overline{x_1^a}$ and the Dq-brane is specified by the coordinates $\overline{x_2^a}$and $\overline{x_2^r}$

In the case of mixed ND coordinates, the boundary conditions are

$$\frac{\partial X^r}{\partial|\sigma}(\tau,\sigma)\bigg|_{\sigma=0} = 0 \quad , \quad X^r(\tau,\sigma)|_{\sigma=\pi} = \overline{x_2^r} \qquad (4.1)$$

The particularity of the mixed boundary conditions leads to an appropriate mode expansion given by the relation.

$$X^r(\tau,\sigma) = \overline{x_2^r} + i\sqrt{2a'} \sum_{n\in z_{odd}} \frac{2}{n} \propto_{\frac{n}{2}}^r \exp(-i\,\frac{n}{2}\tau)\cos\left(\frac{n}{2}\sigma\right), \qquad (4.2)$$

Where the summation over one only covers the odd integers, which implies half-integer indices for the vibration modes .

However, the expression of

$$X^r(\tau,\sigma) \pm\ X^r(\tau,\sigma) = \sqrt{2a'} \sum_{n\in z_{odd}} \frac{2}{n} \propto_{\frac{n}{2}}^r \exp(-i\frac{n}{2}(\tau \pm \sigma), \qquad (4.3)$$

Remains identical to that of the components a or i so that from the commutator

$$[X^r(\tau,\sigma), X^s(\tau,\sigma')] = i(2\pi \propto')\delta(\sigma,\sigma')\delta^{rs} \quad , \qquad (4.4)$$

We demonstrate the relationship

$$\left[\propto_{\frac{n}{2}}^r, \propto_{\frac{n}{2}}^s\right] = \frac{m}{2}\delta^{rs}\delta_{m+n,0}\,, \qquad (4.5)$$

4.1. Mass operator

The mass operator receives contributions from all coordinate types:NN, NDandDD. The expression of the product $2p^+p^-$takes the form:

$$2p^+p^- = \frac{1}{\propto'}\Bigg(\propto' p^i p^i + \frac{1}{2}\propto_0^a \propto_0^a + \sum_{n=1}^{\infty} \propto_{-n}^i \propto_n^i + a_{NN} + \sum_{n=1}^{\infty} \propto_{-n}^i \propto_n^i + a_{DD}$$

$$+ \sum_{m\in z_{odd}^+} \propto_{-\frac{m}{2}}^r \propto_{\frac{m}{2}}^r + a_{ND}\Bigg), \qquad (4.6)$$

Where

$$a_{NN} = -\frac{(q-1)}{24} \qquad (4.7)$$

$$a_{DD} = -\frac{(d-p)}{24} \qquad (4.8)$$

It remains to calculate the contribution a_{ND}, from the following development

$$\frac{1}{2}\sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} = \sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} + \frac{1}{2}\sum_{m\in z_{odd}^{+}}^{\infty} \left[\propto^{r}_{-\frac{m}{2}}, \propto^{r}_{\frac{m}{2}}\right]$$

$$= \sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} + \frac{1}{4}(p-q)\sum_{m\in z_{odd}^{+}}^{\infty} m, \qquad (4.9)$$

The summation over the odd integers is calculated as follows

$$\sum_{n=1}^{\infty} n = \sum_{m\in z_{odd}^{+}} n + \sum_{m\in z_{even}^{+}} n$$

$$= \sum_{m\in z_{odd}^{+}} n + 2\sum_{n=1}^{\infty} n$$

$$\rightarrow \sum_{m\in z_{odd}^{+}} n = -\sum_{n=1}^{\infty} n = \frac{1}{12}$$

So

$$\frac{1}{2}\sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} = \sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} + \frac{1}{48}(p-q)$$

$$= \sum_{m\in z_{odd}^{+}}^{\infty} \propto^{r}_{-\frac{m}{2}} \propto^{r}_{\frac{m}{2}} + a_{ND} \qquad (4.10)$$

Finally we obtain the expression of the mass operator in the following form:

$$M^2 = \left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left(N - \frac{1}{24}(d-1) + \frac{1}{16}(p-q)\right), \qquad (4.11)$$

Or

$$N = \sum_{n=1}^{\infty}\sum_{i=2}^{q} n a_n^{i+} a_n^{i} + \sum_{k\in z_{odd}^{+}}\sum_{r=q+1}^{p} \frac{k}{2} a_{\frac{k}{2}}^{r+} a_{\frac{k}{2}}^{r} + \sum_{m=1}^{\infty}\sum_{a=p+1}^{d} m a_m^{a+} a_m^{a}, \qquad (4.12)$$

$$\begin{cases} N_i = \sum_{n=1}^{\infty}\sum_{i=2}^{q} n a_n^{i+} a_n^i, \\ N_r = \sum_{k\in z_{odd}^{+}}\sum_{r=q+1}^{p} \frac{k}{2} a_{\frac{k}{2}}^{r+} a_{\frac{k}{2}}^{r} \\ N_a = \sum_{m=1}^{\infty}\sum_{a=p+1}^{d} m a_m^{a+} a_m^a \end{cases} \quad (4.13)$$

4.2. Spectrum

We propose to determine the nature of the physical states (scalar, vector, and tensor). These states are represented as follows

Ground state

$$M^2|p^+,\overrightarrow{p_T}\rangle = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left[-\frac{1}{24}(d-1) + \frac{1}{16}(p-q)\right]\right\}|p^+,\overrightarrow{p_T}\rangle \quad (4.14)$$

This scalar is generally massive, but can be tachyonic or massless depending on the separation between the two branes and the value$(p-q)$.

Level 1

$$\boldsymbol{M^2\left(a_{\frac{1}{2}}^{+s}|p^+,\overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left[-\frac{1}{24}(d-1) + \frac{1}{16}(p-q) + \frac{1}{2}\right]\right\}\left(a_{\frac{1}{2}}^{+s}|p^+,\overrightarrow{p_T}\rangle\right)} \quad \mathbf{(4.15)}$$

Level 2

1) $M^2\left(a_1^{+b}|p^+,\overrightarrow{p_T}\rangle\right) =$

$$\left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left[-\frac{1}{24}(d-1) + \frac{1}{16}(p-q) + 1\right]\right\} M^2\left(a_1^{+b}|p^+,\overrightarrow{p_T}\rangle\right)(4.16)$$

2) $\boldsymbol{M^2\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t}|p^+,\overrightarrow{p_T}\rangle\right) =}$

$$\left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left[-\frac{1}{24}(d-1) + \frac{1}{16}(p-q) + 1\right]\right\}\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.17)$$

3) $M^2\left(a_1^{+j}|p^+, \overrightarrow{p_T}\rangle\right) =$

$$\left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi \propto'}\right)^2 + \frac{1}{\propto'}\left[-\frac{1}{24}(d-1) + \frac{1}{16}(p-q) + 1\right]\right\}\left(a_1^{+j}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.18)$$

Level 3

1) $$M^2\left(a_1^{+b} a_{\frac{1}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}A\right\}\left(a_1^{+b} a_{\frac{1}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.19)$$

2) $$M^2\left(a_1^{+j} a_{\frac{1}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}A\right\}\left(a_1^{+j} a_{\frac{1}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.20)$$

3) $$M^2\left(a_{\frac{3}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}A\right\}\left(a_{\frac{3}{2}}^{+s}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.21)$$

4) $$M^2\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t} a_{\frac{1}{2}}^{+u}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}A\right\}\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t} a_{\frac{1}{2}}^{+u}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.22)$$

And $A = -\frac{1}{24}(d-1) + \frac{1}{16}(p-q) + \frac{3}{2}$

Level 4

1) $$M^2\left(a_2^{+b}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_2^{+b}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.23)$$

2) $$M^2\left(a_1^{+b} a_1^{+c}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_1^{+b} a_1^{+c}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.24)$$

3) $$M^2\left(a_2^{+j}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_2^{+j}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.25)$$

4) $$M^2\left(a_1^{+j} a_1^{+k}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_1^{+j} a_1^{+k}|p^+, \overrightarrow{p_T}\rangle\right), \quad (4.26)$$

5) $$M^2\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t} a_{\frac{1}{2}}^{+u} a_{\frac{1}{2}}^{+v}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t} a_{\frac{1}{2}}^{+u} a_{\frac{1}{2}}^{+v}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.27)$$

6) $$M^2\left(a_{\frac{1}{2}}^{+s} a_{\frac{3}{2}}^{+t}|p^+, \overrightarrow{p_T}\rangle\right) = \left\{\left(\frac{\bar{x}_2^a - \bar{x}_1^a}{2\pi\propto'}\right)^2 + \frac{1}{\propto'}B\right\}\left(a_{\frac{1}{2}}^{+s} a_{\frac{3}{2}}^{+t}|p^+, \overrightarrow{p_T}\rangle\right) \quad (4.28)$$

7) $$M^2\left(a_1^{+j}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}|p^+,\overrightarrow{p_T}\rangle\right)=\left\{\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\alpha'}\right)^2+\frac{1}{\alpha'}B\right\}\left(a_1^{+j}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}|p^+,\overrightarrow{p_T}\rangle\right), \qquad (4.29)$$

8) $$M^2\left(a_1^{+b}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}|p^+,\overrightarrow{p_T}\rangle\right)=\left\{\left(\frac{\bar{x}_2^a-\bar{x}_1^a}{2\pi\alpha'}\right)^2+\frac{1}{\alpha'}B\right\}\left(a_1^{+b}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}|p^+,\overrightarrow{p_T}\rangle\right), \qquad (4.30)$$

And $B=-\frac{1}{24}(d-1)+\frac{1}{16}(p-q)+2$

4.3. Degeneracy

Level 1

$$\boldsymbol{a_{\frac{1}{2}}^{+s}|p^+,\overrightarrow{p_T}\rangle \to \alpha_S=(p-q)}$$

Level 2

$$\boldsymbol{\left\{a_1^{+b}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}\right\}|p^+,\overrightarrow{p_T}\rangle \to \beta_S=(p-q)\frac{(p-q)(p-q-1)}{2}},$$

$$\boldsymbol{a_1^{+j}|p^+,\overrightarrow{p_T}\rangle \to \beta_V=(q-1)}.$$

Level 3

$$\left\{a_1^{+b}a_{\frac{1}{2}}^{+s},a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}a_{\frac{1}{2}}^{+u},a_{\frac{1}{2}}^{+u}\right\}|p^+,\overrightarrow{p_T}\rangle \to \gamma_S=\left\{\begin{matrix}(p-q)+\dfrac{(p-q)(p-q-1)}{2}+\\ \dfrac{(p-q)(p-q-1)(p-q-2)}{6}+\\ (d-p)(p-q)+(p-q)\end{matrix}\right\}$$

$$a_1^{+j}a_{\frac{1}{2}}^{+s}|p^+,\overrightarrow{p_T}\rangle \to \gamma_V=(q-1)(p-q)$$

Level 4

$$\left\{\begin{matrix}a_2^{+b}\\ a_1^{+b}a_1^{+c}\\ a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}a_{\frac{1}{2}}^{+u}a_{\frac{1}{2}}^{+v}\\ a_{\frac{1}{2}}^{+s}a_{\frac{3}{2}}^{+t}\\ a_1^{+b}a_{\frac{1}{2}}^{+s}a_{\frac{1}{2}}^{+t}\end{matrix}\right\}|p^+,\overrightarrow{p_T}\rangle \to \delta_S=\left\{\begin{matrix}(d-p)+(d-p)\frac{(d-p)(d-p-1)}{2}\\ +(p-q)+\frac{(p-q)(p-q-1)}{2}+\frac{(p-q)(p-q-1)(p-q-2)}{6}\\ +\frac{(p-q)(p-q-1)(p-q-2)(p-q-3)}{24}+(p-q)^2\\ +(d-p)(p-q)+(d-p)\frac{(p-q)(p-q-1)}{2}\end{matrix}\right\}$$

$$\begin{Bmatrix} a_2^{+j} \\ a_1^{+j} a_{\frac{1}{2}}^{+s} a_{\frac{1}{2}}^{+t} \\ a_1^{+j} a_1^{+b} \end{Bmatrix} |p^+, \overrightarrow{p_T}\rangle \to \delta_V = \begin{Bmatrix} (q-1) + (q-1)(p-q) + (q-1)\dfrac{(p-q)(p-q-1)}{2} \\ (q-1)(d-p) \end{Bmatrix}$$

$$a_1^{+j} a_1^{+k} |p^+, \overrightarrow{p_T}\rangle \to \delta_{T_2} = (q-1) + \frac{(q-1)(q-2)}{2}$$

For large separations, the mass of each level is always positive, and the states are scalars or vectors according to the different types of indices. in addition, we haven't a massive gauge fields. The number of states for each level is grouping in (table 4.3)

0	1	2	3	4
1	$\propto_{tot} = (p-q)$	$\beta_{tot} = d + \frac{1}{2}p^2 - pq - \frac{1}{2}p + \frac{1}{2}q^2 + \frac{1}{2}q - 1$	$\gamma_{tot} = \frac{1}{3}p - \frac{1}{3}q - \frac{1}{2}p^2 + pq - \frac{1}{2}q^2 + \frac{1}{6}p^3 - \frac{1}{2}p^2 q + \frac{1}{2}pq^2 - \frac{1}{6}q^3 + dp - dq$	$\delta_{tot} = -1 - \frac{23}{12}p + \frac{1}{2}d + \frac{1}{2}dq^2 - \frac{1}{2}dq + \frac{83}{24}p^2 - dpq - \frac{7}{12}p^3 \frac{1}{12}q^3 + \frac{5}{4}p^2 q + \frac{1}{24}q^4 + \frac{1}{24}p^4 - \frac{1}{6}pq^3 - \frac{1}{6}p^3 q + \frac{1}{6}p^2 q^2 - \frac{3}{4}pq^2 + \frac{23}{12}q - \frac{77}{12}pq + \frac{71}{12}q^2 + \frac{1}{2}d^{2+}\frac{1}{2}dp + \frac{1}{2}dp^2$

Table 4.3:Degeneracy of first four excited levels in open bosonic string between parallel Dp- Dq-branes

4.4. Partition function

It is now a question of confronting the results of the table above with the function of partition to check the coherence of this model.

$$f(x) = Trx^N = \sum_{n'_i, n'_r, n'_a} x^{\langle n'_i, n'_r, n'_a | N_i + N_r + N_a | n'_i, n'_r, n'_a \rangle} \qquad (4.31)$$

$$= \left[\prod_{n=1}^{\infty}\prod_{i=2}^{q}\sum_{n'_i=0}^{\infty} x^{nn'_i}\right]\left[\prod_{n\in z^+_{odd}}\prod_{r=q+1}^{p}\sum_{n'_r=0}^{\infty} x^{\frac{n}{2}n'_r}\right]\left[\prod_{n=1}^{\infty}\prod_{a=p+1}^{q}\sum_{n'_a=0}^{\infty} x^{nn'_a}\right]$$

$$= \prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{q-1}\prod_{n\in z^+_{odd}}\left(\frac{1}{1-x^{\frac{n}{2}}}\right)^{q-p}\prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{q-p}$$

$$= \prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{q-1}\prod_{n=1}^{\infty}\left(\frac{1}{1-x^{\frac{n}{2}}}\right)^{q-p}\prod_{n=1}^{\infty}\left(\frac{1}{1-x^n}\right)^{q-p} \qquad (4.32)$$

Where the product over odd integers is reduced to a product over integers after a few adjustments.

Let's do a development of $f(x)$, by asking $X = \sqrt{x}$

$$f(X) = 1 + (p-q)X + \left(d + \frac{1}{2}p^2 + \frac{1}{2}q^2 - \frac{1}{2}p + \frac{1}{2}q - pq - 1\right)X^2 + \left(\frac{1}{6}p^3 - \frac{1}{6}q^3 - \frac{1}{2}q^2 + -\frac{1}{2}p^2q + -\frac{1}{2}pq^2 - \frac{1}{2}p^2 + \frac{1}{3}p - \frac{1}{3}q + pq - +dp - dq\right)X^3 + \left(-1 + \frac{1}{24}q^4 + \frac{1}{24}p^4 - \frac{7}{12}p^3 + \frac{1}{12}q^3 - \frac{1}{6}pq^3 - \frac{1}{6}p^3q - \frac{23}{21}p + \frac{1}{2}d + \frac{1}{2}dq^2 - \frac{1}{2}dq + \frac{83}{24}p^2 - dpq + \frac{5}{4}p^2q + \frac{1}{4}p^2q^2 - \frac{3}{4}pq^2 + \frac{23}{12}q - \frac{77}{12}pq + \frac{71}{12}q^2 + \frac{1}{2}d^2 + \frac{1}{2}dp + \frac{1}{2}dp^2\right)X^4 \ldots \quad (4.33)$$

The coefficients of this polynomial correspond exactly to the degrees of degeneracy obtained in (Table4.3).

If $p = q$ we obtain

$$f(X) = 1 + (d-1)X^2 + \left(\frac{1}{2}d + \frac{1}{2}d^2 - 1\right)X^4 + \cdots \qquad (4.34)$$

And when $d = 25$ the partition function is that of an ordinary string

$$f(X) = 1 + 24X^2 + 324X^4 + \cdots \qquad (4.35)$$

General conclusion

The purpose of this memorandum is the study of open bosonic string in presence of D-branes. Three configurations of D-branes are given:

- First is open bosonic string on a D-branes involving coordinates of type NN and DD where only those of type NN take a Lorentz indices. The study of the spectrum made it possible to show the consistency of this model, as a result the tachyonic state is always present, on the other hand there are new types of states which, that is possibility of both massless and massive scalar states and massive vector states.
- The second is open bosonic string between two Dp-branes in parallel also implying the same types of coordinates than previously, to be specific the NN and DD types, as well the study of the spectrum has made it possible to prove the consistency of this model, except that now for this case, the distance between the two branes gives a positive contribution to the mass operator which leads to the possibility(with a particular choice of this distance) to definitively eliminate the tachyon, in addition the disappearance of massless vector states. The Virasoro algebra was derived and the central term is similar to their in ordinary case.
- finally The open bosonic string between two Dp-Dq-branes in parallel, now implying in addition to the coordinates of the NN and DD type new mixed coordinates ND type which (like the DD case) is not a Lorentz index whereas the mode indices are now half integers. In addition to the positive contribution to the mass operator due to the distance between the two branes a second positive term reliant on the parameters p and q (in particular on the number of ND type coordinates) gives a second contribution to the mass.

Annex A:

Calculation of some masses

2nd excited level

$$M^2\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) = \frac{1}{\propto'}\left(-1+\sum_{n=0}^{\infty}\sum_{i=2}^{p} n a_n^{+i}\, a_n^i + \sum_{m=0}^{\infty}\sum_{a=p-1}^{d} m a_m^{+a}\, a_m^a\right)\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)$$

$=\frac{1}{\propto'}\left[\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) + \sum_{i=2}^{p} a_1^{+i}\, a_1^i a_1^{+j} a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + \sum_{a=p-1}^{d} a_1^{+a}\, a_1^a a_1^{+j} a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right]$

$=\frac{1}{\propto'}\left[\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) + \sum_{i=2}^{p} a_1^{+i}\, a_1^i a_1^{+j} a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + \sum_{a=p-1}^{d} a_1^{+a}\, a_1^a a_1^{+j} a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right]$

By using of (1.52), we get

$$M^2\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) = \frac{1}{\propto'}\left[\left(-a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) + a_1^{+i}\,\eta^{ij} a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+j}a_1^i a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right]$$

$$= \frac{1}{\propto'}\left[\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) + a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+j}\left(\eta^{ij} + a_1^{+j}a_1^i\right)|p^+,\overrightarrow{p_T}\rangle\right]$$

$$= \frac{1}{\propto'}\left(a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right) \qquad (A.1)$$

3rd excited level

1) $M^2\left(a_3^{+j}|p^+,\overrightarrow{p_T}\rangle\right) =$
$\frac{1}{\propto'}\left(-1+\sum_{n=0}^{\infty}\sum_{i=2}^{p} n a_n^{+i}\, a_n^i + \sum_{m=0}^{\infty}\sum_{a=p-1}^{d} m a_m^{+a}\, a_m^a\left(a_3^{+j}|p^+,\overrightarrow{p_T}\rangle\right)\right.$

$$= \frac{1}{\propto'}\left(-a_3^{+j} + \sum_{i=2}^{p} a_1^{+i}\, a_1^i a_3^{+j} + \sum_{i=2}^{p} 2a_2^{+i}\, a_2^i a_3^{+j} + \sum_{i=2}^{p} 3a_3^{+i}\, a_3^i a_3^{+j} + \sum_{a=p-1}^{d} a_1^{+a}\, a_1^a a_3^{+j} + \sum_{a=p-1}^{d} 2a_2^{+a}\, a_2^a a_3^{+j} + \sum_{a=p-1}^{d} 3a_3^{+a}\, a_3^a a_3^{+j})|p^+, \overrightarrow{p_T}\rangle\right)$$

$$= \frac{1}{\propto'}\left(-a_3^{+j}|p^+, \overrightarrow{p_T}\rangle\right) + 3a_3^{+i}\left(\eta^{ij} + a_3^{+j}a_3^i\right)|p^+, \overrightarrow{p_T}\rangle + 3a_3^{+}a_3^{+j}a_3^a|p^+, \overrightarrow{p_T}\rangle$$

$$= \frac{1}{\propto'}\left(-a_3^{+j}|p^+, \overrightarrow{p_T}\rangle\right) + 3a_3^{+i}\eta^{ij}|p^+, \overrightarrow{p_T}\rangle + 3a_3^{+i}a_3^{+j}a_3^i|p^+, \overrightarrow{p_T}\rangle$$

$$= \frac{1}{\propto'}\left(-a_3^{+j}|p^+, \overrightarrow{p_T}\rangle\right) + 3a_3^{+i}|p^+, \overrightarrow{p_T}\rangle$$

$$= \frac{1}{\propto'}\left(2a_3^{+j}|p^+, \overrightarrow{p_T}\rangle\right), \qquad (A.2)$$

2) $M^2\left(a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle\right) =$
$\frac{1}{\propto'}\left(-1 + \sum_{n=1}^{\infty}\sum_{i=2}^{p} \eta\, a_n^{+i}\, a_n^i + \sum_{n=1}^{\infty}\sum_{a=p-1}^{d} m\, a_m^{+a} a_m^a\right)\left(a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle\right)$

$$= \frac{\mathbf{1}}{\propto'}\left(-\boldsymbol{a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle}\right) + \sum_{\boldsymbol{i=2}}^{\boldsymbol{p}} \boldsymbol{a_1^{+i} a_1^i a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle} + \sum_{\boldsymbol{a=p-1}}^{\boldsymbol{d}} \boldsymbol{a_1^{+a} a_1^a\, a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle}$$

$$= \frac{\mathbf{1}}{\propto'}(-\boldsymbol{a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle} + \sum_{\boldsymbol{i=2}}^{\boldsymbol{p}} \boldsymbol{a_1^{+i} a_1^i a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle})$$

$$= \frac{\mathbf{1}}{\propto'}\left(-\boldsymbol{a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle} + \sum_{\boldsymbol{i=0}}^{\boldsymbol{p}} \boldsymbol{a_1^{+i}\left(\eta^{ii} a_1^{+i} a_1^{+i}\right) a_1^{+j} a_1^{+k}|p^+, \overrightarrow{p_T}\rangle}\right)$$

$$= \frac{\mathbf{1}}{\propto'}\left(-\boldsymbol{a_1^{+i}\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle} + \boldsymbol{a_1^{+i} a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle} + \boldsymbol{a_1^{+i} a_1^{+i} a_1^i\, a_1^{+j}\, a_1^{+k}|p^+, \overrightarrow{p_T}\rangle}\right)$$

$$= \frac{\mathbf{1}}{\propto'}\left(\boldsymbol{a_1^{+i} a_1^{+i}\left(\eta^{ij} + a_1^{+j} a_1^i\right) a_1^{+k}|p^+, \overrightarrow{p_T}\rangle}\right)$$

$$= \frac{1}{\propto'}\left(a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+i}a_1^{+j}a_1^{i}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right)$$

$$= \frac{1}{\propto'}\left(a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+j}a_1^{+k}(\eta^{ik} + a_1^{+k}a_1^{i})|p^+,\overrightarrow{p_T}\rangle\right)$$

$$= \frac{1}{\propto'}\left(a_1^{+i}a_1^{+j}a_1^{+k}\big|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+j}a_1^{+i}\eta^{ik}|p^+,\overrightarrow{p_T}\rangle + a_1^{+i}a_1^{+j}a_1^{i}a_1^{+k}a_1^{i}|p^+,\overrightarrow{p_T}\rangle\right)$$

$$= \frac{1}{\propto'}\left(2a_1^{+i}a_1^{+j}a_1^{+k}|p^+,\overrightarrow{p_T}\rangle\right), \qquad (A.3)$$

Annex B

Calculation of mean values

Calculation of $\langle 0|[l_1, l_1]|0\rangle$

Note that $|0\rangle$ being a physical state

$$l_n|0\rangle = 0\ ,\ n \geq 0 \qquad (B.1)$$

Let's then calculate

$$\langle 0|[l_1\ ,\ l_{-1}]|0\rangle = \langle 0|l_1\, l_{-1}|0\rangle$$

$$= \| l_{-1}|0\rangle\|^2 \qquad (B.2)$$

$$l_{-1}|0\rangle = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{-1-n}^{i}\alpha_{n,i}|0\rangle$$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{-1} \alpha_{-1-n}^{i}\alpha_{n,i} + \alpha_{-1}^{i}\alpha_{0,i} \quad + \sum_{n=1}^{\infty} \alpha_{-1-n}^{i}\alpha_{n,i} \quad \right)|0\rangle$$

$$= \frac{1}{2}\left(\sum_{n'=1}^{\infty} \alpha_{-1+n'}^{i}\alpha_{n',i}\right)|0\rangle$$

$$= \frac{1}{2}\sum_{n'=1}^{\infty} \alpha_{-n'}^{i}\alpha_{-1+n',i}\ \ |0\rangle$$

$$= 0$$

So

$$\left\langle 0\middle|\left[l_{1,}\, l_{-1}\right]\middle|0\right\rangle = 0 \qquad (B.3)$$

From the expression (3.28) we have :

$$\left[l_{1,}\, l_{-1}\right] = 2l_0 + A(1) \qquad (B.4)$$

And from equation (1.44), we find that :

$$A(1) = 0 \qquad (B.5)$$

Calculation of$\langle 0|[l_2\ ;\ l_{-2}\ ]|0\rangle$

$$\langle 0\big|[l_{2,}\, l_{-2}]\big|0\rangle = \langle 0|l_2 l_{-2}|0\rangle$$

$$l_{-2}|0\rangle = \frac{1}{2}\ \sum_{n=-\infty}^{\infty} \alpha^i_{-2-n}\alpha_{n,i}\,|0\rangle$$

$$= \frac{1}{2}\left(\ \sum_{n=-\infty}^{-2} \alpha^i_{-2-n}\alpha_{n,i} + \alpha^i_{-1}\alpha_{-1,i} + \alpha^i_{-2}\alpha_{0,i} + \sum_{n=0}^{\infty} \alpha^i_{-2-n}\alpha_{n,i}\right)|0\rangle$$

$$= \frac{1}{2}\left(\ \sum_{n=-\infty}^{-2} \alpha^i_{n}\alpha_{-2-n,i} + \alpha^i_{-1}\alpha_{-1,i}\right)|0\rangle$$

$$= \frac{1}{2}\alpha^i_{-1}\alpha_{-1,i}|0\rangle$$

So:

$$\langle 0|[l_2, l_{-2}]|0\rangle = \big||l_{-2}|0\rangle|\big|^2$$

$$= \left|\left|\frac{1}{2}\alpha^i_{-1}\alpha_{-1,i}|0\rangle\right|\right|^2$$

$$= \frac{1}{4}\langle 0|\alpha^i_1\alpha_{1,i}\alpha^j_{-1}\alpha_{-1,j}|0\rangle \qquad (B.6)$$

Using the relation (1.31) to order the modes, we get:

$$\langle 0|[l_2, l_{-2}]|0\rangle = \frac{1}{4}\big\langle 0\big|\alpha^i_1\big(\alpha^j_{-1}\alpha_{1,j} + \eta^j_i\big)\alpha_{-1,j}\big|0\big\rangle$$

$$= \frac{1}{4}\big\langle 0\big|\big[\alpha^i_1\alpha^j_{-1}\big(\alpha_{1,j}\alpha_{1i} + \eta_{ij}\big) + \eta^j_i\big(\alpha_{-1,j}\alpha^i_1 + \eta^i_j\big)\big]\big|0\big\rangle$$

$$= \frac{1}{2}\big\langle 0\big|\eta^j_i\eta^i_j\big|0\big\rangle$$

$$= \frac{(p-1)}{2} \qquad (B.7)$$

$$\langle 0|[l_2,\ l_{-2}]|0\rangle = \big\langle 0\big|\big(4l_0 + A(2)\big)\big|0\big\rangle$$

$$= A(2)$$

$$= \frac{(p-1)}{2} \qquad (\mathrm{B}.8)$$

From the relation (1.41) and the results (B.7) and (B.8) we obtain :

$$c_3 = -c_1$$

$$= \frac{(p-1)}{12}$$

$$\text{For DD: } \frac{(p-1)}{12} \to \frac{(d-p)}{12}$$

Virasoro's algebra then takes the following form:

$$[l_n^{tot}, l_m^{tot}] = (n-m)l_{n+m}^{tot} + \frac{(d-1)}{12}n(n^2-1)\delta_{n+m,0} \qquad (\mathrm{B}.9)$$

References :

1- Bartonz wiebach ,A forst core in string theory barton zwiebach 2004
2- Michel Rausch de traubenberg, introduction aux cordes et aux supercordes 1988
3- M. A. V´azquez-Mozo, Open String Thermodynamics and D-Branes 1996 arXiv:hep-th/9607052v2
4- L´arusThorlacius ,Introduction to D-branes 1997 arXiv:hep-th/9708078v1
5- M. C. B. Abdalla , E. L. Graça , Entropy of Bosonic Open String and Boundary Conditions arXiv:hep-th/0201243v2
6- Jeffrey A. Harvey and Per Kraus , D-Branes as Unstable Lumps in Bosonic Open String Field Theory 2000 , arXiv:hep-th/0002117v2
7- J. Polchinski, TASI Lectures on D-Branes, hep-th/9611050
8- S. Sethi and M. Stern, D-Brane Bound State Redux, hep-th/9705046.
9- Jeffrey A. Harvey and Per Kraus , D-Branes, Washington Taylor and Barton Zwiebach 2004 , arXiv:hep-th/0311017v3