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Thème:

**Tree level spinor helicity amplitude formalism for the
standard model of electroweak and strong interactions**

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Dedication

I would like to dedicate this thesis first and foremost to my parents who were always supportive of me and always pushed me to reach higher. To my friends Hamza and Amine who gave me comfort when I most needed it and who gave me an escape when it seemed hard to go on. To Yasmine who was always here for me through everything, the good and the bad, finally and most importantly I'd like to give a much earned gratitude to my colleagues with whom I share the joy of being here at last, also, to my well esteemed teachers who always provided me with knowledge and guidance and for that I'll be eternally grateful.

Thank you ever so much.

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In loving memory of Pr. Mustapha Bentaiba.

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Abstract

In this thesis we study the spinor helicity formalism (SHF). This formalism is essentially based on Weyl spinors, where we write the physical quantities appearing in scattering amplitudes in terms of these spinors. Here we study two distinct cases: massless particles and massive ones and build the formalism for each case. After that, we apply SHF to study a few scattering processes including different sides of techniques necessary to acquire.

We will also plot Feynman diagrams for scattering processes via the FeynArts package; illustrate how the amplitudes for these processes are obtained by using the FeynCalc package and showcase the SpinorsExtras package(sub-package of S@M) via a simple example to obtain the amplitudes in terms of spinor products.

Key Words: Spinor Helicity Formalism, Tree Level Amplitude, Scattering Amplitude.

ملخص

في هذه المذكرة سنستعرض طريقة دالة عزم اللف الذاتي. هذه الطريقة تعتمد أساسا على دوال اللف عديمة الكتلة حيث نكتب المقادير الفيزيائية التي تظهر في نوبات الانتشار بدلالة هذه الدوال. سوف نقوم بدراسة حالتين: جسيمات عديمة الكتلة و جسيمات ذات كتلة و ندرس الطريقة بدقة في كل حالة. بعد ذلك سنستعرض أمثلة تطبيقية تتفاوت في الصعوبة و تغطي جانبا مختلفا من تقنيات حساب النوبات.

سوف نقوم أيضا برسم مخططات فاينمان و نرى كيف تبرمج طريقة دالة عزم اللف الذاتي باستعمال برمجية Mathematica و حزمه المكملة.

Abbreviation and Convention

Abbreviation

<i>SM</i>	<i>Standard Model</i>
<i>CSM</i>	<i>Classical Standard Model</i>
<i>QCD</i>	<i>Quantum ChromoDynamics</i>
<i>QED</i>	<i>Quantum ElectroDynamics</i>
<i>EW</i>	<i>ElectroWeak</i>
<i>GF</i>	<i>Gauge Fixing</i>
<i>SHF</i>	<i>Spinor Helicity Formalism</i>
<i>MHV</i>	<i>Maximum Helicity Violating</i>
<i>LHC</i>	<i>Large Hadron Collider</i>
<i>LCD</i>	<i>Light Cone Decomposition</i>
<i>SLCD</i>	<i>Simultaneous Light Cone Decomposition</i>

Convention

we used :

- The metric signature : $(-1, 1, 1, 1)$

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Introduction

It is one of mankind's greatest blessings that we are born curious. Curiosity can often lead to amazing feats and one of the questions that stimulates one's curiosity is: where are we in this universe? The development of this question over thousands of years took us to another question: what are the laws under which this universe exists? The objective behind this question is to try and establish well funded theories that would make us understand the laws of nature better. On that note, there is only two major theories that got us close to that goal. The first being Einstein's General Relativity [1] to describe the ample objects such as planets, and the second being the standard model (SM) [2, 3, 4] which studies the smallest fundamental objects.

When approaching the subject of particle physics, one must fully understand what he's stepping into. To talk about particle physics is to dive directly into the smallest particles of matter and the ongoing interaction between them. The SM is a unison of two separate theories: QCD (Quantum Chromo Dynamics) [5] and Electroweak theory (EW) [6].

The experimental side of particle physics is centered around measuring the number of events occurring inside the collider which is given by the product of the scattering cross section σ_{event} and the luminosity L . Obtaining the differential cross section $\frac{d\sigma}{d\Omega}$ is a stepping stone to obtain the number of events. This quantity is proportional to the square of the amplitude of the processes occurring inside the collider:

$$\frac{d\sigma}{d\Omega} \propto |M|^2 \quad (1)$$

Throughout this thesis our interest resides in calculating the square of the scattering amplitude. The classical method to calculate amplitudes is called "trace method"(it was developed in the early 60's). However with growing collision energies, multiple external leg processes are more occurring which makes this method complicated and inadequate by computer and just impossible to do by hand. This is because such processes contain a large amount of terms. And not only does one have to carry out the calculation to obtain all the amplitudes but also has to calculate the square of the sum of these amplitudes. Generally after carrying out the heavy simplification we are left with a simple small expression because many terms cancel each other out. A better alternative method for calculating amplitudes is called

"Spinor Helicity Formalism" (SHF) [7, 8]. Our objectives for this thesis are:

- To study in detail the SHF and understand how to apply it in physical processes to obtain the squared amplitudes.
- To present it in a comprehensible introductory manner which makes the thesis easy to exploit.

This thesis will be organized as follows :

- Chapter 1 : we briefly present the SM and exhibit its lagrangian and explicitly write down each term and briefly explain what it stands for.
- Chapter 2 : we construct the objects used to perform calculations in this formalism by using Weyl spinors [9] and use an efficient notation to write the main physical quantities appearing in amplitudes. We study two separate cases; massless case of SHF which is how this formalism was originally introduced. However the experiment enforces the extension of SHF to include massive particles.
- Chapter 3 : we apply SHF in a variety of examples each containing a different side in order to cross over as much calculation techniques as possible. Such examples are Bhabha scattering, pair annihilation process and Higgs production process which is a three particle final state case.
- Finally we arrive to the conclusion of our work where we present the challenges we overcame throughout this thesis along with perspectives for the near future.

Chapter 1

Standard Model

*"A deep emotion
comes from true devotion
to the right notion
of spin in motion."*

R.Gastmans

The SM is a theory initially introduced by Weinberg [10], Glashow [11] and Salam [12] to describe the EW interaction for leptons. It was modified to include the strong nuclear interaction by Glashow, Iliopoulos and Maiani [13]. It's a quantum field theory which is based on a local gauge symmetry of the group $SU(3)_c \times SU(2)_I \times U(1)_Y$, where the subscripts c , I and Y denote the color charge, the weak isospin and the hypercharge respectively. The SM stipulates that the main constituents of our universe are elementary fermions -leptons and quarks that is- classified into three generations :

$$\begin{aligned} 1^{st} \text{ generation} &\rightarrow \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (\nu_e, e)_R; \begin{pmatrix} u \\ d \end{pmatrix}_L, (u, d)_R \\ 2^{nd} \text{ generation} &\rightarrow \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, (\nu_\mu, \mu)_R; \begin{pmatrix} c \\ s \end{pmatrix}_L, (c, s)_R \\ 3^{rd} \text{ generation} &\rightarrow \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, (\nu_\tau, \tau)_R; \begin{pmatrix} t \\ b \end{pmatrix}_L, (t, b)_R \end{aligned} \tag{1.1}$$

The interactions among these particles are carried by bosonic messenger particles : gluons "g" for the color interaction -more commonly known as the strong nuclear interaction-, photons "γ" for the electromagnetic interaction, and the massive vector bosons "Z" and "W±" for the weak interaction.

This chapter consists of an overview of the SM lagrangian and the Feynman rules for calculating amplitudes at tree level.

1.1 The Standard Model lagrangian

Technically the SM can be expressed in several different formulations, however, despite the appearance, the lagrangian formulation is one of the easiest and most compact ways of presenting the theory.

The classical SM (CSM) lagrangian is written as a sum of multiple terms [14], we have:

$$\mathcal{L}_C = \mathcal{L}_{gauge} + \mathcal{L}_{Fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \quad (1.2)$$

We shall present all of these terms and pinpoint what each term stands for below.

1.1.1 Gauge Field lagrangian

The gauge group $SU(3)_c \times SU(2)_I \times U(1)_Y$ has twelve generators: eight T_s^α 's that form the $SU(3)$ algebra, three T^a 's that form the $SU(2)$ algebra, and a single Y that generates the $U(1)$ algebra. The generators T_s^α , T^a and Y satisfying the Lie brackets ¹ :

$$[T_s^\alpha, T_s^\beta] = if^{\alpha\beta\lambda} T_s^\lambda, \quad [T^a, T^b] = i\epsilon^{abc} T^c, \quad [Y, Y] = 0$$

where ϵ^{abc} is totally anti-symmetric tensor and $f^{\alpha\beta\lambda}$ are the structure constants of $SU(3)$ [14].

Hence there are three independent constants in \mathcal{L}_{gauge} : (g_s) is the QCD gauge coupling constant related to $SU(3)_c$ subgroup, (g) and (g') are respectively the $SU(2)_I$ and $U(1)_Y$ gauge coupling constants.

Gauge field dynamics are governed by the gauge field lagrangian :

$$\mathcal{L}_{Gauge} = -\frac{1}{4} G_{\mu\nu}^\alpha G^{\alpha\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad (1.3)$$

where $G_{\mu\nu}^\alpha$, $W_{\mu\nu}^a$ and $B_{\mu\nu}$ are the field strength for the strong interaction, the weak

¹ α, β and λ varying from 1 to 8, on the other hand a, b and c take the values from 1 to 3.

interaction and the electromagnetic interaction respectively, and are given by :

$$G_{\mu\nu}^\alpha = \partial_\mu G_\nu^\alpha - \partial_\nu G_\mu^\alpha + g_s f^{\alpha\beta\lambda} G_\mu^\beta G_\nu^\lambda \quad (1.4)$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c \quad (1.5)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (1.6)$$

G_μ^α , W_μ^a and B_μ are respectively the gluon fields, $SU(2)$ and $U(1)$ gauge fields.

1.1.2 Fermionic lagrangian

In order to determine the fermionic contribution to the SM lagrangian, we associate to every fermion intervening in the theory a spinor (ψ_i), thus the fermionic contribution is accounted for via the lagrangian \mathcal{L}_F [14], given by :

$$\mathcal{L}_F = i \sum_{k=1}^3 \left[\bar{\Psi}_L^{k,l} \gamma^\mu D_{L,\mu}^l \Psi_L^{k,l} + \bar{\Psi}_R^{k,l} \gamma^\mu D_{R,\mu}^l \Psi_R^{k,l} + \bar{\Psi}_L^{k,q} \gamma^\mu D_{L,\mu}^q \Psi_L^{k,q} + \bar{\Psi}_R^{k,q} \gamma^\mu D_{R,\mu}^q \Psi_R^{k,q} \right] \quad (1.7)$$

where the sum over (k) denotes a sum over the three generations of both leptons denoted by (l) and quarks denoted by (q), the covariant derivatives are as follows :

$$\begin{aligned} D_{L,\mu}^l &= D_{\Phi,\mu} = \partial_\mu - ig W_\mu^a T^a - ig' Y B_\mu \\ D_{R,\mu}^l &= \partial_\mu - ig' Y B_\mu \\ D_{L,\mu}^q &= \partial_\mu - ig_s G_\mu^\alpha T_s^\alpha - ig W_\mu^a T^a - ig' Y B_\mu \\ D_{R,\mu}^q &= \partial_\mu - ig_s G_\mu^\alpha T_s^\alpha - ig' Y B_\mu \end{aligned} \quad (1.8)$$

Written in this manner the fermionic lagrangian is invariant under the local gauge transformations of the $SU(3) \otimes SU(2) \otimes U(1)$ group. This lagrangian is also invariant under global transformation of the Poincaré group. It is important to point out that the fermionic lagrangian does not include in an explicit way the mass terms because then the lagrangian would not be gauge invariant, instead the mass of fermions must be introduced in a way that preserves this property. The only way to do this is to introduce the mass via the Higgs mechanism [14]. For this we need to introduce the Higgs lagrangian.

1.1.3 Higgs lagrangian

In the SM the Higgs field is represented by a complex scalar isotopic doublet:

$$\Phi = \begin{bmatrix} \varphi^\dagger \\ \frac{v+H+i\varphi_Z}{\sqrt{2}} \end{bmatrix} \quad (1.9)$$

The Higgs field lagrangian is introduced via the following lagrangian :

$$\mathcal{L}_s = (D_{\Phi,\mu}\Phi)^\dagger (D^{\Phi,\mu}\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \quad (1.10)$$

Φ is the isotopic doublet for the complex scalar field.

μ and λ are free parameters for the model.

1.1.4 Yukawa lagrangian

The fermions interact with the higgs field to acquire their mass. This interaction is embedded in the SM via the Yukawa lagrangian [14].

$$\begin{aligned} \mathcal{L}_Y = & - \sum_{k=1}^3 \left[\lambda_k^{l,up} \bar{\Psi}_L^{k,l} (i\sigma^2\Phi^*) \Psi_{R,up}^{k,l} + \lambda_k^{l,down} \bar{\Psi}_L^{k,l} \Phi \Psi_{R,down}^{k,l} + (h.c) \right] \\ & - \sum_{k=1}^3 \left[\lambda_k^{q,up} \bar{\Psi}_L^{k,q} (i\sigma^2\Phi^*) \Psi_{R,up}^{k,q} + \lambda_k^{q,down} \bar{\Psi}_L^{k,q} \Phi \Psi_{R,down}^{k,q} + (h.c) \right] \end{aligned} \quad (1.11)$$

where : $\lambda_k^{l,up}$, $\lambda_k^{l,down}$, $\lambda_k^{q,up}$ and $\lambda_k^{q,down}$ are the Yukawa coupling constants.

So the classical SM lagrangian is the sum of the previously mentioned terms

$$\mathcal{L}_c = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \quad (1.12)$$

However since the SM is a quantum field theory so the fields must be quantified. This procedure means that we must introduce a field operator for every field intervening and it must be done in a way that preserves gauge invariance; this implies adding two new terms to the CSM lagrangian.

The canonical quantization of a non abelian gauge field lagrangian yields an inconsistency in the theory which is the fact that the time component of the gauge field \hat{G}_0^a vanishes which means that the conjugate momentum $\hat{\Pi}_0^a$ will also vanish while the Lie brackets stip-

ulate that :

$$\left[\hat{G}_0^a(x), \hat{\Pi}_0^a(y) \right]_{x_0=y_0} = i\delta^3(\vec{x}-\vec{y}) \quad (1.13)$$

The solution to this problem lies within ignoring the classical implications of gauge invariance and adding a gauge fixing term given by :

$$\mathcal{L}_{GF} = \frac{1}{2\xi_G} (\partial \cdot G^a)^2 \quad (1.14)$$

However this creates an even bigger problem, the lagrangian is no longer gauge invariant.

Feynman found a solution in 1963 [15] which consisted of introducing new bosonic fields which verify the fermionic Lie brackets called "ghosts". In 1967 Faddeev and Popov [16] used path integrals to justify Feynman's strange proposition, the second term added into the lagrangian depends on these ghosts

$$\mathcal{L}_{ghost} = i(\partial^\mu \chi_1^a) D_\mu^{ab} \chi_2^b \quad (1.15)$$

where χ_1^a, χ_2^b are the Faddeev-Popov ghosts, $D_\mu^{ab} = \delta^{ab} \partial^\mu - g_s f^{abc} G_\mu^c$ is the covariant derivative while a, b and c are the adjoint representation indices.

Adding the gauge fixing term and the ghost term to the classical lagrangian breaks invariance under local classical gauge transformations and extends the invariance to local ghost transformations, this extension is called the BRS transformations [17].

Eventually the quantized SM lagrangian will be the sum of the classical lagrangian and the ghost term and the gauge fixing term [14] :

$$\mathcal{L}_{SM} = \mathcal{L}_c + \mathcal{L}_{ghost} + \mathcal{L}_{GF} \quad (1.16)$$

1.2 Feynman Rules

A Feynman diagram is a graphical representation for the quantum amplitudes of a given process, composed of three elements external lines, internal lines and vertices. To each one of these elements we associate a mathematical expression dictated by the SM lagrangian, the product of these expressions-with the appropriate order- will yield the amplitude of the depicted process. How one tells which mathematical expression is the right one to use is answered by Feynman Rules. These rules can be derived but we will just list the ones we will use throughout this thesis [18, 8].

In order to simplify the process we will classify Feynman rules by the sectors in which they intervene -QCD and EW sectors that is- below :

1.2.1 Feynman rules for the Electroweak theory

The EW sector Feynman rules intervening throughout this work are listed as follows

- Propagators

A photon propagator is represented with a wavy line:

$$\mu \overset{\gamma}{\text{~~~~~}} \nu \quad \frac{-i}{p^2+i\epsilon} [\mathbf{g}_{\mu\nu} - (1-\xi) \frac{p_\mu p_\nu}{p^2}] \quad (1.17)$$

where $g^{\mu\nu}$ is Minkowski's spacetime metric and p_μ is the photon 4-momentum .

We will mostly work in Feynman gauge, $\xi = 1$:

$$\mu \overset{\gamma}{\text{~~~~~}} \nu \quad \frac{-i}{p^2+i\epsilon} \mathbf{g}_{\mu\nu} \quad (1.18)$$

The same representation goes for the Z vector boson

$$\mu \overset{Z}{\text{~~~~~}} \nu \quad \frac{-i \mathbf{g}_{\mu\nu}}{k^2 - M_Z^2 + i\epsilon} \quad (1.19)$$

As for the Higgs boson and the rest of the other scalar particle :

$$\text{---} \overset{\vec{p}}{\text{---}} \text{---} \quad \frac{-i \mathbf{g}_{\mu\nu}}{k^2 - M_H^2 + i\epsilon} \quad (1.20)$$

A spinor propagator with mass m is a continuous line with an arrow:

$$i \overset{\vec{p}}{\text{-----}} j \quad \frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon} \delta_{ij} \quad (1.21)$$

The arrow points to the right for a particle and to the left for an antiparticle, and the δ_{ij} is added to conserve flavor

- External lines

When a photon is an incoming or an outgoing particle it contributes by a polarisation vector $\epsilon_\mu(p)$

and is represented graphically by :

$$\text{~~~~~} \bullet \quad \epsilon_\mu(p) \quad (\text{incoming}) \quad (1.22)$$

$$\bullet \text{~~~~~} \quad \epsilon_\mu^*(p) \quad (\text{outgoing}) \quad (1.23)$$

As for fermions, we represent them by :

$$\text{-----} \rightarrow \bullet \quad u^s(p) \quad (1.24)$$

$$\bullet \text{-----} \rightarrow \quad \bar{u}^s(p) \quad (1.25)$$

$$\text{-----} \leftarrow \bullet \quad \bar{v}^s(p) \quad (1.26)$$

$$\bullet \text{-----} \leftarrow \quad v^s(p) \quad (1.27)$$

$u^s(p)$ and $v^s(p)$ are spinors(for electron and positron respectively) satisfying the Dirac equation:

$$(\not{p} - m)u^s(p) = 0, \quad \bar{u}^s(p)(\not{p} - m) = 0 \quad (1.28)$$

$$(\not{p} + m)v^s(p) = 0, \quad \bar{v}^s(p)(\not{p} + m) = 0 \quad (1.29)$$

where s is an index numbering the two spin degrees of freedom of the spinor. The spinors $\bar{u}^s(p)$ and $\bar{v}^s(p)$ are defined by:

$$\bar{u}^s(p) = u^s(p)^\dagger \gamma^0, \quad \bar{v}^s(p) = v^s(p)^\dagger \gamma^0 \quad (1.30)$$

- Vertices

Gauge couplings to fermions are as follows:² :

$$\begin{array}{c} \psi_f \\ \swarrow \\ \psi_f \end{array} \begin{array}{c} \text{~~~~~} \\ Z_\mu \end{array} \quad i \frac{g}{\cos\theta_W} \gamma_\mu (g_V^f - g_A^f \gamma_5) \quad (1.31)$$

$$\begin{array}{c} \psi_f \\ \swarrow \\ \psi_f \end{array} \begin{array}{c} \text{~~~~~} \\ A_\mu \end{array} \quad ieQ_f \gamma_\mu \quad (1.32)$$

² Q_f is the charge of the fermion, $Q_f = -1$ for an electron.



$$i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \quad (1.33)$$

Sometimes it is more useful to write these couplings in terms of the left and right projectors:

$$P_L = \frac{1-\gamma_5}{2}, \quad P_R = \frac{1+\gamma_5}{2} \quad (1.34)$$

we then get:

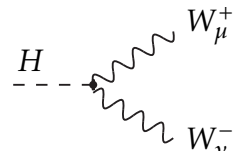
$$(g_V^f - g_A^f \gamma_5) = (g_L^f P_L + g_R^f P_R) = (g_- \gamma_- + g_+ \gamma_+) \quad (1.35)$$

Where : $\gamma_- = P_L$ and $\gamma_+ = P_R$.

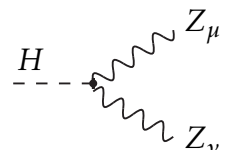
Higgs boson couplings



$$-i \frac{g}{2} \frac{m_f}{m_W} \quad (1.36)$$




$$i g m_W g_{\mu\nu} \quad (1.37)$$



$$i \frac{g}{\cos(\theta_W)} m_Z g_{\mu\nu} \quad (1.38)$$

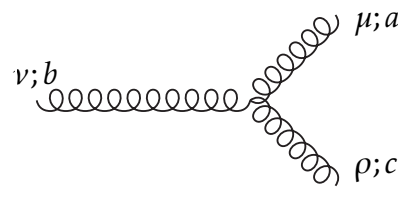
1.2.2 QCD Feynman rules

In this part we list the rules used throughout this work for performing amplitude calculations in color interactions. The gluon propagator looks similar to that of the photon's with the addition of the Kronecker delta coming from the color interaction.



$$\frac{-i}{p^2+i\epsilon} [g_{\mu\nu} - (1-\xi) \frac{p_\mu p_\nu}{p^2}] \delta_{ab} \quad (1.39)$$

The three-gluon vertex is given by the following formula :



$$gf^{abc} [g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu] \quad (1.40)$$

All momenta are taken to be incoming.

Remember that in this part we only mentioned Feynman rules used in chapter 3.

Chapter 2

Spinor Helicity Formalism

"The mere neglect,
of a mass effect
is quite suspect
and often not correct."

R.Gastmans

Scattering amplitudes are at the very center of high energy physics right at the intersection between the SM and collider physics. Currently the main trends in collider physics are the ones specializing in colliding Hadrons, this era started with the Tevatron. It is now residing at the Large Hadron Collider (LHC) [19] at a collision energy of approximately 13 TeV designed to push the SM to its limits and it has so far passed every test marvelously. However there are quite a few questions the SM still fails to answer, in order to get closer to answering these questions higher energy collisions are needed. The challenge here does not only lie in developing stronger colliders but also stronger calculating techniques, if we are to collide hadrons at a higher energy scale then it is expected to encounter multiple finite state particle processes. So here's the rub, the standard go to method when calculating scattering amplitudes is the trace evaluation method¹ [20].

This method is extremely convenient and easy to handle when a process yields two particles or even three. However the method gets increasingly difficult and somewhat uncomfortable, until reaching a point at which this method is just unusable. This is mainly due to the fact that calculating the square of an amplitude implies having to calculate the mix terms product, the result

¹See appendix B

eventually will be quite simple but the process is still a big mess. With these problems in mind, the need for a more efficient formalism is urgent deal. Among the many promising formalisms, SHF is the one that stands out the most [7]. The SHF is a relatively new method developed in the 1980s [8] which eliminates the need to calculate mix terms, in other words it makes the square of a sum equal to the sum of the squares.

In this chapter we will see how this formalism works. We will also showcase the standard notation for this formalism and set the formulas we work with in both the massive and massless cases.

2.1 Weyl Spinors

The first time P. Dirac wrote down his relativistic equation for spin half fermions, he had the electron in mind, this paper was published in 1928 and was met with a mixture of reactions but one thing is certain it was quite the breakthrough.

A year later H. Weyl showed that for a massless fermion a simpler equation with less degrees of freedom would suffice; and as big a breakthrough the Dirac equation was, time showed that Weyl's work on massless fermion fields was a stepping stone for what was to come. We start from the explicit Dirac equation and we take the $m \rightarrow 0$ limit to get :

$$i\hbar\frac{\partial\psi}{\partial t} = -i\hbar c\alpha^j\partial_j\psi \quad (2.1)$$

with the absence of mass, α^j matrices are no longer required to be 4 dimensional and would satisfy the equation with instead two dimensions only, thus we can just use Pauli matrices instead, as for the left hand side of the equation the matrix $\sigma^0 = \mathbb{1}$. By taking both members of the equation to one side we find the Weyl equation :

$$\sigma_\mu p^\mu\psi = 0 \quad (2.2)$$

where

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.3)$$

2.1.1 Helicity, Chirality and Spin

Spin is a quantity which was first postulated by Ralph Kronig as an idea to W. Pauli and then formally published, in late 1925, by S. Goudsmit and G. Uhlenbeck. Simply put, spin is the intrinsic angular momentum of the particle, and like the name suggests it's an intrinsic property of a particle, meaning it can never change in value.

For a particle of momentum \vec{P} and angular momentum \vec{J} , helicity is defined as the projection

of the intrinsic angular momentum \vec{S} of the particle onto the momentum axis of said particle [18]

$$h = \frac{2\vec{S}\cdot\vec{P}}{|\vec{P}|} \quad (2.4)$$

the factor "2" is introduced so that the eigenvalues of h remain integer for any particle. For a fermion obeying the Dirac equation -that is $\vec{S} = \frac{\vec{\sigma}}{2}$ - helicity can be written as :

$$h = \frac{\vec{\sigma}\cdot\vec{P}}{|\vec{P}|} \quad (2.5)$$

This operator clearly commutes with Dirac Hamiltonian. So this means that helicity is time invariant. Another interesting property is that helicity is invariant scalar under space rotations. As for boosts though, helicity may well change sign for a particularly boosted observer.

The Dirac representation is a direct sum of the irreducible left and right unitary representations of the Lorentz group: $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$. This makes a Dirac spinor a doublet of a left and a right handed Weyl spinors:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad (2.6)$$

We define:

$$P_L = \frac{1 - \gamma_5}{2}, \quad P_R = \frac{1 + \gamma_5}{2} \quad (2.7)$$

as projection operators for the spinors. These projectors are said to be "left" and "right" handed projection operators verifying $P_R^2 = P_R$, $P_L^2 = P_L$ and $P_R + P_L = 1$ and:

$$P_R \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}, \quad P_L \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \quad (2.8)$$

Chirality is defined from a mathematical perspective as the eigenvalue of γ_5 matrix given by :

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \quad (2.9)$$

this matrix satisfies anti-commutation brackets with all Dirac matrices and $\gamma_5^\dagger = \gamma_5$ along with $(\gamma_5)^2 = \mathbb{1}$ ². The γ_5 matrix commutes only with products containing an even number of Dirac matrices. So this means that γ_5 will not commute with the Dirac Hamiltonian and consequently for the massive case chirality is not time invariant.

If we look closer at helicity and chirality, we'd realize that these two are strangely related. Helicity is conserved but not Lorentz invariant whereas chirality is Lorentz covariant but changes in time and both of these shortcomings are due to the massive character of the particles. In the

²See appendix A.

massless limit however, the difference between these two vanishes completely, helicity and chirality become completely indistinguishable.

2.1.2 Notation for Weyl Spinors

We saw earlier that Weyl spinors belong to the irreducible representation of the Lorentz group, this property makes these objects very useful in many contexts as they are somewhat seen as building blocks for every field there is.

So now we're going to construct a concise notation for products and contractions of Weyl spinors, we will denote the left handed Weyl spinors with a normal Greek letter, while the right handed ones will be a tilded Greek letter, that is :

- ψ_α represents the left handed spinors
- $\tilde{\psi}^{\dot{\alpha}}$ represents the right handed spinors.

where $\alpha, \dot{\alpha}=1,2$. We now introduce the 2×2 totally anti symmetric tensor:

$$\varepsilon^{\alpha\beta} = -\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.10)$$

One can always use the matrices to verify that $\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = \delta_\alpha^\gamma$. This tensor is to be seen γ -function wise- as the $g^{\mu\nu}$ of Weyl spinors, however we must be careful with raising and lowering indexes as :

$$\psi^\alpha \chi_\alpha = \varepsilon^{\alpha\beta} \varepsilon_{\alpha\gamma} \psi_\beta \chi^\gamma = -\varepsilon^{\beta\alpha} \varepsilon_{\alpha\gamma} \psi_\beta \chi^\gamma = -\delta_\gamma^\beta \psi_\beta \chi^\gamma = -\psi_\gamma \chi^\gamma \quad (2.11)$$

In addition to making manipulations much simpler, this tensor helps us define the Weyl inner products:

- Left handed inner product is given by :

$$\psi\chi = \psi^\alpha \chi_\alpha = \psi_\alpha \varepsilon^{\alpha\beta} \chi_\beta = -\chi_\beta \varepsilon^{\alpha\beta} \psi_\alpha = \chi_\beta \varepsilon^{\beta\alpha} \psi_\alpha = -\chi_\alpha \psi^\alpha = -\chi\psi \quad (2.12)$$

- Right handed inner product is given by :

$$\tilde{\psi}\tilde{\chi} = \tilde{\psi}^{\dot{\alpha}} \tilde{\chi}_{\dot{\alpha}} = -\tilde{\chi}_{\dot{\alpha}} \tilde{\psi}^{\dot{\alpha}} = -\tilde{\chi}^{\dot{\alpha}} \tilde{\psi}_{\dot{\alpha}} = -\tilde{\chi}\tilde{\psi} \quad (2.13)$$

The next thing to introduce is the matrices intervening in the Weyl representation. The Dirac γ matrices are replaced by Pauli matrices $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. Now that all of these changes took place, let's review the kinetic term of the Dirac lagrangian $\psi = (\lambda, \chi)$ is:

$$\mathcal{L}_{kin} = i\bar{\psi}\not{\partial}\psi = i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + i\tilde{\chi}^\dagger \sigma^\mu \partial_\mu \tilde{\chi} \quad (2.14)$$

using the spinors with their component indexes, the lagrangian takes the form :

$$\mathcal{L}_{kin} = i\tilde{\lambda}^{\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}\alpha}^{\mu} \partial_{\mu} \lambda^{\alpha} + i\chi^{\alpha} \sigma_{\alpha\dot{\alpha}}^{\mu} \partial_{\mu} \tilde{\chi}^{\dot{\alpha}} \quad (2.15)$$

with $\tilde{\lambda} \equiv \lambda^{\dagger}$ and $\chi \equiv \tilde{\chi}^{\dagger}$.

Finally we write two very useful formulas :

$$g_{\mu\nu} \sigma_{\alpha\dot{\alpha}}^{\mu} \sigma_{\beta\dot{\beta}}^{\nu} = 2\varepsilon_{\alpha\beta} \varepsilon_{\dot{\alpha}\dot{\beta}} \quad (2.16)$$

$$\varepsilon_{\alpha\beta} \varepsilon_{\dot{\alpha}\dot{\beta}} \sigma^{\mu\beta\dot{\beta}} = \bar{\sigma}_{\dot{\alpha}\alpha}^{\mu} \quad (2.17)$$

2.2 Massless case

Originally, the SHF was introduced for the study of massless initial state particle processes [21], where many multiple finite state particle amplitudes were derived. In this section we're going to illustrate the original SHF.

2.2.1 A simpler more convenient notation

Let p and q be two four-momentum, while λ and χ their corresponding spinors. We define the notation:

$$|p\rangle^{\dot{\alpha}} = \tilde{\lambda}^{\dot{\alpha}} , \quad \langle q|_{\dot{\alpha}} = \tilde{\chi}_{\dot{\alpha}}$$

$$|p]_{\alpha} = \lambda_{\alpha} , \quad [q]^{\alpha} = \chi^{\alpha} \quad (2.18)$$

so next we rewrite the inner products in terms of helicity spinors:

$$\langle pq \rangle = \langle p|_{\dot{\alpha}} |q\rangle^{\dot{\alpha}} = \tilde{\lambda}_{\dot{\alpha}} \tilde{\chi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_{\dot{\alpha}} \tilde{\chi}_{\dot{\beta}} = -\tilde{\lambda}^{\dot{\beta}} \tilde{\chi}_{\dot{\beta}} \quad (2.19)$$

$$(2.20)$$

$$[pq] = [p]_{\alpha} [q]^{\alpha} = \lambda^{\alpha} \chi_{\alpha} = \varepsilon_{\alpha\beta} \lambda^{\alpha} \chi^{\beta} = -\lambda_{\beta} \chi^{\beta}$$

Here, the notation $\langle pq \rangle$ is simply an abbreviation for the contraction of dotted spinor indices, while $[pq]$ is for the undotted spinor indices. Whether the spinors are left or right handed is determined by the shape of the brackets, angled brackets are left handed helicity spinors and squared brackets are right handed helicity spinors, these spinor products satisfy :

$$\langle pq \rangle = -\langle qp \rangle , \quad [pq] = -[qp] \quad (2.21)$$

they are anti-symmetric, on the other side there are no $\langle pq \rangle$ -brackets, because the indices cannot contract directly to form a Lorentz scalar, and as a particular result:

$$\langle pp \rangle = [pp] = 0 \quad (2.22)$$

and for real momenta we have:

$$\begin{aligned} [p]^\alpha &= (|p\rangle^{\dot{\alpha}})^* \quad , \quad \langle p|_{\dot{\alpha}} = (|p]_{\alpha})^* \\ [pq]^* &= \langle qp \rangle \end{aligned} \quad (2.23)$$

These formulas help us deduce many simplifications while calculating amplitudes.

2.2.2 Vectors

The SHF's strength comes from the fact that spin-1 particles transform in the $(\frac{1}{2}, \frac{1}{2})$ Lorentz representation. This allows us to write the 4-momentum of any particle as bispinors

$$p_\mu \rightarrow p_{\alpha\dot{\alpha}} = p_\mu \sigma_{\alpha\dot{\alpha}}^\mu = \begin{pmatrix} -p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & -p^0 - p^3 \end{pmatrix} \quad (2.24)$$

more generally we have 4 relations :

$$p_{\alpha\dot{\alpha}} = (\sigma^\mu)_{\alpha\dot{\alpha}} p_\mu \quad , \quad p^{\dot{\alpha}\alpha} = \bar{\sigma}^{\mu\dot{\alpha}\alpha} p_\mu \quad , \quad p^\mu = \frac{1}{2} \sigma^{\mu\dot{\alpha}\alpha} p_{\alpha\dot{\alpha}} \quad , \quad p^\mu = \frac{1}{2} \bar{\sigma}^{\mu\alpha\dot{\alpha}} p^{\dot{\alpha}\alpha} \quad (2.25)$$

which provide an easy conversion from the 4 vector representation to the $(\frac{1}{2}, \frac{1}{2})$ Lorentz representation and vice versa. In the case of lightlike momenta like massless gauge theories, we have $\det(p^{\dot{\alpha}\alpha}) = 0$, which is a very important constraint that is key to many simplifications to come.

It's pretty obvious that the 2×2 matrices can be written as an outer product :

$$p_{\alpha\dot{\alpha}} = -\lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \quad , \quad p^{\dot{\alpha}\alpha} = -\tilde{\lambda}^{\dot{\alpha}} \lambda^\alpha \quad (2.26)$$

since the determinant is nil, the product of two massless vectors $p_{\alpha\dot{\alpha}} = -\lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ and

$q_{\alpha\dot{\alpha}} = -\chi_{\alpha}\tilde{\chi}^{\dot{\alpha}}$, gives :

$$\begin{aligned}
p \cdot q &= \left(\frac{1}{2} \sigma^{\mu\dot{\alpha}\alpha} p_{\alpha\dot{\alpha}} \right) \left(\frac{1}{2} \sigma_{\mu}^{\dot{\beta}\beta} q_{\beta\dot{\beta}} \right) \\
&= \frac{1}{4} \sigma^{\mu\dot{\alpha}\alpha} \sigma_{\mu}^{\dot{\beta}\beta} p_{\alpha\dot{\alpha}} q_{\beta\dot{\beta}} \\
&= -\frac{1}{2} \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon^{\alpha\beta} \lambda_{\alpha} \tilde{\lambda}^{\dot{\alpha}} \chi_{\beta} \tilde{\chi}^{\dot{\beta}} \\
&= -\frac{1}{2} \langle pq \rangle [qp]
\end{aligned} \tag{2.27}$$

in order to find this result we used (2.16). The same formula is derived via another method in appendix C.

Eventually one realizes that spinor inner product are just square roots of factor lorentzian inner products up to a phase factor. Thus we can just use the bracket notation for spinors related to a particular momentum be it boson or fermion, that is :

$$\langle pq \rangle = i\sqrt{2p \cdot q} e^{i\phi}, \quad [qp] = i\sqrt{2p \cdot q} e^{-i\phi} \tag{2.28}$$

and the bispinors take a more intuitive form in Dirac notation :

$$p^{\dot{\alpha}\alpha} = -|p\rangle[p|, \quad |p_{\alpha\dot{\alpha}} = -|p\rangle\langle p| \tag{2.29}$$

2.2.3 Dirac spinors

We can now proceed with the next step, which is expressing massless Dirac spinors in the Weyl basis. Dirac spinors are four component spinors, so we can express them in terms of spinor helicity notation as follows:

$$u_{-}(p) = v_{+}(p) = \begin{pmatrix} |p]_{\alpha} \\ 0 \end{pmatrix}, \quad \bar{u}_{+}(p) = \bar{v}_{-}(p) = \left([p|^{\alpha} \quad 0 \right) \tag{2.30}$$

$$u_{+}(p) = v_{-}(p) = \begin{pmatrix} 0 \\ |p\rangle^{\dot{\alpha}} \end{pmatrix}, \quad \bar{u}_{-}(p) = \bar{v}_{+}(p) = \left(0 \quad \langle p|_{\dot{\alpha}} \right)$$

where the subscript \pm indicates the helicity.

The Weyl representation is useful when dealing with massless particles. We defined the gamma matrices in the Weyl basis as follows:

$$\gamma_{\alpha\dot{\alpha}}^{\mu} = \begin{pmatrix} 0 & \sigma_{\alpha\dot{\alpha}}^{\mu} \\ \bar{\sigma}^{\mu\dot{\alpha}\alpha} & 0 \end{pmatrix} \tag{2.31}$$

so we can write :

$$\not{p} = p_\mu \gamma^\mu = \begin{pmatrix} 0 & p_\mu \sigma_{\alpha\dot{\alpha}}^\mu \\ p_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha} & 0 \end{pmatrix} = \begin{pmatrix} 0 & p_{\alpha\dot{\alpha}} \\ p^{\dot{\alpha}\alpha} & 0 \end{pmatrix} \quad (2.32)$$

The Dirac 4-spinors satisfy a spin completeness relation, for $m = 0$ we have :

$$\sum_{s=\pm} u_s(p) \bar{u}_s(p) = \sum_{s=\pm} v_s(p) \bar{v}_s(p) = -\not{p} \quad (2.33)$$

This relation can be expressed in SHF as follows:

$$-\not{p} = |p\rangle[p| + |p]\langle p| \quad (2.34)$$

For a process with n external particles, the momentum conservation $\sum_{i=1}^n p_i^\mu = 0$ encoded in spinor notation ³ is :

$$\sum_{q=1}^n \langle pq \rangle [qp] = 0 \quad (2.35)$$

The following relations are obtained by using Eq (2.16):

$$[p\gamma^\mu q] = \langle p\gamma^\mu q \rangle = 0 \quad (2.36)$$

and the Gordon identity

$$\langle p\gamma^\mu q \rangle = \langle p\sigma^\mu q \rangle = [q\bar{\sigma}^\mu p] = [q\gamma^\mu p] \quad (2.37)$$

Finally, Fierz rearrangement :

$$\langle p\gamma^\mu q \rangle \langle r\gamma_\mu s \rangle = 2\langle pr \rangle [qs]. \quad (2.38)$$

The difference between manipulating heavy Dirac spinors and Weyl spinors is clearly visible now. The most important advantage of this method is that we do not need to find explicit representations for the angle and square spinors; we can simply work abstractly with $|p\rangle$ and $|p]$ and later relate the results to the momentum vectors, these twistors are the pillars of what we call the "spinor helicity formalism".

2.2.4 Polarization vectors

When dealing with scattering processes containing bosonic external lines, there's always polarization vectors showing up in the amplitude formula. So the idea remains the same which is writing the polarization vectors in terms of helicity spinor products.

³consider all particles outgoing.

As seen in [21] the polarization vectors ϵ_λ^μ can be written in spinor helicity notation as follows:

$$\epsilon_+^\mu(k, q) = -\frac{\langle q\gamma^\mu k \rangle}{\sqrt{2}\langle qk \rangle}, \quad \epsilon_-^\mu(k, q) = -\frac{\langle k\gamma^\mu q \rangle}{\sqrt{2}[qk]} \quad (2.39)$$

In the two previous expressions, q denotes an arbitrary massless reference momentum. For every polarization vector in an amplitude, an arbitrary massless reference momentum can be chosen, and must not be aligned with k^μ -that is $q \cdot k \neq 0$ -. The two polarization vectors are related by complex conjugation:

$$(\epsilon_\pm^\mu)^* = -\epsilon_\mp^\mu \quad (2.40)$$

The square of the amplitudes we calculate must be in accord with results found by other methods, for this to be possible the new polarization vectors must satisfy a few properties such as normalization and completeness relation. Below we're going to demonstrate the completeness relation, the rest of the properties will be mentioned in Appendix C.

The polarization vectors obey the standard completeness relation [22]

$$\begin{aligned} \sum_{\lambda=\pm} \epsilon_\lambda^\mu(k, q) \epsilon_\lambda^\nu(k, q)^* &= \epsilon_+^\mu(k, q) \epsilon_+^\nu(k, q)^* + \epsilon_-^\mu(k, q) \epsilon_-^\nu(k, q)^* \\ &= -\frac{\langle q\gamma^\mu k \rangle \langle k\gamma^\nu q \rangle}{2\langle qk \rangle \langle qk \rangle^*} - \frac{\langle k\gamma^\mu q \rangle \langle q\gamma^\nu k \rangle}{2[qk][qk]^*} \\ &= -\frac{\langle q\gamma^\mu k \rangle \langle k\gamma^\nu q \rangle}{2\langle qk \rangle [kq]} - \frac{\langle k\gamma^\mu q \rangle \langle q\gamma^\nu k \rangle}{2[qk] \langle kq \rangle} \\ &= \frac{1}{4kq} (\langle q\gamma^\mu k \rangle \langle k\gamma^\nu q \rangle + \langle k\gamma^\mu q \rangle \langle q\gamma^\nu k \rangle) \\ &= \frac{1}{4kq} [\text{Tr}(\gamma^\mu \not{k} \gamma^\nu \not{q}) + \text{Tr}(\gamma^\mu \not{q} \gamma^\nu \not{k})] \\ &= \frac{1}{4kq} \text{Tr}(\gamma^\mu \not{q} \gamma^\nu \not{k}) = -g_{\mu\nu} + \frac{q^\mu k^\nu + q^\nu k^\mu}{kq} \end{aligned} \quad (2.41)$$

where (2.27), (C.13) and (A.8) were used.

The slashed polarization vectors can also be written in terms of spinor products as follows:

$$\not{\epsilon}_+(k, q) = \frac{\sqrt{2}}{\langle qk \rangle} (|k\rangle \langle q| + |q\rangle \langle k|), \quad \not{\epsilon}_-(k, q) = \frac{\sqrt{2}}{[qk]} (|k\rangle [q| + |q\rangle [k|]) \quad (2.42)$$

2.3 Massive case

However if one wants to approach reality, then massive initial state processes must be taken into account, especially if we are to study the EW SM and QCD with massive quarks. In this section we're going to build massive helicity spinors [23] and use them to derive the amplitudes the same way we did in the massless case.

2.3.1 Light cone decomposition (LCD)

The idea that comes to mind is simple, since the SHF is built in a way to deal with massless fields then we must build the massive fields from massless building blocks. So which fields can be used as building blocks and are indeed massless? the answer is Weyl spinors.

The technique used for the construction of the massive fields is called Light Cone Decomposition (LCD) [24] we start off by writing the lightlike momentum r^μ as: ($r^\mu = p^\mu - \alpha q^\mu$). Now r^μ being lightlike means by definition that $r^2 = 0$, so we have:

$$\begin{aligned} 0 &= (p^\mu - \alpha q^\mu)(p_\mu - \alpha q_\mu) = p^2 - 2\alpha p_\mu q^\mu \\ \implies \alpha &= \frac{p^2}{2p \cdot q} \end{aligned} \quad (2.43)$$

where $q^2 = 0$ and $p^2 = -m^2$. Thus the massive 4-momentum is given by :

$$p^\mu = r^\mu + \frac{p^2 q^\mu}{2r \cdot q} \quad (2.44)$$

2.3.2 Massive Dirac spinors

Let's consider the positive and negative energy solutions of the Dirac equation. Keep in mind we're writing these solutions in the form of 2 two-dimensional Weyl spinors, that is ⁴

$$u_s = \begin{pmatrix} \lambda_a \\ \tilde{\chi}^{\dot{a}} \end{pmatrix} \quad (2.45)$$

The Dirac equation is explicitly written as :

$$(\not{p} + m) u_s(p) = \begin{pmatrix} m & p_{\alpha\dot{\alpha}} \\ p^{\dot{\alpha}\alpha} & m \end{pmatrix} \begin{pmatrix} \lambda_a \\ \tilde{\chi}^{\dot{a}} \end{pmatrix} = 0 \quad (2.46)$$

⁴ $s = \pm$

which yields the following system :

$$\begin{aligned} p_{\alpha\dot{\alpha}}\tilde{\chi}^{\dot{\alpha}} + m\lambda_a &= 0 \\ p^{\dot{\alpha}\alpha}\lambda_a + m\tilde{\chi}^{\dot{\alpha}} &= 0. \end{aligned} \quad (2.47)$$

Let's apply the LCD to express the massive bispinors $p_{\alpha\dot{\alpha}}$ and $p^{\dot{\alpha}\alpha}$ in terms of massless bispinors $(r_{\alpha\dot{\alpha}}, q_{\alpha\dot{\alpha}})$:

$$\begin{aligned} p_{\alpha\dot{\alpha}} &= r_{\alpha\dot{\alpha}} + \omega q_{\alpha\dot{\alpha}} \\ p^{\dot{\alpha}\alpha} &= r^{\dot{\alpha}\alpha} + \omega q^{\dot{\alpha}\alpha} \end{aligned} \quad (2.48)$$

where $\omega = \frac{p^2}{2r \cdot q} = -\frac{m^2}{2r \cdot q}$ and using (2.29) we get :

$$\begin{aligned} p_{a\dot{a}} &= -|r]_a \langle r|_{\dot{a}} - \omega |q]_a \langle q|_{\dot{a}} \\ p^{\dot{a}a} &= -|r\rangle^{\dot{a}} [r|^a - \omega |q\rangle^{\dot{a}} [q|^a \end{aligned} \quad (2.49)$$

inserting (2.49) in (2.47) we get :

$$\begin{aligned} (|r]_a \langle r|_{\dot{a}} + \omega |q]_a \langle q|_{\dot{a}})\tilde{\chi}^{\dot{a}} &= m\lambda_a \\ (|r\rangle^{\dot{a}} [r|^a + \omega |q\rangle^{\dot{a}} [q|^a)\lambda_a &= m\tilde{\chi}^{\dot{a}} \end{aligned} \quad (2.50)$$

The solutions for the two last equations are as follows:

$$u_+ = \begin{pmatrix} \frac{m}{[rq]} |q]_a \\ |r\rangle^{\dot{a}} \end{pmatrix}, \quad u_- = \begin{pmatrix} |r]_a \\ \frac{m}{\langle rq\rangle} |q\rangle^{\dot{a}} \end{pmatrix} \quad (2.51)$$

with $\lambda_a = \frac{m}{[rq]} |q]_a$, $\tilde{\chi}^{\dot{a}} = |r\rangle^{\dot{a}}$ for the positive energy solution u_+ and $\lambda_a = |r]_a$, $\tilde{\chi}^{\dot{a}} = \frac{m}{\langle rq\rangle} |q\rangle^{\dot{a}}$ for the negative energy solution u_- .

The equation (2.46) admits 2 solution, the same goes for anti-spinors v_{\pm} and \bar{u}_{\pm} and \bar{v}_{\pm} . So we have a total of 8 solutions given by :

$$\begin{aligned} u_-(p) &= |r] + \frac{m}{\langle rq\rangle} |q\rangle, & u_+(p) &= \frac{m}{[rq]} |q] + |r\rangle \\ v_+(p) &= |r] - \frac{m}{\langle rq\rangle} |q\rangle, & v_-(p) &= -\frac{m}{[rq]} |q] + |r\rangle \\ \bar{u}_-(p) &= \frac{m}{[qr]} [q| + \langle r|, & \bar{u}_+(p) &= [r| + \frac{m}{\langle qr\rangle} \langle q| \\ \bar{v}_+(p) &= -\frac{m}{[qr]} [q| + \langle r|, & \bar{v}_-(p) &= [r| - \frac{m}{\langle qr\rangle} \langle q| \end{aligned} \quad (2.52)$$

2.3.3 Massive gauge bosons

The final brick in this particular wall is the massive polarization vectors, the only thing we need to do now is to write them down in terms of spinor notation, and proceed to applications

$$\epsilon_+^\mu(k) = \frac{\langle q\gamma^\mu r \rangle}{\sqrt{2}\langle rq \rangle}, \quad \epsilon_-^\mu(k) = \frac{\langle r\gamma^\mu q \rangle}{\sqrt{2}[qr]} \quad (2.53)$$

and the longitudinal mode is given via LCD by

$$\epsilon_0^\mu(k) = \frac{1}{m}r^\mu + \frac{m}{2r \cdot q}q^\mu \quad (2.54)$$

encoding the last equation in spinor notation, the longitudinal mode becomes:

$$\epsilon_0^\mu(k) = \frac{1}{2m}\langle r\gamma^\mu r \rangle + \frac{m}{4r \cdot q}\langle q\gamma^\mu q \rangle \quad (2.55)$$

with:

$$\langle r\gamma^\mu r \rangle = 2r^\mu, \quad \langle q\gamma^\mu q \rangle = 2q^\mu \quad (2.56)$$

Chapter 3

Application

"It's nevertheless
an endless mess
before one can asses
nature's inventiveness."

R.Gastmans

Now that we saw how to construct all the physical quantities in terms of SHF, it is time to see how to use the SHF to calculate the square of the amplitudes in a few examples.

In this chapter we're going to start with a simple two body Higgs decay, we move on then to treat Bhabha scattering, after that we study a case containing polarization vectors and the example we chose is an electron-positron annihilation process. We then study the case of a slightly complicated process occurring at the LHC, which is the Higgs production process. Finally we study a case of gluon scattering process in QCD.

3.1 2-body Higgs decay

For the sake of keeping calculations simple we start off with a simple two-body Higgs decay

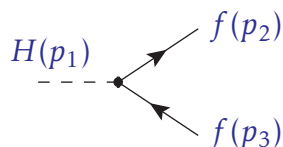


Figure 3.1: Feynman diagram for $h(p_1) \rightarrow f(p_2)\bar{f}(p_3)$

Where f and \bar{f} are a fermion and an anti fermion.

The amplitude for this process can be derived from Feynman rules and is given by :

$$M_{\lambda_2\lambda_3} = \frac{m_f}{v} \bar{u}_{\lambda_2}(p_2)v_{\lambda_3}(p_3) \quad (3.1)$$

where $p_1^2 = -m_h^2$, $p_2^2 = p_3^2 = -m_f^2$ and λ is an index which represents the helicity of the particles. In the massive case. We shall use simultaneous light cone decomposition (**SLCD**) to express the massive momenta p_2 and p_3 in terms of massless momenta (r_2, q_2, r_3, q_3) . The massive momenta take the form: :

$$\begin{aligned} p_2 &= r_2 - \frac{m_f^2}{2r_2q_2}q_2 \\ p_3 &= r_3 - \frac{m_f^2}{2r_3q_3}q_3 \end{aligned} \quad (3.2)$$

We can choose $r_2 = q_3$ and $r_3 = q_2$. We have one Feynman diagram with 2 fermionic external lines, each particle has two helicity states $\lambda = \pm$. It appears we would need to compute 4 amplitude M_{+-} , M_{-+} , M_{--} and M_{++} , but using **SLCD** to fix massless momenta reduces the number of non vanishing amplitudes. In this case M_{-+} and M_{+-} vanish. The nonzero amplitudes are M_{--} and M_{++} while the first is linked to the second by complex conjugation, that is :

$$M_{++} = M_{--}^* \quad (3.3)$$

So we have only one amplitude to compute:

$$\begin{aligned} M_{--} &= \frac{m_f}{v} \bar{u}_-(2)v_-(3) = \frac{m_f}{v} \left(\frac{m_f}{[q_2q_3]}[q_2] + \langle q_3 | \right) \left(-\frac{m_f}{[q_2q_3]}|q_3\rangle + |q_2\rangle \right) \\ &= \frac{m_f}{v} \left(-\frac{m_f^2}{[q_2q_3][q_2q_3]}[q_2q_3] + \langle q_3q_2 \rangle \right) \\ &= \frac{m_f}{v[q_2q_3]}(s_{q_2q_3} - m_f^2) \end{aligned} \quad (3.4)$$

as we stated in chapter 2 the $\langle pq \rangle$ -brackets vanish. And for the second non vanishing amplitude we use complex conjugation to obtain :

$$M_{++} = M_{--}^* = \frac{m_f}{v\langle q_3q_2 \rangle}(s_{q_2q_3} - m_f^2) \quad (3.5)$$

Finally the averaged squared amplitude is :

$$\langle |M|^2 \rangle = 2|M_{--}|^2 = 2 \frac{m_f^2}{v^2 s_{q_2 q_3}} (s_{q_2 q_3} - m_f^2)^2 \quad (3.6)$$

The mathematica code below contains the code for calculating the amplitude for the two body Higgs decay process via the SpinorsExtras package [25] which coincides with the result we obtained.

```

1 << Spinors '
2 Needs["SpinorsExtras"]
3 DeclareLVector[p1, p2, p3]
4 MP[p1, p1] ^= -mH^2;
5 MP[p2, p2] ^= -mf^2;
6 MP[p3, p3] ^= -mf^2;
7 (*MP[r1, r1]^=MP[r2, r2]^=0;*)
8 $Assumptions = mH > 0 && mf > 0
9 A[{p3_, +1}, {p2_, -1}] := -mf/v Spba[SpM[p3, 1], SpM[p2, +1]]
10 A[{p3_, -1}, {p2_, +1}] := -mf/v Spab[SpM[p3, 1], SpM[p2, +1]]
11 A[{p3_, -1}, {p2_, -1}] := -mf/v Spaa[SpM[p3, 1], SpM[p2, +1]]
12 A[{p3_, +1}, {p2_, +1}] := -mf/v Spbb[SpM[p3, 1], SpM[p2, +1]]
13 diags = {A[{p3, +1}, {p2, -1}], A[{p3, -1}, {p2, +1}],
14         A[{p3, -1}, {p2, -1}], A[{p3, +1}, {p2, +1}]}
15 diagsLCD1 = LightConeDecompose[diags, p2] // Refine
16 subs = {SpAssoc[p2] -> SpRef[p3], SpAssoc[p3] -> SpRef[p2]}
17 diagsLCD = % /. subs // Simplify
18 subs2 = {SpRef[p3] -> Sp[3], SpRef[p2] -> Sp[2]}
19 fin = diagsLCD /. subs2 // Simplify // Together // ConvertSpinorsToS

```

This code was inserted in order to provide an example on how to use the SpinorsExtras extension of S@M package [26] in order to calculate scattering amplitudes.

3.2 Bhabha scattering

It's the process $e^-e^+ \rightarrow e^-e^+$ which yields two diagrams [27]

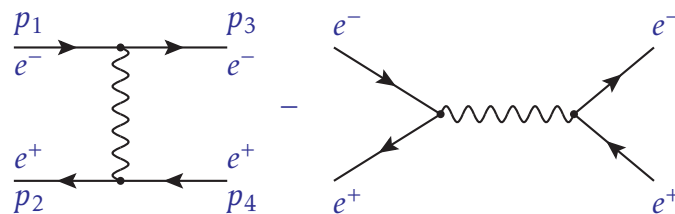


Figure 3.2: Feynman diagrams for Bhabha scattering

corresponding to the amplitudes :

$$\begin{aligned} M_1(\lambda_1 \lambda_2; \lambda_3 \lambda_4) &= \frac{e^2}{t} \bar{u}_{\lambda_3}(p_3) \gamma^\mu u_{\lambda_1}(p_1) \bar{v}_{\lambda_2}(p_2) \gamma_\mu v_{\lambda_4}(p_4) \\ M_2(\lambda_1 \lambda_2; \lambda_3 \lambda_4) &= -\frac{e^2}{s} \bar{v}_{\lambda_2}(p_2) \gamma^\mu u_{\lambda_1}(p_1) \bar{u}_{\lambda_3}(p_3) \gamma_\mu v_{\lambda_4}(p_4) \end{aligned} \quad (3.7)$$

where $s = -(p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$ are the Mandelstam invariants and the λ_i represent the fermion helicities.

We have 2 Feynman diagrams each with 4 external lines, each particle has two helicity states $\lambda = \pm$. The first thing to notice is that the amplitude is zero if two external fermion lines that meet at a vertex have the same helicity. This is because we get zero if we sandwich the product of an odd number of gamma matrices between two twistors of the same helicity, explicitly $\langle p \gamma^\mu q \rangle = [p \gamma^\mu q] = 0$. We will get a nonzero result for the tree-level amplitude only if two of the helicities are positive, and two are negative. This means from 16 possible helicity amplitudes $M(\lambda_1 \lambda_2; \lambda_3 \lambda_4)$ only 6 give a non zero tree level amplitude : $M(++; ++), M(+--; +-), M(+--; -+), M(--; --), M(-+; -+), M(-+; +-)$. Moreover, the first three are linked to the last three by complex conjugation, thus :

$$M(++; ++)^* = M(--; --), \quad M(+--; +-)^* = M(-+; -+), \quad M(+--; -+)^* = M(-+; +-)$$

So we have only three amplitudes to compute.

We use SLCD to express the massive momenta p_1, p_2, p_3 and p_4 in terms of massless momenta r_i and q_i ($i = 1, \dots, 4$) :

$$p_i = r_i - \frac{m^2}{2r_i q_i} q_i \quad (3.8)$$

with $p_i^2 = -m^2$ ($i = 1, \dots, 4$) and $r_i^2 = q_i^2 = 0$. Using SLCD to fix the massless momenta will be crucial to reduce the number of helicity amplitudes, let us initiate with M_1 and set:

$$r_1 = q_3 \quad r_3 = q_1 \quad r_2 = q_4 \quad r_4 = q_2$$

So the nonzero helicity amplitudes for M_1 are $M(++; ++), M(+--; +-), M(--; --)$ and $M(-+; -+)$

$$\begin{aligned} M_1(++; ++) &= \frac{e^2}{t} \bar{u}_+(p_3) \gamma^\mu u_+(p_1) \bar{v}_+(p_2) \gamma_\mu v_+(p_4) \\ &= \frac{e^2}{t} \left[\left([r_3] + \frac{m}{\langle q_3 r_3 \rangle} \langle q_3 | \right) \gamma^\mu \left(\left(\frac{m}{[r_1 q_1]} |q_1 \rangle + |r_1 \rangle \right) \left(-\frac{m}{[q_2 r_2]} [q_2] + \langle r_2 | \right) \gamma_\mu \left(|r_4 \rangle - \frac{m}{\langle r_4 q_4 \rangle} |q_4 \rangle \right) \right] \\ &= \frac{e^2}{t} \left[\left([q_1] + \frac{m}{\langle q_3 q_1 \rangle} \langle q_3 | \right) \gamma^\mu \left(\frac{m}{[q_3 q_1]} |q_1 \rangle + |q_3 \rangle \right) \left(-\frac{m}{[q_2 q_4]} [q_2] + \langle q_4 | \right) \gamma_\mu \left(|q_2 \rangle - \frac{m}{\langle q_2 q_4 \rangle} |q_4 \rangle \right) \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{e^2}{t} \left[\left([q_1 \gamma^\mu q_3] + \frac{m^2}{\langle q_3 q_1 \rangle [q_3 q_1]} \langle q_3 \gamma^\mu q_1 \rangle \right) \left(\frac{m^2}{[q_2 q_4] \langle q_2 q_4 \rangle} [q_2 \gamma_\mu q_4] + \langle q_4 \gamma_\mu q_2 \rangle \right) \right] \\
&= \frac{e^2}{t} \langle q_3 \gamma^\mu q_1 \rangle \langle q_4 \gamma^\mu q_2 \rangle \left[1 - \frac{m^2}{s_{24}} + \frac{m^4}{s_{13} s_{24}} - \frac{m^2}{s_{13}} \right]
\end{aligned} \tag{3.9}$$

using Fierz rearrangement we find :

$$M_1(++;++) = 2 \frac{e^2}{t} \langle q_3 q_4 \rangle [q_1 q_2] \left[1 + \frac{m^4 - (s_{13} + s_{24}) m^2}{s_{13} s_{24}} \right] \tag{3.10}$$

Using relations (2.36), (2.38), we proceed in the same way and we get :

$$M_1(+--;+-) = 2 \frac{e^2}{t} \langle q_3 q_2 \rangle [q_1 q_4] \left[1 + \frac{m^4 - (s_{13} + s_{24}) m^2}{s_{13} s_{24}} \right] \tag{3.11}$$

$$M_1(-+;--+) = 2 \frac{e^2}{t} \langle q_1 q_4 \rangle [q_3 q_2] \left[1 + \frac{m^4 - (s_{13} + s_{24}) m^2}{s_{13} s_{24}} \right] \tag{3.12}$$

$$M_1(--;---) = 2 \frac{e^2}{t} \langle q_1 q_2 \rangle [q_3 q_4] \left[1 + \frac{m^4 - (s_{13} + s_{24}) m^2}{s_{13} s_{24}} \right] \tag{3.13}$$

The SLCD procedure for M_2 diagram is set by imposing :

$$r_1 = q_2 \quad r_3 = q_4 \quad r_2 = q_1 \quad r_4 = q_3$$

A similar development to the one above easily yields

$$\begin{aligned}
M_2(+--;+-) &= -\frac{e^2}{s} \bar{v}_-(p_2) \gamma^\mu u_+(p_1) \bar{u}_+(p_3) \gamma_\mu v_-(p_4) \\
&= -2 \frac{e^2}{s} \langle q_2 q_3 \rangle [q_1 q_4] \left[1 + \frac{m^4 + (s_{12} + s_{34}) m^2}{s_{12} s_{34}} \right]
\end{aligned} \tag{3.14}$$

$$M_2(-+;--+) = -2 \frac{e^2}{s} \langle q_1 q_4 \rangle [q_2 q_3] \left[1 + \frac{m^4 + (s_{12} + s_{34}) m^2}{s_{12} s_{34}} \right] \tag{3.15}$$

$$M_2(+--;+-) = -2 \frac{e^2}{s} \langle q_2 q_4 \rangle [q_1 q_3] \left[1 + \frac{m^4 + (s_{12} + s_{34}) m^2}{s_{12} s_{34}} \right] \tag{3.16}$$

$$M_2(-+;--+) = -2 \frac{e^2}{s} \langle q_1 q_3 \rangle [q_2 q_4] \left[1 + \frac{m^4 + (s_{12} + s_{34}) m^2}{s_{12} s_{34}} \right] \tag{3.17}$$

taking into account $m \rightarrow 0$ and $q \rightarrow p$ we get :

$$\begin{aligned}
M(+--;+-) &= M_1(+--;+-) + M_2(+--;+-) \\
&= 2e^2 \left[\frac{\langle 32 \rangle [14]}{t} - \frac{\langle 23 \rangle [14]}{s} \right]
\end{aligned} \tag{3.18}$$

$$M(++;++) = M_1(++;++) = 2\frac{e^2}{t}\langle 34 \rangle [12] \quad (3.19)$$

$$M(+--; --) = M_2(+--; --) = -2\frac{e^2}{s}\langle 13 \rangle [24] \quad (3.20)$$

The rest of the amplitudes is obtained by complex conjugation :

$$\begin{aligned} M(-+; --) &= M(+--; +-)^* \\ M(--; --) &= M(++; ++)^* \\ M(-+; +-) &= M(+--; --)^* \end{aligned} \quad (3.21)$$

The sum of the squares of the non zero amplitudes yields :

$$\begin{aligned} \langle |M|^2 \rangle &= \frac{1}{4} \left[2|M(+--; +-)|^2 + 2|M(++; ++)|^2 + 2|M(+--; --)|^2 \right] \\ &= 2e^4 \left[\frac{s_{23}s_{14} + s_{12}s_{34}}{t^2} + \frac{s_{23}s_{14} + s_{24}s_{13}}{s^2} + \frac{s_{23}s_{14}}{st} \right] \end{aligned}$$

inducing Mandelstam variables :

$$s_{23} = -2p_2p_3 = s_{14} = -u = t + s, \quad s_{12} = -2p_1p_2 = s_{34} = s, \quad s_{13} = -2p_1p_3 = s_{24} = t$$

Our final result found is given by the following formula :

$$\langle |M|^2 \rangle = 2e^4 \left[\frac{(t+s)^2 + s^2}{t^2} + \frac{(t+s)^2 + t^2}{s^2} + 2\frac{(t+s)^2}{st} \right] \quad (3.22)$$

We will re-derive this result in Appendix B by using the trace evaluation method.

3.3 The process $e^-e^+ \rightarrow \gamma\gamma$

In the previous section we only considered gauge bosons in the internal lines. What happens if we have gauge bosons in the external lines? In this section we will show how to use the SHF in the case of amplitudes containing external line gauge bosons :

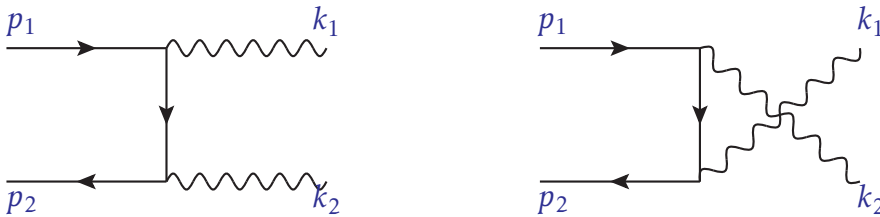


Figure 3.3: Feynman diagrams for $e^-e^+ \rightarrow \gamma\gamma$

In this case the corresponding amplitudes are given by:

$$M_1 = \frac{e^2}{t} \bar{v}(p_2) \epsilon^\mu(k_2) \gamma_\mu(\not{p}_1 - \not{k}_1) \epsilon^\mu(k_1) \gamma_\mu u(p_1) \quad (3.23)$$

$$M_2 = \frac{e^2}{u} \bar{v}(p_2) \epsilon^\mu(k_1) \gamma_\mu(\not{p}_1 - \not{k}_2) \epsilon^\mu(k_2) \gamma_\mu u(p_1)$$

Respectively for M_1 we choose $q_1 = k_1$ and $q_2 = k_2$ and for M_2 the reversed choice, that is, $q_1 = k_2$ and $q_2 = k_1$

We notice from the beginning that there is a simplification to be made, the $\epsilon^\mu(k_i, p_i) \cdot p_i$ ($i, j = 1; 2$) vanishes because of our choice of reference vectors. Using the compact notation $s(p_i, p_j) = \langle p_i p_j \rangle$, the non vanishing helicity amplitudes are as follows :

$$\begin{aligned} M_1(++; --) &= \frac{e^2}{t} \left(-\frac{m}{[k_2 r_2]} [k_2] + \langle r_2 | \right) \left(\frac{\sqrt{2}}{[p_2 k_2]} (|k_2\rangle [p_2] + |p_2\rangle \langle k_2|) (|k_1\rangle [k_1] + |k_1\rangle \langle k_1|) \right) \\ &\quad \left(\frac{\sqrt{2}}{[p_1 k_1]} (|k_1\rangle [p_1] + |p_1\rangle \langle k_1|) \right) \left(\frac{m}{[r_1 k_1]} |k_1\rangle + |r_1\rangle \right) \\ &= -2m \frac{e^2}{t} \frac{[k_2 p_2] \langle k_2 k_1 \rangle [k_1 p_1] \langle k_1 r_1 \rangle}{[k_2 r_2] [p_2 k_2] [p_1 k_1]} \\ &= -2m \frac{e^2}{t} \frac{\langle k_2 k_1 \rangle \langle k_1 r_1 \rangle}{[k_2 r_2]} \end{aligned}$$

$$M(++; --) = M_1(++; --) + M_2(++; --) = -2me^2 \left[\frac{s(k_2, k_1) s(k_1 r_1)}{s^*(r_2, k_2) t} + \frac{s(k_1, k_2) s(k_2 r_2)}{s^*(r_1, k_1) u} \right]$$

$$M(+--; --) = M_1(+--; --) + M_2(+--; --) = -2e^2 s^*(p_2, r_2) s(k_1, k_2) \left[\frac{s(k_2, r_1)}{s^*(k_1, p_2) u} - \frac{s(k_1, r_1)}{s^*(k_2, p_2) t} \right]$$

$$M(+--; ++) = M_1(+--; ++) = -2 \frac{e^2}{t} \frac{s^*(k_2, r_2) s(p_2, k_1) s(k_1, r_1)}{s(p_2, k_2)}$$

$$M(+--; ++) = M_2(+--; ++) = 2 \frac{e^2}{u} \frac{s^*(k_1, r_1) s(p_2, k_2) s^*(p_1, k_2) s(k_2, r_1)}{s(p_2, k_1) s^*(k_1, p_1)}$$

As usual we use the advantage that the helicity amplitudes are complex conjugate of each other :

$$\begin{aligned} M(--;++) &= M(++,- -)^* & , & & M(-+;++) &= M(+,-,-)^* \\ M(-+;+-) &= M(+,-,-)^* & , & & M(-+;-+) &= M(+,-,+)^* \end{aligned} \quad (3.24)$$

by taking into account the limit $m \rightarrow 0$ and $r_i \rightarrow p_i$ with $i = 1, 2$

$$\begin{aligned} M(+,-,-) &= 2e^2 \frac{s(p_2, k_1) s^*(k_2, p_2)}{t}, & M(-+,+-) &= 2e^2 \frac{s(p_2, k_2) s^*(k_1, p_2)}{t} \\ M(+,-,+ -) &= 2e^2 \frac{s(p_2, k_2) s^*(k_1, p_2)}{u}, & M(-+,-+) &= 2e^2 \frac{s(p_2, k_1) s^*(k_2, p_2)}{u} \end{aligned} \quad (3.25)$$

the sum over the square of the amplitudes yields :

$$\begin{aligned} \sum_{\lambda_i} |M(\lambda_1 \lambda_2; \lambda_3 \lambda_4)|^2 &= 2|M(+,-,-)|^2 + 2|M(+,-,+ -)|^2 \\ &= 2 \left[4e^2 \frac{s_{p_2 k_1} s_{p_2 k_2}}{t^2} + 4e^2 \frac{s_{p_2 k_2} s_{p_2 k_1}}{u^2} \right] \\ &= 8e^2 \frac{u^2 + t^2}{ut} \end{aligned} \quad (3.26)$$

with:

$$s_{p_2 k_2} = -2p_2 k_2 = -2p_1 k_1 = t \quad , \quad s_{p_2 k_1} = -2p_2 k_1 = -2p_1 k_2 = u \quad (3.27)$$

3.4 The Higgs production process

Among the many processes for the Higgs productions we're going to study the example $e^-(p_1)e^+(p_2) \rightarrow Z(q_1)Z(q_2)H(k)$. We start by noticing that, in the limit of neglecting the electron mass we have the following Feynman four diagrams for this process at tree level [28] :

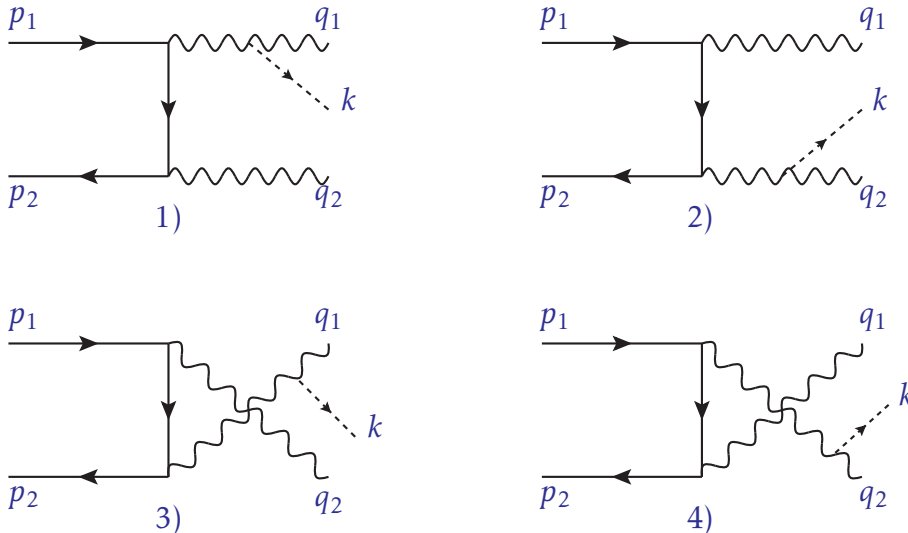


Figure 3.4: Tree level Feynman diagrams for $e^-e^+ \rightarrow ZZH$

These diagrams correspond to the following amplitudes:

$$M_i = \left(\frac{g}{\cos(\theta_W)} \right) \bar{v}(p_2) \Gamma_i u(p_1) \quad (3.28)$$

where θ_W is Weinberg's angle and :

$$\begin{aligned} \Gamma_1 &= \epsilon_\mu(q_1) \epsilon_\nu(q_2) \gamma^\nu (g_V - g_A \gamma_5) (-\not{p}_2 + \not{q}_2) \gamma^\mu (g_V - g_A \gamma_5) \frac{1}{D_e(1) D_Z(1)} \\ \Gamma_2 &= \epsilon_\mu(q_1) \epsilon_\nu(q_2) \gamma^\nu (g_V - g_A \gamma_5) (\not{p}_1 - \not{q}_1) \gamma^\mu (g_V - g_A \gamma_5) \frac{1}{D_e(2) D_Z(2)} \\ \Gamma_3 &= \epsilon_\mu(q_1) \epsilon_\mu(q_2) \gamma^\mu (g_V - g_A \gamma_5) (\not{p}_1 - \not{q}_2) \gamma^\nu (g_V - g_A \gamma_5) \frac{1}{D_e(3) D_Z(3)} \\ \Gamma_4 &= \epsilon_\mu(q_1) \epsilon_\mu(q_2) \gamma^\mu (g_V - g_A \gamma_5) (-\not{p}_2 + \not{q}_1) \gamma^\nu (g_V - g_A \gamma_5) \frac{1}{D_e(4) D_Z(4)} \end{aligned} \quad (3.29)$$

where the denominators of the propagators are :

$$D_e(1) = (-p_2 + q_2)^2 \quad ; \quad D_e(2) = (p_1 - q_1)^2 \quad (3.30)$$

$$D_e(3) = (p_1 - q_2)^2 \quad ; \quad D_e(4) = (-p_2 + q_1)^2 \quad (3.31)$$

$$D_Z(1) = D_Z(3) = (q_1 + k)^2 - M_Z^2 + iM_Z \Gamma_Z \quad (3.32)$$

$$D_Z(2) = D_Z(4) = (q_2 + k)^2 - M_Z^2 + iM_Z \Gamma_Z \quad (3.33)$$

while g_V and g_A are the vectorial and axial coupling constant respectively.

In order to avoid carrying the propagators which do not interfere in this computation we replace them by the constant C_i :

$$M_1(\lambda_1, \lambda_2; \sigma_1, \sigma_2) = C_1 \bar{v}_{\lambda_2}(p_2) \not{\epsilon}_{\sigma_2}(q_2) (g_- \gamma_- + g_+ \gamma_+) (-\not{p}_2 + \not{q}_2) \not{\epsilon}_{\sigma_1}(q_1) (g_- \gamma_- + g_+ \gamma_+) u_{\lambda_1}(p_1)$$

$$M_2(\lambda_1, \lambda_2; \sigma_1, \sigma_2) = C_2 \bar{v}_{\lambda_2}(p_2) \not{\epsilon}_{\sigma_2}(q_2) (g_- \gamma_- + g_+ \gamma_+) (\not{p}_1 - \not{q}_1) \not{\epsilon}_{\sigma_1}(q_1) (g_- \gamma_- + g_+ \gamma_+) u_{\lambda_1}(p_1)$$

$$M_3(\lambda_1, \lambda_2; \sigma_1, \sigma_2) = C_3 \bar{v}_{\lambda_2}(p_2) \not{\epsilon}_{\sigma_1}(q_1) (g_- \gamma_- + g_+ \gamma_+) (\not{p}_1 - \not{q}_2) \not{\epsilon}_{\sigma_2}(q_2) (g_- \gamma_- + g_+ \gamma_+) u_{\lambda_1}(p_1)$$

$$M_4(\lambda_1, \lambda_2; \sigma_1, \sigma_2) = C_4 \bar{v}_{\lambda_2}(p_2) \not{\epsilon}_{\sigma_1}(q_1) (g_- \gamma_- + g_+ \gamma_+) (-\not{p}_2 + \not{q}_1) \not{\epsilon}_{\sigma_2}(q_2) (g_- \gamma_- + g_+ \gamma_+) u_{\lambda_1}(p_1)$$

with :

$$C_i = \left(\frac{g}{\cos(\theta_W)} \right) \frac{M_Z}{D_e(i)D_z(i)}, \quad (i = 1, 2, 3, 4) \quad (3.34)$$

$$g_- = g_A + g_V, \quad g_+ = g_A - g_V.$$

We introduced in chapter 2 the powerful spinor helicity notation for massive gauge bosons; so we use LCD to express massive momenta q_1 and q_2 in terms of massless momenta. The massive momenta take the form:

$$q_1 = r_1 - \frac{M_Z^2}{2r_1 \cdot w_1} w_1, \quad q_2 = r_2 - \frac{M_Z^2}{2r_2 \cdot w_2} w_2 \quad (3.35)$$

We choose for M_1 $r_1 = p_1$ and $r_2 = p_2$, so the polarization vectors take the form :

$$\begin{aligned} \not{\epsilon}_+(q_j) &= \frac{\sqrt{2}}{\langle w_j p_j \rangle} \{ |p_j\rangle \langle w_j| + |w_j\rangle [p_j] \}, \quad \not{\epsilon}_-(q_j) = \frac{\sqrt{2}}{[p_j w_j]} \{ |w_j\rangle \langle p_j| + |p_j\rangle [w_j] \} \\ \not{\epsilon}_0(q_j) &= \frac{1}{M_Z} \not{p}_j + \frac{M_Z}{2r_j \cdot w_j} \not{w}_j, \quad (j = 1, 2). \end{aligned} \quad (3.36)$$

Note that for this special example:

$$\begin{aligned} [p_2 | \not{\epsilon}_+(q_2) = 0, \quad \langle p_2 | \not{\epsilon}_-(q_2) = 0 \\ \not{\epsilon}_+(q_1) | p_1 \rangle = 0, \quad \not{\epsilon}_-(q_1) | p_1 \rangle = 0 \\ \gamma_+ | p \rangle = | p \rangle, \quad \langle p | \gamma_+ = \langle p |, \quad \gamma_- | p \rangle = | p \rangle, \quad [p | \gamma_- = [p | \end{aligned} \quad (3.37)$$

Using SLCD to fix the massless momenta will be crucial to reduce for M_1 the number of Helicity Amplitudes (HAs), to only 4 non vanishing HAs. There are $M_1(-+, -+)$, $M_1(+-, +-)$, $M_1(-+, 0+)$ and $M_1(+-, 0-)$

$$\begin{aligned} M_1(-+, -+) &= 2C_1 g_-^2 M_Z^2 \frac{\langle p_1 w_1 \rangle}{\langle p_2 w_2 \rangle}, & M_1(-+, 0+) &= \frac{C_1 g_-^2 M_Z^3}{\sqrt{2}} \frac{\langle w_2 w_1 \rangle}{\langle p_1 w_1 \rangle \langle p_2 w_2 \rangle} \\ M_1(+-, +-) &= 2C_1 g_+^2 M_Z^2 \frac{[p_1 w_1]}{[p_2 w_2]}, & M_1(+-, 0-) &= \frac{C_1 g_+^2 M_Z^3}{\sqrt{2}} \frac{[w_1 w_2]}{[w_1 p_1][w_2 p_2]} \end{aligned} \quad (3.38)$$

the same choices taken in M_1 are repeated for M_2 while for M_3 and M_4 we take $r_1 = p_2$, $r_2 = p_1$,

so the only non vanishing amplitudes are :

$$\begin{aligned}
 M_2(-+, -+) &= 2C_2g_-^2M_Z^2 \frac{[p_2w_1]}{[p_1w_1]}, & M_2(+-, +0) &= -\frac{C_2g_+^2M_Z^3}{\sqrt{2}} \frac{\langle w_1w_2 \rangle}{\langle p_1w_1 \rangle \langle p_2w_2 \rangle} \\
 M_2(+-, +-) &= 2C_2g_+^2M_Z^2 \frac{\langle p_2w_1 \rangle}{\langle p_1w_1 \rangle}, & M_2(-+, -0) &= -\frac{C_2g_-^2M_Z^3}{\sqrt{2}} \frac{[w_2w_1]}{[w_1p_1][w_2p_2]}
 \end{aligned} \tag{3.39}$$

and

$$\begin{aligned}
 M_3(+-, -+) &= 2C_3g_+^2M_Z^2 \frac{\langle p_2w_2 \rangle [w_2p_2]}{\langle p_2w_1 \rangle [w_1p_2]}, & M_3(-+, 0-) &= -\frac{C_3g_-^2M_Z^3}{\sqrt{2}} \frac{[w_1w_2]}{[w_1p_2][w_2p_1]} \\
 M_3(-+, +-) &= 2C_3g_-^2M_Z^2 \frac{[p_2w_2] \langle w_2p_2 \rangle}{[p_2w_1] \langle w_1p_2 \rangle}, & M_3(+-, 0+) &= -\frac{C_3g_+^2M_Z^3}{\sqrt{2}} \frac{\langle w_2w_1 \rangle}{\langle p_2w_1 \rangle \langle p_1w_2 \rangle}
 \end{aligned} \tag{3.40}$$

thus

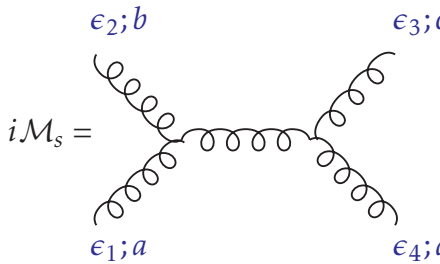
$$\begin{aligned}
 M_4(+-, -+) &= -2C_4g_+^2M_Z^2 \frac{[p_1w_1]}{[p_2w_1]}, & M_4(-+, +0) &= \frac{C_4g_-^2M_Z^3}{\sqrt{2}} \frac{\langle w_2w_1 \rangle}{\langle p_1w_2 \rangle \langle w_1p_2 \rangle} \\
 M_4(-+, +-) &= -2C_4g_-^2M_Z^2 \frac{\langle p_1w_1 \rangle}{\langle p_2w_1 \rangle}, & M_4(+-, -0) &= \frac{C_4g_+^2M_Z^3}{\sqrt{2}} \frac{[w_1w_2]}{[w_2p_1][p_2w_1]}
 \end{aligned} \tag{3.41}$$

Finally the square of the amplitudes

$$\begin{aligned}
 \langle |M|^2 \rangle &= \frac{1}{4} \sum_{\lambda_1 \lambda_2 \sigma_1 \sigma_2} |M(\lambda_1 \lambda_2; \sigma_1 \sigma_2)|^2 \\
 &= \frac{1}{4} (4|M(+-, +-)|^2 + 4|M(+-, -+)|^2 + 2|M(-+, 0+)|^2 \\
 &\quad + 2|M(-+, -0)|^2 + 2|M(-+, 0-)|^2 + 2|M(-+, +0)|^2)
 \end{aligned} \tag{3.42}$$

3.5 Two gluon final state scattering process

In this section we study the example of gluon scattering, the Feynman diagrams and the corresponding amplitudes [18] for $gg \rightarrow gg$ are as follows:



$$\begin{aligned}
 iM_s &= \frac{-ig_s^2}{s} f^{abe} f^{cde} \left[(\epsilon_1 \cdot \epsilon_2)(p_1 - p_2)^\mu + 2\epsilon_2^\mu(p_2 \cdot \epsilon_1) - 2\epsilon_1^\mu(p_1 \cdot \epsilon_2) \right] \\
 &\quad \times \left[(\epsilon_3 \cdot \epsilon_4)(p_3 - p_4)^\mu + 2\epsilon_4^\mu(p_4 \cdot \epsilon_3) - 2\epsilon_3^\mu(p_3 \cdot \epsilon_4) \right]
 \end{aligned}$$

MHV amplitude $M(--, ++)$, with other related by crossing symmetry.

We take all momenta incoming, so we have:

$$s = -(p_1 + p_2)^2 \quad , \quad t = -(p_1 + p_4)^2 \quad , \quad u = -(p_1 + p_3)^2.$$

We choose the reference momentum for $\epsilon^\mu(p_1, r_1)$ and $\epsilon^\mu(p_2, r_2)$ to be $r_1 = r_2 = p_4$ and the reference momentum for $\epsilon^\mu(p_3, r_3)$ and $\epsilon^\mu(p_4, r_4)$ to be p_1 . Then the only polarization contraction that does not vanish is $\epsilon(p_2, p_4) \cdot \epsilon(p_3, p_1)$, and $\epsilon(p_i, r) \cdot p_i = 0$. All of these reduce vastly the calculation.

Let start with s-channel, for the $(--; ++)$ helicity. Only the contraction $\epsilon(p_2, p_4) \cdot \epsilon(p_3, p_1)$ survive, so there is only one term:

$$M_s(--; ++) = \frac{4g_s^2}{s} f^{abe} f^{cde} [\epsilon_-(p_2, p_4) \cdot \epsilon_+(p_3, p_1)] [p_2 \cdot \epsilon_-(p_1, p_4)] [p_3 \cdot \epsilon_+(p_4, p_1)]. \quad (3.44)$$

To proceed, we use the spinor helicity notation of polarization vectors which was introduced in chapter 2. The products like $[\epsilon_-(p_2, p_4) \cdot \epsilon_+(p_3, p_1)]$ are encoded in spinor notation as follows

$$\epsilon_-(p_2, p_4) \cdot \epsilon_+(p_3, p_1) = \frac{\langle 2\gamma^\mu 4 \rangle \langle 1\gamma_\mu 3 \rangle}{2\langle 13 \rangle [42]} = \frac{\langle 21 \rangle [43]}{\langle 13 \rangle [42]}. \quad (3.45)$$

For $p_2 \cdot \epsilon_-(p_1, p_4)$ we have³ :

$$p_2 \cdot \epsilon_-(p_1, p_4) = -\frac{\langle 2\gamma^\mu 2 \rangle \langle 1\gamma_\mu 4 \rangle}{2\sqrt{2}[41]} = \frac{\langle 21 \rangle [42]}{\sqrt{2}[41]}. \quad (3.46)$$

Finally, the product $p_3 \cdot \epsilon_+(p_4, p_1)$ becomes:

$$p_3 \cdot \epsilon_+(p_4, p_1) = -\frac{\langle 3\gamma^\mu 3 \rangle \langle 1\gamma_\mu 4 \rangle}{2\sqrt{2}\langle 14 \rangle} = \frac{[43]\langle 31 \rangle}{\sqrt{2}\langle 14 \rangle}. \quad (3.47)$$

By inserting (3.45), (3.46) and (3.47) in (3.44) we get :

$$M_s(--; ++) = 2g_s^2 f^{abe} f^{cde} \frac{\langle 21 \rangle [34]^2}{[21][14]\langle 14 \rangle}. \quad (3.48)$$

Now we put every thing in terms of $\langle \cdot \rangle$ by using the momentum conservation, we have:

$$\begin{aligned} (p_1 + p_2)^2 = (p_3 + p_4)^2 &\Rightarrow \frac{\langle 12 \rangle [21]}{\langle 34 \rangle [43]} = 1 \\ (p_1 + p_4)^2 = (p_2 + p_3)^2 &\Rightarrow \frac{\langle 14 \rangle [41]}{\langle 23 \rangle [32]} = 1 \end{aligned} \quad (3.49)$$

³Recall that $2p_i^\mu = \langle p_i \gamma^\mu p_i \rangle = \langle i \gamma^\mu i \rangle$.

and we know that $\sum_{k=1}^4 \langle ik \rangle [kj] = 0$, with $(i = 1, j = 3)$ that:

$$\langle 12 \rangle [23] + \langle 14 \rangle [43] = 0 \Rightarrow \frac{\langle 12 \rangle [23]}{\langle 14 \rangle [43]} = -1. \quad (3.50)$$

Put these two last relation in (3.5) we can simplify the result as

$$\begin{aligned} M_s(--;++) &= -2g_s^2 f^{abe} f^{cde} \left(\frac{\langle 21 \rangle [34]^2}{[21][14]\langle 14 \rangle} \right) \left(\frac{\langle 12 \rangle [21]}{\langle 34 \rangle [43]} \right) \left(\frac{\langle 14 \rangle [41]}{\langle 23 \rangle [32]} \right) \left(\frac{\langle 12 \rangle [23]}{\langle 14 \rangle [43]} \right) \\ &= 2g_s^2 f^{abe} f^{cde} \frac{\langle 12 \rangle^3}{\langle 34 \rangle \langle 23 \rangle \langle 14 \rangle}. \end{aligned} \quad (3.51)$$

Next, we multiply the numerator and denominator by $\langle 12 \rangle$ to get :

$$M_s(--;++) = -2g_s^2 f^{abe} f^{cde} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 34 \rangle \langle 23 \rangle \langle 14 \rangle} \quad (3.52)$$

This is our final and simplest expression for $M_s(--;++)$.

Then, we move on to the t -channel diagram, which is $2 \leftrightarrow 4$ and $b \leftrightarrow d$ from the s -channel. With our polarization choice, $\epsilon(p_1, p_4) \cdot p_4 = \epsilon(p_4, p_1) \cdot p_1 = 0$ and therefore :

$$M_t(--;++) = 0 \quad (3.53)$$

Finally, consider the u -channel amplitude, which is $2 \leftrightarrow 3$, $b \leftrightarrow c$ from the s -channel. This does not vanish but gives:

$$\begin{aligned} M_u(--;++) &= \frac{4g_s^2}{u} f^{ace} f^{bde} [\epsilon_+(p_3, p_1) \cdot \epsilon_-(p_2, p_4)] [p_3 \cdot \epsilon_-(p_1, p_4)] [p_2 \cdot \epsilon_+(p_4, p_1)] \\ &= 2g_s^2 f^{ace} f^{bde} \frac{1}{\langle 13 \rangle [31]} \left(\frac{\langle 21 \rangle [34]}{\langle 13 \rangle [24]} \right) \left(\frac{\langle 13 \rangle [34]}{[14]} \right) \left(\frac{\langle 21 \rangle [42]}{\langle 14 \rangle} \right) \end{aligned} \quad (3.54)$$

After some simplifications, the final result is :

$$M_u(--;++) = -2g_s^2 f^{ace} f^{bde} \frac{\langle 21 \rangle^4}{\langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle} \quad (3.55)$$

so, the total matrix element $M = M_s + M_t + M_u$ is

$$M(--;++) = -2g_s^2 \left[f^{abe} f^{cde} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 34 \rangle \langle 23 \rangle \langle 14 \rangle} + f^{ace} f^{bde} \frac{\langle 21 \rangle^4}{\langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle} \right] \quad (3.56)$$

to get the square of the HA's, we have to perform the color sums[18], we have :

$$(f^{abe} f^{cde})^2 = N^2(N^2 - 1) \quad (3.57)$$

$$(f^{abe} f^{cde})(f^{ace} f^{bde}) = \frac{1}{2}N^2(N^2 - 1) \quad (3.58)$$

and

$$\left| \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 34 \rangle \langle 23 \rangle \langle 14 \rangle} \right|^2 = \frac{s^2}{t^2}, \quad \left| \frac{\langle 21 \rangle^4}{\langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle} \right|^2 = \frac{s^4}{t^2 u^2} \quad (3.59)$$

$$\frac{[12]^4}{[12][34][23][14]} \frac{\langle 21 \rangle^4}{\langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle} = \frac{s^3}{t^2 u} \quad (3.60)$$

so that :

$$\sum_{\text{colors}} |M(--;++)|^2 = 4g_s^4 N^2(N^2 - 1) s^4 \left[\frac{1}{t^2 s^2} + \frac{1}{t^2 u^2} + \frac{1}{u^2 s^2} \right] \quad (3.61)$$

With this answer, it is not hard to compute the full $|M|^2$. Since only the MHV amplitude do not vanish, they're all given by some crossing symmetry of this result. For example $M(-+;-+)$ is given by $M(--;++)$ after interchanging s with u . There are six types of two positive and two negative helicities $(--,++)$ and $(++,--)$ yield a factor s^4 , $(-+;-+)$ and $(+-;+-)$ yield t^4 , $(-+;+-)$ and $(+-;-+)$ yield u^4 . The helicity sums is therefore :

$$\sum_{\text{colors, helicities}} |M|^2 = 4g_s^4 N^2(N^2 - 1)(s^4 + t^4 + u^4) \left[\frac{1}{t^2 s^2} + \frac{1}{t^2 u^2} + \frac{1}{u^2 s^2} \right] \quad (3.62)$$

Conclusion

In this thesis, we were interested in SHF for calculating scattering amplitudes in the SM of elementary particles in both electroweak and QCD sectors.

We studied the structure of SHF where we defined the main notions and tools which the formalism is built upon. We also constructed the physical quantities appearing in scattering amplitudes such as 4-vectors, Dirac spinors and polarization vectors in the massless case by using Weyl spinors. We then generalized the formalism to deal with massive particles with the intermediate of LCD and SLCD techniques.

Arriving at the 3rd chapter we successfully used SHF to calculate squares of amplitudes for a variety of examples such as Bhabha scattering, pair annihilation process and Higgs disintegration. However the crown jewel of our work was calculating the scattering amplitudes for the Higgs production process where we managed to reduce the number of amplitudes we had to calculate from 144 to only 16 via an effective choice of reference vectors. This result is a clear proof of the strength of this formalism.

The main challenge we encountered throughout this thesis was getting familiar with the many conventions existing in the literature of this formalism. We had to manipulate our way through different choices of metrics and notations to reconcile a few of the existing conventions. We were able to transit from one convention to another and obtain the same result of the squared amplitude.

The next challenge was implementing the formalism in Mathematica as we had a hard time finding an example for calculating scattering amplitudes. So we created a simple example which shows how to use the SpinorsExtras package to obtain the scattering amplitude in terms of massive spinor products.

The main objectives for future reference is reconciling the FeynCalc and S@M packages. Achieving this objective would be a huge exploit for the high energy physics community since it updates the powerful FeynCalc package to use the SHF method. Further development of the SHF to extend it for the study of complex momenta is also an objective we intend to achieve. Of course eventually the biggest challenge will be using the SHF to tackle the problem at the frontiers of particle physics namely radiative corrections and loop calculations.

Appendix A

Dirac matrices

The conventions in this thesis have been therefore taken from Srednicki's QFT text book which is the "mostly plus" metric [21]

$$g_{\mu\nu} = (-1, 1, 1, 1) \quad (\text{A.1})$$

It is useful to note that :

$$g_{\mu\nu}g^{\mu\nu} = 4 \quad (\text{A.2})$$

The chirality matrix defined as :

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (\text{A.3})$$

obeys :

$$\gamma_5^2 = \mathbb{1} \quad , \quad \{\gamma_5, \gamma_\mu\} = 0 \quad (\text{A.4})$$

Recall the fundamental anticommutator relation for the Dirac matrices :

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = -2g^{\mu\nu} \quad (\text{A.5})$$

Some traces over the Dirac matrices and chirality matrix are:

$$\text{Tr}(\mathbb{1}) = 4, \quad \text{Tr}(\gamma_5) = \text{Tr}(\gamma_5\gamma^\mu\gamma^\nu) = 0 \quad (\text{A.6})$$

$$\text{Tr}(\gamma^\mu\gamma^\nu) = -4g^{\mu\nu}, \quad \text{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\varepsilon^{\mu\nu\rho\sigma} \quad (\text{A.7})$$

$$\text{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \quad (\text{A.8})$$

$$\text{Tr}(\text{odd number of Dirac matrices}) = 0 \quad (\text{A.9})$$

where $\varepsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor, verifying:

$$\varepsilon^{0123} = \varepsilon^{3210} = 1 \quad , \quad \varepsilon^{0123} = -\varepsilon^{1023} \quad (\text{A.10})$$

Appendix B

Trace method

Now that we have used SHF to reach the square of the amplitude for some processes we're going to go over the trace evaluation method for the Bhabha scattering process. The square of the amplitude in the trace method is the average over all initial spin configurations i , and sum over all final spin configurations f :

$$\langle |M|^2 \rangle \equiv \text{average over initial spins, sum over final spins.} \quad (\text{B.1})$$

For Bhabha scattering, we have

$$\begin{aligned} M_1 &= \frac{e^2}{t} \bar{u}(p_3) \gamma^\mu u(p_1) \bar{v}(p_2) \gamma_\mu v(p_4) \\ M_2 &= -\frac{e^2}{s} \bar{v}(p_2) \gamma^\mu u(p_1) \bar{u}(p_3) \gamma_\mu v(p_4) \end{aligned} \quad (\text{B.2})$$

Summing over final states and averaging over initial states:

$$\frac{1}{4} \sum_{spin} |M|^2 = \frac{1}{4} \sum_{spin} |M_1|^2 + |M_2|^2 + M_1 M_2^\dagger + M_2 M_1^\dagger \quad (\text{B.3})$$

To calculate $|M|^2$ we need to find the complex conjugate of the amplitude. To get this, we first recall that:

$$\gamma_\mu^\dagger \gamma_0 = \gamma_0 \gamma_\mu \quad \text{and} \quad \gamma_0^\dagger = \gamma_0 \quad (\text{B.4})$$

So

$$\begin{aligned} \left(\bar{u}(p_a) \gamma_\mu u(p_b) \right)^\dagger &= \left(u(p_a)^\dagger \gamma_0 \gamma_\mu u(p_b) \right)^\dagger = u(p_b)^\dagger \gamma_\mu^\dagger \gamma_0^\dagger u(p_a) \\ &= u(p_b)^\dagger \gamma_\mu^\dagger \gamma_0 u(p_a) = \bar{u}(p_b) \gamma_\mu u(p_a) \end{aligned} \quad (\text{B.5})$$

in this part we're only going to compute $|M_1|^2$, so we can write M_1^\dagger as :

$$M_1^\dagger = \frac{e^2}{t} \bar{v}(p_4) \gamma_\nu v(p_2) \bar{u}(p_1) \gamma^\nu u(p_3) \quad (\text{B.6})$$

and therefore :

$$|M_1|^2 = \frac{e^4}{t^2} [\bar{u}(p_3) \gamma^\mu u(p_1)] [\bar{v}(p_2) \gamma_\mu v(p_4)] [\bar{v}(p_4) \gamma_\nu v(p_2)] [\bar{u}(p_1) \gamma^\nu u(p_3)] \quad (\text{B.7})$$

The reason we wrote the square of the amplitude this way is to show that it is only a product of the numbers written in brackets. Thus $|M_1|^2$ could also be written by permutation of the brackets :

$$|M_1|^2 = \frac{e^4}{t^2} [\bar{u}(p_3) \gamma^\mu u(p_1)] [\bar{u}(p_1) \gamma^\nu u(p_3)] [\bar{v}(p_2) \gamma_\mu v(p_4)] [\bar{v}(p_4) \gamma_\nu v(p_2)] \quad (\text{B.8})$$

which shows that $|M_1|^2$ is the contraction of the two tensors, one depending only on the initial states, and other depending only on the final state.

The spin sum can be performed with Dirac trace, we are ready now to sum over the spin of particles, using the completeness relation¹ :

$$\sum_{spin} u_\alpha^s(p) \bar{u}_\beta^s(p) = \sum_{spin} \bar{u}_\beta^s(p) u_\alpha^s(p) = -\not{p}_{\alpha\beta} \quad (\text{B.9})$$

using these relations :

$$\begin{aligned} \sum_s \sum_{s'} [\bar{u}^s(p_3) \gamma^\mu u^{s'}(p_1)] [\bar{u}^{s'}(p_1) \gamma^\nu u^s(p_3)] &= \sum_s \bar{u}_\beta^s(p_3) \gamma_{\beta\rho}^\mu (-\not{p}_1)_{\rho\sigma} \gamma_{\sigma\alpha}^\nu u_\alpha^s(p_3) \\ &= (-\not{p}_3)_{\alpha\beta} \gamma_{\beta\rho}^\mu (-\not{p}_1)_{\rho\sigma} \gamma_{\sigma\alpha}^\nu \\ &= \text{Tr}(\not{p}_3 \gamma^\mu \not{p}_1 \gamma^\nu) \end{aligned} \quad (\text{B.10})$$

which is a simple expression we can evaluate using γ -matrix properties² :

$$\begin{aligned} \frac{1}{4} \sum_{spin} |M_1|^2 &= \frac{e^4}{4t^2} \text{Tr}(\not{p}_3 \gamma^\mu \not{p}_1 \gamma^\nu) \text{Tr}(\not{p}_2 \gamma_\mu \not{p}_4 \gamma_\nu) \\ &= \frac{2e^4}{t^2} [(2p_1 p_4)(2p_2 p_3) + (2p_1 p_2)(2p_3 p_4)] \end{aligned} \quad (\text{B.11})$$

(A.8) and (A.2) were used. We can simplify this by making the connection with Mandelstam

¹Note that in this special example $m = 0$

²Recall that Tr denotes the trace of the matrix (the sum of its diagonal elements) $\text{Tr}(A) = \sum_i A_{ii}$.

variables:

$$\begin{aligned}
s &= -(p_1 + p_2)^2 = -(p_3 + p_4)^2 = -2p_1p_2 = -2p_3p_4 \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 = -2p_1p_3 = -2p_2p_4 \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 = -2p_1p_4 = -2p_2p_3 \\
u + s + t &= 0
\end{aligned} \tag{B.12}$$

After some algebraic manipulations, the result is :

$$\frac{1}{4} \sum_{spin} |M_1|^2 = 2e^4 \frac{s^2 + (t+s)^2}{t^2} \tag{B.13}$$

The same thing with M_2 and the mix product. Finally we get :

$$\frac{1}{4} \sum_{spin} |M|^2 = 2e^4 \left[\frac{(t+s)^2 + s^2}{t^2} + \frac{(t+s)^2 + t^2}{s^2} + 2 \frac{(t+s)^2}{st} \right] \tag{B.14}$$

Appendix C

Spinor Helicity

In chapter 2, the SHF was introduced. We list here a set of identities involving spinor products and the spinor representation for polarization vectors. Useful references are [21, 29].

C.1 Spinor product

The solutions of the massless Dirac equation, are denoted by the Dirac bra-ket notation

$$\begin{aligned} u_-(p) = v_+(p) = |p], \quad u_+(p) = v_-(p) = |p\rangle \\ \bar{u}_-(p) = \bar{v}_+(p) = \langle p|, \quad \bar{u}_+(p) = \bar{v}_-(p) = [p| \end{aligned} \tag{C.1}$$

where $u_s(p)$ and $v_s(p)$ ($s = \pm$) respectively stand for the positive and negative energy solutions to the massless Dirac equation $\not{p}u_s(p) = \not{p}v_s(p) = 0$. The spinor products can be constructed as follows

$$\langle ij \rangle = \bar{u}_-(p_i)u_+(p_j) \quad [ij] = \bar{u}_+(p_i)u_-(p_j) \tag{C.2}$$

Spinor product are antisymmetric

$$\langle ij \rangle = -\langle ji \rangle, \quad [ij] = -[ji] \tag{C.3}$$

and thus

$$\langle ii \rangle = [ii] = 0, \quad \langle ij \rangle = [ij] = 0 \tag{C.4}$$

When calculating amplitudes, we will need the complex conjugate of spinor products

$$\langle ij \rangle^* = [ji] \tag{C.5}$$

so

$$\langle ij \rangle [ji] = \langle ij \rangle \langle ij \rangle^* = |\langle ij \rangle|^2 \quad (\text{C.6})$$

The spinor completeness relation with $m = 0$ read $u_+ \bar{u}_+ + u_- \bar{u}_- = -\not{p}$, this can be written in SHN as

$$-\not{p} = |p\rangle\langle p| + |p]\langle p| \quad (\text{C.7})$$

We obtain the following identities by using the projection operators γ_{\pm}

$$\gamma_+ \not{p} = -|p\rangle\langle p|, \quad \gamma_- \not{p} = -|p]\langle p| \quad (\text{C.8})$$

These allow us to derive a connection to the most prevalent kinematic notation

$$\begin{aligned} \langle ij \rangle [ji] &= -\langle i \gamma_+ \not{p}_j i \rangle = -\text{Tr}(\gamma_+ \not{p}_j i \langle i) \\ &= \text{Tr}(\gamma_+ \not{p}_j \gamma_- \not{p}_i) = p_i^\mu p_j^\nu \text{Tr}(\gamma_+ \gamma_\mu \gamma_- \gamma_\nu) \\ &= p_i^\mu p_j^\nu \text{Tr}(\gamma_\mu \gamma_- \gamma_\nu) = 2p_i^\mu p_j^\nu g_{\mu\nu} \\ &= -2p_i p_j = s_{ij} \end{aligned} \quad (\text{C.9})$$

Other useful relation are the Gordon identity

$$\begin{aligned} [i \gamma_\mu i] &= \text{Tr}(\gamma_\mu i \langle i) = -\text{Tr}(\gamma_\mu \gamma_+ \not{p}_i) \\ &= -p_i^\nu \text{Tr}(\gamma_\mu \gamma_+ \gamma_\nu) = -2p_i^\nu g_{\mu\nu} \\ &= 2p_{i\mu} \end{aligned} \quad (\text{C.10})$$

Fierz rearrangement is expressed by :

$$\langle i \gamma^\mu j \rangle \langle k \gamma_\mu l \rangle = 2 \langle ik \rangle [jl] \quad (\text{C.11})$$

The next important identity is the momentum conservation. If a set of momenta are conserved $\sum_{i=1}^n p_i^\mu = 0$, in helicity spinors it becomes:

$$\sum_{j=1}^n \langle ij \rangle [jk] = 0 \quad (\text{C.12})$$

Another useful relation is that

$$\begin{aligned} \langle i\gamma^\mu j \rangle \langle j\gamma_\mu i \rangle &= \text{Tr}(\gamma^\mu j \langle j\gamma^\nu i \rangle \langle i \rangle) = \text{Tr}(\gamma^\mu \gamma_{-\not{p}_j} \gamma^\nu \gamma_{-\not{p}_i}) = \text{Tr}(\gamma^\mu \not{p}_j \gamma^\nu \gamma_{-\not{p}_i}) \\ [j\gamma^\mu i] [i\gamma^\nu j] &= \text{Tr}(\gamma^\mu \not{p}_j \gamma^\nu \gamma_{+\not{p}_i}) \end{aligned} \quad (\text{C.13})$$

C.2 Polarization vectors

If we want to apply the SHF to the processes containing bosonic external line, we need to write polarization vectors in terms of spinor notation

$$\epsilon_+^\mu(k, q) = -\frac{\langle q\gamma^\mu k \rangle}{\sqrt{2}\langle qk \rangle}, \quad \epsilon_-^\mu(k, q) = -\frac{\langle k\gamma^\mu q \rangle}{\sqrt{2}[qk]} \quad (\text{C.14})$$

q is an arbitrary massless reference momentum, the contraction with γ_μ is :

$$\begin{aligned} \not{\epsilon}_+(k, q) &= -\frac{\langle q\gamma^\mu k \rangle \gamma_\mu}{\sqrt{2}\langle qk \rangle} = \frac{\sqrt{2}}{\langle qk \rangle} (|k\rangle \langle q| + |q\rangle \langle k|) \\ \not{\epsilon}_-(k, q) &= -\frac{\langle k\gamma^\mu q \rangle \gamma_\mu}{\sqrt{2}[qk]} = \frac{\sqrt{2}}{[qk]} (|k\rangle [q| + |q\rangle \langle k|) \end{aligned} \quad (\text{C.15})$$

with

$$\langle q\gamma^\mu k \rangle \gamma_\mu = -2(|k\rangle \langle q| + |q\rangle \langle k|) \quad (\text{C.16})$$

The polarization vectors have some interesting properties. Using the definition (C.14) and (C.15), we could simply obtain the following important relation :

$$\begin{aligned} (\epsilon_\pm^\mu)^* &= -\epsilon_\mp^\mu \\ \epsilon_+(k, q) \cdot \epsilon_-(k, q) &= -1 \\ \epsilon_\pm(k, q) \cdot \epsilon_\mp(k, q') &= 0; \text{ if } q = q' \\ \epsilon_\pm(k, q) \cdot q &= 0 \\ \not{\epsilon}_+(k, q)|q\rangle \sim \langle qq\rangle &= 0 \\ [q|\not{\epsilon}_-(k, q) \sim [qq] &= 0 \end{aligned} \quad (\text{C.17})$$

Appendix D

Input for FeynCalc

In this section we're going to implement the Bhabha scattering and pair annihilation processes in mathematica via FeynCalc [30, 31, 32] using the trace method to determine the square of the amplitude, then we're going to implement Bhabha scattering process again in FeynCalc but this time by using SHF to determine the amplitude:

D.1 Input for traces method

Bhabha scattering :

```
1 << FeynCalc `
2 M1 = e^2/t SpinorUBar[p3, 0].GA[mu].SpinorU[p1, 0] SpinorVBar[p2,
3     0].GA[mu].SpinorV[p4, 0]
4 M2 = - e^2/s SpinorVBar[p2, 0].GA[mu].SpinorU[p1, 0] SpinorUBar[p3, 0].GA[
5     mu].SpinorV[p4, 0]
6 MTotal = M1 + M2
7 MTotalC = ComplexConjugate[MTotal] /. mu -> muC
8 MsAvg1 = 1/4 FermionSpinSum[MTotal MTotalC] /. DiracTrace -> Tr //
9     Contract // Simplify
10 MsAvg1 // InputForm
11 MsAvg2 = MsAvg1 // FCE
12 MsAvg2 // InputForm
13 onshell = {SP[p1, p2] -> -s/2, SP[p3, p4] -> -s/2, SP[p1, p3] -> -t/2,
14     SP[p2, p4] -> -t/2, SP[p1, p4] -> -u/2, SP[p2, p3] -> -u/2}
15 MsAvg3 = MsAvg2 /. onshell // Simplify
16 MsAvg = MsAvg3 /. u -> (-s - t)
```

Pair Annihilation :

```
1 << FeynCalc `
```

```

2 dm[mu_] := DiracMatrix[mu]
3 ds[p_] := DiracSlash[p]
4 sp[p_, q_] := ScalarProduct[p, q]
5 prop[p_, m_] := ds[p] + m
6 Line1 := prop[p2, -m].dm[mu].prop[p1 - k1, m].dm[nu].prop[p1, m].dm[
7     nu].prop[p1 - k1, m].dm[mu]
8 Line2 := prop[p2, -m].dm[mu].prop[p1 - k2, m].dm[nu].prop[p1, m].dm[
9     nu].prop[p1 - k2, m].dm[mu]
10 Line12 := prop[p2, -m].dm[nu].prop[p1 - k1, m].dm[mu].prop[p1, m].dm[nu].prop[
11     p1 - k2, m].dm[mu]
12 Line21 := prop[p2, -m].dm[mu].prop[p1 - k2, m].dm[nu].prop[p1, m].dm[mu].prop[
13     p1 - k1, m].dm[nu]
14 ans1 = Simplify[Contract[Tr[Line1]]/t^2]
15 ans2 = Simplify[Contract[Tr[Line2]]/u^2]
16 ans21 = Simplify[Contract[Tr[Line21]]/(t u)]
17 ans = e^4 (ans1 + ans2 + ans12 + ans21)
18 onshell = {sp[p1, p1] -> m^2, sp[p2, p2] -> m^2, sp[k1, k1] -> 0,
19     sp[k2, k2] -> 0}
20 res = ans /. onshell
21 res = res /. m -> 0
22 kin = {sp[p1, p2] -> s/2, sp[p1, k1] -> -t/2, sp[p1, k2] -> -u/2,
23     sp[p2, k1] -> -u/2, sp[p2, k2] -> -t/2, sp[k1, k2] -> s/2}
24 aux = Expand[res /. kin]
25 Print[" !!(\*UnderscriptBox[\(\[Sum]\), \
26     \(\spins\)]\)|M!\(\*SuperscriptBox[\(\|\), \(\2\)]\)= ", aux]
27 final = aux /. s -> (-u - t) // Simplify

```

D.2 Input for helicity amplitude method

Bhabha scattering with SHF

```

1 << FeynCalc '
2 dp[s_] := (1 + s DiracMatrix[5])/2
3 U[p_, s_] := dp[s] . Spinor[p, 0]
4 UBar[p_, s_] := SpinorUBar[p, 0] . dp[-s]
5 UBar[p_, s_] := SpinorUBar[p, 0] . dp[-s]
6 Myds[p_] := U[p, 1] UBar[p, 1] + U[p, -1] UBar[p, -1]
7 GammaGamma[p_, q_, s_] := 2 ( U[q, s] UBar[p, s] ) + 2 ( U[p, -s] UBar[q, -s]
8     )
9 delta[s1_, s2_] := If[s1 == s2, 1, 0]
9 M1[s1_, s2_, s3_, s4_] := delta[s1, s2] UBar[p3, s3] . GammaGamma[p2, p1, s1] .
    U[p4, s4]

```

```

10 res1[s1_, s2_, s3_, s4_] := DiracSimplify[DotSimplify[M1[s1, s2, s3, s4]]/s
11 M2[s1_, s2_, s3_, s4_] := delta[s1, s3] UBar[p2, s2] . GammaGamma[p3, p1, s1] .
    U[p4, s4]
12 res2[s1_, s2_, s3_, s4_] := -DiracSimplify[DotSimplify[M2[s1, s2, s3, s4]]/t
13 vlist = {p1, p2, p3, p4}
14 simp1 = Table[Spinor[vlist[[i]], 0] . Spinor[vlist[[j]], 0] ->
15     sp[vlist[[i]], vlist[[j]]] + spc[vlist[[j]], vlist[[i]]], {i, 1,
16     4}, {j, 1, 4}] /. {sp[p_, p_] -> 0, spc[q_, q_] -> 0}
17 simp2 = Table[ Spinor[vlist[[i]], 0] . DiracMatrix[5] .
18     Spinor[vlist[[j]], 0] -> -sp[vlist[[i]], vlist[[j]]] +
19     spc[vlist[[j]], vlist[[i]]], {i, 1, 4}, {j, 1,
20     4}] /. {sp[p_, p_] -> 0, spc[q_, q_] -> 0}
21 simp = Flatten[{simp1, simp2}];
22 M[s1_, s2_, s3_, s4_] := Expand[res1[s1, s2, s3, s4] + res2[s1, s2, s3, s4] /.
    simp]

```


Appendix E

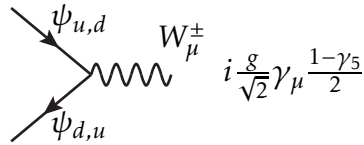
Feynman diagrams with axodraw4j and FeynArts

In this section we show how to use axodraw4j and FeynArts [33] package, illustrated with an example. JaxoDraw is a JAVA program for drawing Feynman diagrams, it has a simple interface allowing all actions to be carried out in a simple manner, in addition to it's simple interface the program allows us to import the work done to multiple programs and allows us to save the work done under many extensions such as XML and EPS

The basics of using axodraw4j are shown in the following LaTeX document :

```
1 \begin{center}
2 \fcolorbox{white}{white}{
3 \begin{picture}(400,70) (0,0)
4 \SetWidth{1.0}
5 \SetColor{Black}
6 \Line[arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset=0.2](100,65)
   (130,40)
7 \Line[arrow,arrowpos=0.5,arrowlength=5,arrowwidth=2,arrowinset=0.2](130,40)
   (100,15)
8 \Photon(130,40)(160,40){4}{4}
9 \Text(180,30)[lb][ $\frac{g}{\sqrt{2}}\gamma_{\mu}\frac{1-\gamma_5}{2}$ ]
10 \Text(115,55)[lb][ $\psi_{u,d}$ ]
11 \Text(115,15)[lb][ $\psi_{d,u}$ ]
12 \Text(155,45)[lb][ $W_{\mu}^{\pm}$ ]
13 \end{picture}
14 }
15 \end{center}
```

After compiling we obtain the following figure



The prevalent rules preparation of a document are :

- Insert the following

```
1 \usepackage {axodraw4j}
```

in the preamble of the .tex file

- put the axodraw4j object you want to insert inside a picture environment

```
1 \fcolorbox {white} {white} {
2 \begin {picture} (x,y) (0,0)
3 .....
4 \end {picture} }
```

here x and y denote the size of the box that is to be inserted in the document and that contains the graph, and $(0,0)$ is the origin.

FeynArts is a *Mathematica* package for the generation and visualization of Feynman diagrams and amplitudes. throughout this thesis we used it along with *FeynCalc* to confirm our results, here's an example of how it works on the *Mathematica* program : We start off with Bhabha scattering example where we see how to obtain Feynman diagram via *FeynArts* package:

```
1 $LoadFeynArts = True
2 topl1 = CreateTopologies[0, 2 -> 2];
3 diag1 = InsertFields[
4   poo, {F[2, {1}], -F[2, {1}]} -> {F[2, {1}], -F[2, {1}]},
5   InsertionLevel -> {Particles}, ExcludeParticles -> {S, V[2]};
6 Paint[diag1]
```

We start off by loading *FeynArts* package with the first command, we then create the topology where the numbers inside the box are respectively the number of loops, the number of particles in the initial state and the number of particles in the final state. After that we use the "InsertFields" command to choose the process with special strings each for a specific particle.

The examples below are respectively for electron-positron annihilation process and two gluon scattering process.

```
1 top2 = CreateTopologies[0, 2 -> 2];
2 diag2 = InsertFields[ass, {F[2, {1}], -F[2, {1}]} -> {V[1], V[1]},
3     InsertionLevel -> {Particles}, Model -> {SM},
4     ExcludeParticles -> {S}];
5 Paint[diag2]
```

```
1 top3 = CreateTopologies[0, 2 -> 2]
2 diag3 = InsertFields[gluon, {V[5], V[5]} -> {V[5], V[5]},
3     Model -> "SMQCD", InsertionLevel -> {Particles}]
4 Paint[diag3]
```

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