الجمهورية الجزائرية الديموقراطية الشعبية

The People's Democratic Republic of Algeria

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

Saad Dahleb Blida 1 University

Faculty of Sciences

Department of mathematics

Master dissertation

Speciality: Operational Research

Strategic Distribution Network Expansion:

RISK AVERSION

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2023/2024

Acknowledgment:

First and foremost, we must thank Allah the Almighty for granting us the courage and good health to complete this work.

We would like to express our gratitude to our supervisor, Mr Boudjemaa, a professor at Saad Dahleb University Blida 1, for agreeing to supervise this work, for his assistance, and his availability throughout its completion.

Finally, we thank our parents and families, who we greatly appreciate for their moral support and assistance, as well as everyone who supported and helped us throughout this endeavor.

Manel & Karima ✍

Dedication

I dedicate this work :

To the source of my strength in my weakness, to those who encouraged me to persevere throughout my life, to my first supporters, my dear mom and dad.

To my dear grandmothers and grandfather.

To my brothers and sisters, my steadfast support that never bends.

To my nieces and nephews, Wissal, Ranim, Mohamed Acil, and Dhirar.

To all the generous family.

To all my friends who have always encouraged me, and to whom I wish more success.

thank you !

Benkradidja Manel ✍

Dedication

I dedicate this work :

To those who believe in me when everyone lets me down,whose prayers were the secret of my success and whose compassion was the balm for my wounds.To the people most dear to my heart, my parents.

To my support, my strength, and my refuge after God, to those who showed me what is more beautiful than life, my brother and sister.

To my friends, who provided me with endless support and understanding, especially during the most challenging times.Your patience and companionship have been invaluable.

To that person........

thank you !

Aidaoui Karima

ملخص
تتناول هذه المذكرة نموذجا لتحسين التخطيط الاستثماري لشبكة توزيع الكهرباء مع مراعاة المخاطر باستخدام نموذج الأمثلية. يهدف النموذج إلى تقليل المخاطر من خلال الخطوات التالية:

- **بناء نموذج تحسين: تطوير نموذج رياضي دقيق لتمثيل المشكلة الاستثمارية بشكل شامل.**
- **حساب التكاليف: حساب التكاليف المرتبطة بدالة الهدف، مثل تكاليف القيمة الشرطية عند المخاطر.**
- **تصور النتائج: إنشاء رسوم بيانية مرئية تعرض الحلول المثلى لمشكلات التخطيط الاستثماري بدقة.**

Abstract
The thesis discusses an optimization model for investment planning in the electrical distribution network, taking risks into account using an optimization approach. The model aims to reduce risks through the following steps:

- **Building an Optimization Model:** Creating an accurate mathematical model to clearly represent the investment problem.
- **Calculating Costs:** Figuring out the costs related to the objective function, like the costs for Conditional Value at Risk (CVaR).
- **Visualizing Results:** Making clear visual charts that show the best solutions for the investment planning problems.

Résumé

Ce mémoire traite d'un modèle d'optimisation pour la planification des investissements dans le réseau de distribution électrique, en tenant compte des risques à l'aide d'une approche d'optimisation. Le modèle vise à réduire les risques en suivant les étapes suivantes :

- **Construction d'un Modèle d'Optimisation :** Création d'un modèle mathématique précis pour représenter clairement le problème d'investissement.
- **Calcul des Coûts :** Détermination des coûts liés à la fonction objective, comme les coûts pour la Valeur à Risque Conditionnelle (CVaR).
- **Visualisation des Résultats :** Création de graphiques visuels clairs montrant les meilleures solutions pour les problèmes de planification des investissements.

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LIST OF ALGORITHMS

ABBREVIATIONS

- **LP:** Linear Programming.
- **NLP:** Non-Linear Programming.
- **MILP:** Mixed Integer-Linear Programming.
- **MINLP:** Mixed Integer Non-Linear Programming.
- **ILP:** Integer Linear Programming.
- **MV:** medium value.
- **B&B:** Branch and Bound.
- **CVaR:** Conditional Value at Risk.
- **HILP:** High-Impact low Probability.
- **HV/MV:** The transformation of hut voltage lowered to a medium value.
- **NLP-BB:** Branch and Bound algorithm for nonlinear programming.
- z_{MINLP}**:** objective function for mixed integer non linear programming.
- **LNS:** Large Neighborhood Search.
- **DER:** Distributed Energy Resources.
- **BIPV:** Building-Integrated Photo Voltaic.
- **AI:** Artificial Intelligence.
- **IT:** Information Technology.
- **ML:** Machine Learning.

- **SOC:** State Of Charge.
- **NP-hard:** Non-deterministic Polynomial-time hard.
- **NP-complete:** Non-deterministic Polynomial-time complete.
- **EENS:** Expected Energy Not Supplied.
- **LOLP:** Loss Of Load Probability.
- **LOLE:** Loss Of Load Expectation.
- **SAIDI:** System Averge Interruption Dduration Index.
- **CAIDI:** Customer Averge Interruption Dduration Index.
- **SAIfI:** System Averge Interruption Frequency Index.
- **CBA:** Cost-Benefit Analysis.
- **NPV:** Net Present Value.
- **IRR:** Internal Rate of Return.
- **RI:** Reliability Index.
- **CapEx:** Capital Expenditure.
- **OpEx:** Operational Expenditure.

INTRODUCTION

Operations research is a discipline that aims to use advanced analytical methods to make optimal decisions in complex situations. It encompasses a wide range of techniques, such as mathematical modeling, optimization, simulation and data analysis, to solve decision-making problems in various fields, including logistics, planning, operations management and many others.In the context of an energy company such as sonelgaz, operational research can play a crucial role in the effective planning and management of its operations related to the distribution of electricity and gas throughout the territory, taking into account factors such as fluctuating demand, operational constraints, maintenance costs and government regulations.

Company presentation

Sonelgaz, short for ”Société Nationale de l'Electricité et du Gaz”; is the Algerian national company responsible for managing electricity and gas services across the country. Since its establishment in 1969, Sonelgaz has evolved into a key player in the energy sector in Algeria, playing a vital role in providing energy to individuals, businesses, and industries in the country. Sonelgaz's main areas of operation include:

1. Electricity Production.
2. Electricity and Gas Distribution.
3. Commercialization.

Electricity and gas distribution

Sonelgaz,the National Electricity and Gas Company, stands as a cornerstone within Algeria's energy sector. Its distribution domain, entrusted with supplying electricity and gas to millions of households, businesses, and industries nationwide, represents a crucial aspect of its operations. Nonetheless, this domain faces its own set of challenges and issues, demanding constant attention to ensure reliable and high-quality energy provision to its customers.

Faced with the challenges experienced in Sonelgaz‘s distribution domain , it becomes imperative to find a backup scheme that minimizes economic losses. By implementing an innovative and reliable backup plan, Sonelgaz could mitigate the financial implications of these issues while ensuring continuity of service for its customers.

General Description Of The Problem

The planning of electric distribution network expansion poses a major challenge for energy sector companies, specially in the face of the transition to more sustainable energy sources and the constant growth in electricity demand This evolution requires an adaptation of the traditional planning approach in order to effectively integrate the operational constraints, risks and uncertainties inherent in the large-scale expansion of networks. In this context, adopting a scalable and risk management approach becomes crucial to ensure the reliability and resilience of energy infrastructure in the long term, The central question that arises is:How to design a scalable approach for planning the expansion of electricity distribution networks, while taking into account the associated risks and ensuring the reliability of the system? This question raises complex challenges in terms of modeling, optimization and decision-making, particularly with regard to risk management and maximizing system reliability. Thus, research in this area seeks to develop innovative methods to efficiently and robustly plan the expansion of electricity distribution networks, taking into account the multiple constraints and uncertainties inherent in this process.

”A Scalable Approach to Large Scale Risk-Averse Distribution Grid Expansion Planning” by” Alexandre Moreira, Miguel Heleno, Alan Valenzuela, Joseph H. Eto, Senior, Jaime Ortega, and Cristina Botero”[37]

This study proposes a practical methodology to determine the optimal combinations of investments in new line segments and storage devices, balancing the risks associated with high-impact, low-probability (HILP) events and the reliability concerning routine outages. A mixed-integer linear programming (MILP) model was used, incorporating various factors such as efficiency, working hours, and different emergency scenarios. To implement the model, Python and Pyomo were used, and CPLEX Solver was employed for its robustness in solving complex problems.

The methodology was tested using a real distribution network from the Commonwealth Edison (ComEd) Reliability Program in Illinois, USA, demonstrating its efficiency in handling large and complex networks, including 54-node and 2055-node systems. The results highlighted the model’s ability to reduce problem-solving time and achieve a balance between reliability and cost. This approach proves to be scalable and applicable in real-world operational environments, making it a valuable tool for planning and developing future electrical distribution networks.

hypotheses

Here are some hypotheses for the given problematic:

1. **Technological Scalability Hypothesis:** This hypothesis posits that incorporating adaptable and scalable technologies in the expansion planning process enhances the flexibility and efficiency of distribution networks. By leveraging technologies that can scale with growing demands and changes in network conditions, operators can optimize resource allocation and operational efficiency.

2. **Risk Integration Hypothesis:** The integration of risk assessment methodologies into the expansion planning framework improves decision-making by accounting for potential hazards and uncertainties. By quantifying risks associated with different expansion scenarios, planners can prioritize investments that minimize risks and maximize system reliability.

3. **Reliability Optimization Hypothesis:** Emphasizing reliability criteria and using advanced optimization algorithms in the planning process enhances the overall performance and resilience of the distribution system. This hypothesis suggests that by optimizing for reliability metrics such as outage rates or system uptime, planners can design networks that better withstand disruptions and meet customer expectations.

4. **Data-Driven Planning Hypothesis:** Leveraging comprehensive data analytics and machine learning techniques enables better forecasting and decision-making in expansion planning while reducing risks. By analyzing historical data and real-time information, planners can make informed decisions that optimize network performance, anticipate future demand, and mitigate potential failures.

5. **Cost-Risk Trade-off Hypothesis:** Balancing investment costs with risk mitigation strategies leads to cost-effective expansion plans that maintain system reliability within acceptable risk thresholds. This hypothesis acknowledges the trade-off between upfront investment costs and ongoing operational risks, advocating for strategies that optimize this balance to achieve long-term cost-efficiency and reliability.

6. **Adaptive Planning Hypothesis:** Implementing adaptive planning strategies that can dynamically adjust to changing conditions and uncertainties promotes resilience and reliability in distribution networks. This hypothesis emphasizes the importance of flexibility in planning processes, allowing adjustments based on evolving technological advancements, regulatory changes, and environmental factors to ensure sustainable network performance.

Objective Function:

1. **Minimization of Investment Costs and Imbalance Costs:** This objective function aims to minimize the total investment cost in new line segments and storage devices, as well as the imbalance costs in the base case and a convex combination between the expected value and the Conditional Value at Risk (CVaR) of imbalance costs associated with a set of failure scenarios.

2. **Minimization of the Expected Value of Load Loss Cost and its CVaR:** This approach considers not only the minimization of the expected value of the load loss cost but also the CVaR of this cost for a range of failure scenarios. This allows capturing the influence of high-impact events even if they have low probabilities

3. **Components of the Total Cost Objective Function:** The objective function aims to minimize the total cost of the system by integrating several specific cost components. The overall objective function Ctotal is defined as the sum of the following costs:

    - Fixed line costs (C_{line})
    - Fixed and variable battery costs ($C_{battery}$)
    - Imbalance costs ($C_{imbalance}$)
    - Load shedding and island operation costs ($C_{reductioncharge}$)
    - Conditional Value at Risk (CVaR) costs (C_{CVaR})

Constraints:

- **Binary Investment Variables Constraints:** These constraints ensure that the decision variables, which determine which candidate assets will receive investments, are binary. This helps to identify the optimal choice among the available alternatives [37].

- **State of Charge (SOC) Constraints for Storage Devices:** These constraints ensure that the SOC of storage devices follows a predetermined schedule and stays within specified limits, ensuring the safe and efficient use of storage devices [37].

- **Loss of Load Constraints:** These constraints aim to ensure that the loss of load remains within maximum limits in various scenarios, guaranteeing the system's ability to meet demand even during failures [37].

- **Power Flow Constraints:** These linear constraints represent the physical properties of the distribution grid, ensuring that power flows remain within capacity limits and adhere to Kirchhoff's circuit laws for both existing and prospective lines [37].

- **Voltage Bounds Constraints:** These constraints impose voltage limits on each bus within the distribution grid to ensure the safe and stable operation of the system [37].
- **SOC Variation and Capacity Limits Constraints:** These constraints address the SOC variation over time and impose limits on the charging and discharging capacities of both existing and candidate storage devices, including charging and discharging rates [37].
- **Investment Combinations and Line Installation Constraints:** These constraints ensure that only one investment combination is chosen and link installed lines with investment decisions, clarifying the relationship between investment decisions and the actual configurations of the grid [37].
- **Operational Constraints under Failure Scenarios:** These constraints deal with operational decisions under various failure scenarios, including the use of storage device capacities and maintaining the balance between supply and demand [37].

The thesis is composed of four chapters which are developed as follows:

Chapter 1: Definitions and basic concepts

In this chapter, we delve into the definitions and basic concepts in our field. We clarify the terms and fundamental principles that underpin our research, thus establishing a common language for the rest of our study.

Chapter 2: Optimization Models

In this chapter, We will examine in detail different optimization models and the various methods used to solve them, emphasizing how they improve decision-making in different settings.

Chapter 3: Foundations of Problem: Modeling, Analysis and Strategics

This chapter explores fundamental topics in distribution networks and risk management, alongside mathematical problem formulation. It focuses on basic concepts such as network planning, risk management, and risk aversion, as well as mathematical formulation techniques and optimization methods. The study aims to define goals, constraints, and improvement methods for electrical distribution networks using practical and scalable methodologies.

Chapter 4: Case Study

In this chapter, the discussion centers on a series of ongoing studies. The chapter begins by detailing the data set used, followed by presenting various case studies. Next, the optimal model employed to achieve specific objectives is introduced, and the anticipated outcomes are analyzed. Additionally, a comparison is provided between the utilization of Cplex and CBC solvers. Subsequently, the final network configuration is examined, offering a graphical interpretation of the findings. The process of selecting network branches is also explored, outlining the rationale behind choosing primary branches. Finally, a Monte Carlo simulation is conducted, and the results are evaluated using both Cplex and CBC solvers.

CHAPTER 1

DEFINITIONS AND BASIC CONCEPTS

There are several mathematical concepts and techniques that are fundamentally used in the field of planning and expanding electrical distribution networks, and this is what we find in section 1.1 and section 1.2

1.1 Technical definition

Electrical Network: An electrical network is designed to carry energy from the source of production to the user.

A network comprises:

1. Electrical nodes where the facilities are connected, these are the substations.
2. Transmission lines (overhead or underground).
3. Voltage and current transformers.
4. Devices to protect the network.

Distribution: Distribution is the final processes that links the power generation plants to the customer. This voltage is lowered to a medium value by transformer substations (HV/MV substation). From these substations, lines or cables are connected to transformer substation located close to the lines used, either to supply MV directly to subscribers or to public distribution substations (MV/LV substation) where the voltage is lowered to a value that allows it to be used directly by subscribers for the various uses of electrical energy (lighting, etc...).

Distribution Grid: is the final stage of the electrical grid that distributes electricity to homes, industry and other end-users.

Capacity: The capacity of an electrical distribution line refers to the maximum amount of electricity it can reliably carry without overload or excessive loss. This capacity is often expressed in terms of the maximum current that the line can support.

Resilience: The resilience of an electrical network refers to its ability to maintain stable and reliable operation in the face of disturbances such as equipment failures, extreme weather conditions, or malicious attacks.

Failure: A failure in an electrical network occurs when a component, such as a transmission line or a transformer, ceases to function properly, resulting in a disruption of electrical supply.

Risk Aversion: Risk aversion is the tendency to prefer certain outcomes with lower returns over uncertain out comes with higher return. In the context of network planning, this may translate to a preference for less risky solutions, even if they involve higher costs, to ensure the reliability and stability of the network.

Storage Devices: Are computer hardware components used to store and retrieve digital information.

1.2 Mathematical definition

Graph theory plays a fundamental and extensive role in the planning and expansion of distribution grids, providing the mathematical frameworks and tools necessary for modeling, analyzing, and solving various problems in this field. The network is represented as a graph to illustrate energy flow and directions. Shortest path algorithms are used to determine efficient routes for energy distribution, while enhancing network reliability through adding redundant paths and identifying critical nodes. Additionally, it contributes to optimizing energy flow and efficiently expanding the network using spanning trees and branching techniques, with the capability to detect and isolate faults using graph techniques and dynamically adapting to changes in load and generation patterns.

Convex combination: Is a linear combination of points where all coefficients are non-negative and sum to 1.

Vertices: A finite set of points.

Edges: Finite number of lines.

Graph: Is made up of vertices and edges.

1. If the two extremities of an edge are equal, the edge is said to be a loop.
2. Two different edges can have the same extremities.

   view figure 1.1

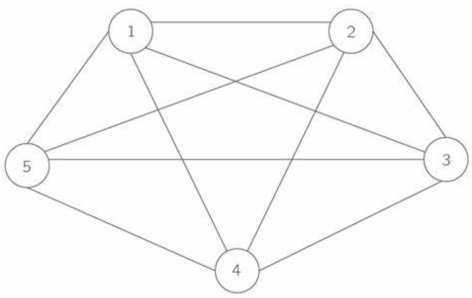


Figure 1.1: A graph with 5 vertices and 10 edges

A Directed Graph: Is a graph where each edge is oriented by an arrow. a directed edge goes from an initial extremity to a final extremity.

Bipartite Graphs: A graph whose nodes can be divided into two distinct sets such that no edge connects two nodes from the same set. Bipartite graphs are often used to model binary relationships between sets of data.

Expansion Graph: An extended graph that includes both the nodes and edges of the existing distribution graph, as well as potential new installations for network expansion, such as new transformers, substations, and distribution lines. This graph allows for modeling various possible configurations for network expansion.

A Path: Is a sequence of edges joined end to end, connecting two vertices called path extremities.

An Oriented Path: Is a sequence of oriented edges such that the final edge of an edge is equal to the initial edge of the next edge.

Shortest Paths: A shortest paths in a graph are the paths between two nodes that require the fewest number of edges. These paths are important for determining efficient routes for transporting electricity through the distribution network.

Tree: is an indirect graph in which any two vertices are connected by exactly one path, or equivalently connect a cyclic indirect graph. For example the figure 1.2

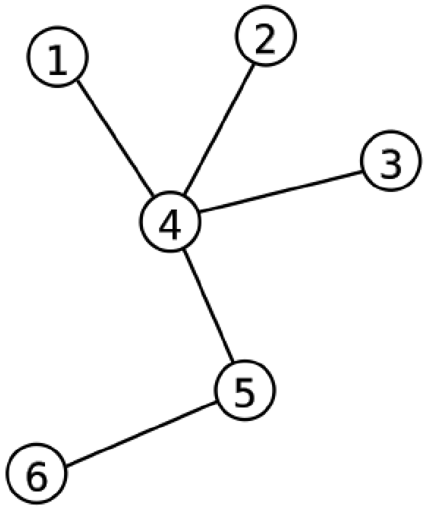


Figure 1.2: A labeled tree with 6 vertices and 5 edges

Nodes: In a graph, nodes represent individual entities in the network, such as transformers, substations, or other connection points. Each node can be considered as a starting or ending point for the flow of energy in the network.

Flow: A flow is a function that assigns a value to each edge, representing the amount of flow passing through that edge. The flow must satisfy certain constraints, such as not exceeding the capacity of any edge and maintaining flow conservation at each intermediate vertex (except for the source and sink).

Flow network: Is a directed graph where each edge has a capacity and is typically used to model a system where quantities can flow from one node to another. Flow networks are extensively used in various real-world applications, including transportation systems, computer networks, and hydraulic systems. the key components of a flow network:Vertices (Nodes), Edges (Links), Capacity, Source and Sink, flow.

Maximum Flow: In a graph network, maximum flow refers to the maximum amount of flow (e.g., electricity in an electrical network) that can be transported from a source node to a destination node while respecting the capacities of the edges and the constraints of the network. view the figure 1.3

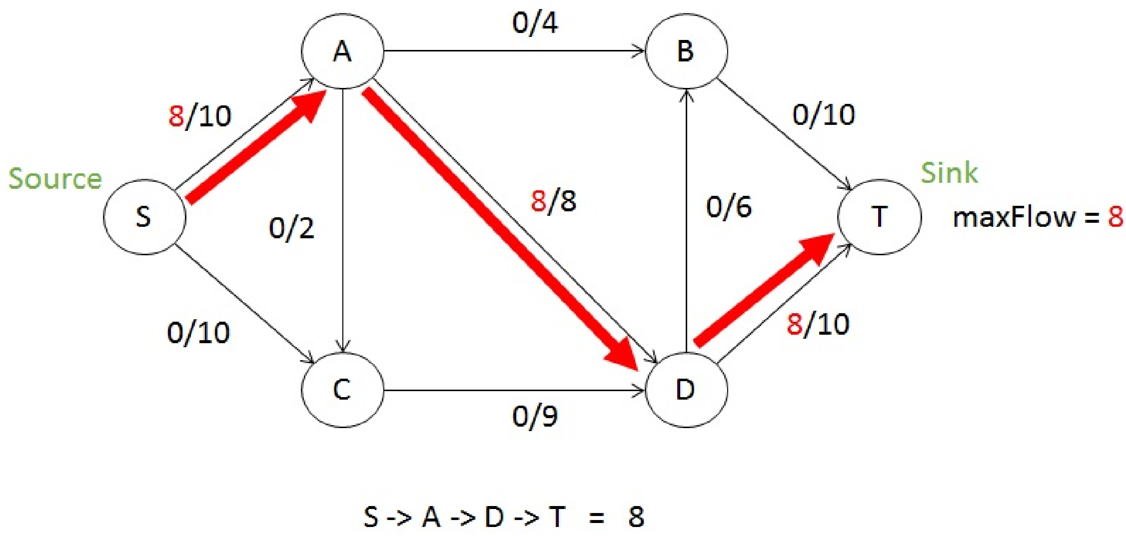


Figure 1.3: the maximum flow

Cut: The cut is often denoted as (S,T) where S contains the source vertex s and possibly other vertices, and T contains the sink vertex t and possibly other vertices. Edges in the flow network that have one endpoint in S and the other end point in T constitute the "cut edges.

Minimum Cut: A partition of a graph into two sets of nodes such that the number of edges crossing the partition is minimized. Minimum cuts are used in various contexts, including network segmentation and resilience planning. view figure 1.4

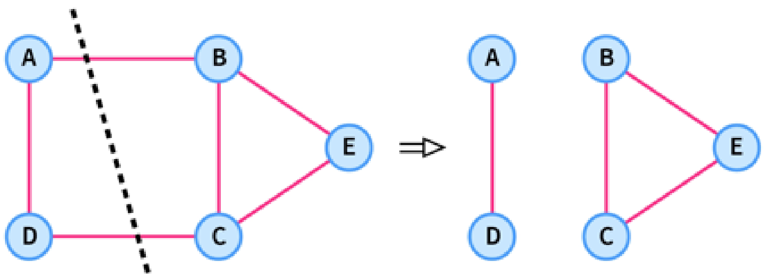


Figure 1.4: A minimum cut

By clarifying the key definitions, we have established a common language that will facilitate in-depth discussions and subsequent analyses. Thus, this chapter serves not only as a starting point but also as a reference point for our study.

CHAPTER 2

OPTIMIZATION MODELS

Optimization models are mathematical tools used to find the best solution to a problem by maximizing or minimizing an objective function while considering constraints. These models come in various forms such as linear programming, integer programming and nonlinear programming [30]. They are crucial in lead optimization processes in drug discovery, where properties of molecules are improved based on specific criteria [34]. Researchers have developed generic models for optimization, utilizing methods like simulated-annealing and hill-climbing to find optimal solutions efficiently [45]. Additionally, a structured set of data and templates are used to generate prescriptive models that are then translated into technical and business prescriptive domains, ultimately leading to the creation and solving of optimization models for specific problems [14].

2.1 Types of mathematical optimization models

In general, the more basic the assumptions made about the components of the optimization model, the more effective the approaches are for resolving such issues.

The model can be expressed in general form as:

Minimize

$$f(\mathbf{x})$$

subject to

$$g_i(\mathbf{x}) \leq 0 \quad \text{for } i = 1, \ldots, m$$

$$h_i(\mathbf{x}) = 0 \quad \text{for } i = 1, \ldots, \ell$$

$$\mathbf{x} \in X,$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function, $g : \mathbb{R}^n \to \mathbb{R}^m$ is a collection of m inequality constraints and $h : \mathbb{R}^n \to \mathbb{R}^l$ is a collection of l equality constraints.

In fact, every inequality constraint can be represented by an equality constraint by making $h_i(x) = g_i(x) + x_{n+1}$ and augmenting the decision variable vector $x \in \mathbb{R}^n$ to include the slack variable x_{n+1}. However, since these constraints behave very differently from an algorithmic standpoint, we will explicitly represent both whenever necessary.

In the next section we will present the general types of optimization models:

2.1.1 Linear Programming problem

Linear programming(**LP**), also known as linear optimization, is a method used to achieve the best outcome in a mathematical model by optimizing a linear objective function subject to linear equality and inequality constraints [4] [16].It involves finding a vector that minimizes a function while satisfying given constraints,commonly applied in mathematics, economics,business and engineering fields like transportation and manufacturing [4][16].Linear programming is crucial for optimizing resource allocation and management, offering solutions to maximize profit, minimize costs, or reduce resource use [42] [13].

Linear programming are problems that can be expressed in standard form as:

find a vector:

$$\mathbf{x}$$

that minimizes:

$$\mathbf{c}^\mathsf{T}\mathbf{x}$$

subject to:

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$

and

$$\mathbf{x} \geq 0$$

where the components of $\mathbf{x}$ are the variable to be determined, $\mathbf{c}$ and $\mathbf{b}$ are given vectors, and $\mathbf{A}$ is a given matrix [4].

Linear programming problem formulated in standard form using nonnegative slack variables is given as follows [39]:

$$P = \{x \in \mathbb{R}^n : A_1x \leq b_1, A_2x \geq b_2, x \geq 0\}$$

It can be represented as a polyhedral set in standard form by introducing nonnegative slack variables $s_1 \geq 0$ and $s_2 \geq 0$, making the set:

$$P = \left\{(x, s_1, s_2) \in \mathbb{R}^{n+|b_1|+|b_2|} : A_1x + s_1 = b_1, A_2x - s_2 = b_2, (x, s_1, s_2) \geq 0\right\}$$

where $|u|$ represents the cardinality of vector u [39]

Characteristics of LP problem:

Regardless of the way one defines linear programming, certain basic requirements are necessary before this technique can be employed to distribution network problems. These include:

1. **Well Defined Objective Function for Distribution Networks:** A clearly defined objective must be stated, this may involve maximizing efficiency by optimizing resource utilization, minimizing costs while using limited productive factors, or optimizing the distribution of resources over a specific time period.

2. **Alternative Courses of Action in Distribution Networks:** There must be alternative courses of action available. For instance, decisions could involve selecting between various combinations of equipment and manpower, or allocating manufacturing capacity in specific ratios to produce different products.

3. **Additivity of Resources and Activities in Distribution Networks:** Additivity here ensures that the total resources utilized across different activities equal the sum of resources used by each individual activity, promoting efficient resource allocation without interactions between activities.

4. **Linearity of Objective Function and Constraints in Distribution Networks:** Linear programming necessitates that both the objective function and constraints governing it should be linear. Without adherence to linearity, this technique cannot be effectively applied.

5. **Non-negativity of Decision Variables in Distribution Networks:** In the context of distribution networks, all decision variables should be non-negative as negative activities or variables are typically nonsensical.

6. **Divisibility of Activities and Resources in Distribution Networks:** Activities and resources must be divisible, allowing for fractional quantities and ensuring continuous resource utilization and output.

7. **Finiteness of Activities and Resources in Distribution Networks:** The optimal solution cannot be computed if there are infinitely many alternative activities and resource restrictions. Typically, distribution network problems involve a finite number of activities and constraints.

8. **Proportionality of Activity Levels to Resources in Distribution Networks:** Proportionality assumes linear relationships between activities and resources. For example, doubling output requires doubling the necessary resources, implying constant resource productivity and returns to scale.

9. **Single-valued Expectations in Distribution Networks:** It means that resources, activities, etc., are known with certainty, facilitating a deterministic programming model for optimizing distribution networks

Linear Programming Methods

The linear programming problem can be solved using different methods, we will discuss the two most important techniques called the graphical method and simplex method.

1. **Simplex method:** simplex method is one of the most popular methods to solve linear programming problems. It is an iterative process to get the feasible optimal solution. In this method, the value of the basic variable keeps transforming to obtain the maximum value for the objective function. The algorithm for linear programming simplex method is provided below:

   - Step 1: Establish a given problem. i.e write the inequality constraints and objective function.
   - Step 2: Convert the given inequalities to equations by adding the slack variable to each inequality expression.
   - Step 3: Create the initial simplex tableau. Write the objective function at the bottom row. Here, each inequality constraint appears in its own row. Now, we can represent the problem in the form of an augmented matrix, which is called the initial simplex tableau.
   - Step 4: Identify the greatest negative entry in the bottom row, which helps to identify the pivot column. The greatest negative entry in the bottom row defines the largest coefficient in the objective function, which will help us to increase the value of the objective function as fastest as possible.
   - Step 5: Compute the quotients.To calculate the quotient, we need to divide the entries in the far right column by the entries in the first column, excluding the bottom row. The smallest quotient identifies the row. The row identified in this step and the element identified in the step will be taken as the pivot element.
   - Step 6: Carry out pivoting to make all other entries in column is zero.
   - Step 7: If there are no negative entries in the bottom row, end the process. Otherwise, start from step 4.
   - Step 8: Finally, determine the solution associated with the final simplex tableau.

2. **Graphical method:** The graphical method is used to optimize the two-variable linear programming. If the problem has two decision variables, a graphical method is the best method to find the optimal solution. In this method, the set of inequalities are

subjected to constraints. Then the inequalities are plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersecting region will help to decide the feasible region. The feasible region will provide the optimal solution as well as explains what all values our model can take. Let us see an example here and understand the concept of linear programming in a better way [13].

2.1.2 Non-Linear Programming

Nonlinear programming (NLP) is a mathematical optimization technique used to solve problems where the relationships between decision variables, constraints, and the objective function are nonlinear[31]. NLP problems can be more complex and challenging to solve than linear programming (LP) problems, and they may require different algorithms, such as the gradient method, the Newton method, or the penalty method [39]. NLP can be used to model problems such as portfolio optimization, machine learning, engineering design, and economics. To determine if a problem requires linear or nonlinear programming, one can look at the mathematical expressions that define the problem. If the expressions involve only linear terms, such as constants, coefficients, and variables, then the problem is linear. If the expressions involve nonlinear terms, such as powers, roots, logarithms, trigonometric functions, or products of variables, then the problem is nonlinear [31]. Sometimes, a non linear problems can be converted to a linear problem by applying some transformations or approximations[51].

This problem is nonlinear due to the presence of nonlinear functions in the definition of the objective function $f(x)$ or in the constraints $g_i(x)$ and $h_i(x)$. When these functions contain components such as x_i^2, $\sin(x_i)$ or e^{x_i}, they are not linear because they include terms like powers, trigonometric functions, or exponential that cannot be represented by simple linear equations.

Consider the following nonlinear programming problem:

Minimize

$$f(\mathbf{x})$$

subject to

$$g_i(\mathbf{x}) \leq 0 \quad \text{for } i = 1, \ldots, m$$

$$h_i(\mathbf{x}) = 0 \quad \text{for } i = 1, \ldots, \ell$$

$$\mathbf{x} \in \mathbb{X},$$

where $f, g_1, \ldots, g_m, h_1, \ldots, h_\ell$ are functions defined on $\mathbb{R}^n, \mathbb{X}$ is a subset of $\mathbb{R}^n$, and $\mathbf{x}$ is a vector of n components $x_1, \ldots, x_n$. The above problem must be solved for the values of the variables $x_1, \ldots, x_n$ that satisfy the restrictions and meanwhile minimize the function f [12].

The function f is usually called the objective function, or the criterion function. Each of the constraints $g_i(\mathbf{x}) \leq 0$ for $i = 1, \ldots, m$ is called an inequality constraint, and each of the constraints $h_i(\mathbf{x}) = 0$ for $i = 1, \ldots, l$ is called an equality constraint, A vector $\mathbf{x} \in \mathbb{X}$ satisfying all the constraints is called a feasible solution to the problem. The collection of all such solutions forms the feasible region [12].

The difference between LP and NLP

Linear programming is a method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships whereas nonlinear programming is a process of solving an optimization problem where the constraints or the objective functions are nonlinear.

2.1.3 Integer Linear Programming

Integer programming is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers[35]. It is a subfield of discrete optimization, and is used in various fields such as operations research, computer science and economics. The problem can be formulated as follows:

maximize/minimize

$$c^{\mathsf{T}} x$$

subject to

$$Ax \leq b$$

$$x \geq 0$$

$$x \in \mathbb{Z}^n$$

where $c \in \mathbb{R}^n$,$A \in \mathbb{R}^{n \times m}$ and $c \in \mathbb{R}^m$.

Integer programming has various applications, such as scheduling, resource allocation and network design[38]. It is also used in combinatorial optimization problems, such as the knapsack problem, the traveling salesman problem and the vertex cover problem[49]. These problems are known to be NP-hard, and integer programming provides a framework for solving them optimally or approximately[35].

Algorithm for ILP

There are various methods to solve integer programming problems, including the branch-and-bound method,branch-and-cut, and the cutting plane method[21].

- **Branch-and-Bound method:** The branch-and-bound method is a systematic search algorithm that explores the solution space by branching on integer variables and bounding the objective function [21].

- **The cutting plane method:** The cutting plane method is a technique that adds valid inequalities to the linear programming relaxation to cut off fractional solutions [21].

- **Branch-and-Cut method:**The Branch and Cut method is an optimization algorithm used to solve integer linear programming (ILP) problems. It combines the Branch and Bound algorithm with the use of cutting planes [48]. This method involves running a branch and bound algorithm and using cutting planes to tighten the linear programming relaxations. The Branch and Cut method is generally more efficient than the pure Branch and Bound method, as the cutting planes help to reduce the size of the search tree[48][44]. It has been successfully applied to solve a wide range of optimization problems, including mixed-integer linear programs, submodular functions, large-scale symmetric traveling salesman problems, and more[44][43].

Algorithm 1 Branch and Cut algorithm

1: Add the initial ILP to L, the list of active problems
2: Set $x^* =$ **null** and $v^* = -\infty$
3: **while** L is not empty **do**
4:     Select and remove (dequeue) a problem from L
5:     Solve the LP relaxation of the problem.
6:     **if** the solution is infeasible **then**
7:         Go back to Step 3 (while)
8:     **else**
9:         Denote the solution by x with objective value v.
10:         **if** $v \leq v^*$ **then**
11:             Go back to Step 3
12:         **end if**
13:         **if** x is integer **then**
14:             Set $v^* \leftarrow v$, $x^* \leftarrow x$ and go back to Step 3
15:         **end if**
16:         **if** desired, search for cutting planes that are violated by x **then**
17:             If any are found, add them to the LP relaxation and return to Step 3.2
18:         **end if**
19:         Branch to partition the problem into new problems with restricted feasible regions. Add these problems to L and go back to Step 3
20:     **end if**
21: **end while**
22: Return x^*

where:

- L: List of active problems (ILP instances) being considered for solution.

- x^*: Current optimal integer solution.
- v^*: Objective function value associated with x^*, the current optimal solution.
- x: Represents a feasible solution to the ILP problem.
- v: Objective function value associated with a feasible solution x.

Solving Linear Equations, Linear Programming, and Integer Linear Programming: We review the hierarchical and methodological approach to solving linear equations,

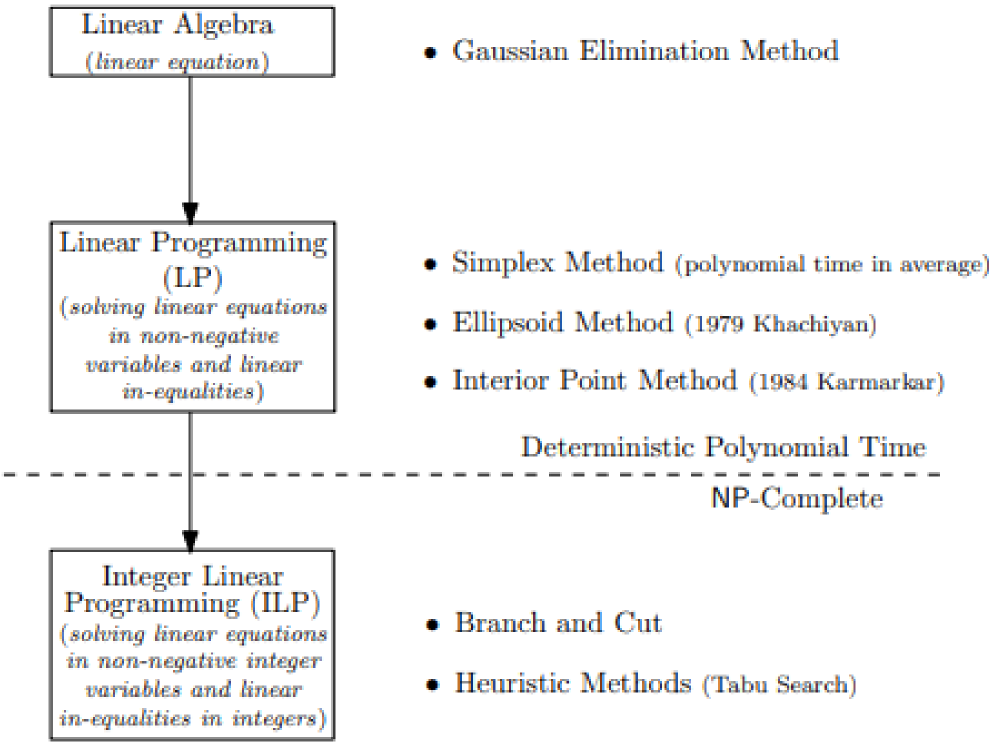


Figure 2.1: The difference in time complexity of linear programs and integer linear programs is highlighted by the dashed line.

linear programming, and integer linear programming, as illustrated in the Figure 2.1.

1. **Linear Algebra**: Linear algebra deals with solving linear equations using the Gaussian elimination method.
    - **Gaussian Elimination Method**: This method is used to transform a system of linear equations into a triangular or echelon form, which simplifies the solution of the system.
2. **Linear Programming (LP)**: Linear programming aims to solve linear equations with non-negative variables and linear inequalities. The methods used to solve linear programming problems include:

- **Simplex Method**: This method is very efficient and has polynomial time in average cases.
- **Ellipsoid Method**: Discovered by Khachiyan in 1979, this method is used to examine solution points within a variable ellipsoid.
- **Interior Point Method**: Discovered by Karmarkar in 1984, it is one of the effective polynomial methods for solving linear programming problems.

Linear programming can be solved in deterministic polynomial time.

3. **Integer Linear Programming (ILP)**: Integer linear programming aims to solve linear equations with non-negative integer variables and linear inequalities in integers. This problem is classified as NP-Complete, which means finding an efficient solution is a significant challenge. The methods used to solve integer linear programming problems include:

   - **Branch and Cut Method:** This method is used to divide the problem into subproblems and solve them iteratively.
   - **Heuristic Methods:** Such as Tabu Search, which are approximate methods aimed at finding good solutions in a reasonable time.

2.1.4 Mixed Integer Linear Programming

Mixed-Integer linear programming (MILP) is an optimization problem in which some or all of the variables are restricted to be integer. MILP problems consist of a linear objective function, linear constraints and integer restrictions on some or all of the variables [50]. MILP is part of the broader scope of Mixed-Integer Nonlinear Programming (MINLP), which handles both discrete and continuous variables with non-convex functions, presenting challenges in optimization [46].

The general form of mixed integer linear programming is:

$$\begin{aligned} \text{maximize} \quad & c^\top x + d^\top y \\ \text{subject to} \quad & A_1 x + A_2 y \leq b \\ & x \in \mathbb{R}^{n_x}_{\geq 0},\ y \in \mathbb{Z}^{n_y}_{\geq 0} \end{aligned}$$

where $b \in \mathbb{R}^m$, $c \in \mathbb{R}^{n_x}$, $d \in \mathbb{R}^{n_y}$, $A_1 \in \mathbb{R}^{m \times n_x}$, and $A_2 \in \mathbb{R}^{m \times n_y}$ are given.

Key Concepts of MILP

1. **Decision Variables:**
   - **Continuous Variables:** These variables can take any real value within a given range [39].
   - **Integer Variables:** These variables are restricted to integer values [39].
   - **Binary Variables:** A special case of integer variables that can only take values 0 or 1, often used to represent yes/no decisions [39].
2. **Objective Function:** The objective function in MILP is a linear function that we aim to either maximize or minimize [39].
3. **Constraints:** Constraints in MILP are linear inequalities or equalities that the decision variables must satisfy [39].

Formulation of MILP:

To formulate a MILP problem, follow these steps [39]:

- **Define Decision Variables:** Identify the variables that represent the decisions to be made, specifying which are continuous, integer, or binary.
- **Construct the Objective Function:** Develop a linear objective function that reflects the goal of the problem, whether it is to maximize profit, minimize cost, etc.
- **Establish Constraints:** Formulate the constraints that the decision variables must satisfy. These constraints can include resource limitations, capacity restrictions, and logical conditions.

Solving MILP:

1. **Presolving Methods:** Employed before the branch-and-cut process to simplify the problem [39]. Presolving techniques include:
   - **Reducing Problem Size:** Fixing variables and eliminating redundant constraints.
   - **Strengthening LP Relaxation:** Tightening bounds and coefficients where the LP relaxation of an integer programming problem $\min\{c^\top x : x \in P \cap \mathbb{Z}^n\}$ with $P = \{x \in \mathbb{R}^n_+ : Ax \leq b\}$ is the linear programming problem$\min\{c^\top x : x \in P\}$. An LP relaxation is indeed a relaxation, as it expands the feasible region by relaxing the integrality constraints while keeping the objective function unchanged.
   - **Detecting Infeasibility and Redundancy:** Using constraint activities to identify and remove infeasible or redundant parts of the problem.

These methods help in reducing the computational burden by making the problem easier to solve.

2. **Branch and Bound method:** The Branch and Bound(B&B) method is an algorithmic technique for solving integer and combinatorial optimization problems, including Mixed- Integer Linear Programming (MILP). Algorithm 2 is a step-by-step for the Branch and Bound method

Algorithm 2 B&B algorithm for MILP

Initialize best solution $\leftarrow \infty$ (for minimization problems)
Initialize queue with the root node
while queue is not empty **do**
    current node $\leftarrow$ SELECT NODE(queue)
    SOLVE RELAXATION(current node)
    $z \leftarrow$ optimal value of relaxation
    $x^* \leftarrow$ solution of relaxation
    if $z \geq$ best solution **then**
        PRUNE(current node)
    else
        if x^* is integer feasible **then**
            best solution $\leftarrow \min(\text{best solution}, z)$
        else
            choose variable x_i with fractional value in x^*
            create left node with constraint $x_i \leq \lfloor x_i^* \rfloor$
            create right node with constraint $x_i \geq \lceil x_i^* \rceil$
            add left node and right node to queue
        end if
    end if
end while
return best solution

where z is the variable that holds the optimal solution value (in this case, for the relaxation problem) in each iteration of the algorithm, x^* is the solution to the relaxation or linearized problem at the current node.

3. **Branch and Cut algorithm:** The Branch and Cut algorithm is a powerful method used to solve Mixed-Integer Linear Programming (MILP) problems. It combines elements of branch and bound (to explore the solution space) with cutting planes (to strengthen the linear relaxation of the problem) [39]. The algorithm 3 is a structured outline of the Branch and Cut algorithm for MILP:

Algorithm 3 Branch and Cut Algorithm for MILP

Initialize best solution $\leftarrow \infty$ // For minimization problems
Initialize queue with the root node
while queue is not empty **do**
    current node $\leftarrow$ select node from queue
    Solve relaxation at current node to get z and x^*
    if $z \geq$ best solution **then**
        Prune current node
    else
        if x^* is integer feasible **then**
            best solution $\leftarrow \min(\text{best solution}, z)$
        else
            Choose variable x_i with fractional value in x^*
            Create left node with constraint $x_i \leq \lfloor x_i^* \rfloor$
            Create right node with constraint $x_i \geq \lceil x_i^* \rceil$
            Add left node and right node to queue
        end if
    end if
end while
return best solution

where:

- x_i is chosen based on its fractional value in x^* when branching.
- x^* is the current solution vector obtained from solving the relaxation.
- z is compared with best solution to determine whether to prune a node or update the best solution found so far.

4. **Heuristics:** These are approximate methods used to find feasible solutions quickly [39]. They can be used within the branch-and-bound framework to provide good starting solutions or to improve bounds.

    - **Primal Heuristics:** Such as diving heuristics and local searches, aim to find feasible integer solutions by exploring the neighborhood of the LP relaxation solutions.
    - **Large Neighborhood Search (LNS):** A heuristic that explores large neighborhoods of the current solution to find better solutions.

5. **Decomposition Methods:** Used for large-scale MILP problems by breaking them down into smaller, more manageable subproblems [39].

    - **Dantzig-Wolfe Decomposition:** Reformulates the problem to exploit its structure and solve it iteratively.

- **Benders Decomposition:** Separates the problem into a master problem and sub-problems, solving them iteratively and exchanging information between them [39].

2.1.5 Mixed Integer Non-Linear Programming

Mixed-Integer Nonlinear Programming (MINLP) is a mathematical optimization technique that combines the numerical difficulties of handling nonlinear functions with the challenge of optimizing in the context of non-convex functions and discrete variables[26]. MINLP problems are characterized by the presence of both continuous and integer variables, and nonlinear relationships between these variables and the objective function. MINLP is used to model complex real-world problems in various fields, such as engineering design, finance, and logistics[26][5].

MINLP problems can be solved using various algorithms, including the modified Sequential quadratic programming (SQP) algorithm [22], which is used in the mixed integer nonlinear solver h02da in the NAG Library[5]. This solver can handle both convex and non-convex problems and is useful in scientific, engineering, and financial applications where fully nonlinear models are needed[5].

Basic elements of MINLP methods

The basic concept underlying algorithms for solving (MINLP) is to generate and refine bounds on its optimal solution value. Lower bounds are generated by solving a relaxation of (MINLP), and upper bounds are provided by the value of a feasible solution to (MINLP). Algorithms differ in the manner in which these bounds are generated and the sequence of sub-problems that are solved to generate these bounds. However, algorithms share many basic common elements[28], which are described next.

The general form of mixed integer nonlinear programming is:

$$\text{maximize } f(x,y) \tag{2.1}$$

subject to

$$g_i(x,y) \leq 0 \quad \forall i \in \{1,\ldots,m\} \tag{2.2}$$

$$h_j(x,y) = 0 \quad \forall j \in \{1,\ldots,p\} \tag{2.3}$$

where f is the nonlinear objective function, g_i and h_j are the constraint functions, x are the continuous variables and y are the integer variables.

Key Concepts of MINLP

1. **Decision Variables:**
   - Continuous Variables: These variables can take any real value within a given range [39].
   - Integer Variables: These variables are restricted to integer values [39].
   - Binary Variables: A special case of integer variables that can only take values 0 or 1 [39].
2. **Objective Function:** The objective function in MINLP can be nonlinear, and the goal is to either maximize or minimize it [39].
3. **Constraints:** Constraints in MINLP are nonlinear inequalities or equalities that the decision variables must satisfy[39].

Formulation of MINLP

To formulate a MINLP problem [39], follow these steps:

- **Define Decision Variables:** Identify the variables that represent the decisions to be made, specifying which are continuous, integer or binary.
- **Construct the Objective Function:** Develop a nonlinear objective function that reflects the goal of the problem.
- **Establish Constraints:** Formulate the constraints that the decision variables must satisfy. These constraints can include nonlinear relationships, resource limitations and logical conditions.

Solving MINLP:

1. **Branch-and-Bound algorithm for nonlinear programming** In NLP-BB, the lower bounds come from solving the sub-problems (NLPR (l_I, u_I)). Initially, the bounds (L_I, U_I) (the lower and upper bounds on the integer variables in (MINLP)) are used, so the algorithm is initialized with a continuous relaxation whose solution value provides a lower bound on z_{MINLP}. The variable bounds are successively refined until the subregion can be fathomed. Continuing in this manner yields a tree $\mathscr{L}$ of sub-problems. A node N of the search tree is characterized by the bounds enforced on its integer variables: $N \stackrel{\text{def}}{=} (l_I, u_I)$. Lower and upper bounds on the optimal solution value $z_L \leq z_{\text{MINLP}} \leq z_U$ are updated through the course of the algorithm. Algorithm 1 gives pseudo-code for the NLP-BB algorithm for solving (MINLP) [28].

Algorithm 4 The NLP Branch and Bound algorithm for solving MINLP

1: **Initialize**

2:
$$\mathscr{L} \leftarrow \{(L_I, U_I)\}, z_U = \infty, x^* \leftarrow \text{ NONE.}$$

3: **Terminate?**
Is $\mathscr{L} = \emptyset$? If so, the solution x^* is optimal.

4: **Select** Choose and delete a problem $N^i = \left(l_I^i, u_I^i\right)$ from $\mathscr{L}$.

5: **Evaluate.** Solve NLPR $\left(l_I^i, u_I^i\right)$. If the problem is infeasible, go to step 1(Initialize) , else let $z_{\mathrm{NLPR}\left(l_I^i, u_I^i\right)}$ be its optimal objective function value and $\hat{x}^i$ be its optimal solution.

6: **Prune.**

7: **if** $z_{\mathrm{NLPR}}\left(\ell_I^i, u_I^i\right) \geq z_U$, go to step 1 **then** .

8: **end if**

9: **if** $\hat{x}^i$ is fractional, go to step 5.

10: **else**let $z_U \leftarrow z_{\mathrm{NLPR}\left(l_I^i, u_I^i\right)}, x^* \leftarrow \hat{x}^i$, and delete from $\mathscr{L}$ all problems with $z_L^j \geq z_U$. Go to step 1 **then**

11: **end if**

12: **Divide.** Divide the feasible region of N^i into a number of smaller feasible sub-regions, creating nodes $N^{i_1}, N^{i_2}, \ldots, N^{i_k}$. For each $j = 1, 2, \ldots, k$, let $z_L^{i_j} \leftarrow z_{\mathrm{N.PR}\left(l_I^i, u_i^i\right)}$ and add the problem N^{i_j} to $\mathscr{L}$. Go to 1 .

where

- $\mathscr{L}$: The list of sub-problems to be solved.
- (L_I, U_I): The lower and upper bounds of the feasible region for the solution.
- z_U: The current value of the best known solution (upper bound).
- z_L: The current value of the best known solution (lower bound).
- x^*: The variable that holds the current optimal solution.
- N^i: Represents a subproblem selected from the list $\mathscr{L}$.
- (l_I^i, u_I^i): Tuple representing the lower and upper bounds of the feasible region for a specific subproblem N^i.
- $z_N LPR$: The optimal objective function value obtained when solving a nonlinear programming problem (NLPR).
- $z_{\mathrm{NLPR}}(l_I^i, u_I^i)$:: The optimal objective function value obtained from solving the subproblem N^i with bounds (l_I^i, u_I^i).
- $\hat{x}_i$: The optimal solution obtained for the subproblem N^i.
- $z_L^{i_j}$: The optimal objective function value associated with subproblem N^{i_j} after it has been divided into smaller sub-regions.

Branch-and-Bound Method: This method extends the branch-and-bound approach to handle nonlinearities in the objective function and constraints [39]. Algorithm 5 is Branch-and-Bound for MINLP Pseudocode.

Algorithm 5 Branch-and-Bound for MINLP Pseudocode

```
Initialize the root node with the original MINLP.
Solve the nonlinear programming (NLP) relaxation of the current node.
if the solution is integer and better than the current best then
    Update the best solution.
end if
if the solution is not integer then
    Select a variable to branch on.
    Create two new nodes with additional constraints (branching).
end if
Prune nodes that cannot improve the current best solution.
Repeat until all nodes are processed.
```

2. **Outer Approximation Method:** This method solves a sequence of MILP subproblems and NLP subproblems iteratively.

3. **Generalized Benders Decomposition:** This method decomposes the problem into a master problem and subproblems, solving them iteratively and exchanging information between them [39].

4. **Heuristics:** These are approximate methods used to find feasible solutions quickly.

    - **Primal Heuristics:** Such as diving heuristics and local searches, aim to find feasible integer solutions by exploring the neighborhood of the NLP relaxation solutions.
    - **Large Neighborhood Search (LNS):** A heuristic that explores large neighborhoods of the current solution to find better solutions.

5. **Hybrid Methods:** Combining different optimization techniques to leverage their strengths [39].

    - **Hybrid Branch-and-Cut and Outer Approximation:** Integrating cutting planes into the outer approximation framework.
    - **Metaheuristic-Guided Decomposition:** Using metaheuristics like genetic algorithms to guide the decomposition process.

2.1.6 Comparison between Optimization models

The differences between optimization models can be categorized in the following table:

Aspect	**LP**	**ILP**	**MILP**	**NLP**
Variables	Only continuous	Only integer	Mixed continuous and integer	Continuous, can include integer
Complexity	Solvable in polynomial time using algorithms like simplex or interior point methods[27] [33]	NP-hard, making it computationally intensive, especially for large instances [47]	Generally harder to solve than LP due to the added complexity of discrete variables, but more versatile than ILP	Most complex due to nonlinearity, may have multiple local optima, making it difficult to solve globally [27][40]
Objective Function and Constraints	Linear	Linear	Linear	Nonlinear
Flexibility	Limited to linear problems with continuous variables	Suitable for problems requiring integer solutions	Can approximate non-convex problems with arbitrary accuracy using binary variables [40]	High flexibility for handling nonlinear problems
Solution Quality	Provides optimal solutions efficiently but may not handle discrete decision variables or nonlinearity	Can provide exact solutions if solved optimally but the solution time may increase exponentially with problem size	Can provide exact or near-optimal solutions for a wide range of optimization problems with discrete decisions	Can provide high-quality solutions for problems where linear approximations are not sufficient
Applicability	Suitable for many practical problems due to its efficiency[27] [33]	Suitable for optimization problems where decision variables are required to be integer values [47]	Popular choice for energy system optimization problems[27]	Suitable for problems with nonlinear objective functions or constraints

Aspect	LP	ILP	MILP	NLP
Nonlinearity Handling	Not applicable	Not applicable	More flexible, can handle non-convex problems with the use of binary variables [40]	Designed to handle nonlinearity
Presence of Multiple Local Optima	Single global optimum [40][27]	Single global optimum [40][27]	Single global optimum [40][27]	Multiple local optima [40][27]

Table 2.1: the comparison between optimization models

To conclude this chapter dedicated to optimization models and methods, we have explored a range of essential approaches and tools for tackling complex decision and planning problems. These optimization models offer powerful and effective solutions for maximizing or minimizing objective functions while adhering to a set of constraints. This chapter serves not only to introduce these essential concepts but also to provide a solid foundation for more advanced analyses and practical applications to come.

CHAPTER 3 MATHEMATICAL MODELING OF THE PROBLEM

In an electricity-dependent world, the efficient and reliable management of electrical networks is crucial. Electrical distribution networks must meet growing demands for capacity, flexibility, and resilience against failures. The challenges posed by the integration of renewable energies, increasing demand, and the need to minimize service interruptions make the planning and optimization of these networks more complex and essential.

To address the question posed in the first chapter, we present this study in this chapter which aims to provide a comprehensive theoretical and practical framework for understanding existing work and developing a mathematical model for the planning of electrical distribution networks. It explores the fundamental concepts of distribution networks, resilience, and risk aversion, as well as mathematical modeling and optimization techniques. The goal is to enable readers to grasp the underlying principles of electrical network planning, understand scalability challenges, and implement effective optimization algorithms to address the problems encountered in this field.

3.1 Fundamental Concepts of Distribution Networks

3.1.1 Distribution Network Planning

Distribution network planning is a strategic process aimed at designing an efficient and cost-effective distribution network to meet customer demand. This process is essential to ensure that the electrical infrastructure can meet future needs while optimizing costs and minimizing environmental impacts [36][37]. The accompanying figure 3.1 represents an electrical distribution network that includes several switches and circuit breakers, illustrating a practical example of such a network. The network consists of six points numbered from 1 to 6, which represent different locations in the electrical network where loads or power sources are connected. The horizontal and vertical lines in the diagram represent wires that connect these points together, with each line carrying a specific electrical current denoted by the symbols I_2, I_3, I_4, I_5, and I_6. Circuit breakers, represented by the symbols B_1 to B_5, are devices used to protect the electrical circuit from overcurrents, each connected between two points allowing electrical current to pass through. The arrows in the image indicate the direction of the electrical current flowing through each point. For instance, the downward arrow at point 2 shows that the current I_2 flows downwards from point 2. The connections between points are as follows: Point 1 is connected to point 2 via breaker B_1 with current I_2; Point 2 to point 3 via breaker B_2 with current I_3; Point 3 to point 4 via breaker B_3 with current I_4; Point 4 to point 5 via breaker B_4 with current I_5; and Point 3 to point 6 via breaker B_5 with current I_6. This detailed representation highlights the critical components and their interconnections within an electrical distribution network, emphasizing the importance of strategic planning in ensuring the network's efficiency and reliability.

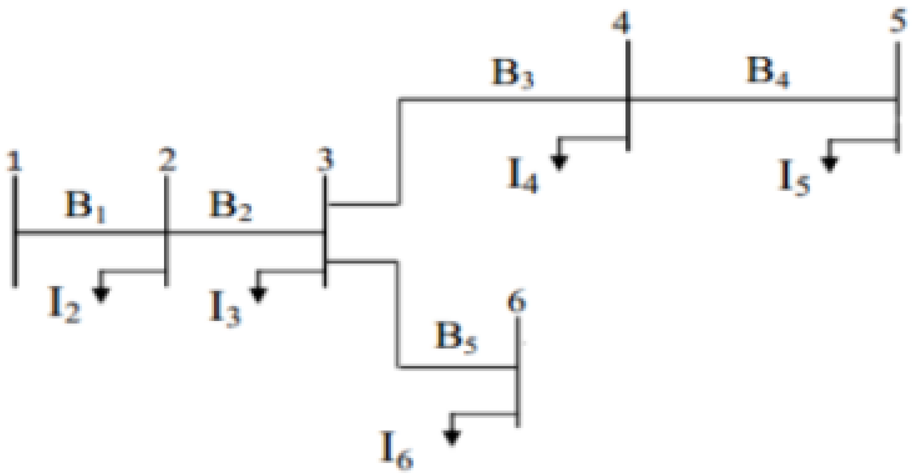


Figure 3.1: Single line diagram of distribution network

Objectives and Considerations of Distribution Network Planning

1. **Reliability and Continuity of Service:**

    - Minimize service interruptions and guarantee stable power supply for all consumers.
    - Integrate alternative paths and redundant equipment to prevent widespread outages and ensure rapid recovery after disruptions.

2. **Economic Efficiency:**

    - Minimize investment and operating costs, while maximizing economic benefits. This includes optimizing resources to achieve the best value for money.
    - Use existing infrastructure efficiently while planning new investments strategically to maximize their utility and lifespan.

3. **Quality of Electricity:**
    - Ensure voltage and frequency levels remain within specified limits to protect end-user equipment and ensure optimal performance.
    - Minimize harmonics, voltage variations and other disturbances which can affect the quality of the electricity supplied.
4. **Flexibility and Adaptability:**
    - Effectively integrate renewable energy sources, taking into account their variability and intermittency.
    - Plan for the future integration of emerging technologies such as smart grids, electric vehicles and energy storage systems.
5. **Security and Compliance:**
    - Ensure compliance with local, national and international regulations regarding performance and safety.
    - Ensure safe operations for personnel and the public, including protection against voltage surges, short circuits and other electrical hazards.

Typical Constraints

1. **Power Flow Constraints:** Power flow constraints are essential to ensure the safe and reliable operation of the distribution network. These constraints limit the amount of power that can be transported over distribution lines without exceeding the thermal capacities of conductors. Proper management of power flow is crucial to prevent overheating and potential damage to the infrastructure [36][37].
2. **Management Techniques:** Techniques such as active network management and the use of advanced control devices help optimize power flows and avoid overloads. Real-time monitoring and dynamic adjustment of power flows ensure that the network operates within safe limits [36][37].
3. **Voltage and Current Limits:** Voltage and current limits must be maintained to ensure power quality and avoid damage to electrical equipment. Fluctuations in voltage can affect the performance of appliances and disrupt customer satisfaction. Maintaining stable voltage and current levels is crucial for the overall reliability of the distribution network [36][37].
4. **Technological Solutions:** The installation of voltage regulators, automatic tap changers and demand management systems allows for precise control of voltage levels.

These technologies help maintain a consistent power supply and prevent issues related to voltage sags or surges [36][37].

5. **Capacity Constraints:** The capacities of distribution lines, substations, and other equipment must be sufficient to meet the projected peak demand. Planning must account for future growth to prevent potential overloads and ensure that the network can handle maximum load conditions without failures.

6. **Predictive Approaches:** Using advanced forecasting models helps anticipate future demand and plan necessary investments to expand network capacity. This includes upgrading existing infrastructure and building new facilities to accommodate load growth.

7. **Operational Constraints:** Operational constraints involve the management of distributed energy resources (DER) and energy storage systems. These constraints include technical limitations and the need for rapid response to demand fluctuations. Effective operation of DER and storage systems is crucial for balancing supply and demand in real-time.

8. **Smart Management:** Adopting smart grid technologies enables more efficient and responsive management of operational constraints. Smart grids integrate advanced sensors, communication networks and data analytic to enhance the coordination and control of DER and storage systems[36][37].

9. **Budget and Cost Constraints:** Budget and cost constraints are critical factors in distribution network planning. Planners must optimize investments to achieve the best return on investment while staying within budgetary limits. Cost-effective solutions are necessary to ensure the financial sustainability of the distribution network.

10. **Financing Strategies:** Innovative financing strategies, such as public-private partnerships and performance-based incentives, can provide the necessary resources for network expansion and modernization projects. Effective budget management and cost control measures ensure that projects are completed on time and within budget[36][37].

The ultimate goal of distribution network planning is to develop a plan adaptable to changing business conditions while meeting customer service requirements in a cost-effective and environmentally responsible manner. By integrating the latest technological advancements and considering economic and environmental factors, planners can create robust and sustainable networks.

Evolution of Energy System Planning with Renewable Integration and Smart Technologies

Incorporating traditional methods into modern energy planning frameworks is crucial for ensuring the sustainability, resilience,and efficiency of energy systems. Traditional approaches , such as demand forecasting, capacity planning, reliability studies, and cost optimization, have historically served as foundational pillars in energy planning strategies[36][37].

1. **Demand Forecast:**
   - Using historical electricity consumption data to predict future needs.
   - Taking into account economic, demographic and industrial factors to estimate demand growth.
2. **Capacity Planning:**
   - Provide safety margins to ensure that the network can meet the maximum expected demand.
   - Reinforcement of distribution lines, transformers and substations to meet growing demand.
3. **Reliability Studies:**
   - Analysis of outage scenarios to ensure that the network can withstand individual equipment outages without significant loss of service.
   - Use of measurements for the evaluation of the reliability and performance of the electrical system.
4. **Cost Optimization:**
   - Evaluation of investment projects based on their long-term profitability.
   - Allocation of financial resources optimally to maintain and improve the network.

By integrating traditional methods 3.1.1 with modern technologies, energy planners can develop comprehensive strategies that optimize resource utilization, enhance grid reliability, and mitigate environmental impacts. This holistic approach fosters sustainable energy systems capable of meeting present and future energy demands[36][37].

These modern methodologies, prominent examples encompass:

1. **Integration with Solar Energy:** Incorporating solar energy technologies, such as building-integrated photo voltaic (BIPV) and advancements in solar thermal systems, diversifies the energy mix and reduces reliance on fossil fuels.

2. **Adoption of Modular Architecture and Data Management:** Leveraging modular IT and data architectures facilitates the seamless integration of renewable energy sources and smart technologies. This enables real-time monitoring, control, and optimization of energy usage, enhancing system flexibility and scalability.

3. **Integration with Artificial Intelligence and Machine Learning:** Harnessing the power of artificial intelligence (AI) and machine learning (ML) algorithms optimizes energy production, distribution, and consumption patterns. AI-driven analyses of vast datasets improve predictive capabilities, enabling proactive management of energy resources and grid stability.

4. **Implementation of Energy Storage Solutions:** Deploying energy storage technologies, such as batteries and pumped hydro, addresses intermittency issues associated with renewable energy sources. Energy storage systems enhance grid resilience by storing excess energy during periods of low demand for use during peak hours,ensuring a reliable energy supply.

3.1.2 Fundamentals of risk management

Definition:

The definition of risk in The oxford English Dictionary[19] is as follows:”a chance or possibility of danger, loss, injury or other adverse consequences” and ”at risk” is defined as ”exposed to danger.” In this context, risk denotes negative outcomes. Nevertheless, taking a risk can lead to a positive result. Another perspective is that risk is associated with uncertainty regarding the outcome [23]. And thus, risk management can be defined as: the process of identifying, evaluating, and prioritizing risks, followed by the coordinated and economical application of resources to minimize, monitor, and control the probability or impact of unfortunate events or to maximize the realization of opportunities[11][24]. It involves anticipating what might not go to plan and putting in place actions to reduce uncertainty to a tolerable level[11].

The main steps of risk management include[11][24]:

1. **Risk identification:** Analyzing activities, processes, and environments to identify potential risks from various sources, both internal and external.

2. **Risk analysis and assessment:** Establishing the probability of a risk event occurring and its potential impact. Risks are then evaluated and ranked according to prominence and consequence.

3. **Risk mitigation and monitoring:** Developing and implementing methods and options to reduce threats, such as risk avoidance, transfer, sharing, or reduction. Continuous monitoring and adaptation of the risk management process is crucial.

Effective risk management helps organizations protect their assets, promote a risk-aware culture, support strategic decision-making, and increase the likelihood of business continuity and success[24][29]. It is a fundamental discipline for enterprises operating in complex and rapidly evolving business environments[29].

Resilience and Reliability

Concept of Resilience in Electrical Distribution Networks: The resilience of electrical distribution networks refers to their ability to withstand, quickly recover from, and adapt to major disruptions such as natural disasters, technical failures, or intentional attacks. Key resilience indicators include robustness, recovery time, recovery cost, and adaptive capacity.

1. **Resilience Indicators:**

    - **Robustness:** Measures the network's ability to resist initial disruptions without failure. This can be evaluated by the reliability of system components and the redundancy of infrastructure. For example, reinforced structures and redundant lines can enhance the network's robustness.
    - **Recovery Time:** Represents the duration required to restore service after a disruption. Networks with effective planning and management can significantly reduce recovery time. Techniques such as automatic reconfiguration and the use of microgrids can help minimize downtime.
    - **Recovery Cost:** Includes the financial costs of repairing and restoring service after a disruption. This encompasses the replacement of damaged equipment, labor costs, and economic losses due to service interruptions.
    - **Adaptive Capacity:** Refers to the network's ability to evolve and adapt to new conditions and threats. This includes integrating new technologies such as renewable energy sources and energy storage systems, as well as improving management and maintenance practices.

2. **Strategies to Improve Resilience:**

    - **Investment in Infrastructure:** Strengthening existing infrastructure and building new, more resilient infrastructure to withstand natural disasters and other threats. This includes installing underground cables in storm-prone areas and constructing more robust substations.
    - **Energy Storage Technologies:** Integrating energy storage systems to ensure continuous power supply during disruptions and facilitate quick recovery. Batteries and other storage systems can provide backup power and stabilize the grid.

- **Micro-grids and Decentralized Systems:** Developing micro-grids capable of operating independently during main grid outages. Micro-grids can be quickly restored and continue to provide electricity to critical areas.
- **Advanced Planning and Management:** Using modeling and simulation tools to anticipate potential disruptions and develop contingency plans. Proactive planning can identify vulnerabilities and enable preventive measures to enhance resilience.
- **Policy and Regulatory Reforms:** Implementing policies and regulations that promote investments in resilience and encourage the adoption of resilient technologies. Financial incentives and subsidies can support these efforts.

Reliability: On the other hand, focuses on routine failures and is generally evaluated using classical reliability indices(SAIDI – SAIFI – CAIDI – MAIFI 3.1.2). These indices are designed to capture common outages and are often used for ex-post evaluation. The traditional methodologies used by the industry to plan improvements in distribution systems do not account for the risk associated with HILP (High-Impact, Low-Probability) events, which are much less predictable and far more impactful than routine events.

HILP Events: HILP events are incidents that, although rare, have extremely serious consequences on energy systems. Here are some typical characteristics and examples of HILP events:

Characteristics of HILP Events:

1. Low probability: These events rarely occur, making their prediction and modeling difficult.
2. High impact: When they occur, they can cause significant damage to infrastructures, widespread service disruptions, and significant economic costs.
3. Complexity of management: The unpredictable nature and severity of these events require robust and flexible approaches for resilience and recovery.

Examples of HILP Events:

1. Natural disasters:
   - Earthquakes.
   - Hurricanes and severe wind storms.
   - Major floods.
   - Wildfires.

2. Technological and industrial incidents:
   - Major power grid failures (blackouts).
   - Explosions or accidents at critical facilities.
3. Cyberattacks:
   - Coordinated attacks against critical energy infrastructures.
4. Pandemics:
   - Large-scale epidemics affecting human resources and infrastructure operations.

Importance of considering HILP Events: HILP events present unique challenges for planning and managing energy systems. Considering these events in planning processes is crucial for several reasons:

1. Preparedness and resilience:
   - Developing infrastructures capable of withstanding extreme events and quickly recovering after their occurrence.
   - Integrating resilience strategies into the design and operation of energy systems.
2. Investment and planning:
   - Justifying investments in long-term resilience measures by demonstrating the benefits in terms of risk reduction.
   - Using stochastic optimization techniques to account for the uncertainty and variability of HILP events in investment decisions.
3. Policy and regulation:
   - Collaborating with regulators to develop standards and guidelines that account for HILP risks.
   - Educating stakeholders on the need for resilient investments and the long-term benefits for the security and reliability of energy systems.

To bridge the gap between resilience and reliability, it is necessary to develop analytical methodologies that support utilities investment decisions. These methodologies must be able to capture the benefits of long-term risk mitigation and demonstrate to regulators the added value in terms of resilience of various investment options.

types of risk in distribution grid

The types of risk management approaches can be categorized as follows:

1. **Risk-Averse Investment Plans:** Three distinct levels of risk aversion in investment strategies are identified: risk-neutral ($\lambda = 0$), medium risk aversion ($\lambda = 0.5$), and high risk aversion ($\lambda = 1$). These strategies vary in their approach to minimizing the Conditional Value at Risk (CVaR) associated with the cost of load loss. The highest level of risk aversion focuses exclusively on reducing CVaR[36].

2. **Incorporation of Risk-Based Objectives:** Integrating risk-based objectives in planning processes is emphasized as crucial. This integration addresses both routine failures related to reliability and extreme events related to resilience. Traditional risk-neutral metrics, such as Expected Energy Not Served (EENS), are insufficient for capturing the impacts of High-Impact Low-Probability (HILP) events, highlighting the need for risk-aversion strategies[36].

3. **Consideration of Reliability and Resilience:** The proposed methodology allows planners to assign different levels of importance to reliability and resilience according to their risk aversion. This approach includes investments in both line segments and storage devices, providing a comprehensive risk management strategy for large-scale distribution systems[36].

Traditional metrics and limitations

Traditional metrics for risk management in distributions networks expansion planning are essential for ensuring that the grid can meet future demands while minimizing potential risk. These metrics typically cover various aspects of reliability, economic performance and system robustness.

A selection of traditional metrics:

1. **Expected Energy Not Supplied (EENS):**

   This metric quantifies the anticipated amount of energy that will not be delivered to customers due to system failures or inadequacies. It is crucial for evaluating the reliability and resilience of the distribution grid, ensuring that necessary improvements can be made to minimize disruptions.

2. **Loss of Load Probability (LOLP):**

   The LOLP represents the probability that the system's demand will exceed the available supply capacity at any given time. This measure is essential for assessing the adequacy of the system's capacity to meet consumer demand, helping to identify potential areas of vulnerability.

3. **Loss of Load Expectation (LOLE):**

   LOLE indicates the expected number of hours or days in a specific period during which the load surpasses the available capacity. This metric provides valuable insights into the frequency of potential supply shortfalls, aiding in strategic planning to enhance grid reliability.

4. **System Average Interruption Frequency Index (SAIFI):**

   SAIFI measures the average number of interruptions a customer would experience over a set period. This index is key to understanding the reliability of the distribution system from the customer's perspective, highlighting areas where service improvements are needed.

5. **System Average Interruption Duration Index (SAIDI):**

   SAIDI calculates the total duration of interruptions for the average customer over a specified period. It is a vital measure of system reliability, reflecting the average outage duration customers face and indicating where infrastructure enhancements may be necessary.

6. **Customer Average Interruption Duration Index (CAIDI):**

   CAIDI measures the average time required to restore service to customers per interruption. This index focuses on the efficiency of the utility in restoring service after an outage, emphasizing the importance of rapid response and repair times.

7. **Frequency of Interruptions:**

   This metric counts the number of times service is interrupted over a certain period, helping utilities identify areas with frequent service disruptions and prioritize maintenance and upgrades accordingly.

8. **Duration of Interruptions:**

   The total time that service is interrupted during a given period is measured by this metric, assisting in evaluating the impact of outages on customers and informing strategies to reduce interruption durations.

9. **Cost-Benefit Analysis (CBA):**

   CBA is a financial assessment that compares the costs of expanding the distribution grid to the benefits gained, such as reduced outages and increased capacity. It ensures economic viability and helps prioritize investments based on their expected returns.

10. **Net Present Value (NPV):**

NPV calculates the present value of cash inflows minus the present value of cash outflows over the lifecycle of a grid expansion project. This measure determines the overall value and financial feasibility of the project, guiding investment decisions.

11. **Internal Rate of Return (IRR):**

    IRR is the discount rate that makes the NPV of an expansion project zero. It is used to evaluate the profitability of an investment, indicating the potential returns relative to the cost of the project.

12. **Risk of Cascading Failures:**

    This metric assesses the probability and impact of a failure in one part of the grid causing subsequent failures in other parts. Understanding this risk helps in designing a more robust and resilient distribution network.

13. **Reliability Index (RI):**

    RI is a composite measure of system reliability that often incorporates factors like SAIFI, SAIDI, and CAIDI. It provides a comprehensive overview of the system's reliability performance, guiding improvements and strategic planning.

14. **Capital and Operational Expenditure (CapEx and OpEx):**

    These metrics measure the investment required for infrastructure expansion and the ongoing costs of operating and maintaining the expanded grid. Ensuring that financial resources are used efficiently and effectively is crucial for sustainable grid development.

Role of risk management

The role of risk management in electricity distribution networks is essential to assess, anticipate and manage risks linked to the reliability and resilience of the network. Risk management aims to identify potential risks, assess their probability and impact, and implement strategies to mitigate them. In the context of electricity distribution networks, risk management makes it possible to take into account extreme low probability events (HILP) such as natural disasters, which can have a major impact on the network. By integrating risk-based metrics, such as CVaR (Conditional Value at Risk), into the planning of the expansion of distribution networks, risk management makes it possible to better anticipate and manage these extreme events, thus strengthening the resilience of the network and ensuring its reliability in the face of various failure scenarios.

3.2 Risk Aversion

Risk aversion in distribution network expansion refers to a strategy that considers uncertainties such as load demand variations and extreme weather events like hurricanes when planning network upgrades. This approach aims to enhance resilience by strategically placing distributed generation (DG) units, reinforcing existing lines, and implementing operational measures like DG rescheduling and load curtailment. By incorporating risk assessment into expansion planning, utilities can optimize investments in new line segments and storage devices to mitigate the impact of high-impact low-probability events while improving overall reliability [8].

In the context of distribution grid planning, risk aversion plays a critical role in decision-making processes. Planners must consider variability in electricity demand due to factors like population growth, economic changes, and technological advancements (e.g., increased use of electric vehicles). Renewable energy sources, such as solar and wind, introduce variability in supply due to their intermittent nature [2].

Ensuring that the grid remains reliable under various conditions, including peak demand and extreme weather events, is paramount. Planners need to account for the potential failure of grid components and the cascading effects that such failures might have [3].

Regulatory and environmental concerns must also be considered. Regulations regarding emissions, land use, and other environmental impacts can introduce uncertainties in project timelines and costs. Compliance with regulatory requirements can pose significant challenges [7].

Economic considerations include cost overruns, changes in funding, and fluctuations in energy prices. Investment decisions must balance upfront costs against long-term operational savings and benefits [1].

Additionally, a risk-averse strategy for the optimal placement and sizing of photovoltaic inverters in distribution networks under uncertainties like solar irradiance and load variations has been proposed, highlighting the importance of considering risk measures in decision-making processes [6].

3.2.1 Implementation Strategies

Implementing risk-averse strategies in distribution network planning involves considering high-impact, low-probability (HILP) events, uncertainty in load demand and renewable energy generation, and balancing risk with reliability. Strategies include system hardening, investing in smart grid technologies, distributed generation (DG) placement, line hardening, DG rescheduling, topology reconfiguration, microgrid forming, and priority-based load curtailment [37] [9]. The proposed risk-averse approaches utilize stochastic optimization frameworks, information-gap decision theory (IGDT), the $\varepsilon - constraint$ method, and fuzzy decision-making to find robust solutions against uncertainties, optimize hardening schemes within budget constraints, and achieve resilience in distribution networks [37]. These strategies aim to enhance grid resilience by explicitly incorporating risks of extreme weather events, modeling HILP event impacts, and providing flexibility to analyze trade-offs between risk-neutral and risk-averse planning strategies.

Advanced analytic and modeling involve utilizing advanced simulation and forecasting tools to model various risk scenarios and their potential impacts on the grid [20]. Grid modernization includes investing in smart grid technologies that enhance monitoring, control, and automation capabilities, allowing for more responsive and adaptive grid management [17]. Diversification of energy sources entails integrating a mix of traditional and renewable energy sources to spread risk and enhance supply reliability [41]. Stakeholder engagement is crucial, involving engaging with stakeholders to understand their concerns and incorporate their insights into risk management strategies [52]. Regular reviews and updates involve continuously reviewing and updating risk management plans to reflect new information, technologies, and evolving risks [18]. In summary, risk aversion is a crucial aspect of distribution grid planning that helps ensure reliability, economic stability, regulatory compliance, and long-term sustainability. By adopting a risk-averse approach, utilities can better navigate uncertainties and deliver consistent, reliable electricity to end-users[32].

3.2.2 Types of risk averse

1. **Risk-Neutral Plan ($\lambda = 0$):** The "Risk-Neutral Plan" ($\lambda = 0$) is a network management approach focused on minimizing costs without considering extreme failure scenarios. This method primarily addresses routine outages.

   Failure Modeling:

   - **Routine Outages:** The plan concentrates on frequent, low-severity outages. Rare and catastrophic events are not significantly accounted for. Failure management relies on average failure scenarios, reducing investment costs in resilience infrastructure.
   - **Single Failure State:** Each outage scenario is simplified to a single failure state, easing modeling but potentially underestimating impacts of more complex and extreme outages.
   - **Power Flow Constraints:** Managed in a simplified manner, without using complex optimal power flow (OPF) models for each failure state. The system is assumed to handle outages while maintaining acceptable load flow limits.

Value of Lost Load (VoLL) The VoLL is used to evaluate costs associated with outages. Here are the specific values observed in the documents for this plan:

- **VoLL of 1.50\$/kWh:**
  (a) Annual average unserved energy: 6.09 MWh
  (b) CVaR1% of annual unserved energy: 17.05 MWh
  (c) Worst-case scenario of annual unserved energy: 23.21 MWh
- **VoLL of 5.00\$/kWh:**
  (a) Annual average unserved energy: 4.18 MWh
  (b) CVaR1% of annual unserved energy: 14.05 MWh
  (c) Worst-case scenario of annual unserved energy: 22.49 MWh

Investment Strategy

- Minimal Investments: Investments are minimized, focusing on immediate needs and routine outages. This includes mainly maintenance and minor network improvements. There are no significant investments in resilience infrastructure against extreme events.
- No Preparation for Extreme Events: Since the plan does not account for extreme scenarios, there are no substantial expenditures for energy storage systems or strengthening infrastructure for natural disasters.

2. **Medium Risk Aversion Plan ($\lambda = 0.5$):** The "Medium Risk Aversion Plan" ($\lambda = 0.5$) is a network management approach that considers both routine outages and more extreme failure scenarios. This allows for a better balance of costs and risks by investing more in network resilience.

Failure Modeling

- Inclusion of Extreme Scenarios: Unlike the risk-neutral plan, this plan includes extreme outage scenarios. Rare but potentially catastrophic events are considered in planning.
- Risk Weighting: Modeling includes equal weighting between the expected value of lost load and Conditional Value at Risk (CVaR). This helps evaluate not only average outage costs but also costs in worst-case scenarios.
- Complex Power Flow Constraints: Power flow constraints are managed more sophisticated, including optimized models for each failure state. This allows for better preparation and response to outages.

Value of Lost Load (VoLL) The VoLL for the medium risk aversion plan shows a reduction in outage impacts due to additional investments:

- VoLL of 1.50$/kWh:
  (a) Annual average unserved energy: Reduced compared to the risk-neutral plan
  (b) CVaR1% of annual unserved energy: Reduced
  (c) Worst-case scenario of annual unserved energy: Reduced
- VoLL of 5.00$/kWh:
  (a) Annual average unserved energy: Reduced compared to the risk-neutral plan
  (b) CVaR1% of annual unserved energy: Reduced
  (c) Worst-case scenario of annual unserved energy: Reduced

Investment Strategy

- Moderate Investments: Investments are higher than in the risk-neutral plan, including substantial network improvements and energy storage systems. These investments aim to strengthen resilience against both frequent and extreme outages[36].
- Preparation for Extreme Events: A significant portion of investments is directed towards preparing for rare but severe events, such as natural disasters. This includes reinforcing critical infrastructure and adding storage capacity to handle peak loads and prolonged interruptions[36].

Resilience and Reliability: The moderate risk aversion plan improves network resilience and reliability by considering a wide range of failure scenarios. Additional investments allow for better management of routine outages and an effective response to extreme events, reducing the likelihood of prolonged and costly outages[36].

3. **High Risk Aversion Plan ($\lambda = 1$):** The "High Risk Aversion Plan" is a network management strategy that emphasizes minimizing the impacts of extreme outages by investing significantly in resilience and disaster preparedness.

Failure Modeling:

- Extreme Outage Scenarios: This plan includes detailed modeling of extreme outage scenarios. This involves a rigorous consideration of rare but severe events, such as natural disasters, cyberattacks, or other large-scale incidents.
- Maximum Risk Weighting: The weighting is heavily focused on Conditional Value at Risk (CVaR), meaning investment and management decisions are made with the worst-case scenarios in mind. This reduces the risk of extreme losses.

- Optimized Power Flow Constraints: Power flow models are optimized for each failure state, ensuring the network can respond effectively even under intense stress conditions. Detailed simulations are used to predict and manage power flow constraints in critical situations.

Value of Lost Load (VoLL): VoLL values for the high risk aversion plan show significant reductions in impacts due to massive resilience investments:

- VoLL of 1.50$/kWh:
  (a) Average annual unserved energy: Reduced compared to risk-neutral and moderate risk aversion plans
  (b) CVaR1% of annual unserved energy: Significantly reduced
  (c) Worst-case scenario of annual unserved energy: Substantially mitigated
- VoLL of 5.00$/kWh:
  (a) Average annual unserved energy: Reduced compared to risk-neutral and moderate risk aversion plans.
  (b) CVaR1% of annual unserved energy: Significantly reduced.
  (c) Worst-case scenario of annual unserved energy: Substantially mitigated.

Investment Strategy:

- Significant Investments: Investments are considerable, including advanced technologies for network resilience, large-scale energy storage systems, and the reinforcement of critical infrastructure to withstand extreme events.
- Maximum Preparation for Extreme Events: A large portion of investments is dedicated to preparation and response for rare but severe events. This includes developing detailed contingency plans, implementing robust backup systems, and continuously improving infrastructure.
- Resilience Technologies: Investments include the adoption of advanced technologies such as real-time energy management systems, smart grids, and advanced protection devices to minimize outage impacts.

Resilience and Reliability: The high risk aversion plan aims to maximize network resilience and reliability. Substantial investments and detailed preparations significantly reduce the risks associated with outages, ensuring continuity of service even under the most extreme conditions.

3.2.3 Techniques for measuring risk aversion

Techniques for measuring risk aversion, play a crucial role in managing financial and operational risks, particularly in network planning. These metrics provide a quantitative basis for understanding and mitigating potential losses, ensuring that strategic decisions are informed by a comprehensive risk assessment. Among these techniques are[36]:

1. **Value at Risk (VaR):**

    - **Definition:** VaR is a measure of exposure to financial or operational risk. It represents the maximum amount that can be lost with a certain probability (e.g., 95%) within a given time frame (e.g., one day).
    - **Calculation** VaR is calculated by determining the maximum amount that can be lost with a certain probability. For example, if the probability is 95%, VaR is the maximum amount that can be lost with a 95% probability.
    - **Application:** In the context of distribution network planning, VaR is used to assess exposure to the risk of network disruption. For example, VaR can be used to determine the maximum amount that can be lost in the event of a network disruption with a certain probability.

2. **Conditional Value at Risk (CVaR):**

    - **Definition:** CVaR is a measure of exposure to financial or operational risk that considers not only the probability but also the magnitude of the risk. It represents the maximum amount that can be lost with a certain probability (e.g., 95%) within a given time frame (e.g., one day), taking into account the magnitude of the risk.
    - **Calculation:** CVaR is calculated by determining the maximum amount that can be lost with a certain probability, taking into account the magnitude of the risk. For example, if the probability is 95%, CVaR is the maximum amount that can be lost with a 95% probability, considering the magnitude of the risk.
    - **Application:** In the context of distribution network planning, CVaR is used to assess exposure to the risk of network disruption, considering not only the probability but also the magnitude of the risk. For example, CVaR can be used to determine the maximum amount that can be lost in the event of a network disruption with a certain probability, taking into account the magnitude of the risk.

3.3 Mathematical Formulation

Based on the provided code snippets and the context of optimization and network modeling, here is a detailed list of sets, indexes, variables, and parameters:

3.3.1 Sets and Indexes

1. **Sets:**

   - L_c: Set of candidate lines.
   - H: Set of batteries.
   - D: Set of typical days.
   - N_{load}: Set of loads (buses).
   - T: Set of time periods.
   - S: Set of scenarios.
   - Ψ_N: Set of indexes of all nodes of the distribution grid.
   - Ψ_{SS}: Set of indexes of nodes that are substations of the distribution grid.
   - Ω: Set of indexes of failure scenarios.
   - $\Omega_{\text{resilience}}$: Set of indexes of failure scenarios associated with resilience.
   - Ω_{routine}: Set of indexes of routine failure scenarios.
   - C: Set of indexes of failure states.

2. **Indexes:**

   - l: Index of lines.
   - h: Index of batteries.
   - d: Index of typical days.
   - n: Index of loads (buses).
   - t: Index of time periods.
   - s: Index of scenarios.
   - g: Index of generators.
   - s: Index of storage units.
   - c: Index of failure state.
   - e: Index of the islands formed under a contingency state.
   - j: Index of investment decision.
   - t_0: Index of the first time period of a day type.

3.3.2 Variables

- C_{total}: Total cost.
- C_{line}: Line cost.
- C_{battery}: Battery cost.
- $C_{\text{imbalance}}$: Imbalance cost.
- $C_{\text{reduction charge}}$: Load shedding and island operation cost.
- C_{CVaR}: Conditional Value at Risk cost.
- $c_{\text{fix},l}$: Fixed cost for line l.
- $x_{\text{fix},l}$: Binary variable indicating whether line l is installed.
- $c_{\text{sd fix},h}$: Fixed cost for battery h.
- $c_{\text{sd var},h}$: Variable cost for battery h.
- $x_{\text{sd},h}$: Decision variable for battery h.
- $x_{\text{sd var},h}$: Decision variable for variable cost of battery h.
- $p_{\text{in max},h}$: Maximum input capacity of battery h.
- w_d: Weight for day d.
- p_f: Penalty factor.
- c_{imb}: Imbalance cost.
- $\delta^-_{n,t,d}$: Negative imbalance variable for load n at time t on day d.
- $\delta^+_{n,t,d}$: Positive imbalance variable for load n at time t on day d.
- λ: Weighting parameter.
- $s_{\text{prob},s}$: Probability of scenario s.
- $l_{\text{tds},t,d,s}$: Load shedding for time t, day d, and scenario s.
- $\phi_{\text{cvar},t,d,s}$: Scenario variable for CVaR.
- $P_{g,t}$: Power generated by generator g at time t.
- $S_{s,t}$: Power supplied from storage s at time t.

- D_t: Demand at time t.
- x_g: Binary variable indicating investment in generator g.
- y_s: Binary variable indicating investment in storage s.
- $\Delta^+_{n,t,d}$: Positive imbalance in bus n at time period t of day type d.
- $\Delta^-_{n,t,d}$: Negative imbalance in bus n at time period t of day type d.
- $\zeta_{t,d}$: CVaR auxiliary variable representing the value at risk at time period t of day type d.
- $\psi_{\text{CVaR},t,d,s}$: CVaR auxiliary variable.
- $f_{l,t,d}$: Flow in line l at time period t of day type d.
- $g_{\text{Tr},n,t,d}$: Injection via substation n at time period t of day type d.
- $L^\dagger_{t,d,s}$: Load shedding at time period t of day type d of scenario s.
- $L_{j,e,c}$: Load shedding in island e for relevant investment j under failure state c.
- $p_{\text{in},h,t,d}$: Charging of storage device h at time period t of day type d.
- $p_{\text{out},h,t,d}$: Discharging of storage device h at time period t of day type d.

3.3.3 Parameters

- α_{cvar}: Confidence level for CVaR.
- $c_{\text{fix},l}$: Fixed cost for line l.
- $c_{\text{sd fix},h}$: Fixed cost for battery h.
- $c_{\text{sd var},h}$: Variable cost for battery h.
- $p_{\text{in max},h}$: Maximum input capacity of battery h.
- w_d: Weight for day d.
- p_f: Penalty factor.
- c_{imb}: Imbalance cost.
- λ: Weighting parameter.
- $s_{\text{prob},s}$: Probability of scenario s.
- ReserveMargin: Predefined percentage to ensure reliability.

- $C_{\text{inv},g}$: Investment cost of generator g.
- $C_{\text{inv},s}$: Investment cost of storage s.
- Budget: Budget for investments.
- α_{CVaR}: CVaR parameter.
- δ: Number of hours in a time period t.
- η: Round trip efficiency of batteries.
- ρ: Probability of scenario s.
- C_{Imb}: Cost of imbalance.
- $C_{L,\text{fix},l}$: Fixed investment cost of candidate line l.
- $C_{SD,\text{fix},h}$: Fixed investment cost of candidate storage device h.
- $C_{SD,\text{var},h}$: Variable investment cost of candidate storage device h.
- $D_{\text{peak},i}$: Peak demand of bus i.
- $D_{n,t,d}$: Demand of bus n at time period t of typical day d.

3.3.4 Scenario-Based Approach

Using Scenarios to Capture Uncertainty in Demand and Outages: The scenario-based approach is a method used to model and manage uncertainty in complex systems such as electricity distribution networks. This method involves creating different possible scenarios that represent potential variations in key parameters, such as electricity demand and network outages.

1. **Capturing Uncertainty in Demand**
   - **Variation in Demand:** Scenarios can represent different trajectories of electricity demand growth, taking into account factors such as demographic changes, the adoption of new technologies (like electric vehicles and smart appliances), and changes in consumer behavior.
   - **Historical Data and Projections:** Use historical data and future projections to create scenarios that cover a range of possibilities, from low to extremely high demand.
2. **Capturing Uncertainty in Outages**

- **Types of Outages:** Include various types of outages in the scenarios, such as those caused by extreme weather events, equipment failures, or cyberattacks.
- **Frequency and Duration:** Model the frequency and duration of outages to reflect the uncertainties and variabilities observed in historical data and risk assessments.

Methods for Generating and Selecting Representative Scenarios

1. **Generating Scenarios:**

   - **Historical Data Analysis:** Use historical data on demand and outages to identify trends and patterns. This information can be used to generate plausible future scenarios[36].
   - **Stochastic Models:** Develop stochastic models that incorporate probability distributions for key variables (e.g., electricity demand, outages). These models can generate a large number of possible scenarios[36].
   - **Monte Carlo Simulation:** Utilize Monte Carlo simulation techniques to explore a wide range of scenarios by varying random parameters according to their probability distributions[36].

2. **Selecting Representative Scenarios**

   - **Clustering:** Apply clustering techniques to group similar scenarios and select a representative number of scenarios that capture the diversity of possible futures. For example, algorithms like k-means can be used to cluster scenarios into homogeneous groups[36].
   - **Dimensionality Reduction:** Use dimensionality reduction methods, such as principal component analysis (PCA), to identify scenarios that explain most of the variability in the data[36].
   - **Selection Criteria:** Define selection criteria based on planning objectives and acceptable risks. Criteria may include coverage of extreme cases, representation of average cases, and consideration of high-impact scenarios[36].

How are Outage Scenarios Modeled in Distribution Networks The modeling of breakdown scenarios in distribution networks is done as follows:

1. **Considering (Routine Failures) and High Impact Low Probability (HILP Events):** Scenarios include both routine failures and rare but high-impact events, such as natural disasters.

2. **Generating Failure Scenarios from Historical Data:** Failure scenarios are generated from historical data on network reliability, considering different levels of severity.

3. **Modeling Network State in Scenarios:** A binary variable y_{lmtd} is used to represent the state of the network (whether line l is available or not) for each scenario node m, hour t, and day d. This variable reflects line failures due to routine failures and rare events.

4. **Transition Probabilities Between Scenarios:** Each failure scenario is associated with a transition probability π'_{md} which reflects the likelihood of reaching this scenario. Scenarios associated with rare but high-impact events (HILP) have much lower transition probabilities.

5. **Considering the Impact of Failures in the Objective Function:** The model aims to minimize a convex combination of the expected value and the CVaR (Conditional Value at Risk) of operational costs, including the energy not supplied due to failures. This allows capturing the impact of rare but high-impact events, in addition to routine failures.

3.3.5 Objective Function

Theoretical Formulation The overall objective function C_{total} is defined as the sum of the following costs:

- Fixed line costs C_{line}
- Fixed and variable battery costs C_{battery}
- Imbalance costs $C_{\text{imbalance}}$
- Load shedding and island operation costs $C_{\text{reduction charge}}$
- Conditional Value at Risk (CVaR) costs C_{CVaR}

Thus, the objective function is as follows:

$$C_{\text{total}} = C_{\text{line}} + C_{\text{battery}} + C_{\text{imbalance}} + C_{\text{reduction charge}} + C_{\text{CVaR}} \tag{3.1}$$

Cost Components

1. **Line Costs C_{line}** The fixed line costs are calculated as follows:

$$C_{\text{line}} = \sum_{l \in L_c} c_{\text{fix},l} \cdot x_{\text{fix},l} \tag{3.2}$$

where $c_{\text{fix},l}$ is the fixed cost for line l and $x_{\text{fix},l}$ is the binary variable indicating whether line l is installed.

2. **Battery Costs C_{battery}**

   Battery costs include both fixed and variable costs:

$$C_{\text{battery}} = \sum_{h \in H} (c_{\text{sd fix},h} \cdot x_{\text{sd},h} + c_{\text{sd var},h} \cdot x_{\text{sd var},h} \cdot p_{\text{in max},h}) \tag{3.3}$$

   where $c_{\text{sd fix},h}$ and $c_{\text{sd var},h}$ are the fixed and variable costs for battery h, respectively, $x_{\text{sd},h}$ and $x_{\text{sd var},h}$ are decision variables, and $p_{\text{in max},h}$ is the maximum input capacity of battery h.

3. Imbalance Costs $C_{\text{imbalance}}$

   Imbalance costs are calculated as follows:

$$C_{\text{imbalance}} = \sum_{d \in D} w_d \cdot pf \cdot c_{\text{imb}} \cdot \sum_{n \in N_{\text{load}}} \sum_{t \in T} (\delta^-_{n,t,d} + \delta^+_{n,t,d}) \tag{3.4}$$

   where w_d is the weight for day d, pf is a penalty factor, c_{imb} is the imbalance cost, and $\delta^-_{n,t,d}$ and $\delta^+_{n,t,d}$ are the imbalance variables for load n at time t and day d.

4. **Load Shedding and Island Operation Costs $C_{\text{reduction charge}}$,**

   These costs are calculated as follows:

$$C_{\text{reduction charge}} = (1-\lambda) \cdot pf \cdot c_{\text{imb}} \cdot \sum_{d \in D} w_d \cdot \sum_{t \in T} \sum_{s \in S} s_{\text{prob},s} \cdot l_{\text{tds},t,d,s} \tag{3.5}$$

   where λ is a weighting parameter, $s_{\text{prob},s}$ is the probability of scenario s, and $l_{\text{tds},t,d,s}$ is the load shedding for time t, day d, and scenario s.

5. **CVaR Costs C_{CVaR}**

   CVaR-related costs are formulated as follows:

$$C_{\text{CVaR}} = \lambda \cdot pf \cdot c_{\text{imb}} \cdot \sum_{d \in D} w_d \cdot \sum_{t \in T} \left(\zeta_{t,d} + \sum_{s \in S} \frac{s_{\text{prob},s}}{1-\alpha_{\text{cvar}}} \cdot \phi_{\text{cvar},t,d,s} \right) \tag{3.6}$$

   where α_{cvar} is the confidence level for CVaR, $\zeta_{t,d}$ is an auxiliary variable, and $\phi_{\text{cvar},t,d,s}$ is a scenario variable.

In summary, the objective function C_{total} aims to minimize the total costs associated with the management and optimization of the system by integrating several specific cost components. This formulation allows for comprehensive and optimized modeling of the system's associated costs.

3.3.6 Constraints

The constraints of the model typically include the following elements:

1. **Power Balance Constraints** These constraints ensure that supply meets demand at all times:

$$\sum_g P_{g,t} + \sum_s S_{s,t} = D_t \quad \forall t$$

   where:

   - $P_{g,t}$ is the power generated by generator g at time t.
   - $S_{s,t}$ is the power supplied from storage s at time t.
   - D_t is the demand at time t.

2. **Capacity Constraints** These constraints impose limits on generation, storage, and transmission capacities:

$$P_{g,t} \leq P_g^{\max} \quad \forall g, \forall t$$

$$S_{s,t} \leq S_s^{\max} \quad \forall s, \forall t$$

   where:

   - $P_g^{\max}$ is the maximum capacity of generator g.
   - $S_s^{\max}$ is the maximum capacity of storage s.

3. **Operational Constraints** These constraints include minimum and maximum generation levels, ramp rates, etc.:

$$P_g^{\min} \leq P_{g,t} \leq P_g^{\max} \quad \forall g, \forall t$$

$$|P_{g,t} - P_{g,t-1}| \leq \text{Ramp}_g \quad \forall g, \forall t$$

   where:

   - $P_g^{\min}$ is the minimum generation level of generator g.
   - Ramp_g is the ramp rate of generator g.

4. **Reliability Constraints** These constraints ensure the reliability and stability of the system under various scenarios:

$$\sum_g P_{g,t} + \sum_s S_{s,t} \geq (1 + \text{Reserve Margin}) \times D_t \quad \forall t$$

where:

- Reserve Margin is a predefined percentage to ensure reliability.

5. **Investment Constraints** These constraints limit investments in new infrastructure based on budget or other criteria:

$$\sum_g C_g^{\text{inv}} \times x_g + \sum_s C_s^{\text{inv}} \times y_s \leq \text{Budget}$$

where:

- C_g^{inv} is the investment cost of generator g.
- x_g is a binary variable indicating investment in generator g.
- C_s^{inv} is the investment cost of storage s.
- y_s is a binary variable indicating investment in storage s.

3.3.7 Scalability Methodologies

1. **Large-Scale Model Scalability Challenges:**

   - Algorithmic Complexity: Large-scale models can exhibit high algorithmic complexity, meaning that computation time significantly increases with problem size. This can make obtaining solutions within reasonable timeframes challenging.
   - Memory Consumption: Large models may require significant memory to be solved, which can pose resource management issues, especially on machines with limited memory capacities.
   - Balance Between Accuracy and Speed: Obtaining precise solutions for large-scale models can be extremely costly in terms of computation time. Finding a balance between solution accuracy and the time required to obtain them is a key challenge.
   - Adaptability to Changes: Large-scale models often need to be readjusted to account for new data or parameter changes. Ensuring that the model can be quickly updated without compromising existing performance is an additional challenge.

- Algorithm Scalability: Algorithms used to solve models must be able to adapt to increasingly large datasets without sacrificing efficiency. Some algorithms may not be sufficiently scalable to effectively handle large-scale problems.

2. **Model Complexity Reduction Techniques:**

   - Scenario Decomposition: This approach involves dividing the model into smaller sub- problems, each corresponding to a specific scenario. By solving these sub-problems sequentially or in parallel, the overall model complexity can be reduced.
   - Scenario Aggregation: Instead of considering each scenario individually, similar scenarios can be grouped or aggregated, reducing the total number of scenarios to be considered. This simplifies the overall model while retaining important problem characteristics.

3. **Heuristic Approaches for Timely Solutions:**

   - Meta-heuristics: Meta-heuristics, such as genetic algorithms, simulated annealing, or ant colony algorithms, provide approximate but often high-quality solutions within reasonable time-frames for large-scale problems.
   - Cutting Plane Techniques: Cutting plane techniques, such as Gomory cuts or Chvátal Gomory cuts, can be used to reduce the effective size of the model by adding constraints that eliminate certain non-optimal solutions.
   - Approximate Solution Methods: Instead of seeking the optimal solution, these methods aim to find a satisfactory solution within a reasonable time-frame using efficient search strategies, such as local search or tabu search.

3.3.8 Optimizing Electrical Distribution Networks

The setting takes place within an electrical distribution network, encompassing various components such as transmission lines and transformers. This network is subject to various constraints and objectives, including notably operational cost reduction and ensuring power supply reliability. To address this complex issue, we used a developed Python script based on the 'Pyomo' library dedicated to modeling and optimization solving,which can be summarized in the next plan 3.2:

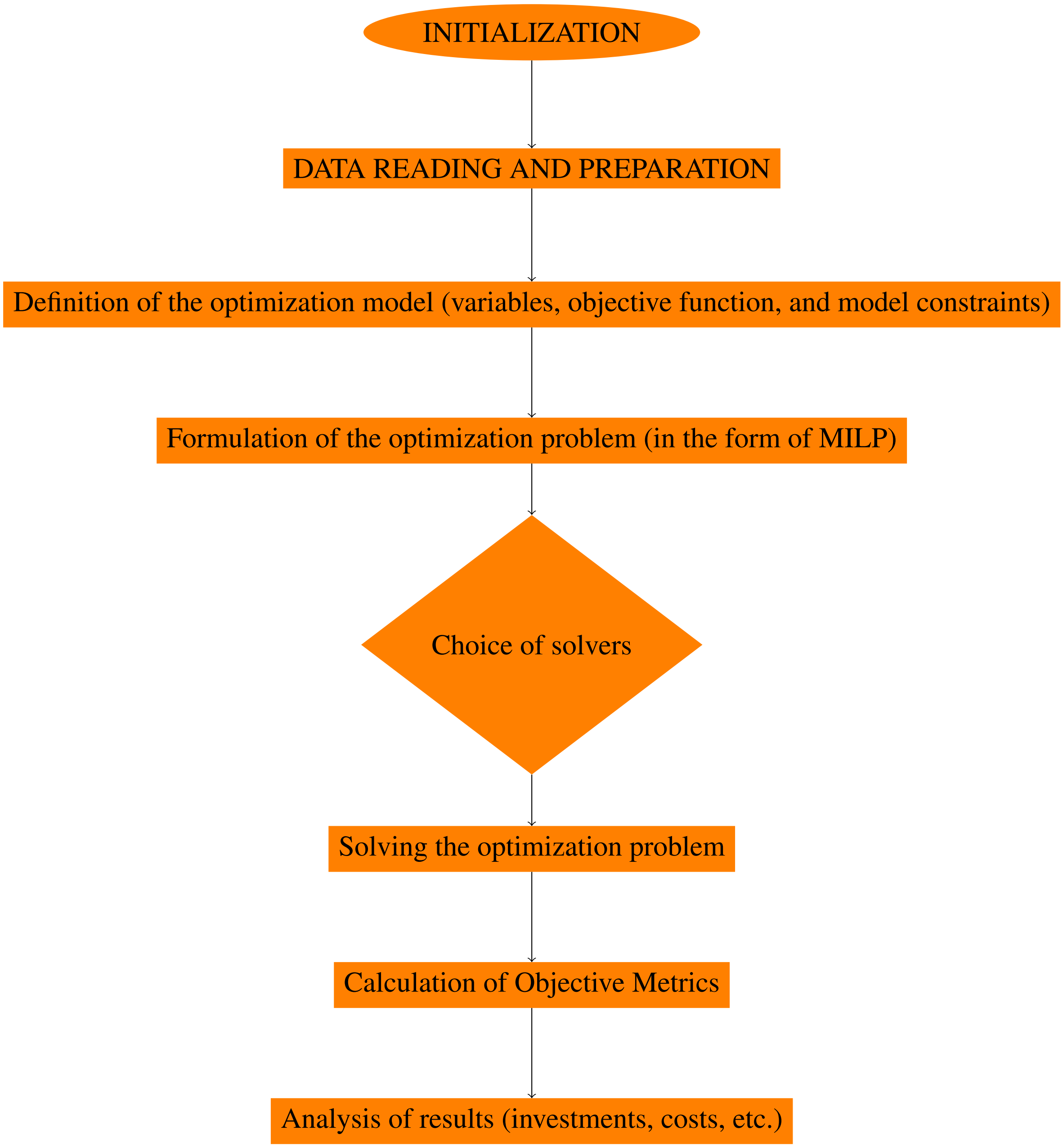


Figure 3.2: Investment Optimization Process Flowchart

This script starts by initializing a logging system using the 'logging' module, recording messages in a specified file. Then, it imports the required modules, including 'pandas' for data manipulation, as well as 'Solver Factory' from Pyomo allowing for the instantiation of optimization solvers. Two main functions are defined: 'main()' and 'run investment()'. The 'main()' function acts as an entry point, where data is read from a specific directory via the 'read data alternative()' function. The 'run investment()' function constructs and solves the optimization model using a class called 'Expansion Planning'. After solving the model, the results (objective costs and found solutions) are saved in a journal file as well as in CSV files to facilitate access and visualization.

Why We Chose MIlP Over Other Methods? Choosing MIlP over other methods is due to several reasons that make it particularly suitable for planning the expansion of distribution power grids, especially when considering risk and resilience:

- Handling Discrete and Continuous Decisions: MIP allows for both discrete (binary/integer) and continuous decision variables, which is essential for accurately modeling investment decisions (such as whether to install new lines) and operational variables (like power flows).
- Complex Constraint Modeling: MIP can model a wide range of constraints, including those on power flow, capacities, and voltages. This flexibility is crucial for representing the physical and operational limits of the power grid accurately.
- Global Optimality: MIP finds globally optimal solutions, which is important in investment planning to ensure cost-effectiveness and reliability. Suboptimal decisions can lead to higher costs or reduced system performance.
- Integration of Risk Measures: MIP effectively incorporates advanced risk measures such as Conditional Value at Risk (CVaR). This enables the model to handle uncertainties and provide robust solutions that enhance system resilience.
- Scalability and Efficiency: Advances in optimization algorithms and computational power have made MIP capable of solving large-scale problems efficiently, making it suitable for complex grid expansion scenarios involving numerous variables and constraints.
- Industry Acceptance: MIP is widely used and accepted in the industry for various optimization problems, ensuring that the methods and solutions are aligned with industry standards and practices.

Comparison with Other Methods

- Linear Programming (LP): Suitable for continuous variables but cannot handle discrete decisions necessary for investment planning.
- Nonlinear Programming (NLP): Can model nonlinear relationships but may struggle with global optimality and increased problem complexity.

MIP is chosen for its ability to handle the complexity of grid expansion planning, model a variety of constraints, achieve global optimality, integrate risk measures, and its broad acceptance in the industry.

Methodology Used in the selection of candidate branches

1. Scenario Analysis: The article uses a scenario-based approach to determine the candidate branches. Scenarios are defined to represent different load conditions and outages in the distribution network.

2. Stochastic Modeling: Stochastic modeling is employed to account for uncertainty in demand and outages. This modeling helps identify the network branches that are critical for maintaining network reliability and robustness under various outage scenarios.

3. Mixed-Integer Linear Programming (MILP): A MILP model is formulated to optimize network expansion. The model includes binary investment variables for each candidate branch. The model's solutions determine which branches are selected for expansion based on their impact on reducing imbalance costs and improving network resilience.

4. Capacity and Flow Constraints: Line capacity and power flow constraints are incorporated into the model to ensure that the selected candidate branches effectively improve network performance.

Graphical visualization: After solving the electrical network optimization problem, a crucial step is to visualize the obtained results. For this purpose, the code use a data set representing the network's topology, including transmission lines, substations, and loads, stored in specific CSV files ('branches.csv', 'substations.csv', 'loads.csv'), read using the 'pandas' library. These data are then used to instantiate the 'Network' class, defined in the 'network.py' file, responsible for creating and visualizing the electrical network. By calling the 'visualize()' method of this class, we generate a graphical representation of the electrical network, using the 'matplotlib' library. This visual representation provides an overview of the network's changes after optimization, thus revealing the improvements made and the system's efficiency. Finally, the graph is displayed using 'plt.show()'.

Modeling provides us with a powerful means to understand, analyze, and solve complex problems across various domains. It allows us to simplify reality while preserving the essentials, thus facilitating decision-making and strategic planning. By encapsulating interactions between variables and identifying causal relationships, it enables us to predict the consequences of different actions and strategies. This chapter serves as a cornerstone of our journey, laying the necessary groundwork for in-depth analysis and effective solution formulation. By understanding the underlying mechanisms of problems, we are better equipped to propose strategic interventions and make informed decisions.

CHAPTER 4

CASE STUDY

In this chapter, we apply the Monte Carlo simulation method for planning network expansion with risk consideration.

We will study two cases of an electrical distribution system consisting of 54 buses [10]. We use this system to illustrate the method of planning the expansion of distribution networks while taking into account the risks of failures and the costs associated with investments.

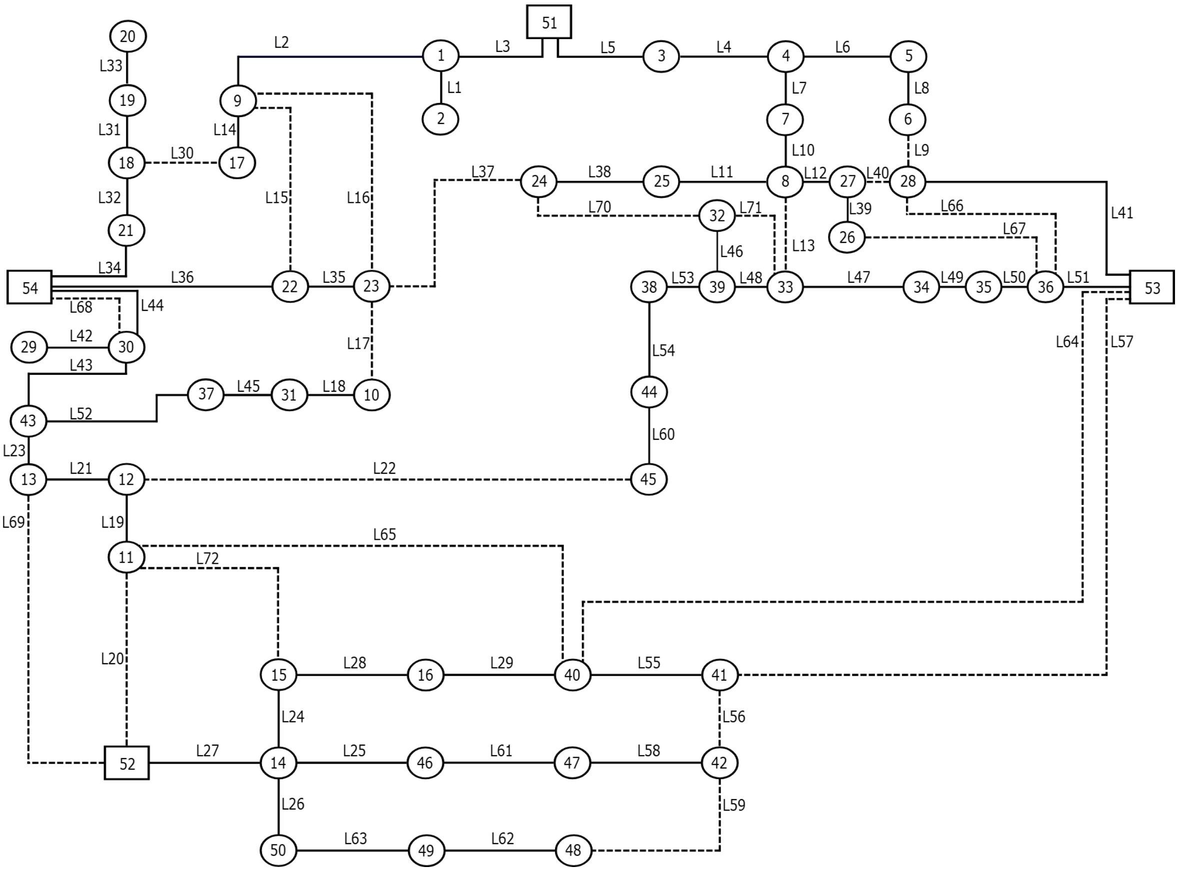


Figure 4.1: 54 Bus System

This figure4.1 illustrates an electrical distribution network with various nodes (or buses), lines (or branches), and substations. Here is a detailed explanation of the elements in the figure:

1. **Nodes (Buses):**

   The nodes, represented by numbered circles from 1 to 54, are the points where electricity is consumed or redistributed. Each node can represent a load, a connection point, or a substation.

   Example: Node 1 is connected to line L2 and node 9.

   Some nodes have multiple connections, indicating their importance in the network structure.

   Example: Node 8 has several connected lines (L10, L12, L13), making it a crucial connection point.

2. **Substations:**

   Substations are represented by squares and are numbered 51, 52, 53, and 54. They play a crucial role in voltage transformation and electricity distribution across the network.

   Example: Substation 51 is connected to node 1 via line L3.

   They are strategically placed to optimize electricity distribution. They transform and regulate voltage before distributing electricity to connected nodes.

   Example: Substation 54 serves several nodes, including nodes 54, 21, and 22 via lines L34, L36, and L35.

3. **Lines (Branches):**

   The lines (branches) connect the nodes and can be existing (solid lines) or proposed (dashed lines). Each line has a unique number for easy identification.

   - Solid Lines: Represent existing lines.

     Example: Line L1 connects nodes 2 and 9.

   - Dashed Lines: Indicate candidate lines for addition.

     Example: Line L14 is a candidate line between nodes 9 and 17.

     The dashed lines represent redundancy and expansion options for the network. Adding these lines could improve the network's resilience and capacity.

   Example: Line L65 could provide a new connection between nodes 11 and 15, thereby improving the network's flexibility.

4. **Line Numbers (L):**

Each line is numbered (L1 to L72) for precise identification and to facilitate network analysis.

Example: Line L10 connects nodes 7 and 8.

Load Distribution:

The figure shows how loads are distributed across the network, with several nodes aligned along certain main branches.

Example: Nodes 3, 4, 5, and 6 are aligned along the same main branch (L3, L4, L6, L7), showing a linear distribution of electricity.

4.1 Data Used

The data is structured into several CSV files, each containing specific information about different aspects of the electrical network [10]. Here is a detailed description of columns and data types for each category:

1. **Branches and Branch Candidates :**

Branches In the context of electrical systems, "branches" refer to the connections or pathways that allow the flow of electric current between different nodes (or buses) in the network

Branch_candidates are transmission lines that could be added to a power grid to improve its capacity, reliability, or efficiency. Each candidate is evaluated based on several criteria.

Branches and branch candidates describe existing and potential transmission lines in the network.

- from_bus: The starting point (node) of the branch in the grid.
- to_bus: The ending point (node) of the branch.
- max_ka: The maximum current (in kilo amperes) the branch can carry, indicating its capacity.
- Z_ohm_km: The impedance per kilometer, affecting how much voltage drop and power loss occurs along the line.
- r_len_km: The physical length of the branch in kilometers.
- c_fix_usd: The fixed cost associated with constructing the branch, given in US dollars.

- OH: Indicates whether the branch is an overhead line (1 if true) or underground (0 if false).
- lifetime: The expected operational lifespan of the branch in years.

2. **Substations:**

   These data provide information about substations, each associated with an electrical node and specific transmission capacity.

   - substation_index: Unique identifier for each substation.
   - bus: Identifier of the electrical node associated with the substation.
   - g_tr_max_kw: Maximum transmission capacity (in kilowatts).

3. **Loads:**

   Loads describe electrical demand connected to different nodes of the network.

   - Bus: Node number to which the load is connected.
   - peakDemand: Peak electricity demand over a specific period.
   - nCustomers: Number of customers connected to the network or electrical station.

4. **Hourly_profiles:** this data can be used to analyze temporal patterns of energy consumption and the associated costs throughout the day. It can help in:Understanding how energy demand changes throughout the day, Planning energy costs and managing budgets, Making operational decisions based on variable energy costs.

   - type:
     - demand_profile: Describes the energy demand profile throughout the day.
     - costs_dol_kWh: Describes the cost of energy per kilowatt-hour throughout the day.
     - battery_soc: Represents the state of charge of the battery throughout the day.
   - day: The specific day to which the data pertains. It can represent different days of the week or consecutive days.
   - t0 to t23: Each column represents a specific hour of the day. The values in these columns represent either the energy demand or the energy cost for that hour.
   - day_weights: weighs the day to represent scenarios over a year.

5. **Events List:**

   Events include incidents related to branches and substations, allowing analysis of event frequency and duration.

- frequency: Frequency of the event occurrence.
- duration: Duration of the event.
- Branches: Numbers of branches involved in the event.
- substations: Associated substations (if available).

6. **Coordinates:**

   Coordinates provide information about nodes and their locations in a two-dimensional space.

   - bus: Node number.
   - Longitude (x): East or west distance from the Greenwich Meridian.
   - Latitude (y): North or south distance from the Equator.

7. **Days:**

   Days record information about weighting or specific measures over a series of days.

   - days: Day number from a starting point (day 0).
   - weight: Weighting or specific measure recorded each day.

8. **General Parameters:**3.3.5

   General parameters are used in optimization analyses.

   - lambda: Weighting factor or compromise coefficient in optimization problems.
   - alpha_cvar: Confidence level for Conditional Value at Risk (CVaR).
   - pf: Power factor, measuring the efficiency of electrical energy use.
   - c_imb_usd_kwh: Cost of imbalance in USD per kWh.
   - bigM: Large constant used in mathematical optimization models.
   - sbase_mva: Base power in mega-volt-amperes (MVA).
   - vbase_kv: Base voltage in kilovolts (kV).
   - discount_rate: Discount rate used to discount future cash flows to their present value.

9. **Storage Candidates:**

   It contains information about energy storage options

   - H: Storage candidate identifier.
   - bus: Identifier of the electrical bus associated with the storage candidate.
   - p_in_max_kw: Maximum input power (kilowatts).

- p_out_max_kw: Maximum output power (kilowatts).
- s_charge: Charging capacity.
- eff: Efficiency.
- c_SD_fix_usd: Fixed storage cost (in USD).
- c_SD_var_usd_kwh: Variable storage cost per kilowatt-hour (in USD).
- sd_max: Maximum storage capacity.
- lifetime: Storage system's lifetime (in years).

4.2 Case Studies

4.2.1 Case 1

The system consists of:

- 50 load nodes.
- 1 substation.
- 53 branches (existing lines).
- 19 branch candidates (candidate lines).
- 4 storage candidates (candidates nodes for the installation of storage devices).

4.2.2 Case 2

The system consists of:

- 50 load nodes.
- 4 substations.
- 50 branches (existing lines).
- 22 branch candidate (candidate lines).
- 4 storage candidates (candidates nodes for the installation of storage devices).

The figure 4.2 and 4.3 are initial configuration of the network for case 1 4.2.1 and case 2 4.2.2.

Figure 4.2: Initial configuration of the network for case 1

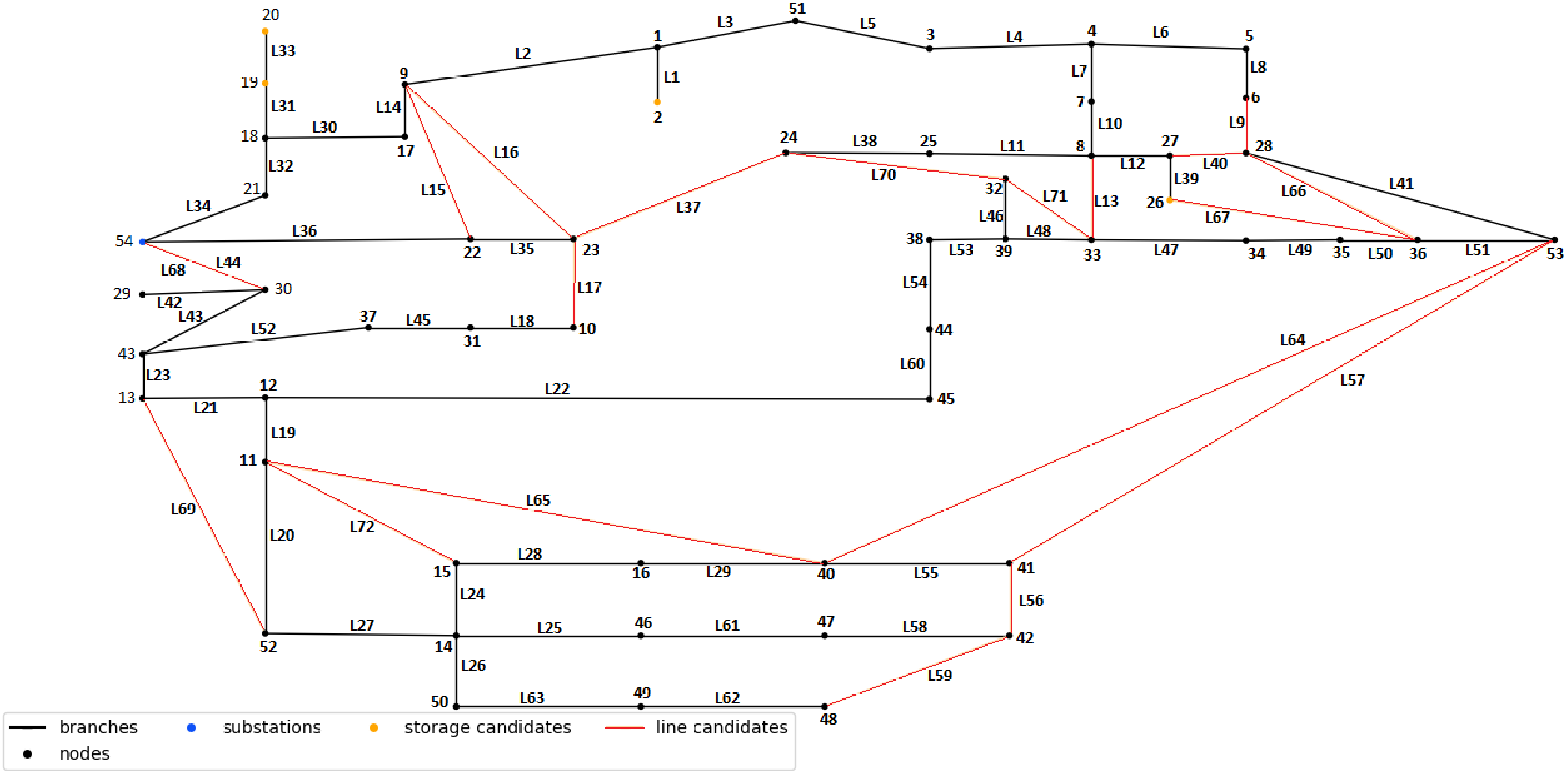
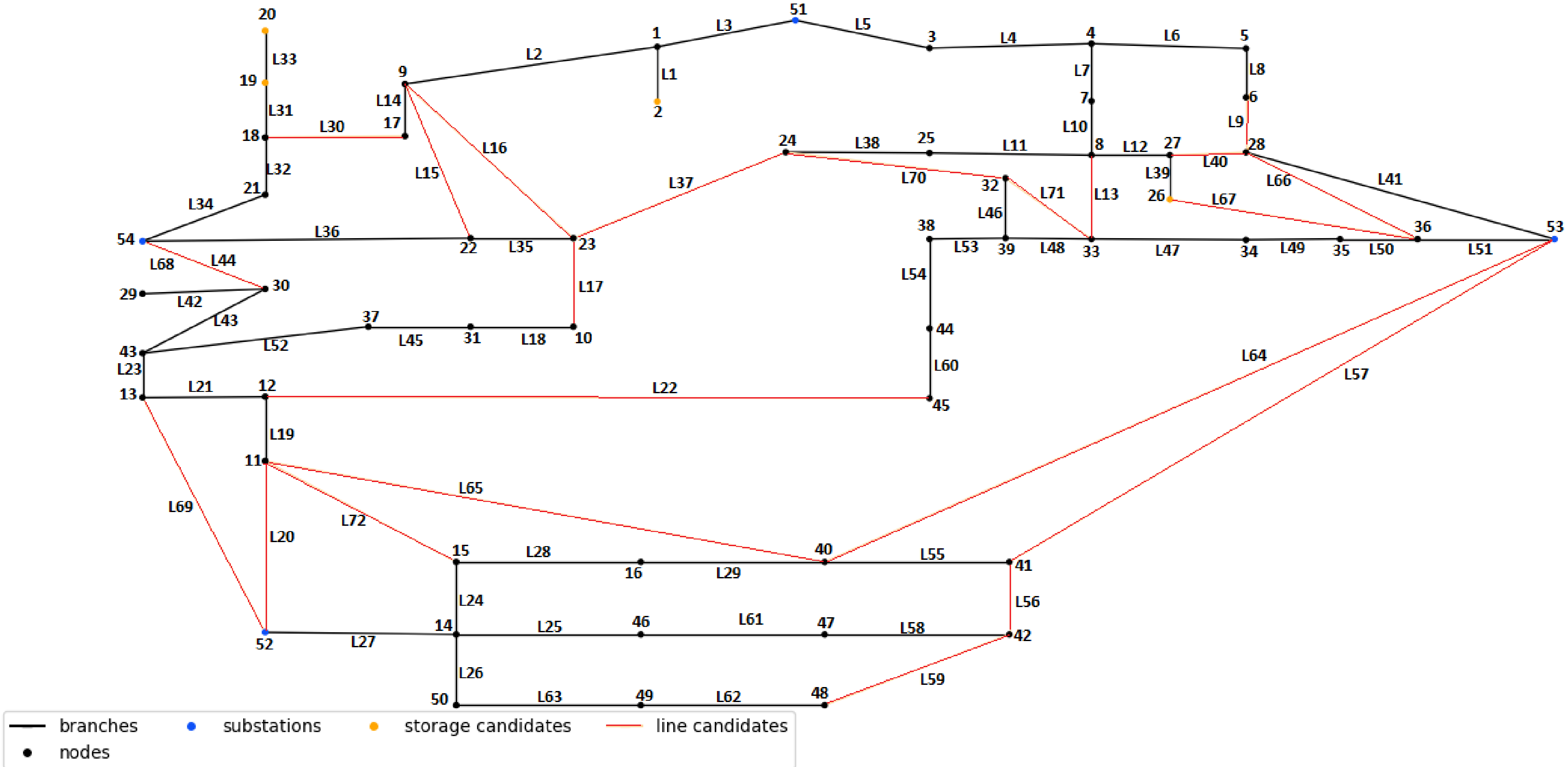


Figure 4.3: Initial configuration of the network for case 2



4.3 Optimization Model

The optimal model used in this study is based on MILP, developed using the Python code framework discussed in the previous chapter. Focused on optimizing the expansion of the energy distribution network by considering risk and resilience factors. The main objective of the model is to minimize:

- **Investment Costs:** This includes costs associated with installing new lines (line_inv) and energy storage devices (storage_inv), aiming to meet increasing demand and ensure system reliability.

The model incorporates several constraints and specific variables, including:

- **Power Flow Constraints:** Ensure that power flows through the network remain within stability limits, even under failure conditions simulated by scenario-based analyses (scenarios_time).
- **Installation constraints:** Limitations on installing new transmission lines (candidate_branches), and energy storage solutions (candidates_storage), considering physical limitations and budget constraints.
- **Capacity Constraints:** Boundaries on the capacity of energy storage devices (capacity) and transmission lines (f_max_ka), ensuring operational feasibility and grid performance.
- **Voltage Constraints:** Maintain voltage levels (v_min, v_max) across different nodes (bus), crucial for protecting equipment and ensuring customer satisfaction.
- **Decision Variables:** Include decisions on where and when to install new lines (line_inv) and energy storage devices (storage_inv), optimizing infrastructure investments based on anticipated demand growth and reliability improvements.

Cases	variables	Linear Constraints
Case 1	69392	102968
Case 2	64959	93635

Table 4.1: Comparison of Case Studies: Variables and Constraints

4.4 Result and Analysis

All these results are obtained from the same environment, which consists of a 64-bit system with an Intel Atom N2600 processor running at 1.60 GHz, 2.00 GB of RAM, and a device

ID of 6E8A5C4F-0EDB-416F-96C9-3EAB799E0DCB, along with a product ID of 00330-50000-00000-AAOEM. This device does not support touch or stylus input for its display. These specifications ensure consistent and reliable performance measurements and computational tasks.

4.4.1 Optimization using Cplex solver

Cplex solver

CPLEX, developed by IBM, stands as a cornerstone in optimization technology, renowned for its versatility and efficiency in tackling complex decision-making challenges across various industries. This optimization solver excels in solving linear programming (LP), mixed-integer linear programming (MILP), quadratic programming (QP), and other mathematical models with precision and speed. It leverages advanced algorithms such as simplex, interior-point, and barrier methods, coupled with heuristic techniques and cutting-edge strategies like cutting planes and efficient branching, ensuring rapid convergence and high-quality solutions even for large-scale problems [25].

Applications:

- Logistics and Supply Chain: Optimize transportation routes, scheduling, and inventory management.
- Finance: Asset allocation, portfolio optimization, risk management.
- Energy and Utilities: Grid optimization, resource allocation, demand forecasting.
- Manufacturing: Production planning, facility layout, supply chain integration.

Cplex results

To solve our optimization model, we used the IBM ILOG CPLEX Optimizer 22.1.0.0. The following sections 4.2 summarize the steps and results obtained during the evaluation and optimization processes for the two cases:

details	Case 1	Case 2
Solution Time	239.24 sec	106.22 sec
Number of Iterations	17193	2359
Nodes explored	41	0
Battery Costs (bat costs)	30493.25	31602.29
CVaR Costs (cvar costs)	398866.13	77763.51
Imbalance Costs (imb costs)	0.00	0.00
Line Costs (line costs)	416012.92	419317.52
Load Shedding Island Costs (load shedding island costs)	19943.30	3888.18
Total Objective Cost (obj)	655910.89	491745.65

Table 4.2: Table of CPLEX Optimization Results for Case 1 and Case 2

1. **Solution Time:**

   - Case 1:239.24 sec
   - Case 2:106.22

   The solution time is significantly shorter for case 2. The presence of 4 substations in case 2 compared to one in case 1 could simplify the optimization problem by reducing the complexity of energy supply paths, allowing the solver to find an optimal solution more quickly.

2. **Number of Iterations:**

   - case 1:17193
   - case 2 :2359

   Case 1 requires a much higher number of iterations. This could be due to the more constrained network configuration with only one sub-station requiring more computations to optimize energy flows and system stability.

3. **Number of Nodes Explored:**

   - Case 1: 41
   - Case 2: 0

   The solver has to explore additional nodes in case 1, indicating a more exhaustive and complex search, whereas in case 2, the solver found a solution without needing to explore additional nodes, suggesting a more direct and less complex solution.

4. **Battery Costs (bat costs):**

   - Case 1: 30493.25
   - Case 2: 31602.29

Battery costs are slightly higher in case 2, due to different optimization of storage device placement to meet network requirements with more substations.

5. **CvaR Costs (cvar costs):**
   - case 1: 398866.13
   - case 2: 77763.51

   Cvar costs are significantly higher in case 1 compared to case 2. This indicates the in case 1, potential risk scenarios beyond a certain probability threshold lead to much higher costs. The difference could be attributed to the network structure and risk management capability in each case. Case 2, with more substations and candidate branches, likely offers better resilience against failures, thus reducing costs associated with risks.

6. **Imbalance Costs (imb costs):**
   - case 1: 0.00
   - case 2: 0.00

   Imbalance costs are often used in energy management models or other systems where it's crucial to balance supply and demand, or maintain certain quantities in a specific equilibrium. If imbalance costs are zero, it could indicate that your model has found a solution where all balance constraints have been meet met, and therefore, no costs have been incurred to correct imbalances.

7. **Line Costs (line costs):**
   - case 1 : 416012.92
   - case 2 : 419317.52

   Line costs are slightly higher in case 2. This increase is attributed to the need to install or upgrade more lines to accommodate the configuration with four substations instead of one. The increase in the number of candidate branches in case 2 (22 compared to 19 in case 1)could also contribute to these additional costs. Despite the higher line costs, this configuration likely allows for better distribution and greater network resilience.

8. **Load Shedding Island Costs (load shedding island costs):**
   - case 1 : 19943.30
   - case 2: 3888.18

Load shedding island costs are much higher in case 1 compared to case 2. This suggests that case 1 requires more load shedding to maintain network stability, which can be attributed to a less optimized and resilient configuration. Case 2, with its four substations and additional candidates branches, is better equipped to manage overload or failure conditions without needing load shedding as frequently.

9. **Total Objective Cost (obj):**

   - case 1 : 655910.89
   - case 2: 491745.65

   The total objective cost is significantly higher in case 1 compared to case 2. This indicates that the optimization model for case 1 resulted in higher operational and investment costs compared to case 2. This difference can be attributed to several factors, including difference in network configuration, the number of substations and candidate branches, as well as how costs associated with batteries, risk (cvar), lines and load shedding islands are managed and optimized.

4.4.2 Optimization using CBC solver

CBC solver

The CBC (Coin-or Branch and Cut) solver is a robust, open-source optimization tool developed as part of the COIN-OR (Computational Infrastructure for Operations Research) project. It is specifically designed to solve Mixed-Integer Linear Programming (MILP) problems. As an open-source alternative to commercial solvers, CBC provides researchers and practitioners with a cost-effective and flexible solution for complex optimization tasks [15].

Features:

- Free and open-source, offering great flexibility and customization.
- Less performant than commercial solvers for very large instances.
- Easily integrates with other open-source tools and modeling libraries like PuLP.

Applications:

- Ideal for researchers and Educators: The open-source nature of CBC makes it an ideal tool for academic research and teaching.
- budget-constrained organizations: For organizations that need robust optimization capabilities but cannot afford expensive commercial solvers, CBC offers a compelling

alternative. It is particularly beneficial for nonprofits, startups, and small businesses that need to optimize resources efficiently.

- Prototype Development: CBC is often used in the development phase of optimization projects. Its flexibility and ease of integration allow developers to build and test models before potentially transitioning to commercial solvers for production-scale problems if higher performance is required.

CBC results:

details	Case 1	Case 2
Solution Time	884.10 sec	3819.55sec
Number of Iterations	11961	22721
Node explored	0	85
Battery Costs (bat costs)	30493.25	31602.29
CvaR Costs (cvar costs)	398816.14	77763.51
Imbalance Costs (imb costs)	0.00	0.00
Line Costs (line costs)	416012.92	419317.52
Load Shedding Island Costs (load shedding island costs)	19943.31	3888.18
Total Objective Cost (obj)	655910.9	491745.65

Table 4.3: Table of CBC Optimization Results for Case 1 and Case 2

1. **Solution Time:**

   - Case 1 : 884.10 sec
   - Case 2: 3819.55sec

   the solution time is significantly longer for Case 2 with CBC, suggesting increased complexity for CBC to deal with a system with more substations and candidate branches.

2. **Number of Iterations:**

   - Case 1 : 11961
   - Case 2: 22721

   For CBC, Case 2 requires many more iterations, reflecting the increased difficulty for the solver to converge to an optimal solution with a more complex setup.

3. **Nodes Explored:**

   - Case 1 :0
   - Case 2 :85

CBC, explores many more nodes for Case 2, suggesting extensive exploration of the search space due to the complexity added by the additional substations and candidate branches.

4. **Costs:** The presence of multiple substations in Case 2 improves the resilience and flexibility of the system, thereby reducing total cost.

4.5 Comparative analysis between Cplex & CBC :

	Case 1		**Case 2**	
Metrics	**CPLEX**	**CBC**	**CPLEX**	**CBC**
Solution Time	239.24 sec	884.10 sec	106.22 sec	3819.55 sec
Number of Iterations	17193	11961	2359	22721
Node explored	41	0	0	85
Battery Costs	30493.25	30493.25	31602.29	31602.29
CVaR Costs	398866.14	398816.14	77763.51	77763.51
Imbalance Costs	0.00	0.00	0.00	0.00
Line Costs	416012.92	416012.92	419317.52	419317.52
Load Shedding Island Costs	19943.31	19943.31	3888.18	3888.18
Total Objective Cost (obj)	655910.9	655910.9	491745.65	491745.65

Table 4.4: Comparison table of metrics for cases 1 and 2 using CPLEX and CBC

1. **Comparison in terms of difference (Solution time- Number of iterations -Node explored):** The difference between the two solvers results from the following reasons:

   Different Algorithms:

   - **CPLEX:** Uses advanced and optimized algorithms to solve MILP problems, such as the simplex algo- rithm, the interior point algorithm, and advanced techniques like branching and cutting planes.
   - **CBC:** As an open-source solver, it may not benefit from the same levels of algorithmic optimzation.

   Heuristics and Optimization Techniques:

   - **CPLEX:** Integrates advanced heuristics, pre-processing techniques, and local search methods that ac- celerate convergence to the optimal solution.
   - **CBC:** May not have these features at such a sophisticated level, resulting in longer computation times.

Tree Search Approach:

- **CPLEX:** Uses a very thorough branch-and-bound search method. It explores more nodes to ensure that no potentially better solution is missed. This thorough exploration helps reduce the search space more rigorously and guarantees an optimal solution.
- **CBC:** Might adopt a more restricted or heuristic strategy in node exploration, reducing the total num- ber of nodes explored. This approach can speed up the resolution time but at the cost of less exhaus- tive exploration.

Cutting Techniques:

- **CPLEX:** Implements advanced and often more numerous cutting techniques, which add new nodes to the decision tree. These cuts help refine the lower and upper bounds and explore more potential scenarios but also increase the number of nodes explored.
- **CBC:** May use fewer or less aggressive cutting techniques, which limits the number of nodes added and explored in the decision tree.

Commercial vs Open Source:

- **CPLEX:** Is a commercial product developed by IBM, benefiting from many years of research, develop- ment, and continuous optimization.
- **CBC:** As an open-source solution, is developed by a community and may not have the same resources for continuous improvements.

2. **Comparison in terms of equality (costs):**

   - The mathematical model of the optimization problem is well formulated, with clearly defined constraints and objectives.
   - In MILP problems, if the model exhibits convex or quasi-convex properties, the optimal solutions will be identical regardless of the solver used. Both solvers will converge to the same optimal solution due to the nature of the problem.
   - The problem does not have equivalent alternative solutions (i.e., multiple optimal solutions with the same costs). This means there is a unique optimal solution that optimally satisfies all constraints.
   - Modern solvers like CPLEX and CBC are designed to operate with high numerical precision. Even though the solving methods may vary, the accuracy of calculations ensures that the final computed costs remain identical.

- Both solvers apply methods to explore the solution space and converge to optimality. Although the number of iterations and explored nodes may vary, the algorithms guarantee convergence to the same optimal costs.

4.6 Final configuration of the network

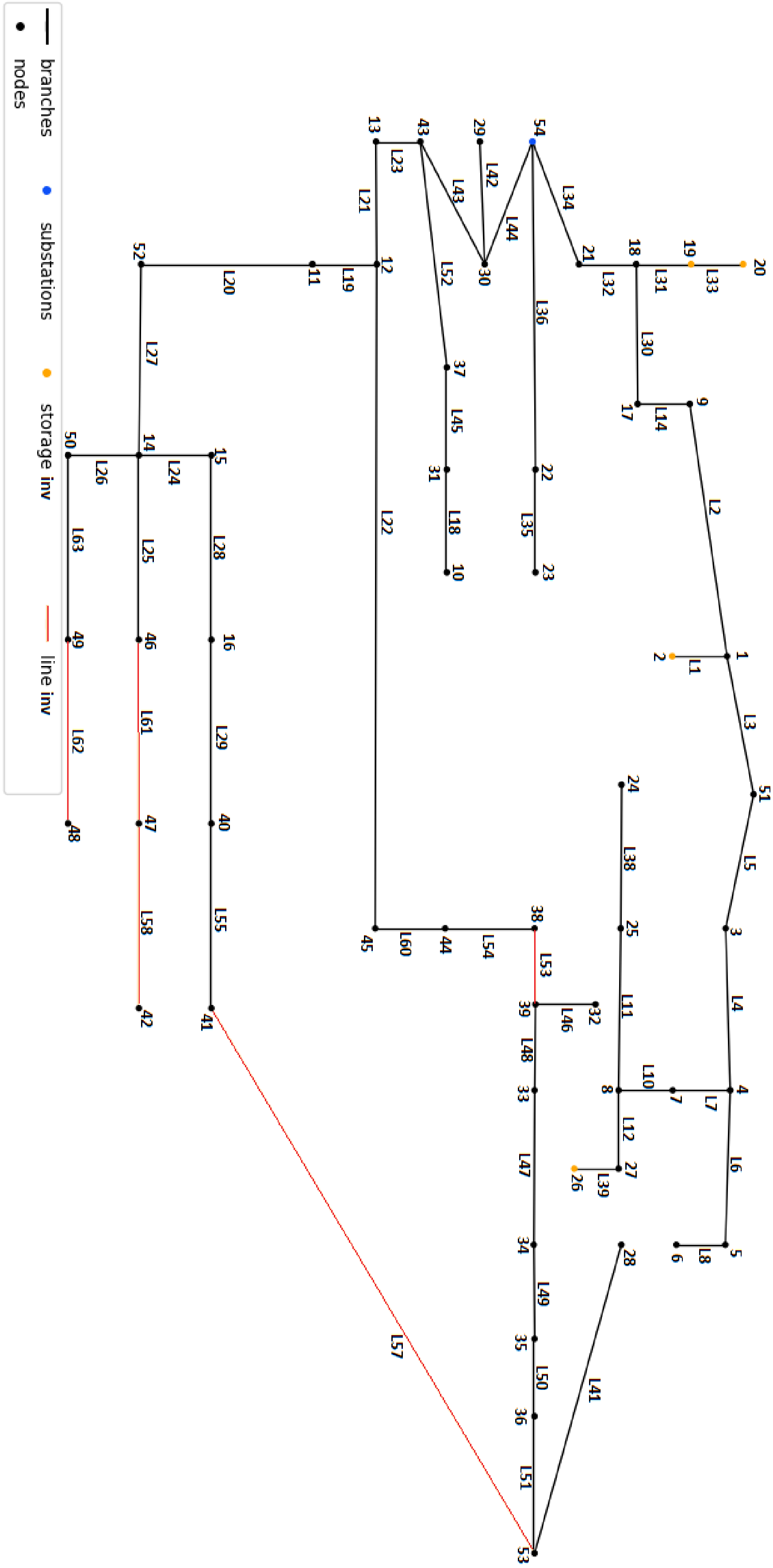


Figure 4.4: Final configuration of the network for case 1

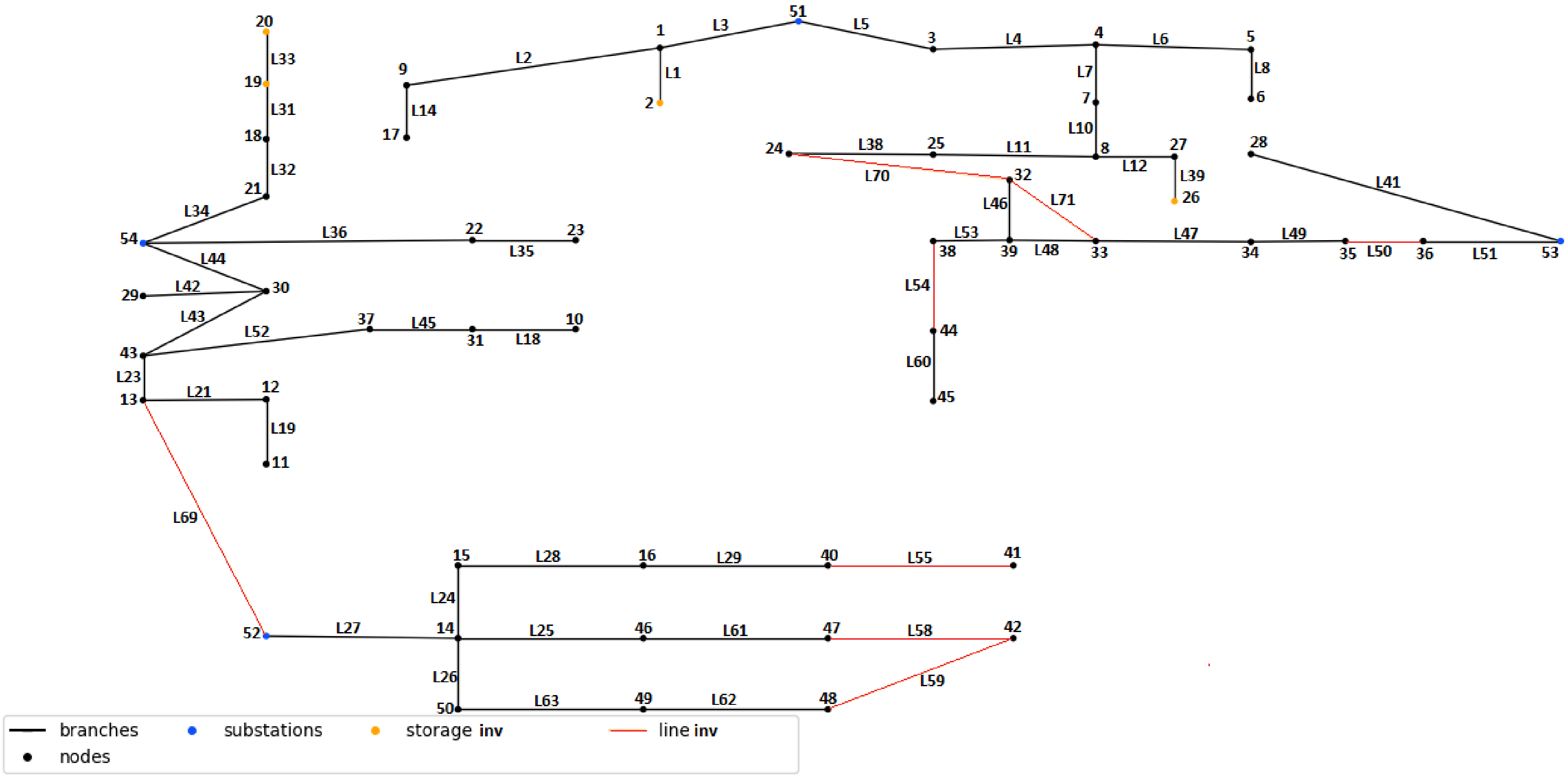


Figure 4.5: Final configuration of the network for case 2

The figures 4.4 and 4.5 are final configuration of the network for case 1 4.2.1 and case 24.2.2.

4.6.1 Graphical interpretation:

both solvers produced similar results for each case we studied. Here is a detailed description of the graphs obtained for the 2 cases:

- Branches (in black): Existing lines that connect different nodes in the network.
- Substations (in blue): Critical points in the network, acting as distribution centers for energy.
- Nodes (in black): Connection points where multiple branches meet.
- Storage candidates (in orange): Potential locations for installing storage capacities.
- Candidates for new lines (in red): Potential locations for new lines to enhance network distribution.

4.6.2 Branch Candidates Selection Process:

- Identification of Critical Branches: Branches that are identified as critical for different fault scenarios and load conditions are selected as candidates.
- Investment Optimization: The MILP model optimizes investments in candidate branches by minimizing investment costs and maximizing network reliability.
- Scenario Evaluation: Each candidate branch is evaluated under various scenarios to determine its potential impact on network resilience.

4.6.3 Why did the model choose primary branches within invested branches?

Choosing primary branches within invested branches is a strategic decision that relates to risk management, efficiency, and business continuity. Here are some reasons for this choice:

- preserving stability and security of the core network during expansion which reduces the risk of negatively impacting the network.
- minimizing the impact of failures.
- preserve the quality of services and products and the organization's credibility.
- reducing costs and financial risks associated with expansion.
- facilitate network management and control over charges.

what happens if we choose branch candidate randomly?

Choosing candidate branches randomly for expanding an electrical distribution network can have several negative impacts on the network's operation and performance.

1. Reduced Network Efficiency :

   - Under utilization of Resources: Randomly selected branches may not match the actual needs of the network, leading to under utilization of available resources.
   - Improper Optimization: Critical network points requiring upgrades might be overlooked, while resources could be wasted on less important segments.

2. Reliability and Stability Issues :

   - Increased Vulnerabilities: Without strategic analysis, new branches may not enhance network redundancy and resilience, making the system more vulnerable to outages.
   - Distribution Instability: Poor planning can result in uneven load distribution, creating over- load or under utilization points.

3. Increased Costs :

   - Inefficient Investments: Financial resources might be allocated to branches that do not significantly improve the network, thereby increasing costs without proportional benefits.
   - Maintenance and Repair Costs: Improperly positioned new branches could require more frequent maintenance and repairs, increasing operational costs.

4. Performance and Service Quality Issues:

   - Degraded Service Quality: End-users may experience voltage fluctuations and service interruptions if new branches are not optimized to meet demand.
   - Load Management Challenges: Random placement can complicate electricity flow and load management, reducing overall network efficiency.

5. Environmental and Regulatory Impact :

   - Non-compliance with Regulations: New branches must adhere to local standards and regulations. Random selection may lead to violations and potential sanctions.
   - Environmental Impact: Suboptimal planning can result in negative environmental impacts, such as habitat destruction or excessive resource consumption.

Deduction: In this context, case 2 appears to be more convergent for both solvers, as it achieves shorter solution times with fewer iterations, and in the case of CPLEX, without node exploration. This suggests enhanced efficiency in converging towards the optimal or near-optimal solution for case 2 compared to case 1 in both solvers.

4.7 Monte Carlo Simulation

Monte Carlo simulations are a class of computational algorithms that rely on repeated random sampling to obtain numerical results. The central idea is to use randomness to solve problems that might be deterministic in principle. This technique is widely used in various fields such as finance, engineering, supply chain management, and scientific research.

Key Concepts of Monte Carlo Simulations

- **Random Sampling:** The process involves generating random variables to simulate the process or system being studied.
- **Repetition:** The simulation runs many iterations (or trials) to approximate the desired quantity.
- **Probability Distribution:** The random variables are often drawn from specific probability distributions relevant to the problem.

4.7.1 Results with Cplex Solver

After running the program with Cplex solver 10 times for case 2, we obtained the following results 4.5:

Simulations	Solution Time	Battery Costs	CVaR Costs	Imb Costs	Line Costs	Load Shedding	obj
Sim 1	106.22	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 2	67.17	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 3	61.77	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 4	91.53	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 5	55.56	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 6	46.13	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 7	46.94	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 8	47.30	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 9	47.66	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 10	53	31602.29	77763.51	0.0	419317.52	3888.18	491745.65

Table 4.5: Results with Cplex Solver

- The variance in solution time stems from the stochastic nature of optimization and the complexity of the model. Each Monte Carlo simulation introduces random variations in input data or simulation conditions, which can influence how the CPLEX solver processes the problem and converges towards an optimal solution.
- However, the fact that the costs, including the total objective cost (obj), remain identical to the first simulation underscores that the model quickly achieves a stable solution in terms of costs. This is encouraging as it indicates robust results that are resilient to the variations introduced by Monte Carlo in this context.

4.7.2 Results with CBC Solver

For CBC solver, we could only perform 2 simulations for case 2 due to:

- Longer Computation Time: The CBC solver typically exhibits longer computation times compared to CPLEX. If each simulation of Monte Carlo requires significant time to converge towards a solution, this could limit the number of simulations that can be executed.
- Computational Resources: The capabilities of the computing infrastructure, including available memory and processing capacity, can also constrain the number of simulations feasible with the CBC solver. Limited resources may restrict the ability to perform a large number of Monte Carlo simulation.

And below are results 4.6 :

Simulations	**Solution Time**	**Battery Costs**	**CVaR Costs**	**Imb Costs**	**Line Costs**	**Load Shedding**	**obj**
Sim 1	3819.55sec	31602.29	77763.51	0.0	419317.52	3888.18	491745.65
Sim 2	4218.88sec	31602.29	77763.51	0.0	419317.52	3888.18	491745.65

Table 4.6: Results with CBC Solver

Increased Solution Time:

1. Algorithmic Complexity: The solver deals with more complex scenarios or searches for optimal solutions in a larger or more complex solution space.

2. Resource Utilization: As each iteration progresses, the solver might consume more computational resources (e.g., memory, CPU cycles), leading to longer times for subsequent iterations. This could be due to accumulated overheads or the need for more extensive computations to handle slightly varied problem.

In conclusion, a range of current studies has been explored in the field of electric network expansion planning with a focus on risk management. The introduction of the data used and presentation of several case studies were followed by the introduction of an optimal model to achieve specific objectives and the analysis of expected results and outputs. Additionally, a comprehensive comparison was provided between the use of Cplex and CBC software, highlighting their capabilities and multiple uses in this context. The final network configuration was reviewed, providing a visual interpretation of the achieved results, along with an explanation of the process of selecting network branches and the reasons behind choosing the primary branches. Finally, a Monte Carlo simulation was conducted and the results analyzed using both Cplex and CBC software. This study represents a significant contribution to exploring the field of electric power network expansion planning with a focus on risk management, opening new avenues for future research and development.

CONCLUSION

To conclude, with rapid advancements in technology and the increasing complexities of electricity networks, there is a growing interest in developing improvement models for investment planning in this field. As energy demands rise and sustainability challenges become more pronounced, mathematical and analytical models play a crucial role in achieving sustainable growth objectives and delivering energy services more efficiently and sustainably.

Looking ahead, future research is expected to focus on advancements in smart grid technology and big data analytics. These developments will enhance the models' capability to effectively handle and analyze large datasets, supporting informed planning and improvement decisions. Addressing these future challenges will necessitate the development of robust risk management strategies to mitigate potential negative impacts from uncontrollable variables.Furthermore, a comprehensive approach integrating technical, economic, and environmental considerations in the planning and operation of energy networks will be essential. This approach aims to promote sustainability and stability in meeting future energy demands.

In summary, this outlook underscores the increasing challenges and opportunities in the field of planning and enhancing electricity distribution networks. It emphasizes the importance of ongoing innovation and development to meet the requirements of the modern and future era.

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