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Random Vibration and the Single Degree-of-Freedom Vibratory System: A Symbolic Quantification of Isolation and Packaging Performance

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Introduction

With twenty years of research, active and semi-active systems have been shown to have certain performance advantages over passive suspensions in certain operating regimes. Chalasani (1986) and Redfield and Karnopp (1988) have shown that, depending on the performance index and weightings, active control improves performance from little to moderately. There are situations where passive control is quite satisfactory and the complexities and cost of more active means may not be warranted.

To further the understanding of the tradeoffs involved and the performance potentials of active suspensions, this paper *symbolically* quantifies the isolation and stroke performance for a one degree-of-freedom vibratory system subject to a stochastic disturbance input acting through the suspension. The system of this paper models that of tracked vehicles and a class of isolation systems quite well. It also gives insight into low and high frequency performance for two degree-of-freedom systems such as a typical suspension model for automobiles, aircraft, and rail vehicles. Because of the nature of the single degree-of-freedom model, issues of handling cannot be readily addressed in this work. The 1 DOF model does not adequately predict dynamic tire forces.

The main contributions of this work are the closed form symbolic solutions developed for optimal suspension response and the demonstration of the marked similarity between the frequency and mean square response of the 1 degree-of-freedom model of this paper and the more involved 2 degree-of-freedom model incorporating a so-called "unsprung mass."

System Model

The generic system model is shown in Fig. 1 with a suspended mass separated from a displacement or velocity disturbance by both passive and active components. The suspended body has mass M and velocity V. The input of interest in this paper is velocity V_0 approximating ground or rail unevenness or a structural disturbance depending on the type of isolation system involved. The disturbance force F is also of general concern but is not a consideration of this work. The suspension consists of a compliance with stiffness K, passive damping with coefficient Bp, and a force actuator with force Fa that is proportional to the mass velocity, V ($Fa = BaV$). This type of active control is termed "active damping" because it acts on the mass as if it were damped to an inertial ground.

It is a simple matter to derive the state equations, Eq. (1). x_s is the suspension stroke or rattle-space and p is the mass momentum.

$$\begin{bmatrix}\dot{x}_s\\ \dot{p}\end{bmatrix} = \begin{bmatrix} 0 & \dfrac{-1}{M} \\ K & \dfrac{-(Ba+Bp)}{M}\end{bmatrix}\begin{bmatrix}x_s\\ p\end{bmatrix} + \begin{bmatrix}1\\ Bp\end{bmatrix}V_0 + \begin{bmatrix}0\\ 1\end{bmatrix}F \qquad (1)$$

These equations will be manipulated for both frequency and RMS response measures to determine optimal performance as a function of the parameters.

Frequency Response

In most any isolation problem, the performance of interest is threefold: (*1*) mass isolation represented by mass acceleration or velocity, (*2*) force transmission to the environment, and (*3*) suspension stroke. In the one DOF model, the force transmitted and the acceleration of the mass are proportional. Because of the lack of an unsprung mass, the 1 DOF model cannot predict tire contact forces over all frequency.

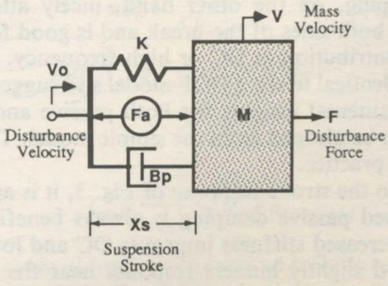

Fig. 1 One DOF isolation system with passive and active damping

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