

**NUMERICAL
COMPUTATION
of
INTERNAL
and
EXTERNAL FLOWS**

Volume 2

*Computational Methods
for Inviscid and
Viscous Flows*

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Volume 2:

Computational Methods for Inviscid and Viscous Flows

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This second volume deals with the applications of computational methods to the problems of fluid dynamics. It complements the first volume to provide an excellent reference source in this vital and fast growing area. The author includes material on the numerical computation of potential flows and on the most up-to-date methods for Euler and Navier-Stokes equations. The coverage is comprehensive and includes detailed discussion of numerical techniques and algorithms, including implementation topics such as boundary conditions. Problems are given at the end of each chapter and there are comprehensive reference lists.

Of increasing interest, the subject has powerful implications in such crucial fields as aeronautics and industrial fluid dynamics. Striking a balance between theory and application, the combined volumes will be useful for an increasing number of courses, as well as to practitioners and researchers in computational fluid dynamics.

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