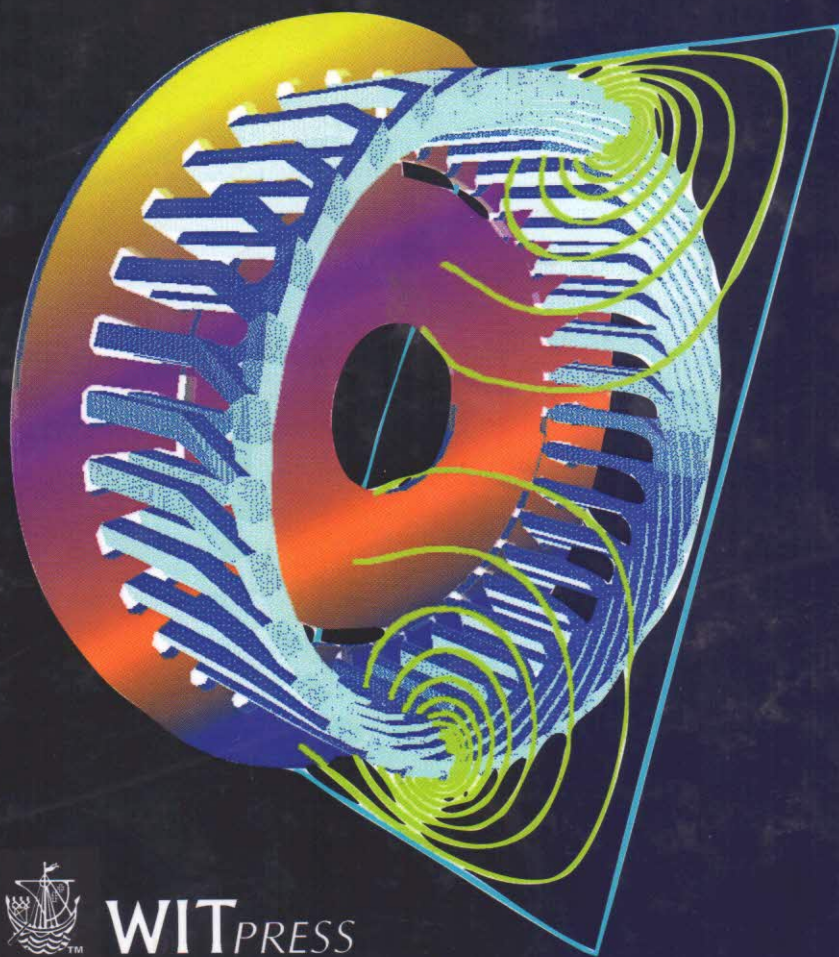


**Numerical Modelling
and Design of
ELECTRICAL MACHINES
and DEVICES**

K. Hameyer and R. Belmans



WITPRESS

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