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Theme:

# Study and conception of a Cartesian Robot 

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## Dedication:

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#### Abstract

The field of Cartesian robots is continuously advancing to address the increasing demands of users in terms of cost, time, flexibility, and quality. This is why Cartesian robot structures are being introduced, as they exhibit exceptional performance in precision, rigidity, and speed.

Our project aims to develop a small Cartesian robot with multiple architectures. The mechanical component responsible for positioning the tool in the horizontal plane is based on the biglide-type robot structure with three degrees of freedom. This design ensures both rigidity and precise positioning.

Initially, our focus was on modeling and simulating the key parts of the machine using SolidWorks and UGS software. Subsequently, we explored various technological solutions to implement the robot within the constraints of our financial resources and what is available in the local market. The construction of the robot was successfully completed, and it underwent testing for simple pick and place operations, validating the proposed concept. We hope that this modest undertaking can serve as a foundation for future projects in the field of robotics.


Keywords: Design, Production, Robotic, Arduino, GRBL, UGS, Solid works, Rdm6.

$$
\begin{aligned}
& \text { يتقام مجال الروبوتات الايكارتية باستمرار لتلبية المتطلبات المتز ايدة للمستخدمين من حيث النككلة والوقت والمرونة } \\
& \text { والجودة. هذا هو اللبب في أنه يتم تقديم هياكل الروبوت الديكارتية، لأنها تظهر أداءً استثنائيًا في الدقة و الصلابة } \\
& \text { والسرعة. } \\
& \text { يهدف مشرو عنا إلى تطوير روبوت ديكارتي صغير بهياكل متعددة. يتمد المكون الميكانيكي المسؤول عن وضع الأداة } \\
& \text { في المستوى الأفقي على هيكل الروبوت من نوع biglide بثلاث درجات من الحرية. يضمن هذا التصميم الصلابة } \\
& \text { وتحديد المو اقع بدقة. }
\end{aligned}
$$


 الانتهاء من بناء الروبوت بنجاح، وخضع للاختبار لعطليات اختيار ومكان بسيطة، والتحقق من صحة اللفهوم المقترح.
نأمل أن يكون هذا العمل المتو اضع بمثابة أساس للمشاريع المستقبلية في مجال الروبوتات.

الكلمات الرئيسية: التصميم والإنتاج، الروبوتات، الأردوينو، GRBL و SolidWorks وUGS و Rdm6.

## Résumé

Le domaine des robots cartésiens progresse continuellement pour répondre aux exigences croissantes des utilisateurs en termes de coût, de temps, de flexibilité et de qualité. C'est pourquoi des structures de robots cartésiens sont introduites, car elles présentent des performances exceptionnelles en termes de précision, de rigidité et de vitesse.

Notre projet vise à développer un petit robot cartésien à architectures multiples. Le composant mécanique responsable du positionnement de l'outil dans le plan horizontal est basé sur une structure de robot de type biglide à trois degrés de liberté. Cette conception garantit à la fois la rigidité et la précision du positionnement.

Dans un premier temps, nous nous sommes concentrés sur la modélisation et la simulation des pièces clés de la machine à l'aide des logiciels SolidWorks et UGS. Par la suite, nous avons exploré diverses solutions technologiques pour mettre en œuvre le robot dans les limites de nos ressources financières et de ce qui est disponible sur le marché local. La construction du robot a été achevée avec succès et il a été testé pour des opérations simples de prise et de dépose, validant ainsi le concept proposé. Nous espérons que cette modeste entreprise pourra servir de base à de futurs projets dans le domaine de la robotique.

Mots-clés : Conception, Production, Robotic, Arduino, GRBL, UGS, SolidWorks, Rdm6.

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## List of notations

$\boldsymbol{\tau}:$ Vector of applied torques
L: Robot Lagrangian
$\boldsymbol{E}$ : Kinetic energy of the system
$\boldsymbol{U}$ : Potential energy of the system
$\boldsymbol{q},(\boldsymbol{q},)^{\cdot} \boldsymbol{q}:$ are position, velocity and acceleration in common space respectively
$\boldsymbol{M}(\boldsymbol{q})$ : is the inertia matrix, it is symmetrical positive definite, its elements are a function of the joint pos
m: mass.
$\boldsymbol{M}_{-} \boldsymbol{j}$ : the moment of external forces exerted on body $C_{-} j$ around $O_{-} j$
$\boldsymbol{g}^{\wedge} \boldsymbol{T}$ : transposed acceleration of gravity
$\mathbf{L}_{-}(\mathbf{0}, \boldsymbol{j})$ : designating the vector with origin $O_{-} 0$ and end $O_{-} j$
$\boldsymbol{S}_{-} \mathbf{j}:$ vector with origin $O_{-} j$ and end at the center of mass of body $C_{\_} j$
$\boldsymbol{C}\left(\boldsymbol{q}, \boldsymbol{q}^{\dot{*}}\right) \boldsymbol{\boldsymbol { q } ^ { \cdot }}$ : is the vector $(n \times 1)$ of Coriolis torques and centrifugal forces
$\boldsymbol{r}$ : the distance between motor and the applied force point
$\boldsymbol{g}$ : gravity
$\boldsymbol{i}$ : components weights
$\boldsymbol{r}$ : Acceleration
$\Delta \boldsymbol{V}$ : the linear speed
$\boldsymbol{F}$ : Axial force [N].
$\boldsymbol{A t}$ : Total bearing surface between screw teeth and nut teeth. in the plane perpendicular to the axis. [mm2].
$\boldsymbol{N}=$ number of screw revolutions per minute.
$\boldsymbol{p}_{\mathrm{as}}=$ thread pitch [mm]
$\boldsymbol{\alpha}=$ thread helix angle
$\boldsymbol{V} \boldsymbol{s} \boldsymbol{t}=$ sliding speed on average diameter. [m/min]
$\boldsymbol{V} \boldsymbol{t r}=$ transfer speed $[\mathrm{m} / \mathrm{min}$
-p: engine pouwer
$N_{D}$ : motor rotational speed
D: driven pulley diameter
d:driver puller parameter
$\boldsymbol{\theta}_{\boldsymbol{m}}$ : the winding angle
$\boldsymbol{P}_{\boldsymbol{b}}:$ Basic belt power
$\boldsymbol{P}_{\boldsymbol{a}}:$ power rating
$\boldsymbol{n}_{\boldsymbol{b}}$ : the number of belts
$f z:$ Additional coefficient for dynamic forces
$\boldsymbol{f}_{\mathrm{h}}:$ Hardness coefficient
$\boldsymbol{f}_{\mathrm{t}}$ : Temperature coefficient.
$\boldsymbol{f}_{1}$ : Coefficient of service life
$\boldsymbol{f}_{\mathrm{n}}$ : Coefficient de nombre de tours
$\boldsymbol{N}$ : speed in rpm
K: for Ball bearing
$\boldsymbol{C}_{\boldsymbol{d y n}}$ : load capacity
$L_{\mathrm{h}}$ : The service Life
$\boldsymbol{M}_{\boldsymbol{f}}:$ Deflection moment
$\boldsymbol{R}_{\boldsymbol{A}}$ Support reaction
$\boldsymbol{R}_{\boldsymbol{B}}$ Support reaction
$\eta$ : efficiency
$\boldsymbol{P}:$ power
$\boldsymbol{f}$ : the coefficient of friction
C: Motor Torque

## General Introduction

In today's rapidly evolving technological landscape, robotics has emerged as a significant field of research and innovation. One prominent category of robots that has gained widespread attention is Cartesian robots. They are mechanical systems designed to perform precise and controlled movements in a three-dimensional Cartesian coordinate system.

The study of Cartesian robots encompasses various aspects, including their design, kinematics, dynamics, control systems, and applications. Researchers and engineers around the world have been actively engaged in exploring the capabilities of these robots and devising advanced techniques for their modeling and programming.

This thesis aims to delve into the extensive research conducted on Cartesian robots, focusing on their studies, modeling techniques, and programming methodologies. By comprehensively understanding the theoretical foundations and practical implementations of Cartesian robots, we aim to contribute to the existing knowledge in the field and explore new possibilities for their utilization in diverse industries.

The thesis will be organized into the following chapters:
In the First chapter, we will provide an overview of Cartesian robots, their history, and their fundamental characteristics. We will explore their advantages, limitations, and typical applications. Additionally, we will discuss the various configurations and types of Cartesian robots, including portal, overhead, and XY tables.

In the second chapter, we will focus on the linear axes that form the core components of Cartesian robots. We will examine the different types of linear actuators, such as ball screws, linear motors, and pneumatic cylinders, and analyze their characteristics, advantages, and limitations. Furthermore, we will discuss the design considerations for linear axes, including factors like precision, speed, and load capacity.

In the third chapter will delve into the geometric, kinematic, and dynamic modeling of Cartesian robots. We will explore the mathematical representations and transformations used to describe the robot's position, orientation, and motion in the Cartesian space. Additionally, we will discuss the development of dynamic models that account for the robot's inertia, forces, and torques.

In the Forth chapter will focus on the sizing of Cartesian robots. We will discuss the methodologies and considerations involved in determining the appropriate size and specifications of the robot for a given application. Topics covered will include payload capacity, workspace analysis, acceleration, and deceleration requirements.

# Chapter 1: background and generalities 

## 1.1) Introduction

Robotics is a branch of engineering that deals with the design, construction, operation, and use of robots. It is a rapidly advancing field that is transforming many areas of industry, healthcare, and research by developing various robots. Furthermore, robotics is a multidisciplinary field combining knowledge from various fields such as mechanical engineering, electrical engineering, computer science, and artificial intelligence to make robots that can perform autonomously or with minimal human intervention a wide range of tasks that are either too dangerous, too difficult, or too tedious for humans.

An industrial robot is a multi-purpose manipulator that can be reprogrammed on two or more axes, it is automatically controlled, reprogrammable and can be either fixed in place or fixed to a mobile platform for use in automation applications in an industrial environment. The first patent for what we would now consider a robot was filed in 1954 by George C. Devol and issued in 1961 [1]. The device comprised a mechanical arm with a gripper that was mounted on a track and the sequence of motions was encoded as magnetic patterns stored on a rotating drum. The first robotics company, Unimation, was founded by Devol and Joseph Engelberger in 1956 and their first industrial robot shown in Fig. 1.1 was installed in 1961.


Figure 1-1 Universal automation, the first Unimation robot working at a General Motors factory [1]

According ISO 8373:2021 [2], an industrial robot is an "actuated mechanism programmable in two or more axes with a degree of autonomy, moving within its environment, to perform intended tasks". Mainly, robotic manipulators are designed to move materials, parts or tools through variable and programmable movements to accomplish various tasks. Indeed, thanks to their flexibility, industrial robots are used in various industrial applications. Their use has been motivated by numerous economical and technical reasons. In fact, the control of such machines using
computer software makes them both flexible in the way they work and versatile in the variety of tasks they can achieve.

Most robots are designed to be flexible and perform various functions. However, the majority of manufacturers offer ranges of robots dedicated to specific applications. These dedicated ranges have specific equipment. For example, welding robots are generally equipped with welding torches; paint robots are particularly waterproof and have special protection; palletizing robots are designed with a more rigid structure, etc.

Among the tasks which robotic manipulators are devoted to, we can distinguish two main categories. The first category concerns tasks involving motions along a prescribed geometric path and corresponds, for example, to continuous welding, glowing, flame cutting or deburring operations. The second category concerns tasks requiring point-to-point (or free) motions as in, for example, pick-and- place operations such as loading and unloading of belt conveyors, tool changes in machinetools and simple assembly operations. Sometimes, intermediate configurations are inserted between the initial and final configurations in order to get a better control of the robot trajectory (e.g.: for obstacle avoidance). Better sensors, cheaper computing power and a higher degree of programmability combined to move robots out of the simple repetitive task phase and give them what thought of as rudimentary intelligence.

## 1.2) Robots classification

Firstly, robots are classified into industrial robot or service robot according to its intended application. Industrial robots are robots for use in industrial automation applications, while a service robot performs useful tasks for humans or equipment excluding industrial automation applications. Secondly, aaccording the last IFR (International Federation of robotics) reports [3], industrial robots are usually (but not exclusively) of one of the following kinematic types: Articulated robot, Cartesian robot, SCARA robot, Cylindrical robot, Parallel robot, or others robots that are not covered by one of the previous classes.

### 1.2.1) Articulated robot

It is a robot whose arm has at least three rotary joints [2], its movement and configuration closely resemble a human arm. The arm is mounted to a base with a twisting joint. The arm itself can feature anywhere from three rotary joints up to ten rotary joints which act as axes, with each additional joint or axis allowing for a greater degree of motion. In fact, the number of axes required is proportional to the
complexity of the task. For example, paint robots in the automotive industry often need more than 6 axes. However, it should also keep in mind that, as a rule, more axes mean less accuracy, thus most articulated industrial robots utilize four or six-axis. Figure 1.2 illustrates some articulated robots and table 1.1 gives their main advantages and disadvantages. The articulated robot arm is the most common kinematic configuration and high motor torques produced by the axes' own weight and relatively long reach can be counterbalanced by weights or springs [4].


Figure 1-2 Examples of articulated robots
table 1-1 Main advantages and disadvantages of articulated robots

\left.| Advantages | Disadvantages |
| :--- | :--- |
| - Good Repeatability and | • Typically requires dedicated |
| important payload |  |
| - High speed controller in addition to line |  |
| master controller like PLC/PC |  |$\right]$| Large working envelope |
| :--- |
| - Great in unique controller, |
| welding and painting |
| applications |

### 1.2.2) Cartesian robot

Basically, a Cartesian Robot or a linear robot is a robot whose arm has three prismatic joints and whose axes are correlated with a cartesian coordinate system, that is a robot with three primary control axes that are all linear and mutually perpendicular to one another. The three sliding joints allow you to move your wrist up-down, in-and-out, and back-forth. In 3D-space, it is incredibly dependable and precise. It's also useful for horizontal movement and piling bins as a robot coordinate
system. A cartesian robot also defined by ISO 8373:2021 [2] as manipulator which has three prismatic joints, whose axes form a Cartesian coordinate system. Figure 1.3 illustrates several cartesian robots and table 1.2 gives their main advantages and disadvantages.


Figure 1-3 Examples of Cartesian robots
table 1-2 Main advantages and disadvantages of Cartesian robots

| Advantages | Disadvantages |
| :---: | :---: |
| - 3 or more axis of movement of almost any length <br> - Scalable <br> - Gearbox and motor can be sized according to range of motion and speeds <br> - Suitable for light to heavy / hanging loads <br> - Flexible and efficient due to linear axes scalability <br> - Inexpensive with good accuracy | - Cannot vary reach into or around obstructions <br> - Linear slides belts rails are not easily sealed against the environment <br> - Not freestanding: stand or frame or other mounting required |

### 1.2.3) SCARA ROBOT

A SCARA robot (Selective Compliance Assembly Robot Arm) is a robot, which has two parallel rotary joints to provide compliance in a plane [2], it is similar to Cartesian robots in that they move in 3 joints or axes. However, in contrast to Cartesian robots, two of the joints of SCARA robots are rotational. They are therefore capable of more
complex movements than the Cartesian robots. They are generally faster and have more flexibility in movement but are less precise than Cartesian robots. Figure 1.4 gives example of SCARA robots and table 1.3 summarizes their main advantages and disadvantages.


Figure 1-4 Examples of SCARA robots
table 1-3 advantages and inconvenient of SCARA robots

| Advantages | Disadvantages |
| :--- | :---: |
| - High Rigidity and High Accuracy: can | • are typically only capable of |
| achieve tolerances lower than 10 microns | carrying a lighter payload. |
| - can operate at a higher speed | Typically, they carry up to 2 kg |
| - Support Tools for Many Different Types | nominal (10 kg maximum |
| of Customers | payload). |
| - can operate in a cleanroom environment. | -doesn't suit all applications, and |
| - The compact design of SCARA Robots | the robot has limited dexterity |
| also makes them more easily re-located in | and flexibility compared to the |
| temporary or remote applications. | full 3D capability of other types |
| - robot suits applications with a smaller | of robots (e.g., six-axis robots). |
| field of operation and where floor space |  |
| is limited |  |

### 1.2.4) SPHERICAL ROBOT

Polar Robots, or spherical robots, have an arm with two rotary joints and one linear joint connected to a base with a twisting joint [2]. The axes of the robot work together to form a polar coordinate, which allows the robot to have a spherical work envelope. Polar Robots are credited as one of the first types of industrial robots to ever be developed. Polar robots are commonly used for die casting, injection molding, welding, and material handling. Figure 1.5 shows some spherical robots and table 1.4 indicates their main advantages and disadvantages. Today these robot types play only
a minor role and are preferably used for pelletizing, loading, and unloading of machines [4].


Figure 1-5 Examples of Spherical robots
table 1-4 advantages and inconvenient of Spherical robots

| Advantages |  |
| :--- | :--- |
| •Simpler control system than | Disadvantages |
| articulated arm. | • relatively poor Repeatability $0.5-1 \mathrm{~mm}$ |
| • Can have long reach. | • Not as flexible as articulated robot |
| • Very good for many welding | arms. |
| applications. | • Older technology. |
| • May be faster than articulated arm | • Often needs a rather large footprint. |

### 1.2.5) PARALLEL ROBOT

Parallel robots are also known as parallel manipulators or generalized Stewart platforms. A parallel robot is a mechanical system that uses several computercontrolled serial chains to support a single platform, or end-effector. Furthermore, a parallel robot can be formed from six linear actuators that maintain a movable base for devices such as flight simulators. These robots prevent redundant movements and to carry out this mechanism, their chain is designed to be short, simple. Figure 1.6 shows some parallel robots and table 1.5 indicates their main advantages and disadvantages. Parallel robots are used in various industrial applications such as: Flight simulators, Automobile simulators, Photonics (optical fibre alignment), ...


Figure 1-6 Examples of parallel robots
table 1-5 advantages and inconvenient of parallel robots

| Advantages | Disadvantages |
| :--- | :--- |
| • Very high speed | • It requires dedicated robot controller |
| - Contact lens shaped working | in addition to line master controller |
| envelope | like PLC/PCs |
| - Excels in high speed, lightweight | • Complex direct kinematics |
| pick and place applications <br> (candy packaging) <br> - High speed and high precision <br> milling machines |  |

### 1.2.6) Cylindrical ROBOT

Cylindrical Robots have a rotary joint at the base and a prismatic joint to connect the links. The robots have a cylindrical-shaped work envelop, which is achieved with rotating shaft and an extendable arm that moves in a vertical and sliding motion. Robots with a cylindrical coordinate system have a relatively simple structure, where one twisting joint is added to two typical linear coordinates. Such type of robots are often used in tight workspaces for simple assembly, for loading-unloading operations, for packing, palletizing, machine tending, or coating applications etc. Flexibility for the robot can be added with coordinates of the wrist assembly system. Figure 1.7 illustrate several cylindrical robots and table 1.6 indicates their main advantages and disadvantages.


Figure 1- 7 Examples of Cylindrical robots
table 1-6 advantages and inconvenient of Cylindrical robots

| Advantages | Disadvantages |  |
| :--- | :--- | :--- |
| • Rigid. | • | Older technology. |
| • Accurate with high repeatability: vary | • | Limited flexibility of movement |
| $0.1-0.5 \mathrm{~mm}$ |  |  |
| • Perfect in applications that require |  |  |
| circular geometry. |  |  |
| - Wide range of sizes |  |  |
| • Inexpensive for their size |  |  |

## 1.3) TYPE OF CARTESIAN ROBOTS

A Cartesian coordinate robot is a major of type of industrial robots known for its linear operations. It has three principal axes of control that move in a straight line rather than rotate and are in right angles to each other ,such is known as the linear arrangement. The job of moving wrist up-down ,in-out,and back-forth is based on the movement of the three sliding joints fixed accordingly. Cartesian robots therefore have a rectangular shape. The link arrangment of the robot offers many advantages against other type of industrial robots such that it simplifies the robot control arm solution. In this section we review several popular designs (Fig. 1.8) of cartesian robots based on architectures proposed by the well known industrial Festo Company [5].


Figure 1-8 main architectures of Cartesian robots proposed by FESTO company [5].

### 1.3.1) 1 D handling systems (Single-axis system)

The single-axis system is characterized by its high mechanical rigidity and sturdy design, which are ideal for long, one-dimensional strokes and large loads. With coordinated stepper and servo motors, servo drives and the integrated energy supply concept, you can get easily a reliable, ready-to-install solution. Figure 1.9 illustrates
one-dimension handling systems. Such a system is a Cost-effective implementation of vertical 2D movements for simple handling tasks such as small parts handling.


Figure 1- 9 Examples of Single-axis system [5]

### 1.3.2) 2 D handling system

Two sub-classes can be distinguished: 2 D linear gantry systems and 2 D gantry systems, acting both in a plan.


Figure 1-10 Examples of 2D linear gantry handling systems [5]
benefits of 2D linear gantry systems (Fig. 1.10) include:

- compact and slim design.
- flexible handling with free movement in the vertical plane even when installation space is limited.
They are particularly useful for:
- rapid processes with high cycle rates.
- Fast repositioning of parts and modules in a large, rectangular working space, such as pick \& place, feeding, stacking, packaging and filling tasks.


Figure 1-11 Examples of 2D gantry handling systems [5]

Key benefits of 2-D gantry systems (Fig. 1.11) include:

- parallel kinematic drive concept ensures low effective loads.
- Universal for low to high payloads.
- For very high requirements in terms of precision and/or very heavy work pieces.
- effective for very long strokes.
- recommended for positioning of end effectors like grippers and vacuum systems or work pieces.

Small version of about $155 \times 110 \mathrm{~mm}$ is frequently used in the electronics industry or in laboratory applications for positioning trays/laboratory microplates. While large version is used for tasks such as the handling of solar wafers with a maximum working space of $1800 \times 2500 \mathrm{~mm}$.

### 1.3.3) 3D handling system

The Cartesian robot for three-dimensional movement in the working space is ideal for very long strokes up to 3000 mm in the x -direction, even with high loads. The combination of several axis modules means it can be used anywhere, for light to heavy work pieces or large payloads. A three-dimensional system that is perfectly tailored to the requirements of a great number of applications. We can distinguish 3D gantry systems and 3D cantilever systems. 3D gantry systems (Fig. 1-12) are reliable and precise thanks to high mechanical rigidity, sturdy design and coupled X-axes. They are extremely precise with a high load capacity, even with very long strokes.


Figure 1-12 Examples of 3D gantry handling systems [5]
The Cartesian cantilever robot (Fig. 1.13) for three-dimensional movements in space is ideal for use in linear assembly units, in assembly tasks and in small parts handling. The cantilever Y-axis allows the work space to be accessed from three sides and makes the best use of the installation space.


Figure 1-13 Examples of 3D cantilever handling systems

## 1.4) Conclusion

Robotic manipulators are an important tool in industry, offering a high level of precision and accuracy that can improve productivity and efficiency. While they have some disadvantages, their advantages make them a valuable investment for many industrial applications. In this chapter, we have presented the various kinematic architectures of industrial robots. A particular attention has been paid to Cartesian robots. Cartesian systems are frequently more effective and economical than traditional robotic systems in many applications. The main component of a Cartesian robot are its linear axes; they are studied in deep in the next chapter.

## Chapter 2: Study of linear axes

## 2.1) Introduction

Linear axes are mechanical systems that are used to move objects in a straight line. They are commonly used in automation and manufacturing applications to precisely position and move equipment, tools, and other objects. They can be designed to move objects in a variety of directions, including horizontal, vertical, and diagonal. They can also be configured as single-axis or multi-axis systems, depending on the number of moving carriages and the directions of motion required. They are typically consisting of a stationary base, a moving carriage, and a linear motion guide that allows the carriage to move smoothly and accurately along a straight path. The carriage is usually powered by a motor, which is controlled by a computer or other control system. Also, linear axes are essential components in many industrial and scientific equipment, including CNC machines, robotics, semiconductor manufacturing, and medical devices. They are highly precise and reliable, and they can be customized to meet specific application requirements.

A linear motion axis is a mechanical system that provides linear motion along a single axis. It is typically used to move loads or equipment in a straight line, either horizontally or vertically. Linear motion axes can be found in a wide range of applications, including manufacturing, automation, robotics, and transportation. [6]

Linear motion axes can be designed to provide a wide range of motion parameters, including speed, acceleration, and positioning accuracy. They can be driven by a variety of power sources, including electric motors, hydraulic systems, or pneumatic systems. The specific design and configuration of a linear motion axis depend on the specific application requirements, load capacity, travel distance, and other factors.

## 2.2) Architecture of a linear axis

The common architecture of a linear motion axis typically includes several key components, which work together to provide linear motion. These components may vary depending on the specific application, but the basic architecture typically includes the following:
$>$ Guidance system: This is a component that provides guidance and support for the moving element of the axis. It may include linear bearings, rails, or other mechanisms that help to maintain accurate and smooth motion.
> Moving element: This is the component that actually moves along the linear guide and provides the desired linear motion. It may include a carriage, slide, or other mechanism that is connected to the load being moved.
$>$ Drive mechanism: This is the component that provides the force or power to move the moving element along the linear guide. It may include a motor,
actuator, or other mechanism that converts rotary motion into linear motion. From an energy point of view, we can distinguish

- Electric Motor Driven Linear Axis: This solution uses an electric motor to provide direct linear motion. The motor consists of a stationary stator and a moving rotor, which are connected to the load being moved. This solution provides high speed and acceleration and is suitable for high-precision applications.
- Pneumatic Driven Linear Axis: This solution uses compressed air to provide linear motion. The air is typically fed into a piston-cylinder arrangement, which moves the load along the linear axis. This solution is cost-effective, provides high speeds, and is suitable for low-load applications.
- Hydraulic Driven Linear Axis: This solution uses hydraulic fluid to provide linear motion. The fluid is typically fed into a piston-cylinder arrangement, which moves the load along the linear axis. This solution provides high power and is suitable for high-load applications.
> Motion transformer mechanism: This is a component that converts the output motion of the drive mechanism into the desired linear motion of the moving element. It may include gears, belts, pulleys, or other mechanisms that help to transmit motion from the drive mechanism to the moving element.
$>$ Control system: This is a component that provides feedback and control over the motion of the linear axis. It may include sensors, controllers, or other mechanisms that monitor and adjust the motion of the axis to achieve the desired performance.

In this chapter, we are going to focus on the guiding system and the transformer mechanism. Guiding systems include components such as rails and bearings, while transformer systems include components such as belts, chains and screws. Figure (2.1) illustrates two linear axes we built at the structure laboratory of our department.


Figure 2-1 Examples of Linear axes

## 2.3) Technological Solutions for Linear Axes

Linear axes might differ in selected technological solution to achieve the desired input-output transformation relationship. Indeed, a linear motion axis typically includes several key components, including a linear guide, a moving element, a drive mechanism, a motion transformer mechanism, and a control system. The linear guide provides support and guidance for the moving element, which is connected to the load being moved. The drive mechanism provides the force or power to move the moving element along the linear guide, while the motion transformer mechanism converts the output motion of the drive mechanism into the desired linear motion of the moving element. The control system provides feedback and control over the motion of the linear axis, ensuring accurate and smooth motion.

There are several technological solutions available to design a linear axis, each with its own advantages and disadvantages. Here are some of the most common solutions:

- Nut-Screw Driven Linear Axis.
- Belt Driven Linear Axis.
- Rack and Pinion Driven Linear Axis

The specific technological solution chosen for a linear axis design depends on the specific application requirements, load capacity, travel distance, and other factors. It is important to consult with a qualified engineer or supplier to determine the best technological solution for a specific linear axis design.

### 2.3.1) Screw-Nut Systems

### 2.3.1.1) Definition

The solution Nut-Screw Driven Linear Axis uses a Nut-screw mechanism to convert rotary motion into linear motion. A screw-nut system is a mechanical device that consists of a threaded screw and a matching nut, which are designed to work together to generate linear motion or force. The screw has a helical thread that runs along its shaft, while the nut has an internal thread that matches the screw's thread. When the screw is rotated, it moves axially through the nut, creating linear motion or force in the direction of the screw's axis. In case of using ball-nut, this solution provides high accuracy and repeatability and is suitable for high loads and speeds. For these reasons, it is widely used in a wide range of applications, including machinery, vehicles, robotics, and construction equipment. [7]

Table 2-1 Screw -Nut characteristics

| Trapezoidal <br> Thread <br> Screw(metric) | Trapezoidal <br> Thread Screw | Round <br> thread screw | Square screw | whiteworth <br> thread gas <br> thread |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

### 2.3.1.2) Calculation of a transmission based on a screw-nut system

The screw and nut system make it possible to transform a rotation movement into a translation movement by combining the movements of both of them. [8]


Figure 2-2 Screw-nut system
A load of mass $(m)$ subjected to a resistive force $\left(F_{\mathrm{r}}\right)$ parallel to its displacement is attached to a nut moving on a pitch screw $(P)$ fixed on the motor shaft. The slides, not shown on the figure, guide the load to impose the translation movement. The nut
moves by one step for each turn of the screw. If the drive shaft turns by an angle $(\theta)$, the load travels the distance:

$$
\begin{equation*}
D=p_{a s} \frac{\theta}{2 \pi} \tag{2.1}
\end{equation*}
$$

By deriving, we get a relation between the linear speed $(V)$ of the load and the angular speed of the motor $(\omega)$ :

$$
\begin{equation*}
V=\frac{p}{2 \pi} \omega \tag{2.3}
\end{equation*}
$$

The power output $\left(P_{s}\right)$ of a screw-nut system depends on the force of the load to be moved, the speed of movement and the efficiency of the system. The power output can be calculated by using this equation:

$$
\begin{equation*}
P_{s}=F V \tag{2.4}
\end{equation*}
$$

the power input $(P e)$ to a motor and its angular velocity $(\omega)$ of rotation. The constant $C$ in this equation represents the motor's electrical torque, or the force that rotates the motor, which is supplied by the electrical power source. The power input can be calculated by using this equation:

$$
\begin{equation*}
P_{e}=C \omega \tag{2.5}
\end{equation*}
$$

$(F)$ is the force exerted by the screw on the nut and $(C)$ the resistant cut exerted by the screw on the motor, the efficiency of the mechanism is defined by :

$$
\begin{equation*}
\eta=\frac{P_{s}}{P_{e}}=\frac{F V}{C \omega} \tag{2.6}
\end{equation*}
$$

which gives:

$$
\begin{equation*}
C=\frac{F V}{\eta 2 \pi} \tag{2.7}
\end{equation*}
$$

the total moment of inertia is $J=J_{\mathrm{m}}+J_{\mathrm{v}}$, if $\left(J_{\mathrm{v}}\right)$ is the moment of inertia of the screw. the Fundamental principle of Dynamics applied to the motor shaft gives:

$$
\begin{equation*}
J \frac{d \omega}{d t}=C-C_{\mathrm{v}} \tag{2.8}
\end{equation*}
$$

The resisting force can be written as $F_{\mathrm{r}}=F c+F f$ with Fc a possible working force and $F f$ an unavoidable friction force in the slides. The fundamental principle applied to the load leads to:

$$
\begin{equation*}
m \frac{d v}{d t}=F-F_{\mathrm{r}} \tag{2.9}
\end{equation*}
$$

From the relations (2.3) to (2.9) , the necessary motor torque is expressed:

$$
\begin{gathered}
C=J \frac{d \omega}{d t}+C_{\mathrm{v}} \\
C=J \frac{d \omega}{d t}+\frac{F P}{\eta 2 \pi} \\
C=J \frac{d \omega}{d t}+\frac{P}{\eta 2 \pi} m \frac{d v}{d t}+\frac{P}{\eta 2 \pi} F_{\mathrm{r}} \\
C=\left(\frac{2 \pi}{P} J+\frac{P}{\eta 2 \pi} m\right) \frac{d v}{d t}+\frac{P}{\eta 2 \pi} F_{\mathrm{r}}
\end{gathered}
$$

there is an optimal value of the pitch which allows to minimize the acceleration torque. it is obtained by deriving the first term of the previous expression as a reference to $p$ :

$$
\left(-\frac{2 \pi}{P^{2}} J+\frac{m}{\eta \times 2 \pi}\right) \frac{d v}{d t}=0
$$

What lead us to:

$$
\begin{align*}
& P^{2}=(2 \pi)^{2} \frac{J \eta}{m} \\
& J=\frac{p^{2} m}{(2 \pi)^{2} \eta} \tag{2.10}
\end{align*}
$$

Here's some of the advantages and disadvantages of screw systems:
Table 2-2 Main advantages and disadvantages of screw-nut system

| Advantages | Disadvantages |
| :--- | :--- |
| This mechanism makes it possible to exert <br> significant forces and pressures. <br> It's also allows fine adjustments. | This mechanism generates a lot of friction. <br> Its fagility can lead to guidance problems. <br> The system is slow unless you have a <br> significant screw step. |

### 2.3.2) Pulleys-Belts Systems

### 2.3.2.1) Definition

The solution belt Driven Linear Axis uses a belt and pulley system to transmit motion from a motor to the moving element. The belt is typically made of highstrength materials such as rubber or polyurethane. This solution is cost-effective and provides high speeds and low noise but may be less accurate than other solutions. The pulley and belt mechanism transmits a rotation movement between two or more pulleys distant from each other. The movement is transmitted via a belt (intermediate member). The pulley and belt mechanism consists of a belt and at least two pulleys. Table 2.3 below shows some type of commonly used belts.


Figure 2-3 Examples of simple Pulley-Belt systems

Table 2-3 Pulley Belts Types

| Type | Scheme | Features |
| :--- | :--- | :--- |
| FLAT BELTS |  | - Very silent <br> - High speed transmission. |
| TRAPEZOIDAL <br> BELTS | - High transmissible power <br> (uses multiple throats) <br> - Poly belts widely used in <br> household appliances |  |
| TIMING BELTS | - Silent transmission without <br> slipping <br> One of the two pulleys must <br> have a flange so that the belt <br> does not Pulleys |  |

The table below describes types of pulley and belt systems, including timing belts, Vbelts, flat belts, each system includes important specifications and characteristics such as belt type, profile, pitch and load capacity. This information allows users to choose the system that suits their specific needs.

### 2.3.2.2) Calculation of a transmission by Belts

The belt is wrapped around the pulleys, and when one pulley is rotated, it causes the belt to move, which in turn causes the other pulley to rotate. This system is used to transmit power or motion from one shaft to another, with the pulleys serving to increase or decrease the speed or torque of the motion


Figure 2-4 transformation of motion by pulleys-belt system

In order to model the motion transformation of the pulleys-belt system of figure 2.5, we consider a mass load $m$ which is subjected to a resistive force $F r$. A pulley of radius $R$ placed on the shaft of the motor drives a toothed belt attached to the load, when the motor turns an angle $\theta$, the mass moves a distance D such as:

$$
\begin{equation*}
D=R \Theta \tag{2.11}
\end{equation*}
$$

By deriving, we get a relation between the linear speed $(V)$ of the load and the angular speed of the motor $(\omega)$ :

$$
\begin{equation*}
V=R \omega \tag{2.12}
\end{equation*}
$$

By following the same equations (2.4),(2.5) and (2.6) from the screw-nut system .C the resisting torque exerted by the pulley on the drive shaft defined by :

$$
\begin{equation*}
C=\frac{F R}{\eta} \tag{2.13}
\end{equation*}
$$

The fundamental principle of dynamics for the motor shaft leads, by including the inertias in Jm, a :

$$
J \frac{d \omega}{d t}=C-C_{\mathrm{r}}
$$

The fundamental principle applied to the load leads to:

$$
m \frac{d v}{d t}=F-F_{r}
$$

we determine the necessary torque for the motor by eliminating the superfluous unknowns between the equations (2.11),(2.12),(2.6) and (2.13) :

$$
\begin{gathered}
C=J \frac{d \omega}{d t}+C_{r} \\
C=J \frac{d \omega}{d t}+\frac{F R}{\eta} \\
C=J \frac{d \omega}{d t}+\frac{R m}{\eta} \frac{d v}{d t}+\frac{R F_{\mathrm{r}}}{\eta} \\
C=\left(\frac{J}{P R}+\frac{R m}{\eta}\right) \frac{d v}{d t}+\frac{R F_{\mathrm{r}}}{\eta}
\end{gathered}
$$

we can optimize the mechanism by minimizing the acceleration torque of the motor (i.e. the torque in the absence of resisting force).for this, we derive the first term of $C$ as a reference to $R$ :
what led us to:

$$
\left(-\frac{J}{R^{2}}+\frac{m}{\eta}\right) \frac{d v}{d t}=0
$$

hence:

$$
\begin{equation*}
J=\frac{R^{2} m}{\eta} \tag{2.14}
\end{equation*}
$$

The table 2.4 represents the advantages and disadvantages of belt-pulleys system.

Table 2-4 Main advantages and disadvantages of belt system

| Advantages | Disadvantages |
| :--- | :--- |
| • Silent transmission | • Limited life |
| • "High" transmission speeds (60 to | • Low transmissible torque for flat |
| $100 \mathrm{~m} / \mathrm{s}$ for flat belts) | straps |
| • Large distance between pulleys <br> possible | Initial belt tension required to <br> ensure adhesion |

By providing a clear overview of the advantages and disadvantages, the table helps engineers and designers make informed decisions and select the most suitable pulley and belt system for their specific requirements. It allows for a comparative analysis of different systems, taking into account factors such as power transmission efficiency, load capacity, speed limitations, and maintenance needs.

### 2.3.3) Rack and Pinion system

### 2.3.3.1) Definition

A rack and pinion linear actuator is made up of a circular gear (the pinion) that engages with a linear gear (the rack). Together, they transform rotational motion into linear motion. The rack is driven in a straight path when the pinion is rotated (Fig. 2.6). Moving the rack linearly, on the other hand, causes the pinion to revolve. This solution provides high accuracy and is suitable for high loads and speeds. Table 2.5 gives some types of rack \& pinion systems


Figure 2-5 Rack-Pinion system

Table 2-5 Types of Rack and Pinion System.

| Type | Model | Features |
| :---: | :---: | :---: |
| Straight teeth |  | - High Precision <br> - High-Load Capacity <br> - Low Backlash |
| Helical teeth |  | - Self-Locking <br> - High Precision <br> - High Efficiency |
| Roller pinion |  | - High Precision <br> - High Speed <br> - Low Noise <br> - High-Load Capacity |

The table presents various rack and pinion systems, including straight, helical, and Roller racks, along with their key specifications and features. It provides insights into the tooth profile, module, pitch, and load capacity of each system, allowing users to evaluate their suitability for specific applications.

### 2.3.5.2) Calculation of a transmission by Rack-Pinion

It consists of a straight bar with teeth (the rack) and a gear (the pinion) that meshes with the teeth on the rack. When the pinion rotates, it causes the rack to move back and forth in a straight line. [8]


Figure 2-6 Rack-Pinion system [9] [10]

A mass load subjected to a resistive force $F r$. A Pinion of radius $R$ placed on the shaft of the motor drives a toothed rack soidaire of the load, when the motor turns an angle, the mass moves a distance:

$$
\begin{equation*}
D=R \Theta \tag{2.15}
\end{equation*}
$$

By deriving, we get a relation between the linear speed $(V)$ of the load and the angular speed of the motor $(\omega)$ :

$$
\begin{equation*}
V=R_{\mathrm{p}} \omega \tag{2.16}
\end{equation*}
$$

By following the same equations (2.4),(2.5) and (2.6) from the screw-nut system .(C) the resisting torque exerted by the pinion on the drive shaft, the efficiency of the mechanism is defined by :

$$
\begin{equation*}
C=\frac{F \times R_{\mathrm{p}}}{\eta} \tag{2.17}
\end{equation*}
$$

The fundamental principle of dynamics for the motor shaft leads, by including the inertias in Jm, a :

$$
\begin{equation*}
J \frac{d \omega}{d t}=C-C_{\mathrm{r}} \tag{2.18}
\end{equation*}
$$

The fundamental principle applied to the load leads to:

$$
\begin{equation*}
m \frac{d v}{d t}=F-F_{\mathrm{r}} \tag{2.19}
\end{equation*}
$$

we determine the necessary torque for the motor by eliminating the superfluous unknowns between the equations (2.15),(2.16),(2.6) and (2.17) :

$$
\begin{gathered}
C=J \frac{d \omega}{d t}+C_{\mathrm{r}} \\
C=J \frac{d \omega}{d t}+\frac{F \times R_{\mathrm{p}}}{\eta} \\
C=J \frac{d \omega}{d t}+\frac{R_{\mathrm{p}} m}{\eta} \frac{d v}{d t}+\frac{R_{\mathrm{p}} F_{\mathrm{r}}}{\eta} \\
C=\left(\frac{J}{P R_{\mathrm{p}}}+\frac{R_{\mathrm{p}} m}{\eta}\right) \frac{d v}{d t}+\frac{R_{\mathrm{p}} F_{\mathrm{r}}}{\eta}
\end{gathered}
$$

we can optimize the mechanism by minimizing the acceleration torque of the motor (i.e. the torque in the absence of resisting force). Thus, we derive the first term of the expression of $C$ versus to $R$ :

$$
\left(-\frac{J}{R_{\mathrm{p}}{ }^{2}}+\frac{m}{\mathrm{n}}\right) \frac{d v}{d t}=0
$$

What led us to: $\quad R_{\mathrm{p}}{ }^{2}=\frac{J \eta}{m}$

$$
\begin{equation*}
\text { Hence, } \quad J=\frac{R_{\mathrm{p}}{ }^{2} m}{\eta} \tag{21}
\end{equation*}
$$

Table 2-6 Main advantages and disadvantages of Rack and Pinion

| Advantages | Disadvantages |
| :--- | :--- |
| $\bullet$ Accuracy of movement | • Noise |
| • High efficiency | • High cost |
| • Low maintenance | • Sensitivity to constraint |
|  | • Length limitation |

## 2.4) Guiding System

A guiding system allows a load to be moved along a linear path with high precision. Numerous technological solutions exist, their common objective is to offer a reduced mechanical clearance, maximum efficiency and long service life. There are many common terms associated are numerous: rail, guide, slider, etc... [9]There are many different types of guides used in robotics, several solutions are exposed bellow.

### 2.4.1) Roller Guide

A guide wheel is a type of wheel, typically used to guide or direct a moving object or material along a specific path or trajectory. Guide rollers (Fig. 2.7) are commonly found in a variety of industrial applications such as conveyor systems, printing presses and packaging equipment.


Figure 2- 7 Roller Guide
Table 2-7 Main advantages and disadvantages of Roller Guide

| Advantages | Disadvantages |  |
| :--- | :--- | :--- |
| • Amazing accuracy and | • Higher cost |  |
| repeatability <br> • Higher load capacity and rigidity. |  |  |

### 2.4.2) Shafts and ball bearings

To reduce linear motion profiles, you need a quick and easy way to do is a good choice, especially if you don't have high tolerance or accuracy requirements. The shaft must be sized to carry the load within acceptable deflection Acceptable Deflection Tolerance - Too much weight and the shaft will bend causing misalignment. mistake. However, these errors are usually very small, on the order of a few thousandths of a millimeter [10]. The table 2.8 represents the advantages and disadvantages of shaft and ball bearing system.


Figure 2-8 Shaft and Ball Bearing
Table 2- 8 Main advantages and disadvantages of shaft and ball bearing

| Advantages | Disadvantages |
| :--- | :--- |
| - Low cost | • Potential for positioning errors. |
| • Easy to install | - Problems at high speed. |
| - Easy to maintain and service. | • Low load capacity. |
| - Low friction. |  |

### 2.4.3) Linear guide

Different types and sizes of linear guides are available depending on specific application needs. Some common types include ball guides, roller guides, and linear guides (Fig. 2.9). Factors such as load capacity, speed, and accuracy requirements determine the type of linear guide that is right for a particular application. The table (2.9) represents the advantages and disadvantages of Linear Guide systems.


Figure 2-9 Linear guides

Table 2-9 Main advantages and disadvantages of Linear Guide

| Advantages | Disadvantages |
| :--- | :--- |
| - High accuracy | - High cost |
| - High load-carrying capacity | - Noise |
| - Low friction | - Limited Travel Length |
| - Low maintenance |  |

2.5) Comparison of Different Types of Linear Motion Guidance Systems

When choosing the best linear guidance system for a linear axis design, there are several factors to consider, including:

- Load capacity: The linear guidance system should be able to handle the weight and size of the load that will be moving along the linear axis.
- Accuracy: The linear guidance system should provide the required level of accuracy for the application.
- Stiffness: The linear guidance system should be stiff enough to resist deflection and maintain accuracy, especially when subjected to high loads.
- Smoothness: The linear guidance system should provide smooth and consistent motion to minimize the risk of vibration and shock.
- Operating environment: The linear guidance system should be able to operate in the intended environment, including temperature, humidity, and exposure to contaminants.
- Maintenance: The linear guidance system should be easy to maintain, with minimal need for lubrication or other forms of maintenance.
- Cost: The linear guidance system should be cost-effective, with a reasonable balance between performance and cost.

As seen before in previous sections, some common types of linear guidance systems include roller, ball bearings, rail.... Each type has its advantages and disadvantages, depending on the specific application requirements. Table 2.10 gives a comparison of different solutions.

For example, recirculating ball bearings must operate with a very low coefficient of friction to avoid jerking during movement. Round rails are generally less expensive, less accurate, less accurate and can support lower loads than profile rails. Profiled rails are generally more expensive than round rails and can be more difficult to align. Therefore, profile rails are generally used in applications with the following requirements: high loads and/or high precision, see table (2.10). Roller guidance systems run on profiles with "V" grooves. Well although they are not as precise as round and profile rails, they are easier to use. easier to use. Their simple design requires little or no maintenance while offering a long service life. They cost less than
round and profile rails, are easier to install and can cover long distances. In this study the choice for the reasons mentioned above, the roller guide system was chosen.

Table 2-10 Comparison of guidance Systems [11]

| Solution | Variant | Load capacity | Speed of movement | Performance |  |  | Mechanical endurance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | rigidity | Clearance | yield |  |
| Roller |  | Medium | High | Good | Weak | Medium | Medium |
| Ball <br> Bearing |  | Medium | High | Good | Very weak | Good | Good |
| Rail | Balls | High | Very high | Very good | Very weak | Very good | Very good |
|  | rolls | Very high | High | excellent | Low | Very good | Very good |

## 2.6) Choosing the Best Linear Guidance System for a linear axis design: A Comparison of Pulley Belts and Screw Nuts

When it comes to linear rail systems, there are a variety of options to choose from which are pulleys and nuts.

In general, pulleys are a good choice when long distances in a straight line are required. They are generally less affected by vibration and can provide greater precision than nuts. This is because they can be designed with a higher number of teeth on the belt, which improves the accuracy of movement. However, when it comes to shorter straight line, screw-nuts are often the better choice. This is because they allow better control over the movement of guided objects and can be more precise over shorter distances.

In some cases, it may be necessary to use a combination of pulleys and nuts in one application. For example, if a system requires both long and short linear motion, it may be best to use pulleys for long distances and nuts for short distances. The choice between pulleys and nuts depends on the specific needs of your application. Factors such as required accuracy, length of linear travel, and potential for vibration should be considered when making a decision

## 2.7) Solutions applied in our Study

In this section, we are going to present several solutions that we have studied, designed and fabricated, in the light of available means at the laboratory of structures of our department of mechanics.

### 2.7.1)First solution

The first axis is a combination of a trapezoidal screw with 100 cm length and one shaft (guiding system) as shown in the following figure (2.10):


Figure 2-10 Solution one
We noticed that this linear axis can generate undesirable vibrations that can affect the overall stability and motion accuracy of the system. These vibrations can be caused by the length of the screw used in the axis.

### 2.7.2) Second solution

In our second solution, we have minimized the length of the screw to $(50 \mathrm{~cm})$ in order to reduce vibrations. we aim to minimize flexing and bending, which are common sources of vibrations in linear axes as shown in the following figure (2.11)


Figure 2-11 Solution two
the successful elimination of vibrations has improved the overall functionality and performance of the system by minimizing the length of screw, allowing it to operate optimally and meet the desired requirements with increased stability and precision.

### 2.7.3) Third solution

in this solution we kept same length of screw in first solution but we added one more shaft which means 2 shafts and 1 screw in the middle as $u$ see the figure (2.12):


Figure 2-12 Solution three
While our solution successfully reduced system vibration, it is important to realize that even with this improvement, vibration can still affect system accuracy. This limitation is especially noticeable in long systems that use screw and nut mechanisms.

### 2.7.4) Fourth solution

By adopting this alternative approach, we have successfully addressed the limitations of the screw-nut system in tall-length applications. We tried The pulley and belt system, combined with the roller guide as shown in the following figure (2.13):


Figure 2-13 Solution four
Combining the pulley and belt system with roller guides increases accuracy and precision, enabling the system to meet desired performance requirements. This solution not only reduces the adverse effects of vibration, but also provides a more robust and efficient mechanism for long length applications, ensuring optimum functionality and durability.

## 2.8) Conclusion

linear axis is a critical component in many industries where high precision, accuracy, and reliability are required. They can improve productivity, reduce costs, and enhance the overall performance of the system, making them an essential tool for many manufacturing, automation, robotics, and aerospace applications. The most important thing about Cartesian robots is modeling in general. This Will Be explained in the next chapter.

# Chapter 3: modeling of a Cartesian robot 

## 3.1) Introduction

Kinematic modeling is an important step not only in the design process of any robot but also during its exploitation phase. Kinematic modelling involves the establishment of both geometric and kinematic relationships governing the kinematics of the considered robot. These models are used to describe and predict the motion of the robot, in joint and Cartesian spaces, allowing engineers and technicians to optimize its performances and ensure that it meets the requirements of the intended application.

The geometric model involves the establishment of a mathematical description of the robot's physical structure. It is used to determine the robot's range of motion, its workspace, and its ability to reach specific points or follow particular paths. Geometric modeling also helps engineers to identify and avoid potential collisions or other issues that could impact the robot's performance or safety. This model is used to determine the robot's joint position, as well as its end-effector position and orientation in space. Kinematic modeling, on the other hand, focuses on the relationship between the robot end-effector motion and its actuators motion from a kinematic point of view (i.e velocity and acceleration).

A Cartesian coordinate robot, is an industrial robot whose three principal axes of control are linear and are at right angles to each other. The three sliding joints correspond to moving the wrist up-down, in-out, back-forth. In this chapter, we are going to develop the kinematic model of 3 degrees of freedom (d.o.f.) robot similar to that of figure 3.1 representing a large class of industrial cartesian robots. The study is based on the DENAVIT HARTENBERG convention.


Figure 3-1 Architecture of studied Cartesian robot

## 3.2) DENAVIT HARTENBERG CONVENTION

The Denavit-Hartenberg (DH) convention is a widely used convention for defining the kinematic parameters of robotic systems. It was introduced by Jacques Denavit and Richard Hartenberg in 1955 as a systematic way of representing the geometry of robotic manipulators. The DH convention defines a set of rules for assigning coordinate frames to the links of a robotic system. Each link is assigned a coordinate frame based on its joint parameters, which include the joint angle, joint displacement, link length, and link twist [1]. The convention uses four parameters to describe the relative orientation and position of adjacent coordinate frames, known as the DH parameters. These parameters are (see fig. 3.2):

1. $\theta$ : The joint angle, which is the rotation about the $z$-axis of the previous frame to align with the z -axis of the current frame.
2. d : The link offset, which is the distance along the z -axis of the previous frame to the intersection with the $x$-axis of the current frame.
3. a: The link length, which is the distance along the $x$-axis of the current frame to the intersection with the $z$-axis of the previous frame.
4. $a$ : The link twist, which is the rotation about the x-axis of the current frame to align with the $x$-axis of the previous frame.

By assigning coordinate frames to each link in a robotic system using the DH convention, it is possible to compute the transformation matrix between any two frames, say $j-1$ and $j$, as follows [1]:

$$
{ }^{j-1} A_{j}=\left[\begin{array}{cccc}
\cos \theta_{j} & -\sin \theta_{j} \cos \alpha_{j} & \sin \theta_{j} \sin \alpha_{j} & a_{j} \cos \theta_{j}  \tag{3.1}\\
\sin \theta_{j} & \cos \theta_{j} \cos \alpha_{j} & -\cos \theta_{j} \sin \alpha_{j} & a_{j} \sin \theta_{j} \\
0 & \sin \alpha_{j} & \cos \alpha_{j} & d_{j} \\
0 & 0 & 0 & 1
\end{array}\right]
$$



Figure 3-2 Definition of standard Denavit and Hartenberg link parameters. [1]

Note that for Denavit-Hartenberg notation, there are only four parameters but there are also two constraints: axis $x_{j}$ intersects $z_{j-1}$ and axis $x_{j}$ is perpendicular to $z_{j-1}$. For this reason, DH convention has become a standard for describing the kinematics of robotic systems due to its simplicity, generality, and ease of implementation. It is widely used in robotics research, education, and industry, and is supported by many software packages and programming languages. However, it is important to note that the DH convention is not always the most appropriate choice for all robotic systems, and alternative conventions may be used depending on the specific requirements of the application.

## 3.3) Geometric model of a 3ddl Cartesian robot

By applying the convention of Devenavit-Hartenberg (DH) for our targeted cartesian robot, we will get the frames of figure 3.3. Note that the coordinate frame $\{\mathrm{j}\}$ is attached to the far (distal) end of link $j$. The $z$-axis of frame $\{j\}$ is aligned with the axis of joint $j+1$.


Figure 3- 3 definition of standard DH link parameters for the Cartesian robot

In consequence, we can establish the following table of DH-parameters:
Table 3-1 parameters table of geometric model

| $\boldsymbol{y} \boldsymbol{j}$ | $\boldsymbol{\alpha}$ | $\boldsymbol{a}$ | $\boldsymbol{d}$ | $\boldsymbol{\theta}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | $a_{1}$ | $d_{1}$ | $\frac{\pi}{2}$ |
| $\mathbf{2}$ | $\frac{\pi}{2}$ | $a_{2}$ | $d_{2}$ | 0 |
| $\mathbf{3}$ | 0 | $a_{3}$ | $d_{3}$ | 0 |

Note that parameters $\left(a_{1}, a_{2}, a_{3}\right)$ are constants and should be identified from the robot design while, parameters $\left(\mathrm{d}_{1}, \mathrm{~d}_{2}, \mathrm{~d}_{3}\right)$ are variable and are controlled by joint actuators.

### 3.3.1) Direct geometric model

The Direct Geometric Model (DGM) is the set of relations that defines the location of the end-effector of the robot as a function of its joint coordinates. For a series structure, it can be represented by homogeneous matrix of relation 3.1.

From the table 3.1 and using the relation 3.1, we get:

$$
{ }^{0} A_{1}=\left[\begin{array}{cccc}
0 & -1 & 0 & 0  \tag{3.2}\\
1 & 0 & 0 & d_{1} \\
0 & 0 & 1 & a_{1} \\
0 & 0 & 0 & 1
\end{array}\right],{ }^{1} A_{2}=\left[\begin{array}{cccc}
1 & 0 & 0 & a_{2} \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & d_{2} \\
0 & 0 & 0 & 1
\end{array}\right],{ }^{2} A_{3}=\left[\begin{array}{cccc}
1 & 0 & 0 & a_{3} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d_{3} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

In consequence, the global transformation matrix is obtained as follows:

$$
{ }^{0} A_{3}={ }^{0} A_{1} \times{ }^{1} A_{2} \times{ }^{2} A_{3}=\left[\begin{array}{cccc}
0 & 0 & 1 & d_{3}  \tag{3.3}\\
1 & 0 & 0 & a_{2}+a_{3}+d_{1} \\
0 & 1 & 0 & d_{2}+a_{1} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

If we suppose that parameters $\mathrm{a}_{1}, \mathrm{a}_{2}$ and $\mathrm{a}_{3}$ are null ${ }^{0} A_{3}$ becomes :

$$
{ }^{0} A_{3}=\left[\begin{array}{lllc}
0 & 0 & 1 & d_{3}  \tag{3.3}\\
1 & 0 & 0 & d_{1} \\
0 & 1 & 0 & d_{2} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

### 3.3.1.2) The inverse geometric model

We have shown how to determine the pose of the end-effector given the joint coordinates and base transforms. A problem of real practical interest is the inverse problem: given the desired pose of the end-effector, what are the required joint coordinates. For example, if we know the Cartesian pose of an object, what joint coordinates does the robot need in order to reach it? This is the inverse kinematics.

In case of a Cartesian robot, we have only one solution, because we have three translation of prismatic joints perpendicular to each other.

Then, let be $\mathbf{P}$ the desired pose of the end-effector:

$$
\boldsymbol{P}=\left[\boldsymbol{p}_{x} \boldsymbol{p}_{y} \boldsymbol{p}_{z}\right]^{T}
$$

From relation (3.3), the coordinates of $O_{3}$ in $R_{0}$, are :

$$
\left\{\begin{array} { l } 
{ p _ { x } = d _ { 3 } }  \tag{3.4a}\\
{ p _ { y } = a _ { 2 } + a _ { 3 } + d _ { 1 } } \\
{ p _ { z } = d _ { 2 } + a _ { 1 } }
\end{array} \quad \text { hence } \quad \left\{\begin{array}{l}
d_{3}=p_{x} \\
d_{1}=p_{y}-a_{2}-a_{3} \\
d_{2}=p_{z}-a_{1}
\end{array}\right.\right.
$$

If we suppose that parameters $\mathrm{a}_{1}, \mathrm{a}_{2}$ and $\mathrm{a}_{3}$ are null relation (3.4) becomes simply:

$$
\begin{equation*}
d_{3}=p_{x}, d_{1}=p_{y}, d_{2}=p_{z} \tag{3.4b}
\end{equation*}
$$

## 3.4) Kinematic model of the Cartesian robot

In the last section we discussed the relationship between joint coordinates and end-effector pose - the manipulator kinematics. Now we investigate the relationship between the rate of change of these quantities - between joint velocity and velocity of the end-effector. This is called Kinematic model or differential kinematics of the manipulator.

The direct kinematic model of a manipulator robot describes the velocities of the operational coordinates as a function of according to the articular speeds. It is noted:

$$
\begin{equation*}
\dot{x}=J(q) \dot{q} \tag{3.5}
\end{equation*}
$$

where $j(q)$ denotes the Jacobian matrix of dimension $(m \times n)$ of the mechanism, equal to $\frac{\partial x}{\partial q}$ and a function of the joint configuration $q$. The same Jacobian matrix is involved in the computation of the direct differential model which gives the elementary variations $d x$ of the operational coordinates as a function of the elementary variations of the joint coordinates $d q$ ) that is [14]:

$$
\begin{equation*}
d x=J(q) d q \tag{3.6}
\end{equation*}
$$

The interest of the Jacobian matrix is multiple [15]
$>$ it is the basis of the inverse differential model, allowing the calculation of a local solution of the joint variables q knowing the operational coordinates $x$;
$>$ in statics, the Jacobian is used to establish the relation between the forces exerted by the end-member on the environment and the forces and torques of the actuators;
$>$ it facilitates the calculation of singularities and the dimension of the accessible operational space of the robot. [16]

The calculation of the Jacobian matrix can be done by deriving the MGD, $X=f(q)$, from the following relation:

$$
\begin{equation*}
J_{i, j}=\frac{\partial f_{i}}{\partial q_{i}} \text { With: } i=1, \ldots \ldots, m ; j=1 \ldots \ldots, n \tag{3.7}
\end{equation*}
$$

where $J_{i j}$ is the element $(i, j)$ of the Jacobian matrix $j$. This method is easy to implement for robots with two or three degrees of freedom. This method is easy to implement for robots with two or three degrees of freedom. [14]

In consequence, the Jacobian Matrix is:

$$
J(q)=\frac{\partial f(q)}{\partial q}=\left(\begin{array}{cccc}
\frac{\partial f_{1}}{\partial q_{1}} & \frac{\partial f_{1}}{\partial q_{2}} & \cdots & \frac{\partial f_{1}}{\partial q_{n}}  \tag{3.8}\\
\frac{\partial f_{2}}{\partial q_{1}} & \frac{\partial f_{2}}{\partial q_{2}} & \cdots & \frac{\partial f_{2}}{\partial q_{n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_{m}}{\partial q_{1}} & \frac{\partial f_{m}}{\partial q_{2}} & \cdots & \frac{\partial f_{m}}{\partial q_{n}}
\end{array}\right)
$$

In case of our robot, we get for $\boldsymbol{q}=\left[\begin{array}{l}d_{1} \\ d_{2} \\ d_{3}\end{array}\right]$ :

$$
\left[\begin{array}{c}
\dot{x}  \tag{3.9}\\
\dot{y} \\
\dot{z}
\end{array}\right]=\left[\begin{array}{c}
\dot{P}_{x} \\
\dot{P}_{y} \\
\dot{z}_{z}
\end{array}\right]=\left[\begin{array}{lll}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right] \cdot\left[\begin{array}{c}
\dot{d}_{1} \\
\dot{d}_{2} \\
\dot{d}_{3}
\end{array}\right] \text {, with } \quad J(q)=\left[\begin{array}{ccc}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right]
$$

If we suppose that parameters $\mathrm{a}_{1}, \mathrm{a}_{2}$ and $\mathrm{a}_{3}$ are null relation (3.4) becomes simply:

$$
\begin{equation*}
\dot{p}_{x}=\dot{d}_{3}, \dot{p}_{y}=\dot{d}_{1}, \dot{p}_{z}=\dot{d}_{2} \tag{3.10}
\end{equation*}
$$

## 3.5) Dynamic model of Cartesian robot:

Dynamic modeling of a Cartesian robot involves developing a mathematical model that describes the motion and behavior of the robot under different conditions, including external forces and torques. This type of modeling takes into account the physical properties of the robot, such as its mass, inertia, and friction, as well as the characteristics of the actuators that control its movements. Also dynamic modeling is an essential tool for designing and controlling Cartesian robots, as it enables engineers to predict the robot's behavior and optimize its performance. By analyzing the dynamic response of the robot to different inputs, we can design control systems that ensure the robot operates safely and accurately in a variety of environments and applications.

There are two problems related to the dynamics of a manipulator that we want to solve. In the first problem, we are given a trajectory point, and we want to find the required vector of the joint pairs. This formulation of the dynamics is useful for the problem of control of the manipulator. The second problem is to calculate how the mechanism will move under the application of a set of joint pairs.

As a conclusion, we can say that dynamic modeling is the mathematical representation of a system that describes its behavior in the internal and external stimulus presented in the system.

Several approaches have been proposed to model robot dynamics. The most frequently used in robotics are [17]:

- The formulation of Lagrange;
- The wording of Newton-Euler.

The Newton-Euler approach is based on the elementary dynamic formulas and on an analysis of forces and moments of constraint acting between the links. As an alternative to the Newton-Euler method, in this section we briefly introduce the Lagrangian dynamic formulation. Whereas the Newton - Euler formulation might be said to be a "force balance" approach to dynamics, the Lagrangian formulation is an "energy-based" approach to dynamics. Of course, for the same manipulator, both will give the same equations of motion. Our statement of Lagrangian dynamics will be brief and somewhat specialized to the case of a serial-chain mechanical manipulator with rigid links [18]

### 3.5.1) The formulation of Lagrange:

In this section, we develop a simple Lagrange method to present the general shape of the dynamic model of robots. First, we consider an ideal system without friction or elasticity, without exerting forces or moments on the environment.

Lagrange's formulation describes the behavior of dynamic system in terms of work and energy stored in the system. Lagrange's equations are commonly written as:

$$
\begin{gather*}
\Gamma_{i}=\frac{d}{d t} \frac{\partial \mathcal{L}}{\partial \dot{q}_{i}}-\frac{\partial \mathcal{L}}{\partial q_{i}}=\tau  \tag{3.11}\\
\mathcal{L}=E-U \tag{3.12}
\end{gather*}
$$

Where:
$\boldsymbol{\tau}$ : Vector of applied torques
$\mathcal{L}$ : Robot Lagrangian
E: Kinetic energy of the system
U: Potential energy of the system
$\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ : are position, velocity and acceleration in common space respectively

### 3.5.2) General form of the Dynamic equations:

### 3.5.2.1) Kinetic energy:

The kinetic energy of the system is a quadratic function of the joint velocities such that [18]:

$$
\begin{equation*}
E=\frac{1}{2} \dot{q}^{T} M(q) \dot{q} \tag{3.13}
\end{equation*}
$$

Where:
$\boldsymbol{M}(\boldsymbol{q})$ : is the inertia matrix, it is symmetrical positive definite, its elements are a function of the joint positions.

Positive definite matrices are those which have the properly that their quadratic form is always a positive scalar. The equation (E) can be seen as analogous to the familiar expression for the kinetic energy of a point mass [18]:

$$
\begin{equation*}
E=\frac{1}{2} m \dot{q}^{2} \tag{3.14}
\end{equation*}
$$

Where:
$\boldsymbol{m}$ : is mass manipulator.

### 3.5.2.2) Potential energy:

since the potential energy is a function of joint positions, the potential energy formula is written as:

$$
\begin{equation*}
U=\sum_{j=1}^{n} U_{j}=\sum_{j=1}^{n}-M_{j} g^{T}\left(\mathcal{L}_{0, j}+S_{j}\right) \tag{3.15}
\end{equation*}
$$

Where:
$\boldsymbol{M}_{\boldsymbol{j}}$ : the moment of external forces exerted on body $C_{j}$ around $O_{j}$
$\boldsymbol{g}^{\boldsymbol{T}}$ : transposed acceleration of gravity
$\mathcal{L}_{0, j}$ : designating the vector with origin $O_{0}$ and end $O_{j}$
$\boldsymbol{S}_{\boldsymbol{j}}$ : vector with origin $O_{j}$ and end at the center of mass of body $C_{j}$
By exploiting the equations 3.11, 3.13 and 3.15 , we obtain the dynamic model of the robot:

$$
\begin{equation*}
\Gamma=M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+G(q) \tag{3.16}
\end{equation*}
$$

Where:
$\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}}$ : is the vector $(n \times 1)$ of Coriolis torques and centrifugal forces, such that :

$$
\begin{equation*}
C \dot{q}=\dot{M} \dot{q}-\frac{\partial E}{\partial q} \tag{3.17}
\end{equation*}
$$

$G=\left[G_{1} \ldots . . G_{n}\right]^{T}$ is the vector of the gravity torques
In the absence of friction and other disturbances, the rigid robot dynamic model can be written as:

$$
\begin{equation*}
M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+G(q)=\tau \tag{3.18}
\end{equation*}
$$

### 3.5.2.3) Dynamic model with friction

Friction plays a major role in limiting the quality of robot performance. Uncompensated friction produces static error, delay and limit cycle behavior.

So, considering friction, Lagrange's equations are commonly written as:
$\Gamma_{i}=\frac{d}{d t} \frac{\partial \mathcal{L}}{\partial \dot{q}_{i}}-\frac{\partial \mathcal{L}}{\partial q_{i}}=\tau-f\left(\tau, \dot{q}_{i}\right) \quad$ for $i=1, \ldots \ldots, n$

### 3.5.3) Dynamic model of Cartesian robot:

After presenting the general notions of dynamic modeling, we can apply Lagrange's formulation to obtain the dynamic model of our Cartesian robot with three degrees of freedom, based on the kinematic model already obtained

However, as our robot has only three prismatic joints (no rotations), the kinetic energy equation will be written as:

$$
\begin{gather*}
E=\frac{1}{2} m v^{2}=\frac{1}{2} \dot{q}^{T} M(q) \dot{q}  \tag{3.20}\\
v=\frac{d}{d t}\left[\begin{array}{l}
x \\
y \\
Z
\end{array}\right]  \tag{3.21}\\
\rightarrow v^{2}=\|v\|^{2}=v^{T} v=\left[\begin{array}{lll}
x & y & z
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]  \tag{3.22}\\
\left\{\begin{array}{l}
v_{1}^{2}=\dot{q}_{1}^{2} \\
v_{2}^{2}=\dot{q}_{1}^{2}+\dot{q}_{2}^{2} \\
v_{3}^{2}=\dot{q}_{1}^{2}+\dot{q}_{2}^{2}+\dot{q}_{3}^{2}
\end{array}\right. \tag{3.23}
\end{gather*}
$$

By replacing the values $v_{1}^{2}, v_{2}^{2}$ and $v_{3}^{2}$ in (3.22) we obtain the kinetic energy of each link:

$$
>E_{1}(q, \dot{q})=\frac{m_{1} \dot{q}_{1}^{2}}{2}
$$

$\Rightarrow E_{2}(q, \dot{q})=\frac{m_{2}\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)}{2}$
$\Rightarrow E_{3}(q, \dot{q})=\frac{m_{3}\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}+\dot{q}_{3}^{2}\right)}{2}$

$$
\begin{equation*}
\Rightarrow \quad E(q, \dot{q})=\frac{m_{1} \dot{q}_{1}^{2}}{2}+\frac{m_{2}\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}\right)}{2}+\frac{m_{3}\left(\dot{q}_{1}^{2}+\dot{q}_{2}^{2}+\dot{q}_{3}^{2}\right)}{2} \tag{3.24}
\end{equation*}
$$

After distribution, we can expect an equation of the following form:

$$
\begin{equation*}
E(q, \dot{q})=\frac{\left(m_{1}+m_{2}+m_{3}\right)}{2} \dot{q}_{1}^{2}+\frac{\left(m_{2}+m_{3}\right)}{2} \dot{q}_{2}^{2}+\frac{\left(m_{3}\right)}{2} \dot{q}_{3}^{2} \tag{3.25}
\end{equation*}
$$

The potential energy $U(q)$ is obtained by considering in this case $h=q_{3}$ and $m=m_{3}$ :

$$
\begin{equation*}
U(q)=m_{3} \cdot g \cdot q_{3} \tag{3.26}
\end{equation*}
$$

After calculating the kinetic and potential energy, we can calculate the Lagrangian $\mathcal{L}$ by using the equation (3.12)

$$
\begin{equation*}
\mathcal{L}(q, \dot{q})=\left(\frac{\left(m_{1}+m_{2}+m_{3}\right)}{2} \dot{q}_{1}^{2}+\frac{\left(m_{2}+m_{3}\right)}{2} \dot{q}_{2}^{2}+\frac{\left(m_{3}\right)}{2} \dot{q}_{3}^{2}\right)-m_{3} \cdot g \cdot q_{3} \tag{3.27}
\end{equation*}
$$

and to determine the dynamic model of our system we follow the steps below:

$$
\frac{\partial \mathcal{L}}{\partial \dot{q}}=\left[\begin{array}{ccc}
\left(m_{1}+m_{2}+m_{3}\right) & 0 & 0  \tag{3.28}\\
0 & \left(m_{2}+m_{3}\right) & 0 \\
0 & 0 & m_{3}
\end{array}\right]\left[\begin{array}{l}
\dot{q}_{1} \\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]
$$

And we have:

$$
\frac{d}{d t} \frac{\partial \mathcal{L}}{\partial \dot{q}}=\left[\begin{array}{ccc}
\left(m_{1}+m_{2}+m_{3}\right) & 0 & 0  \tag{3.29}\\
0 & \left(m_{2}+m_{3}\right) & 0 \\
0 & 0 & m_{3}
\end{array}\right]\left[\begin{array}{l}
\ddot{q}_{1} \\
\ddot{q}_{2} \\
\ddot{q}_{3}
\end{array}\right]
$$

And finally:

$$
\frac{\partial \mathcal{L}}{\partial q}=\left[\begin{array}{c}
0  \tag{3.30}\\
0 \\
m_{3} \cdot g
\end{array}\right]
$$

By grouping the terms, we find the dynamic model of the Cartesian robot:

$$
\left[\begin{array}{l}
F_{1}  \tag{3.31}\\
F_{2} \\
F_{3}
\end{array}\right]=\left[\begin{array}{ccc}
\left(m_{1}+m_{2}+m_{3}\right) & 0 & 0 \\
0 & \left(m_{2}+m_{3}\right) & 0 \\
0 & 0 & m_{3}
\end{array}\right]\left[\begin{array}{l}
\ddot{q}_{1} \\
\ddot{q}_{2} \\
\ddot{q}_{3}
\end{array}\right]+\left[\begin{array}{c}
0 \\
0 \\
m_{3} . g
\end{array}\right]
$$

As we can observe, the represented dynamic model of the equation 3.31 does not include the friction force term.

If we replace by the physical parameters of the robot mentioned in the table below:
Table 3-2 Parameters values

| Notions | value | units |
| :--- | :--- | :--- |
| $\boldsymbol{m}_{\boldsymbol{1}}$ | 0.897 | Kg |
| $\boldsymbol{m}_{\mathbf{2}}$ | 2.87 | Kg |
| $\boldsymbol{m}_{\mathbf{3}}$ | 1.822 | Kg |
| $\mathbf{g}$ | 9.81 | $\mathrm{~m} / \mathrm{s}^{2}$ |

This table give us the final results of the dynamic model:

$$
\left[\begin{array}{l}
F_{1}  \tag{3.32}\\
F_{2} \\
F_{3}
\end{array}\right]=\left[\begin{array}{ccc}
5.58 & 0 & 0 \\
0 & 4.69 & 0 \\
0 & 0 & 1.822
\end{array}\right]\left[\begin{array}{l}
\ddot{q}_{1} \\
\ddot{q}_{2} \\
\ddot{q}_{3}
\end{array}\right]+\left[\begin{array}{c}
0 \\
0 \\
17.87
\end{array}\right]
$$

By following the equation 3.31 we can calculate torque of the motor:
We have:

$$
\begin{equation*}
\tau=\frac{F V}{\eta \omega} \tag{3.33}
\end{equation*}
$$

$r$ : the distance between motor and the applied force point

Thus:

$$
\left[\begin{array}{l}
\tau_{1} \\
\tau_{2} \\
\tau_{3}
\end{array}\right]=\left[\begin{array}{ccc}
\frac{r \theta}{\eta} & 0 & 0 \\
0 & \frac{P_{a s} \theta}{2 \pi \eta} & 0 \\
0 & 0 & \frac{r \theta}{\eta}
\end{array}\right]\left[\begin{array}{l}
F_{1} \\
F_{2} \\
F_{3}
\end{array}\right]
$$

## 3.6) Conclusion

In conclusion, the modeling and simulation study of the Cartesian robot is an important aspect of designing and optimizing the performance of such a system. Using simulation tools, such as SolidWorks, Simulink, or robotic simulation software, enables to test and validate the designs before building the physical system, saving time and money in the design process.

Overall, the geometric, kinematic, and dynamic modeling and simulation study of the robot is a crucial step in designing a high-performance, reliable, and efficient robotic system for a variety of industrial and manufacturing applications. In the next chapter we will apply the sizing of the robot's component.

# Chapter 4: Sizing of Cartesian robot 

## 4.1) Introduction

In this chapter, we delve into the crucial aspects of the Cartesian robot sizing, in particular motors selection, taking into account the resistance encountered in the guiding and the transmission systems. the critical process of Cartesian robot sizing. Determining the appropriate size and dimensions of the robot components is essential to ensure optimal performances and efficiency. We explore factors such as payload requirements, workspace dimensions, speed, and acceleration, all of which contribute to the accurate sizing of the robot. By carefully considering these factors, engineers can design a Cartesian robot that meets the specific application's needs while avoiding issues such as overloading or underutilization.

The robot that we are going to design is a linear Cartesian robot with three degrees of freedom. It must be able to move light objects not exceeding 100 gr , with a linear speed along each axis not exceeding $1 \mathrm{~cm} / \mathrm{s}$. The specifications of the robot involve also the following stokes: $\Delta X=244.85 \mathrm{~mm} \Delta Y=940 \mathrm{~mm}, \Delta Z=468.70 \mathrm{~mm}$.

This organigram bellow represents the steps to follow in this chapter in order to get the final results:


## 4.2) Technological Solutions:

For various technical reasons, we have opted for the following transmission technology solutions. For the horizontal axes of displacement in X and Y , a transmission system based on belts. For the Z axis, we opted for a screw-nut based transmission system.

After applying too many experiences on the linear robots, we achieved 3 robots as a solution:

### 4.2.1) First choice

The first Cartesian robot combines three axes, with two axes featuring belt-pulley guidance and the third axis featuring screw-nut guidance. This robot is equipped with a tool to pick and place objects, making it ideal for material handling applications. The combination of belt-pulley and screw-nut guidance allows for precise and smooth movements, ensuring accurate and efficient placement of objects.


Figure 4-1 Technological Choice $n^{\circ} 01$

### 4.2.2) Second choice

The second Cartesian robot features two axes of belt-pulley guidance and a gripper. This robot is ideal for pick-and-place applications, as the gripper allows for easy and secure manipulation of objects. The use of belt-pulley guidance ensures smooth and precise movements, allowing for accurate placement of objects in a timely manner.


Figure 4- 2 Technological Solution $n^{\circ} 02$

### 4.2.3) Third choice

The third Cartesian robot combines two axes of screw-nut guidance with a tool for surface finishing of pieces. This robot is ideal for applications that require precise and consistent surface finishes, such as polishing or grinding. The use of screw-nut guidance ensures accurate and repeatable movements, allowing for consistent surface finishes across a variety of pieces.


Figure 4- 3 Technological Solution $n^{\circ} 03$

Overall, these Cartesian robots showcase the versatility and flexibility of Cartesian robot technology. By combining different types of guidance systems and end-effectors, these robots can be tailored to a variety of applications, from material handling to surface finishing. in conclusion, the selection of technological solutions was primarily guided by a comparison with existing industrial systems and took into account factors such as cost and market availability.

### 4.2.4) Robot workspace analysis

The workspace of a three-axis Cartesian robot is defined as the area of threedimensional space that the robot can reach with its translational movements along each of the three axes. This is the working area in which the robot can execute its programmed tasks and trajectories. The robot's workspace is limited by the physical dimensions of the robot, the stroke limits of the mechanics, the maximum reach of the end effector (tool or gripper) and the movement limits imposed by the environment in which the robot operates. The volume of the workspace can be calculated from the physical dimensions of the robot and its stroke limits, and is often represented graphically as a cubic volume or prism. Simulation and modelling tools can be used to visualize and optimize the robot workspace, maximizing its efficiency and versatility for specific applications.

The extreme position of the robot axes describes a boundary for the region in which the robot operates. This boundary encloses the work envelope. The size of a work envelope determines the limits of reach. These limits are set-up in the specifications of the robot that we want to make. Our robot has three prismatic joints
whose axes are coincident with the cartesian coordinate system. Thus, the works space has the shape of a rectangular parallelepiped (see Fig. 4.4). The specifications involve the following stokes: $\Delta X=244.85 \mathrm{~mm} \Delta Y=940 \mathrm{~mm}, \Delta Z=468.70 \mathrm{~mm}$.


Figure 4-4 Robot Workspace along ZY


Figure 4- 5 Robot Workspace along X

These limits define the permissible table movements along the XY plane. By position ( $\mathrm{x}=0 / \mathrm{y}=0$, corresponding to the horizontal plane containing the robot), we have a displacement of $+/-290 \mathrm{~mm}$, a stroke of 580 mm along $X$ and $+/-532.5 \mathrm{~mm}$, a stroke of 1065 mm These are clearly shown in figure (4.1).

These limits define the permissible table movements along the Z axis. By position ( $\mathrm{z}=0$, corresponding to the vertical plane containing the robot's robot), we have a displacement of $+/-240 \mathrm{~mm}$ stroke of 480 mm . These are clearly shown in figure (4.2).

## 4.3) Study and calculation of the main machine components

The purpose of this section is to dimension and verify the various functional elements
of our machine to ensure correct operation. We'll be talking about structural calculations
and power calculations. The elements to be dimensioned are

- Trapezoidal screws.
- Trapezoidal Belts
- Bearings.
- Guide shafts.


### 4.3.1) Mass Calculation

The robot Is Made Up of Different Components:

- Normalized Components (Screw, Nuts, Bearing, Rails,). The Weights and Dimensions Listed in Catalogues.
- Drive motors, the weights are also supplied by suppliers directly on the product box, or can be found on their websites.
- Parts specially designed for our machine, whose weights are approximated by SolidWorks CAD software. This takes into account the dimensions of the part and the material of construction.
after entering the parts and defining their materials, solid works allowed us to define the masses of each piece. By taking into account the masses of each component, we were able to make precise calculations to determine the resultant forces on each robot axis. We obtain the Following results in the next tables:

Table 4-1 X axis component weights

| X-AXIS |  |  |  |
| :--- | :--- | :--- | :--- |
| $m_{i}$ | Piece Name | Number of pieces | Mass $(\mathrm{g})$ |
| $\mathrm{m}_{1}$ | Wheel Kit | 2 | 12.64 |
| $\mathrm{~m}_{2}$ | Plate | 3 | 5.17 |
| $\mathrm{~m}_{3}$ | Spacer | 3 | 0.35 |
| $\mathrm{~m}_{4}$ | Screw M5 | 3 | 0.67 |
|  |  |  |  |
|  |  | Total | 43.85 |
|  |  |  |  |
|  |  |  |  |

Table 4- 2 Z axis component weights

| Z-AXIS |  |  | Piece Name |
| :--- | :--- | :--- | :--- |
| $\mathrm{m}_{\mathrm{i}}$ | Motor+pulley+plate | 1 | Number of pieces |
| $\mathrm{m}_{1}$ | Pulley+ Plate | 1 | 649.08 |
| $\mathrm{~m}_{2}$ | Belt | 1 | 13.75 |
| $\mathrm{~m}_{3}$ | $20 \times 20$ Rail | 1 | 5.63 |
| $\mathrm{~m}_{4}$ | 20X40 Rail | 1 | 185.12 |
| m 5 | Pipe holder | 3 | 89.10 |
| m 6 | Bearings | 3 | 7.83 |
| m 7 |  | Total | 2.89 |
|  |  |  |  |

Table 4- 3 Y axis component weights

| Y-AXIS |  |  | Piece Name |
| :--- | :--- | :--- | :--- |
| $\mathrm{m}_{\mathrm{i}}$ | Full Z axis | 1 | Number of pieces |
| $\mathrm{m}_{1}$ | Full X axis | 1 | 1822.59 |
| $\mathrm{~m}_{2}$ | Guiding plates | 2 | 903.92 |
| $\mathrm{~m}_{3}$ | Cbeam | 1 | 69.37 |
| $\mathrm{~m}_{4}$ | Plate 80mm | 1 | 204.98 |
| m 5 | Bearings | 4 | 75.25 |
| m 6 |  | Total | 2.89 |

### 4.3.2) Calculations of Forces, speed

4.3.2.1) Calculations of Forces, speed exerted on the $Z$ axis


Figure 4-7 translation system along $Z$


Figure 4-6 center of gravity along Z


Figure 4- 8 kinematic schema along $Z$

The load F along Z is purely axial and, in addition to the weight of the components $\left(F_{1}\right)$, inertia forces due to acceleration and deceleration $\left(F_{2}\right)$. with the condition that the total forces will be divided on three, as this is supported by the two linear guides and the trapezoidal screw. This gives:

$$
\begin{equation*}
F \mathrm{z}=\frac{1}{3}\left(F_{1}+F_{2}\right) \tag{4.1}
\end{equation*}
$$

With :

$$
\begin{equation*}
F_{1}=\sum \mathrm{m}_{\mathrm{i}} \cdot \mathrm{~g} \tag{4.2}
\end{equation*}
$$

$\boldsymbol{g}$ : gravity
$\boldsymbol{i}$ : components weights

$$
F_{1}=\sum \mathrm{m}_{\mathrm{i}} \cdot \mathrm{~g}=974.84 \times 0.001 \times 9.81=9.56 \mathrm{~N}
$$

$\boldsymbol{r}$ : Acceleration. is given by:

$$
\Upsilon=\frac{\Delta V}{\Delta T}
$$

$\Delta V$ : the linear speed by using formula (2.3):

$$
\Delta V=\frac{p}{2 \pi} \omega=\frac{2}{2 \pi} 314.15=99.99 \mathrm{~mm} / \mathrm{s}=0.099 \mathrm{~m} / \mathrm{s}
$$

Let's take $\Delta=0.1 \mathrm{~s}$, which is a fairly short response time. We get:

$$
\Upsilon=\frac{\Delta V}{\Delta T}=\frac{99.99}{0.1}=999.97 \mathrm{~mm} / \mathrm{s}^{2}=0.99 \mathrm{~m} / \mathrm{s}^{2}
$$

So :

$$
\begin{array}{r}
F_{2}=\sum \mathrm{m}_{\mathrm{i}} \cdot \gamma=0.96 \mathrm{~N} \\
F \mathrm{z}=\frac{1}{3}\left(F_{1}+F_{2}\right)=3.5 \mathrm{~N}
\end{array}
$$

### 4.3.2.2) Forces, speed exerted on the $Y$ axis

The same resonance is used as for the Z axis, forces will be divided by two, as it is supported by the two guide axes.


Figure 4-10 translation system along $Y$


Figure 4- 9 center of gravity along $Y$


Figure 4-11 kinematic schema along Y

$$
\begin{gathered}
F y=\frac{1}{2}\left(F_{1}+F_{2}\right) \quad \text { with } \quad m_{\mathrm{i}}=3157.04 \mathrm{~g} \\
F 1=\sum \mathrm{m}_{\mathrm{i}} \mathrm{~g}=3157.04 \times 0.001 \times 9.81=30.97 \mathrm{~N}
\end{gathered}
$$

$\boldsymbol{F}_{\mathbf{2}}$ Also changes given that it's a function of mass
So :

$$
\begin{gathered}
F_{2}=\sum \mathrm{m}_{\mathrm{i}} \cdot Y \\
\Delta V=R \times \omega=6 \times 314.15=1884.9 \mathrm{~mm} / \mathrm{s}=1.8849 \mathrm{~m} / \mathrm{s} \\
\Upsilon=\frac{\Delta V}{\Delta T}=\frac{1884.9}{0.1}=18849 \mathrm{~mm} / \mathrm{s}^{2}=18.849 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$

$$
\begin{gathered}
F_{2}=3157.04 \times 0.001 \times 18.849=59.50 \mathrm{~N} \\
F y=\frac{1}{3}\left(F^{1}+F^{2}\right)=30.15 \mathrm{~N}
\end{gathered}
$$

### 4.3.2.3) Forces, speed exerted on the $X$ axis

The same resonance is used as for the Y axis, forces will be divided by three, as it is supported by the three roller guide.


Figure 4-13 translation system along X


Figure 4-12 center of gravity along $X$

$$
\begin{gathered}
F x=\frac{1}{3}\left(F_{1}+F_{2}\right) \quad \text { with } \quad m_{\mathrm{i}}=43.85 \mathrm{~g} \\
F_{1}=\sum \mathrm{m}_{\mathrm{i}} \cdot \mathrm{~g}=43.85 \times 0.001 \times 9.81=0.43 \mathrm{~N}
\end{gathered}
$$

$\boldsymbol{F}_{2}$ Also changes given that it's a function of mass
So :

$$
F_{2}=\sum \mathrm{m}_{\mathrm{i}} \cdot \gamma
$$

$\Delta V=R \times \omega=6 \times 314.15=1884.9 \mathrm{~mm} / \mathrm{s}=1.8849 \mathrm{~m} / \mathrm{s}$

$$
\Upsilon=\frac{\Delta V}{\Delta T}=\frac{1884.9}{0.1}=18849 \mathrm{~mm} / \mathrm{s}^{2}=18.849 \mathrm{~m} / \mathrm{s}^{2}
$$

$F_{2}=43.85 \times 0.001 \times 18.849=0.82 \mathrm{~N}$

$$
F x=\frac{1}{3}\left(F^{1}+F_{2}\right)=0.41 N
$$

### 4.3.3) Transmission system calculation

### 4.3.3.1) Calculations on trapezoidal screws

The trapezoidal screw/nut pair has long been used to transform rotary motion into linear motion in a wide range of applications. rotary motion into linear motion. The total power $\left(P_{\mathrm{t}}\right)$ transmitted by the screw to the nut is transformed into power $\left(P_{\mathrm{u}}\right)$. The ratio $\frac{P_{\mathrm{u}}}{P_{\mathrm{t}}}=\eta$ defines the system's efficiency, which depends on the coefficient of friction between the contact surfaces of the screw, nut and thread helix angle.

This is sliding friction. Part of the power is therefore transformed into heat with each movement. It is possible to parameterize this sliding friction to evaluate the proper functioning of the assembly. The criteria is to limit the contact surface pressure on the thread side, to enable smooth sliding between the two surfaces.

We also limit the product $P . V_{\text {st }}\left(P=\right.$ contact surface pressure. and $V_{\text {st }}=$ friction velocity over the average thread diameter) to limit the power lost in the form of heat. as heat. This keeps the temperature of the surfaces in contact in check. This limitation is important because if we use bronze nuts, it's important not to " damage " the lubricant, where as if we use self-lubricating polyamide nuts, which require no extra oil or grease, the temperature must be controlled. the permissible values of the product $P$. $V_{\text {st }}$ are reduced. [19]

### 4.3.3.1.1) Calculation of contact surface pressure " $\boldsymbol{P}_{\mathrm{s}}$ "

The contact surface pressure " $p$ " is calculated using the following formula [19]:

$$
\begin{equation*}
P=\frac{F}{A t} \tag{4.3}
\end{equation*}
$$

$\boldsymbol{F}$ : Axial force $[\mathrm{N}]$.
$\boldsymbol{A t}$ : Total bearing surface between screw teeth and nut teeth. in the plane perpendicular to the axis. [mm2].

In our case we can get $A t$ from the software SolidWorks (2022). $A t=101 \mathrm{~mm}^{2}$

### 4.3.3.1.2) Calculation of sliding speed « $V_{\text {st }}$ "

The sliding speed can be calculated using one of the following formulas [19]:

- if we have already defined the number of revolutions per minute of the screw

$$
\begin{equation*}
V_{\mathrm{st}}=\frac{N p_{\mathrm{as}}}{1000 \sin \alpha} \tag{4.4}
\end{equation*}
$$

$N=$ number of screw revolutions per minute.
$\boldsymbol{p}_{\text {as }}=$ thread pitch [mm]
$\boldsymbol{\alpha}=$ thread helix angle

- if we have already defined the transfer speed of the nut:

$$
\begin{equation*}
V_{\mathrm{st}}=\frac{V_{\mathrm{tr}}}{\sin \alpha} \tag{4.5}
\end{equation*}
$$

$\boldsymbol{V} \boldsymbol{s t}=$ sliding speed on average diameter. [ $\mathrm{m} / \mathrm{min}$ ]
$\boldsymbol{V} \boldsymbol{t r}=$ transfer speed $[\mathrm{m} / \mathrm{min}]$.

NOTE: Screw rotation in revolutions per minute and nut transfer speed are linked by the following formula :

$$
\begin{equation*}
N=\frac{1000 \mathrm{Vtr}}{p_{\text {as }}} \tag{4.6}
\end{equation*}
$$

In our case, we set the number of screw revolutions per minute to 3000 rpm , so we use the equation (4.4) for bronze nuts (which is our case), the product $p . v s t$ is used to draw the graph shown in figuree where three zones are highlighted, each characterized under certain conditions of use, between sliding speed and pressure.


Figure 4-14 Bronze sliding condition [19]
$>$ Zone A: Zone A is the limit $P \cdot V_{\mathrm{st}}=21[\mathrm{~N} / \mathrm{mm} 2 \bullet \mathrm{~m} / \mathrm{min}]$
In this zone, operation is under the best conditions.
Continuous use" is possible because the amount of heat produced between these $P$. $V_{\text {st }}$ limits is fairly
controlled. Nut life is very good.
$>$ Zone B: Zone B is the limit of $P . V_{\mathrm{st}}=80[\mathrm{~N} / \mathrm{mm} 2 \bullet \mathrm{~m} / \mathrm{min}]$.
In this zone, operation takes place under difficult conditions.
Sliding conditions are such that constant lubrication is required to control erosion of bronze
to ensure long nut life. Continuous" use is only possible for limited periods of time periods, as the amount of heat generated is such as to cause the nut to heat up considerably.
on the quality of the lubricant used (the lubricant's contribution to dissipating the heat generated).

In this case, the nut's service life is limited.
$>$ Zone C: Zone C is the limit $P . V_{\text {st }}=250[\mathrm{~N} / \mathrm{mm} 2 \bullet \mathrm{~m} / \mathrm{min}]$.
In this zone, operating conditions are very severe.
With these $P . V_{\text {st }}$ values, working in "continuous use" is not possible. Even with good lubrication lubrication, intense heating and premature wear of the nut occur, as the sliding between the contact surfaces contact surfaces, leading to rapid erosion of the nut.

The coefficient " $f i$ " is used to correct the value of the product " $p . V s t$ ) max" found in graph applying that the sliding speed acceptable at the contact surface pressure value is considered to be the limit given by the "zone" (A, B or C).being the limit given by the "zone" (A, B or C) where we wish to work. To find the permissible $p . V s t$ for the case under study, use the formula

$$
\begin{equation*}
\text { P.Vst. } f \mathrm{i} \tag{4.7}
\end{equation*}
$$

Table 4.4. Safety coefficients with respect to inertial forces [19]

| Load type | $\boldsymbol{f} \boldsymbol{i}$ |
| :--- | :--- |
| Constant loads with controlled acc/dec ramps | From 1 to 0,5 |
| Constant loads with rip starts and stops | From 0,5 to 0,33 |
| Highly variable loads and speeds | From 0,33 to 0,25 |
| Shock and vibration loads | From 0,25 to 0,17 |

Our system's load type is a constant load with controlled acc/dec ramps, so $0.5<f i<$ 1, we'll take $f i=0.7$

### 4.3.3.1.3) Z-axis component calculations:

### 4.3.3.1.3.1) Forces, speed and pressure exerted on the Z-axis:

According to (4.3), the contact surface pressure is

$$
P=\frac{F}{A t}=\frac{3.5}{101}=0.034 \mathrm{~N} / \mathrm{mm}^{2}
$$

helix angle:

$$
\alpha=\tan ^{-1} \frac{p_{\text {as }}}{\pi D}=4.549
$$

According to (4.4), the sliding speed is:

$$
V_{\mathrm{st}}=\frac{1000 \times 3}{1000 \times \sin 4.549}=25.21 \mathrm{~m} / \mathrm{min}
$$

Now let's calculate the product by using Formula (4.7)

$$
P . V_{\text {st. }} f i=0.034 \times 25.21 \times 0.7=0.30 \mathrm{MPa} . \mathrm{m} / \mathrm{min}
$$

In conclusion, according to the Figure (4.14), the screw operates in zone A. Operating conditions are ideal.

### 4.3.3.2) Calculations on Trapezoidal Belts

In order to make the best choice of belt with good performance to our machine, here we'll identify the type trapezoidal from the graph and to calculate we need to know certain data [20].

Bellow we have the following data:
-p: engine pouwer $=3 \mathrm{Kw}$
$-\boldsymbol{N}_{\boldsymbol{D}}:$ motor rotational speed $=3000 \mathrm{tr} / \mathrm{min}$
-D: driven pulley diameter $=12.22 \mathrm{~mm}$
-d: driver puller parameter $=12 \mathrm{~mm}$
-The desired distance between the two pulleys is : $C=980 \mathrm{~mm}$

- First, we calculate the operating engine $P_{s}$ :

$$
\begin{equation*}
P_{s}=P . K_{s} \tag{4.8}
\end{equation*}
$$

The operating coefficient is known in the table (4.5).
We found: $K_{s}=1.4$ because the service life ranges from [16h; 24h] plus it is assumed that the device can be subjected to uniform transmission without jolts.

$$
P_{s}=3 \times 1.4=4.2 \mathrm{Kw}
$$

We can conclude that he types of belt is A because the point with the coordinate( $P_{S}, N_{D}$ ) belongs to the zone of A of the Figure(4.2)

### 4.3.3.2.1) Kinetic study

In this section we will study the kinetic part according to four criteria:
In order to determine the belt linear speed, belt center distance, belt length ( $L p$ ), and winding angle $\left(\theta_{m}\right)$, belt linear speed the following steps were performed.
by using this Formula [20]

$$
\begin{align*}
V=r . \omega & =\frac{\pi D N_{D}}{60}  \tag{4.9}\\
V & =\frac{3.14 \times 12.22 \times 3000}{60}=1.19 \mathrm{~m} / \mathrm{s}
\end{align*}
$$

- Determination of belt length :

$$
\begin{equation*}
L_{p}=2 C+1.57(D+d)+\frac{(d+D)^{2}}{4 C} \tag{4.10}
\end{equation*}
$$

So :

$$
L p=2 \times 980+1.57(12.22+12)+\frac{(12.22+12)^{2}}{4 \times 980}=1998.17 \mathrm{~mm}
$$

From the table (4.6) we get $L p=2013$, andAccording to figure (4.15) we get $K_{L}=1.05$ the winding angle $\theta_{m}$ :

$$
\theta_{m}=180-2 \sin ^{-1}\left(\frac{D-d}{2 \times C}\right)=179.98^{\circ}
$$

And According to figure (4.16) we got $K_{\theta}=0.98$

Basic belt power. From the table (4.7):
$P_{b}=3.5$
Calculation of power rating $P_{a}$ :

$$
\begin{aligned}
& P_{a}=P_{b} \times K_{L} \times K_{\theta} \\
& \qquad P_{a}=3.5 \times 1.05 \times 0.98=3.6 \mathrm{kw}
\end{aligned}
$$

Determination the number of belts:

$$
n_{b}=\frac{P_{s}}{P_{a}}=\frac{3.5}{3.6}=0.97 \approx 1
$$

In conclusion we need only one belt.
Bellow we have the tables and figures that helped us to reach our results:
Table 4- 4 Service Factor Value $K_{s}$

|  | service léger <br> 0 à $6 \mathrm{~h} /$ jour | service normal <br> 6 à $16 \mathrm{~h} /$ jour | service dur <br> 16 à $24 \mathrm{~h} / \mathrm{jour}$ | service très dur <br> en continu |
| :---: | :---: | :---: | :---: | :---: |
| transmission uniforme <br> sans à-coups | 1,0 | 1,2 | 1,4 | 1,6 |
| transmission avec légers <br> à-coups et chocs modérés | 1,1 | 1,3 | 1,5 | 1,8 |
| transmission avec à-coups <br> et chocs élevés * | 1,2 | 1,4 | 1,7 | 2,1 |

* avec des inversions de sens, des démarrages fréquents sous forts couples

Table 4-5 Basic Length Of Trapezoidal Belts $L_{p}$ [20]

| z | $270,295,340,380,405^{*}, 435,465,475^{*}, 485,505,530^{*}, 545,570,610,625^{*}, 635,675,700^{*}, 710,750,780^{*}$, $790,840,895,920^{*}, 940,1000,1055,1080^{*}, 1095,1145,1205,1250,1330^{*}, 1420^{*}, 1540^{*} \ldots$ <br> $790,840,895,920^{*}, 940,1000,1$ 055, $1080^{*}, 1095,1$ 145, 1 205, 1 250, $1330^{*}, 1420^{*}, 1540^{*} \ldots$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | $\begin{aligned} & 415,490,541,585,620,630^{*}, 670,700^{*}, 719,770,790^{*}, 820,871,890^{*}, 933,983,990^{*}, 1049,1100^{*}, 1153, \\ & 1201,1250^{*}, 1303,1353,1405,1430^{*}, 1455,1508,1550^{*}, 1608,1640^{*}, 1709,1750^{*}, 1858,1913, \\ & 1940^{*}, 2013,2050^{*}, 2133,2200^{*}, 2273,2300^{*}, 2393,2480^{*}, 2533,2700^{*}, 2833,3183 \ldots \end{aligned}$ |  |  |  |  |
| B | $613,655,680,729,780,830,881,930^{*}, 980,1000^{*}, 1033,1083,1100^{*}, 1133,1$ 185, $1210^{*}, 1243,1318$, $1370^{*}, 1393,1465,1560^{*}, 1668,1760^{\star}, 1872,1950^{\star}, 2075,2180^{*}, 2283,2300^{*}, 2380,2480,2500^{*}$, $2659,2700^{*}, 2$ 870*, 3 200*, 3 393, 3 600*, 3793,4 060*, 4 430*, 4 820*, 5 043, 5 370*, 5 620, $6070^{*}, 6585$ |  |  |  |  |
| c | 920, 1 075, 1 152, 1 312, 1 462, $1505^{*}, 1662,1760^{*}, 1840,1950^{*}, 2094,2195^{*}, 2348,2420^{*}, 2500,2715^{*}$, $2907,2880^{*}, 3080^{*}, 3$ 312, $3520^{*}, 3720,3964,4060^{*}, 4177,4278,4600^{*}, 5$ 015, $5380^{*}, 5662,6100^{*}$, $6362,6815^{*}, 7035,7600^{*}, 8038,8444,9100^{*}, 10062,10700^{*}$.. |  |  |  |  |
| D | ```2 576, 2 740*, 2 876, 3 100*, 3 226, з 330*, 3 530, 3 730*,4 080*, 4 386, 4 620*, 5 029, 5 400*, 5 676, }6\mathrm{ 100*, 6 370, }6\mathrm{ 840*, }7\mathrm{ 126, }7\mathrm{ 620*, 8 000, 8 405, 9 140*, 10 700*, 11 276, 12 200*, 13 700*, }15\mathrm{ 200*...``` |  |  |  |  |
| E | $4660^{*}, 5040^{*}, 5105,5420^{*}, 5765,6100^{*}, 6505,6850^{*}, 7265,7650^{*}, 8055,8410,8790,9150^{*}, 10035$, $11230,12230^{*}, 13750^{*}, 15280^{*}, 16800^{*} \ldots$ |  |  |  |  |
|  | SPZ | SPA | SPB | SPC |  |
| Streries | $\begin{gathered} 630 \\ \text { à } \\ 3550 \end{gathered}$ | $\begin{gathered} 800 \\ \text { à } \\ 4500 \end{gathered}$ | $\begin{aligned} & 1250 \\ & \text { à } \\ & 8000 \end{aligned}$ | $\begin{gathered} 2000 \\ \text { a } \\ 1250 \end{gathered}$ |  |

Table 4- 6 Basic Power Of Trapezoidal Belts $P_{b}$ [20]

| type | diamètre | vitesse linéaire $V$ de la courroie ( $\mathrm{m} / \mathrm{s}$ ) |  |  |  |  | type courroie | diamètre primitif | vitesse linéaire V de la courroie ( $\mathrm{m} / \mathrm{s}$ ) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 | 10 | 15 | 20 | 25 |  |  | 5 | 10 | 15 | 20 | 25 |
| Z | 50 | 0,45 | 0,72 | 0,85 | - | - | C | 180 | 3,92 | 6,10 | 7,33 | 8,32 | - |
|  | 60 | 0,62 | 1,05 | 1,35 | - | - |  | 210 | 4,59 | 7,38 | 9,40 | 10,86 | 11,76 |
|  | 70 | 0,73 | 1,29 | 1,70 | 1,94 | - |  | 240 | 4,99 | 8,31 | 10,82 | 12,75 | 14,14 |
|  | 80 | 0,83 | 1,48 | 1,97 | 2,30 | 2,41 |  | 280 | 5,50 | 9,27 | 12,26 | 14,70 | 16,50 |
|  | 90 | 0,90 | 1,61 | 2,18 | 2,58 | 2,76 |  | 320 | 5,83 | 9,96 | 13,34 | 16,10 | 18,29 |
|  | 100 | 0,95 | 1,72 | 2,37 | 2,80 | 3,04 |  | 360 | 6,14 | 10,56 | 14,16 | 17,19 | 19,69 |
|  | 110 | 1,00 | 1,82 | 2,48 | 2,99 | 3,27 |  | 430 | 6,55 | 11,25 | 15,32 | 18,68 | 21,43 |
| A | 85 | 1,25 | 2,04 | 2,66 | 3,01 | - | D | 290 | 8,92 | 13,44 | 15,95 | 16,80 | - |
|  | 100 | 1,42 | 2,37 | 3,12 | 3,99 | 4,10 |  | 320 | 9,84 | 15,41 | 18,90 | 20,74 | 20,92 |
|  | 115 | 1,55 | 2,64 | 3,52 | 4,21 | 4,73 |  | 360 | 10,94 | 17,50 | 22,07 | 24,96 | 26,19 |
|  | 130 | 1,65 | 2,85 | 4,04 | 4,60 | 5,22 |  | 400 | 11,80 | 19,20 | 24,61 | 28,33 | 30,42 |
|  | 150 | 1,75 | 3,03 | 4,10 | 4,80 | 5,72 |  | 460 | 12,78 | 21,18 | 27,55 | 32,29 | 34,37 |
|  | 170 | 1,82 | 3,19 | 4,33 | 5,00 | 6,10 |  | 520 | 13,58 | 22,71 | 29,85 | 35,35 | 39,20 |
|  | 190 | 1,87 | 3,30 | 4,54 | 5,55 | 6,39 |  | 580 | 14,16 | 23,96 | 31,64 | 37,76 | 42,80 |
| B | 120 | 2,11 | 3,23 | 4,23 | 4,80 | - | E | 440 | 10,97 | 18,85 | 24,69 | 28,33 | 29,44 |
|  | 140 | 2,35 | 3,95 | 5,02 | 5,83 | 6,37 |  | 480 | 11,89 | 20,65 | 27,39 | 31,92 | 33,91 |
|  | 160 | 2,57 | 4,03 | 5,61 | 6,63 | 7,37 |  | 520 | 12,62 | 22,15 | 29,63 | 34,95 | 37,68 |
|  | 180 | 2,72 | 4,39 | 6,09 | 7,24 | 8,14 |  | 600 | 13,84 | 24,57 | 33,28 | 39,86 | 43,75 |
|  | 200 | 2,81 | 4,81 | 6,42 | 7,71 | 8,75 |  | 700 | 14,94 | 27,26 | 36,66 | 44,28 | 49,35 |
|  | 220 | 2,92 | 4,89 | 6,73 | 8,13 | 9,24 |  | 800 | 15,77 | 28,50 | 39,18 | 47,60 | 53,56 |
|  | 250 | 3,01 | 5,06 | 6,89 | 8,64 | 8,85 |  | 950 | 16,82 | 30,40 | 42,00 | 51,40 | 59,13 |



Figure 4-15 Primary Length [20]


Figure 4-16 Roll Angle [20]

## 4.4) Linear Rail Guide System Calculations

### 4.4.1) Bearing calculations

Bearings at the screw and belt bearings are dimensioned according to two criteria [21]:
$>$ The dynamic load capacity. That is, $C$ bearing $>C$ necessary.
$>$ The service Life. $L_{\mathrm{h}}$ bearing $>L_{\mathrm{h}}$ necessary
For this we will use the following formulas [21]:

$$
\begin{equation*}
C=\frac{f_{\mathrm{z} f_{\mathrm{l}} P}}{f_{\mathrm{t}} f_{\mathrm{n}} f_{\mathrm{h}}} \tag{4.12}
\end{equation*}
$$

. $\boldsymbol{f z}$ : Additional coefficient for dynamic forces. We take $f z=1$.
. $\boldsymbol{f}_{\mathrm{h}}$ : Hardness coefficient. Consider $f_{\mathrm{h}}=1$.
. $\boldsymbol{f}_{\mathrm{t}}$ : Temperature coefficient. In our case $f_{\mathrm{t}}=1$
. $\boldsymbol{f}_{1}$ : Coefficient of service life. We take $L_{\mathrm{h}}$ necessary $=60000$ hours

$$
\begin{equation*}
f_{1}=\sqrt[3]{\frac{L_{\mathrm{h}} \text { necessary }}{500}} \tag{4.13}
\end{equation*}
$$

. $\boldsymbol{f}_{\mathrm{n}}$ : Coefficient de nombre de tours.

$$
\begin{equation*}
f_{1}=\sqrt[3]{\frac{100}{3 N}} \tag{4.14}
\end{equation*}
$$

. $\boldsymbol{N}$ : speed in rpm $=1000$

$$
\begin{equation*}
L_{\mathrm{h}}=\left(\frac{C}{P}\right)^{K} \cdot \frac{10^{6}}{60 N} \tag{4.15}
\end{equation*}
$$

. $\boldsymbol{K}=3$ for Ball bearing

### 4.4.2) Y-axis Bearing Sizing

The load carried by the bearing is divided by four because it is distributed over two guide rails and a trapezoidal Belt.

Knowing that $P=\frac{F_{\gamma}}{4}=\frac{30.5}{4}=7.62 \mathrm{~N}$
$>$ The result is

$$
C=\frac{4.9 \times 1 \times 7.62}{1 \times 0.22 \times 1}=169.71 \mathrm{~N}
$$

> We chose the LM8UU Ball Bearing C $=260 \mathrm{~N}$
$>$ We now check the service life for the chosen bearing.

$$
\begin{aligned}
L_{\mathrm{h}} & =\left(\frac{260}{7.62}\right)^{3} \cdot \frac{10^{6}}{60 \times 3000}=220689 \text { hours } \\
L_{\mathrm{h}} \text { bearing } & =220689 \text { hours } \gg L_{\mathrm{h}} \text { necessary }=60000 \text { hours }
\end{aligned}
$$

### 4.4.3) Z-axis Bearing Sizing

The load carried by the bearing is divided by three because it is distributed over two guide rails and a trapezoidal screw.

Knowing that $P=\frac{F \mathrm{z}}{3}=\frac{3.5}{3}=1.16 \mathrm{~N}$
$>$-The result is

$$
C=\frac{4.9 \times 1 \times 1.16}{1 \times 0.22 \times 1}=25.83 \mathrm{~N}
$$

$>$-We chose the LM8UU Ball Bearing C dyn $=260 \mathrm{~N}$
$>$-We now check the service life for the chosen bearing.

$$
\begin{gathered}
L_{\mathrm{h}}=\left(\frac{260}{3.33}\right)^{3} \cdot \frac{10^{6}}{60 \times 1000}=220689 \text { hours } \\
L_{\mathrm{h}} \text { bearing }=220689 \text { hours } \gg L_{\mathrm{h}} \text { necessary }=60000 \text { hours }
\end{gathered}
$$

## 4.5) deflection Study:

$y$-axis plates are translationally guided on smooth and ball bearings. These rods are mainly subjected to bending, we will follow the following calculation method [22]:


Figure 4-17 Forces applied on axis
With :

$$
c=a+b=L-a
$$

$\boldsymbol{P}$ represents the weight of the elements supported per rod. This weight is represented at the middle of the plate applied to the rod where the deflection is maximum.

Since the system is symmetrical, we deduce that:

$$
R_{A}=R_{B}
$$

$R_{A}$ and $R_{B}$ Support reactions


Figure 4-18 Forces and Reactions applied on axis

From figure (4.18) we found this:

$$
R_{A}=R_{B}=F
$$

After calculation we found :

$$
\begin{array}{cc}
R_{A c}=F & R_{C D}=0 \\
M_{A c}=F x & M_{C D}=F a
\end{array}
$$

M : Deflection moment
to calculate the arrow by using application of the double integrating rule. we get :

$$
\begin{equation*}
E I y^{\prime \prime}=M_{f} \tag{4.16}
\end{equation*}
$$

After calculation we found:

$$
\begin{aligned}
& y_{A C}=\frac{F x}{6 E I}\left(3 a(L-a)-x^{2}\right) \\
& y_{C D}=\frac{F a}{6 E I}\left(3 x(L-a)-a^{2}\right)
\end{aligned}
$$



Figure 4-19 shaft deformation [22]

$$
\begin{array}{r}
f_{c}=f_{d}=\frac{F a^{2}}{6 E I}(3 L-4 a) \\
f_{0}=\frac{F a}{24 E I}\left(3 L^{2}-4 a^{2}\right) \text { with } x_{0}=\frac{L}{2} \tag{4.17}
\end{array}
$$

## DATA:

$F=7.74 N$,
$E=203000 M p a$
$L=980.5 \mathrm{~mm}$,
$a=420 \mathrm{~mm}$,
$I=\frac{\pi}{64} D^{4}=201 \mathrm{~mm}$
Numerical application

$$
f_{0}=\frac{7.74 \times 420}{24 \times 203000 \times 201}\left(3 \times 980.5^{2}-4 \times 420^{2}\right)=7.201 \mathrm{~mm}
$$

## Verification with RDM 6 Software:



Figure 4-20 shaft deformation in software
In conclusion we reached solution to reduce the deflection by adding four supports as it shows in the following figure (4.21):


Figure 4-21 Support solution
4.6) Motor Sizing:

### 4.6.1) On Trapezoidal Screws

To select the drive motors for the trapezoidal screws, it is necessary to determine the torques
of each motor. The formulas are from chapter 2 (2.18):

$$
\begin{gather*}
C=\frac{F p_{\text {as }}}{2 \pi \eta}  \tag{4.18}\\
\eta=\frac{1-f \tan \alpha}{1+\frac{f}{\tan \alpha}} \tag{4.19}
\end{gather*}
$$

the principle of conservation of input/output power is known as following formulas:

$$
\begin{gather*}
P_{e}=C \omega  \tag{4.20}\\
P_{s}=F V \tag{4.21}
\end{gather*}
$$

$\eta$ : efficiency
$P_{e}$ : input power
$P_{s}$ : output power
$\boldsymbol{\alpha}$ : Helix angle
$\boldsymbol{f}$ : the coefficient of friction: in our case, the screw is made of steel and the nut is made of bronze, without lubrication so we take 0.2



Figure 4- 22 efficiency [19]
according to (4.19) we then have

$$
\eta=0.28=28 \%
$$

4.6.1.1) Determining torque and power requirements along the $Z$ axis We have: $F=3.5 \mathrm{~N}$

The torque required to move the rail $(20 \times 40)$ along Z is given by (4.18):

$$
C_{z}=\frac{3.5 \times 2}{2 \times \pi \times 0.28 \times 1000}=0.00397 \mathrm{~N} . \mathrm{m}
$$

The input power required will be:

$$
P_{e Z}=0.00397 \times 314.15=1.24 \mathrm{~W}
$$

The output power required will be:

$$
P_{s z}=0.41 \times 1.8849=0.772809 \mathrm{~W}
$$

We have chosen a Nema 17 stepper motor with a torque of $0.016 \mathrm{~N} . \mathrm{m}$

### 4.6.2) On Trapezoidal Belts

To select the drive motors for the trapezoidal Belts, it is necessary to determine the torques
of each motor. The formulas are as follows

$$
\begin{equation*}
C=\frac{F \cdot R}{\eta} \tag{4.22}
\end{equation*}
$$

for the efficiency we choose $\eta=90 \%$ because on trapezoidal belts the efficiency is between $96 \%$ and $90 \%$.

### 4.6.2.1) Determining torque and power requirements along the $x$ axis

 We have: $\quad F=0.41 \mathrm{~N}$The torque required to move Roller guide along $X$ is given by (4.21):

$$
C_{x}=\frac{0.41 \times 6.1}{0.9 \times 1000}=0.00277 \mathrm{~N} . \mathrm{m}
$$

The input power required will be:

$$
P_{e x}=0.00277 \times 314.15=0.87 \mathrm{~W}
$$

The output power required will be:

$$
P_{s z}=0.41 \times 1.8849=0.772809 \mathrm{~W}
$$

We have chosen a Nema 17 stepper motor with a torque of $0.016 \mathrm{~N} . \mathrm{m}$

### 4.6.2.2) Determining torque and power requirements along the $Y$ axis

We have: $\quad F=30.15 \mathrm{~N}$
The torque required to move the axes Z and X along Y is given by (equation):

$$
C_{y}=\frac{30.15 \times 6.1}{0.9 \times 1000}=0.20 \mathrm{~N} . \mathrm{m}
$$

The input power required will be:

$$
P_{e y}=0.2 \times 314.15=62.83 \mathrm{~W}
$$

The output power required will be:

$$
P_{S Z}=3.5 \times 1.8849=56.829735 \mathrm{~W}
$$

We have chosen a Nema 17 stepper motor with a torque of 0.31 N . m

## 4.5) Static study

The robot's ratio is subject to the load to be moved, to inertial forces caused by during acceleration and deceleration. The static calculation is used to determine the rigidity of the support in order to validate the robot's accuracy of the robot and the stresses applied.
4.5.1) Model Information

| Model name: Assem1 Current Configuration: Default |  |  |
| :---: | :---: | :---: |
| Boss-Extrude1 | Solid Body | Mass:0,555434 kg <br> Volume:0,000205716 m^3 <br> Density: $2700 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ <br> Weight:5,44326 N |
| Boss-Extrude1 | Solid Body | Mass:0,0126366 kg <br> Volume:1,05305e-05 m^3 <br> Density: $1200 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ <br> Weight:0,123838 N |
| Boss-Extrude1 | Solid Body | Mass:0,0126366 kg <br> Volume:1,05305e-05 m^3 <br> Density: $1200 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ <br> Weight:0,123838 N |
| Boss-Extrude1 | Solid Body | Mass:1,15773 kg <br> Volume:0,00042879 m^3 <br> Density: $2700 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ <br> Weight:11,3458 N |
| Boss-Extrude1 | Solid Body | Mass:0,185115 kg <br> Volume:6,85612e-05 m^3 <br> Density: $2700 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ <br> Weight:1,81413 N |

### 4.5.2) Material Properties

We define material of each solid by right-click its icon in the Simulation study tree and select Apply/Edit Material.as it shown in the following table:

| Model Reference | Properties | Components |
| :---: | :---: | :---: |
|  |  | SolidBody 1(Boss-Extrude1)(V-Slot 20x60x1000 Linear Rail-1), <br> SolidBody 1(BossExtrude1)(axe z-1), <br> SolidBody 1(BossExtrude1)(rail 3-1) |
| Curve Data:N/ A |  |  |
|  |  | SolidBody 1(Boss-Extrude1)(V-Slot <br> Gantry Plate 20mm1), <br> SolidBody 1(Boss-Extrude1)(V-Slot Gantry Plate 20 mm 3) |


|  | Thermal <br> expansion <br> coefficient: | $60 /$ Kelvin |  |
| :--- | :--- | :--- | :--- |
| Curve Data:N/A |  |  |  |


| Fixture <br> name | Fixture Image | Fixture Details |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Fixed-1 |  |  | Entities: <br> Type: | 6 edge(s), 6 face(s) <br> Fixed Geometry | | Resultant Forces |
| :--- |
| Components X $\mathbf{Y}$ Z Resultant <br> Reaction force(N) $\mathbf{0 , 0 0 1 7 2 1 1}$ 49,9556 0,00417565 49,9556 <br> Reaction <br> Moment(N.m) $\mathbf{0}$ $\mathbf{0}$ $\mathbf{0}$ $\mathbf{0}$ |

### 4.5.3) Loads and fixtures

We define external loads applied of each piece as $u$ can see at the next table:

| Load name | Load Image | Load Details |
| :---: | :---: | :---: |
| Gravity-1 |  | Reference: Top Plane <br> Values: $00-9,81$ <br> Units: $\mathrm{m} / \mathrm{s}^{\wedge} 2$ |
| Distributed <br> Mass-1 |  | Entities: 2 face(s) <br> Type: Displacement <br> (Direct transfer) <br> Coordinate System: Global cartesian <br> coordinates <br> Translation Values: $--------\quad \mathrm{mm}$ <br> Rotation Values: $---;----\mathrm{deg}$ <br> Reference coordinates: 000 mm <br> Remote Mass: 0.01 kg |


|  |  | Moment of Inertia: Components transferred: | $\begin{aligned} & 0 ; 0 ; 0 ; 0 ; 0 ; 0 \mathrm{~kg} . \mathrm{m}^{\wedge} 2 \\ & \text { NA } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Distributed <br> Mass-2 |  | Entities: <br> Type: <br> Coordinate System: <br> Translation Values: <br> Rotation Values: <br> Reference coordinates: <br> Remote Mass: <br> Moment of Inertia: <br> Components <br> transferred: | 1 face(s) <br> Displacement <br> (Direct transfer) <br> Global cartesian coordinates <br> ---;---; --- mm <br> ---;---; --- deg <br> 000 mm <br> 3.157 kg <br> 0;0;0;0;0;0 kg.m^2 <br> NA |

### 4.5.4) Mesh information

When it comes to mesh we used the standard tool (mesh the model) and we get this tables:

| Mesh type | Solid Mesh |
| :--- | :--- |
| Mesher Used: | Blended curvature-based mesh |
| Jacobian points for High quality mesh | 16 Points |
| Maximum element size | $34,1026 \mathrm{~mm}$ |
| Minimum element size | $1,70513 \mathrm{~mm}$ |
| Mesh Quality | High |
| Remesh failed parts independently | Off |

4.5.4.1) Mesh information - Details

| Total Nodes | 506170 |
| :--- | :--- |
| Total Elements | 266380 |
| Maximum Aspect Ratio | 131,9 |
| \% of elements with Aspect Ratio < 3 | 45,1 |
| Percentage of elements with Aspect <br> Ratio > 10 | 16,7 |
| Percentage of distorted elements | 0 |
| Time to complete mesh(hh;mm;ss): | $00: 02: 22$ |

4.5.5) Study Results


Assem1-Static 1-Stress-Stress1

| Name | Type | Min | Max |
| :--- | :--- | :--- | :--- |
| Displacement1 | URES: Resultant | $0,000 \mathrm{e}+00 \mathrm{~mm}$ | $1,017 \mathrm{e}-$ |
|  | Displacement | Node: 136237 | 03 mm |
|  |  |  | Node: 2169 |



Assem1-Static 1-Strain-Strain1

## Model name: As sem1

Study name: Static 1(-Defaut-)
Plot type: Static displacementD isplacement1

$\ldots$

Assem1-Static 1-Displacement-Displacement1

## Chapter 4: Sizing of Cartesian robot

The displacement calculation validates the rigidity of the axes, and shows that the maximum displacement is at the axis Z . These displacements can affect the robot's precision. In our case its an acceptable value.
The Z axis is the most stressed, with a maximum stress of $7,345.10^{6} \mathrm{~N} / \mathrm{m}^{2}$ this maximum stress is well below the elastic limit of the material, so the axis is resistant to external loads.

## 4.6) Conclusion

this chapter provides a comprehensive framework for robot sizing, covering key aspects such as transmission systems, motor selection, deflection studies, and guidance system evaluation. The knowledge gained from this processare reflected in the design and construction of well-optimized robots that can perform their intended tasks effectively and efficiently.

## Chapter 5: Assembly and programming

## 5.1) Introduction

This chapter will be dedicated to describing the development of the electrical and computer parts of our machine. Our choices naturally gravitated towards widely used open-source solutions within the maker community. In particular, we will be utilizing GRBL, Arduino, and CNC-shield. GRBL is an open-source firmware for a 3-axis microcontroller, specifically designed for the Arduino UNO board with an ATMega 328 chip. It has been around for many years and excels despite its small size of 30 KB , allowing for precise and straightforward control of a 3-axis CNC machine through a CNC-Shield. This is the go-to solution for makers involved in 3-axis CNC projects, and GRBL serves as the foundation for numerous other advanced firmware solutions.

## 5.2) Structure Assembly:

### 5.2.1) X-axis Assembly:

At first we start with the motor assembly with pulley and plat


Second we make assembly between the second Pulley and Plate :


Third we make the assembly of or guiding system


After this assembly of parts, we have to assemble with the rail


The final assembly of X- axis


### 5.2.2) Y-axis Assembly:

Introducing each component on Y -axis


After assembly of each part here is the final result of Y -axis


### 5.2.3) Z-axis Assembly:

Introducing each component on Z-axis



After assembly of each part we got The final result of Z- Axis Assembly:


### 5.2.4) Final shape of the robot

After assembling each axis to get our final structure as $u$ can see in the next figure :


Figure 5-1Final shape of the robot
By also adding these two structures with same axes:


Figure 5- 2 structure $n^{\circ} 2$


Figure 5- 3 structure $n^{\circ} 3$
5.3) Configuring GRBL and controlling the robot with Arduino

The robot is piloted using G-code instruction sequences. GRBL is a practical tool for interpreting G-code and easily controlling a ROBOT with Arduino. GRBL is an open source software or firmware which enables motion control for CNC machines. We can easily install the GRBL firmware to an Arduino and so we instantly get a low cost, high performance CNC controller. The GRBL uses G-code as input, and outputs motion control via the Arduino

It's easier to understand when you look at Figure 5.4:

## Arduino CNC Machine Overview



Figure 5-4 G-code communication to CNC machine

## 5.4) Electronic equipment's

### 5.4.1) Description of the main components used

For the electronics, we'll be using an Arduino Uno board, equipped with a CNC Shield V3 board (figure 5.2). This Shield carries a set of power elements (motor driver) to supply the actuators used.


Figure 5-5 Arduino and the shield used
For the motorization we will use Nema17 motors, which are stepper motors, 200 steps per revolution. These motors are used in $99 \%$ of 3D printer and desktop CNC.


Figure 5-6 Stepper Motor Nema 17
Figure 5- 7Driver DRV8825
The stepper motors are not conventional motors which are plugged into a power supply and which start to rotate as if by magic. They perform a series of steps, to turn, which allows precise control of their speed, their direction of rotation and especially their position.

Now you think it's more complicated than expected to drive a stepper motor, but in fact not. There is an electronic component that will manage for us the pulses to send to the motor. We call it a motor driver.

## Chapter 5: Assembly and programming

The motor driver will receive the orders of speed, direction of rotation and position from the control electronics, in our case the Arduino then transform these orders into a series of electrical pulses which it will send to the motor.

In order to ensure safe operation of the machine, we will utilize limit switches similar to the one depicted in figure 5.5. These are electrical devices that are designed to open and close an electrical circuit. When mounted on moving components, they serve to restrict their movement within specific pre-set limits. Limit switches function similarly to standard switches, and their use in our project serves the purpose of preventing the machine from moving beyond its defined working area. Additionally, they facilitate automatic return of the machine to its starting position (homing).


Figure 5- 8 Limit Switch

And finally, to power all of this, we need a power supply powerful enough mainly to support the load of the motors. Each motor consumes about 5W, which gives us 15 W on a 3-motor assembly.

We used a 24 V 20 A power supply (about 480 W ) which is more than enough for our robot:


Figure 5-9 Power supply 24 V 20 A

### 5.4.2) Assembly of the Parts

The electronics parts as mentioned before are assembled as in the following figure:


Figure 5-10 Wiring diagrams for electrical components

To start, we are going to mount the CNC Shield board on the arduino, as it shown in figure


Figure 5-11 Position for shield


Figure 5-12 Placement of Micro-Step

Then we will place the micro-stepping jumpers, there are 3 possible locations under each motor driver location. Depending on how the jumpers are positioned, we select the micro-stepping division mode, here is the correspondence table for each driver reference:

| DRV8825 |  |  | A4988 |  |  | Mode de micro-stepping |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| J1 | J2 | J3 | J1 | J2 | J3 |  |
| OFF | OFF | OFF | OFF | OFF | OFF | 1 - aucun micro-stepping |
| ON | OFF | OFF | ON | OFF | OFF | $1 / 2$ |
| OFF | ON | OFF | OFF | ON | OFF | $1 / 4$ |
| ON | ON | OFF | ON | ON | OFF | $1 / 8$ |
| OFF | OFF | ON | ON | ON | ON | $1 / 16$ |
| ON | ON | ON |  |  |  | $1 / 32$ |

Table 5-1 Possible jumper positions for driver configuration
We can now place the drivers in their respective locations, on the CNC Shield.
WARNING: Here, you have to watch and position the "Enable" pin (or "En") of the driver on the Enable pin of the CNC Shield, otherwise you can be sure to burn them all.


Figure 5-13 how to correctly place a driver on a shield
On the CNC Shield, next to each motor driver location, there is the name of the axis that this driver will drive ( $\mathrm{X}, \mathrm{Y}$ or Z ), but there is also a 4th driver titled ' A '.

This 4th driver will be able to copy all the movements of one of the 3 other axes, On the side of the CNC Shield, you have places for jumpers with the names of the axis, you will have to place 2 jumpers side by side to select the axis you want to replicate


Figure 5-14 Potential location of motor cables

We just have to connect the power supply and the 4 motors. For power it is simple, there is a terminal block on the CNC Shield, we put the + on the red wire, the - on the black wire ...

Be careful though, do not connect the power supply to the Arduino directly but to the terminal block of the CNC Shield, the Arduino will simply be powered by the USB port.

To connect the motors there is a 4 pin connector next to each driver. There is no particular direction because by turning over the connector, we simply reverse the direction of rotation of the motor.


Figure 5-15 Placement of power supply and motor cables


Figure 5-16 Motors cables assembly in Arduino

The assembly is finished! We can now go on to setting up GRBL. In order to make the robot move we needs steps on programs to set. In our case we have GRBL, UGS

## Chapter 5: Assembly and programming

## 5.5) What is GRBL:

From the image, we can see where the GRBL unfolds in the "overview" of the principle of operation of a CNC machine. It's a firmware that we need to install or upload to the Arduino so it can control the stepper motors of the CNC machine. In other words, the GRBL firmware has the function of translating the G code into motor movement.

### 5.5.1) How to install GRBL

First, Arduino IDE must be installed in the Laptop then we can download the GRBL


Figure 5-17 Download the GRBL
Download it as a ZIP file and then proceed with the following steps:
Open the grbl-master.zip file and extract the files
Open Arduino IçDE, navigate to Sketch >> Include Library >> Add .ZIP Library...


Figure 5- 18 IDE Arduino
Next, navigate to File >> Examples >> grbl >> grblUpload. A new sketch will open and we need to upload it to Arduino board, later we select the Arduino board in our case we select "ARDUINO UNO' 'the COM port and we hit the upload bottom


Figure 5-19 Selection of Grbl

### 5.5.2) GRBL Configuration and controller

At this point we must configure the GRBL to our machine then once we have installed the GRBL firmware the Arduino knows how to read G-code and how to control the machine.

However, in order to send the G-code to the Arduino an interface or a controller software that tell the Arduino what to do. In matter fact there is so many interfaces to work with but we will stick to Universal G-code Sender (UGS).


Figure 5-20 Universal G-code Sender
How to use UGS: Once we downloaded it, we need to extract ZIP file, go to "bin" folder and open file named "ugsplatform". Once its opened we need to configure the machine or the GRBL parameters.


Figure 5-21 Setup Wizard of robot

For that we will use the UGS Setup Wizard which is much more suitable then manually typing commands.

First, navigate machine >> setup wizard, a window will pop, we select the baudrate, which should be 115200, and the port to which our Arduino is connected. Once we connect the Universal G-code Sender with the Arduino, in the next step we can check the direction of moving of the motors.


Figure 5-22Motor wiring test

If needed, we can reverse the direction but in or case all the directions were good or else we can flip them manually on the Arduino CNC shield.

In the next step we can adjust the steps/mm in order the step wizard can do the calculation and give us what value we should update the parameters.


Figure 5-23 STEP CALIBRATION
The default value is 250 steps/mm. this means, if we click the " $x+$ " bottom, the motors will make 250 steps. Now depending on the number of physical steps the motor has, the selected resolution and the transmission type, the machine will move some distance. Using a ruler we can measure the actual movement the machine made and enter that value in the "actual movement" field. Based on this the wizard will calculate and tell us what value we should change the steps/mm parameter.


Figure 5-24 Setting the pitch parameter
In my case the machine moved 25 mm . according to this the wizard suggested to update the steps/mm to a value of 100 steps $/ \mathrm{mm}$

With this value update the Z axis moves correctly same to all the axes as its show in the figure

In the next step we can enable the limit switches and test whether they work properly.


Figure 5- 25 Limit Switches test
Depending whether they are normally open or closed connection, we can also invert them here.

Nevertheless, in the next step we can either enable or disable the homing of the machine depends on your machine.


Figure 5-26 Homing test
Using the "try homing" button the machine will start moving towards the limit end switches. In case it goes the opposite way we can easily invert the direction.

Lastly; we click on finish. Later, we insert the G-code that we prepared to move the machine, in the next step were going to execute our G-code Program:

First, Open >> a window will pop, we select the G-code file that we already programmed. Later, we get this following window. The we excute the code so that the machine start working.


Figure 5-27 G-code Programming

## 5.6) Conclusion

In this chapter, we have introduced the key electrical components used in our project and their wiring. Additionally, we have described the computer part of the machine. Furthermore, we have successfully controlled the stepper motors by uploading GRBL onto Arduino, based on calculations performed by UGS.

## General conclusion

In conclusion, this thesis has covered various aspects of Cartesian robot pick and place systems, including generalities, the study of linear axis, geometric and kinematic modeling, robot sizing, programming, and electrical aspects.

Through careful analysis and simulation, we have determined the optimal size and configuration of the robot, as well as the most appropriate end effector and control system. The study of the linear axis has provided insights into the design, control, and optimization of this critical component of the Cartesian robot pick and place system.

The modeling of the system's geometry and kinematics has enabled us to develop a comprehensive understanding of how the system works and how it can be optimized for specific manufacturing applications. Robot sizing has been determined based on the specific requirements of the application, enabling manufacturers to choose the most appropriate robot for their needs.

The study of programming and electrical aspects has provided insights into the control and operation of the system, and how it can be integrated with other automation systems to optimize manufacturing processes.

Overall, this study provides a valuable contribution to the field of Cartesian robot pick and place systems, advancing our understanding of these systems and providing practical insights into their design, optimization, and implementation. By integrating Cartesian robot pick and place systems with other automation systems, manufacturers can further optimize their manufacturing processes, improving efficiency, productivity, and quality.

Finally, we hope that this modest work will serve as a reference for future projects and encourage them to take a greater interest in the practical side of mechanics of mechanics, as technology continues to advance, we can expect to see even more sophisticated Cartesian robots that can perform increasingly complex tasks, further driving innovation and progress in manufacturing

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## Appendix

## MATERIAL:

Stepper motor:
NEMA 17 stepper motors are widely used in a variety of applications, including 3D printers, CNC machines, robotics, and automation systems. They are known for their high torque at low speeds, precise positioning, and the ability to hold their position without power.

Characteristics:
> rated Voltage: 12 V DC.
$>$ Current: 1.2 A at 4 V .
$>$ Step Angle: 1.8 deg.
$>$ No. of Phases: 4.
> Motor Length: 1.54 inches.
$>4$-wire, 8 -inch lead.
$>200$ steps per revolution, 1.8 degrees.

$>$ Operating Temperature: -10 to $40^{\circ} \mathrm{C}$.
> Price: 3400 DA


## Pulley and idler pulley

The GT2 pulleys are strong yet lightweight, thanks to the aluminum material. This ensures that there is no added weight to the extruder. With these pulleys, we can be sure to eliminate any ringing or unnecessary vibrations and ensure high model quality. hey provide a reliable and accurate means of transmitting power and motion in a wide range of mechanical systems.

Characteristics:
$>$ Use for GT2 6mm Wide Timing Belt
> Material: Aluminum Alloy
$>$ Diameter: 5 mm
> Teeth Qty.: 20 (T)

$>$ Pitch: 2 mm
> Belt Width: 6 mm
> Color: silver
> Price: 300 DA


## Timing belt:

Timing belts are a fantastic way to transfer rotational motion (from a stepper motor) into linear motion (along a rail) and these GT2 belts are excellent for the task. They have a special profile with rounded teeth which reduces backlash. Often used for precision 3D printers and CNC machines.

## Specifications:

$>$ GT2 (GT2-2M)
$>$ Single Sided Belt
$>$ Length: Open ended sold by the Foot (up to 100 feet)
$>$ Pitch: 2 mm
> Belt Width: 5 mm
> Tension Member: Fiberglass
> Color: Black
> Price: 600 for meter


## Wheels:

Solid V Wheels are a strong wheel that resists deflection and distributes loads. It is usually used when heavier loads are required and will compress less than the Dual V Wheel. If many of these wheels are added to a gantry plate it will allow for an even distribution of the load

Package list:
> $1 \times$ Delrin Idler pulley wheel
$>2 \times$ Bearing
$>1 \times$ Precision Shims
> $1 \times 1 / 4^{\prime \prime}$ aluminum spacer
$>1 \times$ Screw (M5*25mm)
> $1 \times$ Nylon lock nut(M5)
Specifications:
> Rockwell Hardness: M80
> Compression Strength: 63MPA
$>$ OD: 23.90 mm
$>$ ID: 16 mm
> Thick: 10.23 mm
> Material: Delrin
> Color: Black
> Price: 600 DA


## Flexible coupling:

These Flexible couplings are used to transmit torque from a stepper motor to a lead screw, thus creating the movement in an actuator

## Specifications:

> Inner diameter: 5 mm inner diameter $/ 8 \mathrm{~mm}$ inner diameter
$>$ Out diameter: 19 mm
$>$ Length: 25 mm
> Material: aluminum
$>$ Color: silver

> Price: 400da

## Metric lead screw:

metric lead screws are a versatile and reliable means of converting rotational motion into linear motion, and they are widely used in many different types of applications that require precise linear motion control

## specification:

$>$ pitch: 2 mm
$>$ diameter: 8 mm
$>$ length: $300 \mathrm{~mm} / 500 \mathrm{~mm} / 1000 \mathrm{~mm}$
$>$ material: Stainless steel lead screw
> lead screw color: silver
$>$ price: $3500 \mathrm{da} / \mathrm{m}$


## Mild steel smooth rod:

Smooth rod 8 mm is a versatile product made from Mild steel which can be used in combination with LM10UU linear ball bearings to achieve linear motion. Due to the material of this bar, the bar may show grooves after longer time of use.

Specifications:
> Material: mild steel
$>$ Bar form: round
$>$ Diameter: 8 mm
> Price: 1300da/m


## Linear ball bearing:

This linear ball bearing is sort of the opposite of the radial ball bearings you may be familiar with. Its intended to slide along an 8 mm linear shaft, rather than rotate around it. These are very slim, and good for when you want to attach a motion carriage onto a railing without adding a lot of weight.

These are very basic bearings, they're meant for a stepper-motion controlled setup so they're not ultra-smooth.

Specification:
> Inside diameter: 8 mm
$>$ Outside diameter: 15 mm
$>$ Length: 24 mm
$>$ Weight: 13 g
> Price: 450da

## Linear rail shaft guide:

The SK8 Supports are usable with our 8mm Smooth Rods, they can simply slide the rail through then tighten the clamp screw at the top to fix it in place.

## Specifications


> Material: aluminum
> Diameter of compatible linear shafts: 8 mm
> Mounting height: 32.8 mm
> Clamping base width: 42 mm
> Clamping mounting screw size: M5
> Clamping screw size: M4
$>$ Weight: 24 g
> Price: 450da

## Linear rail guide:

## V slot

V-Slot Linear Rail is the ultimate solution combining both linear motion and a modular, structural framing system. It's lightweight yet rigid and provides an ultra-smooth track for precise motion. V-Slot Linear Rails are precise, easy to work with and allows you unlimited design control through its modular nature.

## Brief Data:

> Profile size: $20 \times 60 \mathrm{~mm}$.
> Profile Type: V-Slot.
$>$ Length: $1000 \mathrm{~mm} \pm 1.5 \mathrm{~mm}$.
$>$ Center Hole Diameter: approx 4.25 mm . Can be tapped for M5 screws.
> Material: Aluminum 6063 Alloy
$>$ Number of Slot: 8 .
> Coating: Silver Anodized.
> Profile Weight: $1.05 \mathrm{~kg} / \mathrm{m}$
> Price :2700da/m


Profile size: 20x40
Profile Type: V slot
Center Hole Diameter: 4.2 mm
Number of Slot: 6
$>$ Price: 2100da/m
> Profile size: $20 \times 20$
> Profile Type: V slot
$>$ Center Hole Diameter: 4.2 mm
> Number of Slot: 4

$>$ Price: $1200 \mathrm{da} / \mathrm{m}$

## C beam:

As given in the name, C-Beam has a C Shape profile. The C Shape is extremely versatile and gives added strength and functionality. It incorporates the excellent original V-Slot Aluminum Extrusion system from Openbuilds. Because of the V-Slot Aluminum Extrusion system, every side of extrusion can be used as a linear guide. Because C-Beam works hand in hand with V-Slot, all standard V-Slot gantry plates, wheels, mounts, brackets and fasteners are compatible with this extrusion
> Number of Slots: 12
> Size of Slot: 6 Mm (V-SLOT)
> Central Hole Diameter: 4,2 Mm (For M5 Thread)
$>$ Main Material : Aluminum EN AW-6063-T
$>$ Price: 5000da/m



| Scale: 1:2 | CM_MASTER 2 | University Saad Dahleb Blida 01 | Material :PLA |
| :--- | :--- | :--- | :--- |
| - | Part Name : | Guiding Plate | Designed By: <br> Saada Loubna <br> Bensidi Aissa Imad |



| Scale: 1:2 | CM_MASTER 2 | University Saad Dahleb Blida 01 | Material :PLA |
| :--- | :--- | :--- | :--- | :--- |
|  | Part Name : Motor Mount Plate | Designed By: <br> Saada Loubna <br> Bensidi Aissa Imad |  |



| Scale: 1:1 | CM_MASTER 2 | University Saad Dahleb Blida 01 | Material :PLA |
| :--- | :--- | :--- | :--- |
|  | Part Name : Pulley Support | Designed By: <br> Saada Loubna <br> Bensidi Aissa Imad |  |



| Scale: 1:2 | CM_MASTER 2 | University Saad Dahleb Blida 01 | Material :PLA |
| :--- | :--- | :--- | :--- |
|  | Part Name : | Roller Guide Plate | Designed By: <br> Saada Loubna <br> Bensidi Aissa Imad |



| Scale: 1:4 | CM_MASTER 2 | University Saad Dahleb Blida 01 | Material :PLA |
| :--- | :--- | :--- | :--- |
| - | Part Name: Y-axis Guide Plate | Designed By: <br> Saada Loubna <br> Bensidi Aissa Imad |  |










| ITEM NO. | PART NUMBER | QTY. |
| :---: | :---: | :---: |
| 1 | Rail | 1 |
| 2 | Motor Mount Plate Nema 17-1 | 2 |
| 3 | Nema 17 Stepper Motor | 2 |
| 4 | Button Head Socket Cap Screw M3 x 8 mm | 8 |
| 5 | GT2 Timing Pulley 30 Tooth | 2 |
| 6 | Precision Shim $10 \times 5 \times 1$ | 19 |
| 7 | Nylon Insert Lock Nut M5 | 11 |
| 8 | Ball Bearing $5 \times 16 \times 5$ | 22 |
| 9 | Delrin V Wheel | 8 |
| 10 | V-Slot Gantry Plate 20mm | 4 |
| 11 | Aluminum Spacer 6mm | 4 |
| 12 | Eccentric Spacer 6mm | 4 |
| 13 | Low Profile Screw M5 x 30 mm | 8 |
| 14 | Smooth Idler Pulley Wheel | 3 |
| 15 | Nylon Spacer 0.125in | 3 |
| 16 | Low Profile Screw M5 x 25 mm | 3 |
| 17 | Idler Pulley Plate | 3 |
| 18 | Aluminum Spacer 3mm | 3 |
| 19 | Low Profile Screw M5 x 8 mm | 8 |
| 20 | BELT | 1 |
| 21 | rail 3 | 2 |
| 22 | belt 3 | 2 |
| 23 | V-Slot 20x20x500 Linear Rail | 2 |
| 24 | M5 x 65 | 10 |
| 25 | Black Angle Corner Connector | 4 |

Scale: 1:10

## Saada Loubna

 Bensidi Aissa Imad

| ITEM |  |  |
| :---: | :--- | :---: |
| NO. | PART NUMBER | QTY. |
| 1 | V-Slot 40x40x500 | 3 |
| Linear Rail | 2 |  |
| 2 | $20 \times 40$ | 1 |
| 3 | $20 \times 20$ | 2 |
| 4 | V-Slot 20x20x500 |  |
| Linear Rail |  |  |


Scale: $1: 10$

- Some steps of the assembly

- Final assembly of structure one:


