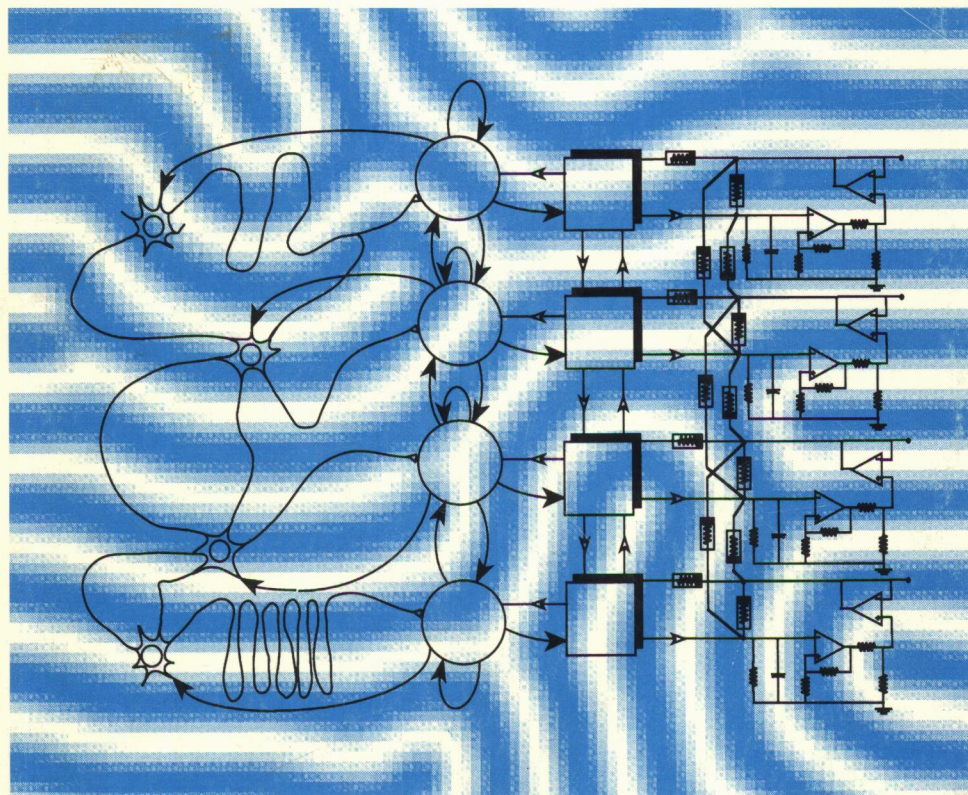


Dynamics and Self-organization of Locally Coupled Neural Networks

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