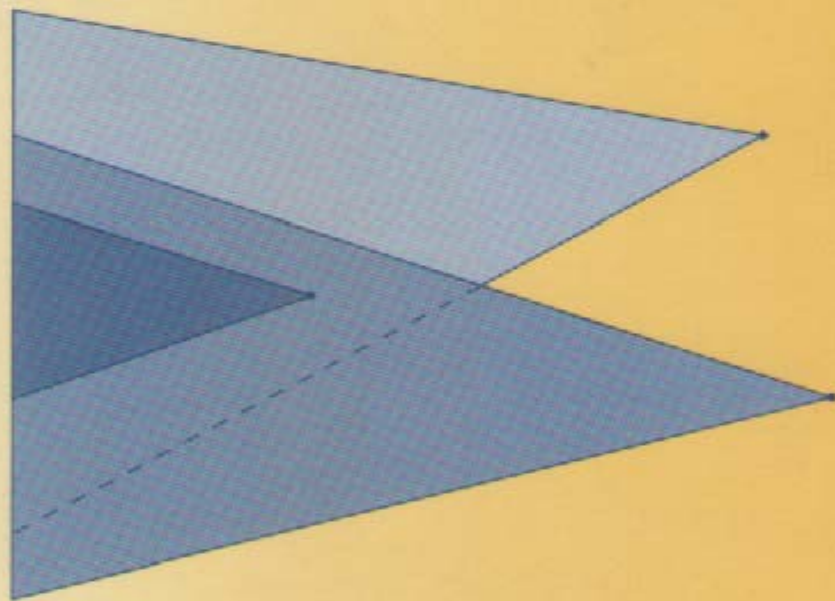


Alan F. Beardon

LIMITS

A New Approach to Real Analysis



Springer

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UNDERGRADUATE TEXTS IN MATHEMATICS

This book is intended as an undergraduate text on real analysis and includes all the standard material such as sequences, infinite series, continuity, differentiation, and integration, together with worked examples and exercises. By unifying and simplifying all the various notions of limit, the author has successfully presented a unique and novel approach to the subject matter that has not previously appeared in book form.

The author defines what is meant by a limit just once, and all of the subsequent limiting processes are viewed as special cases of this one definition. In this way the subject matter attains a unity and coherence that is missing in the traditional approach, and students will be able to fully appreciate and understand the common source of the topics they are studying. These topics are presented as "variations on a theme" rather than essentially different ideas, and this leads to a clearer global view of the subject.

The book is divided into three sections. Part I contains preliminary material on sets, and on real and complex numbers. Part II starts with the definition of a limit and its basic properties, and continues with three basic results: the Intermediate Value Theorem, the Mean Value Inequality, and the Cauchy Criterion—all of which are proved by bisection arguments. The last chapter in this section contains a detailed discussion of infinite series, including a treatment of unordered sums. Part III comprises the standard material in analysis.



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