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**Image Analysis,
Random Fields
and Dynamic
Monte Carlo Methods**

A Mathematical Introduction



Springer

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