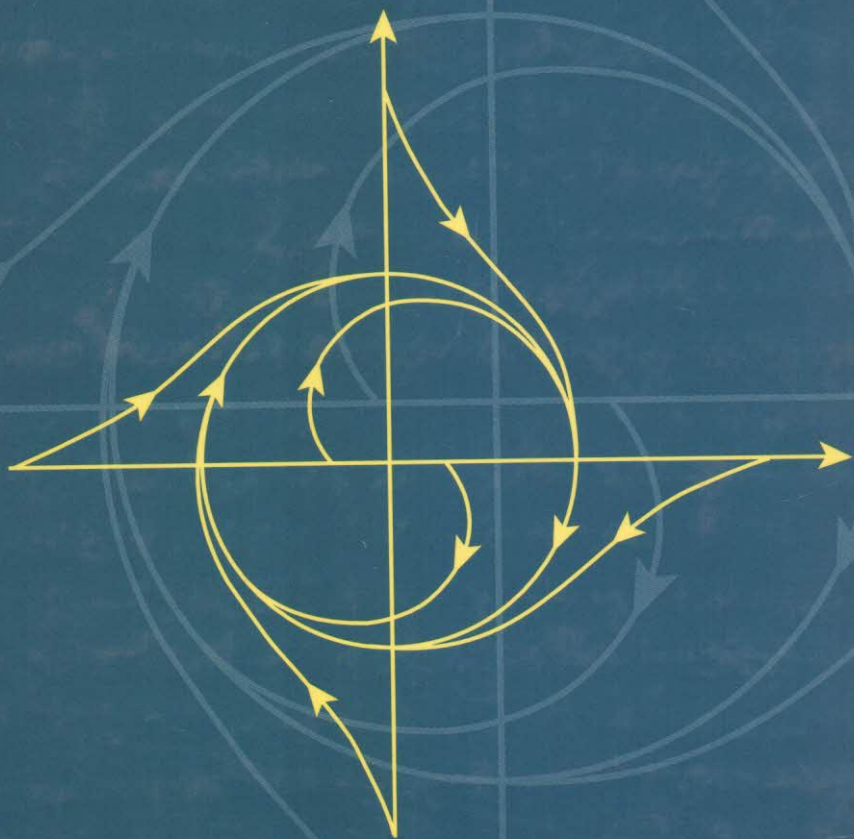


NONLINEAR SYSTEMS

S E C O N D E D I T I O N



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