

Graduate Texts in Mathematics

Serge Lang

Differential and Riemannian Manifolds



Springer

Contents

Preface	v
CHAPTER I	
Differential Calculus	1
§1. Categories	2
§2. Topological Vector Spaces	3
§3. Derivatives and Composition of Maps	6
§4. Integration and Taylor's Formula	10
§5. The Inverse Mapping Theorem	13
CHAPTER II	
Manifolds	20
§1. Atlases, Charts, Morphisms	20
§2. Submanifolds, Immersions, Submersions	23
§3. Partitions of Unity	31
§4. Manifolds with Boundary	36
CHAPTER III	
Vector Bundles	40
§1. Definition, Pull Backs	40
§2. The Tangent Bundle	48
§3. Exact Sequences of Bundles	49
§4. Operations on Vector Bundles	55
§5. Splitting of Vector Bundles	60
CHAPTER IV	
Vector Fields and Differential Equations	64
§1. Existence Theorem for Differential Equations	65
§2. Vector Fields, Curves, and Flows	86

§3. Sprays	94
§4. The Flow of a Spray and the Exponential Map	103
§5. Existence of Tubular Neighborhoods	108
§6. Uniqueness of Tubular Neighborhoods	110
CHAPTER V	
Operations on Vector Fields and Differential Forms	114
§1. Vector Fields, Differential Operators, Brackets	114
§2. Lie Derivative	120
§3. Exterior Derivative	122
§4. The Poincaré Lemma	135
§5. Contractions and Lie Derivative	137
§6. Vector Fields and 1-Forms Under Self Duality	141
§7. The Canonical 2-Form	146
§8. Darboux's Theorem	148
CHAPTER VI	
The Theorem of Frobenius	153
§1. Statement of the Theorem	153
§2. Differential Equations Depending on a Parameter	158
§3. Proof of the Theorem	159
§4. The Global Formulation	160
§5. Lie Groups and Subgroups	163
CHAPTER VII	
Metrics	169
§1. Definition and Functoriality	169
§2. The Hilbert Group	173
§3. Reduction to the Hilbert Group	176
§4. Hilbertian Tubular Neighborhoods	179
§5. The Morse–Palais Lemma	182
§6. The Riemannian Distance	184
§7. The Canonical Spray	188
CHAPTER VIII	
Covariant Derivatives and Geodesics	191
§1. Basic Properties	191
§2. Sprays and Covariant Derivatives	194
§3. Derivative Along a Curve and Parallelism	199
§4. The Metric Derivative	203
§5. More Local Results on the Exponential Map	209
§6. Riemannian Geodesic Length and Completeness	216
CHAPTER IX	
Curvature	225
§1. The Riemann Tensor	225
§2. Jacobi Lifts	233

§5. Application of Jacobi Lifts to $d\exp_x$	240
§4. The Index Form, Variations, and the Second Variation Formula	249
§5. Taylor Expansions	257

CHAPTER X

Volume Forms	261
§1. The Riemannian Volume Form	261
§2. Covariant Derivatives	264
§3. The Jacobian Determinant of the Exponential Map	268
§4. The Hodge Star on Forms	273
§5. Hodge Decomposition of Differential Forms	279

CHAPTER XI

Integration of Differential Forms	284
§1. Sets of Measure 0	284
§2. Change of Variables Formula	288
§3. Orientation	297
§4. The Measure Associated with a Differential Form	299

CHAPTER XII

Stokes' Theorem	307
§1. Stokes' Theorem for a Rectangular Simplex	307
§2. Stokes' Theorem on a Manifold	310
§3. Stokes' Theorem with Singularities	314

CHAPTER XIII

Applications of Stokes' Theorem	321
§1. The Maximal de Rham Cohomology	321
§2. Moser's Theorem	328
§3. The Divergence Theorem	329
§4. The Adjoint of d for Higher Degree Forms	333
§5. Cauchy's Theorem	335
§6. The Residue Theorem	339

APPENDIX

The Spectral Theorem	343
§1. Hilbert Space	343
§2. Functionals and Operators	344
§3. Hermitian Operators	347
Bibliography	355
Index	361

This text provides an introduction to basic concepts in differential topology, differential geometry, and differential equations, and some of the main basic theorems in all three areas: for instance, the existence, uniqueness, and smoothness theorems for differential equations and the flow of a vector field; the basic theory of vector bundles including the existence of tubular neighborhoods for a submanifold; the calculus of differential forms; basic notions of symplectic manifolds, including the canonical 2-form; sprays and covariant derivatives for Riemannian and pseudo-Riemannian manifolds; applications to the exponential map, including the Cartan–Hadamard theorem, and the first basic theorem of calculus of variations. These are all covered for infinite-dimensional manifolds, modeled on Banach and Hilbert spaces, at no cost in complications, and some gain in the elegance of the proofs. In the finite-dimensional case, differential forms of top degree are discussed, leading to Stokes' theorem (even for manifolds with singular boundary), and several of its applications to the differential or Riemannian case.

ISBN 0-387-94338-2
ISBN 3-540-94338-2

