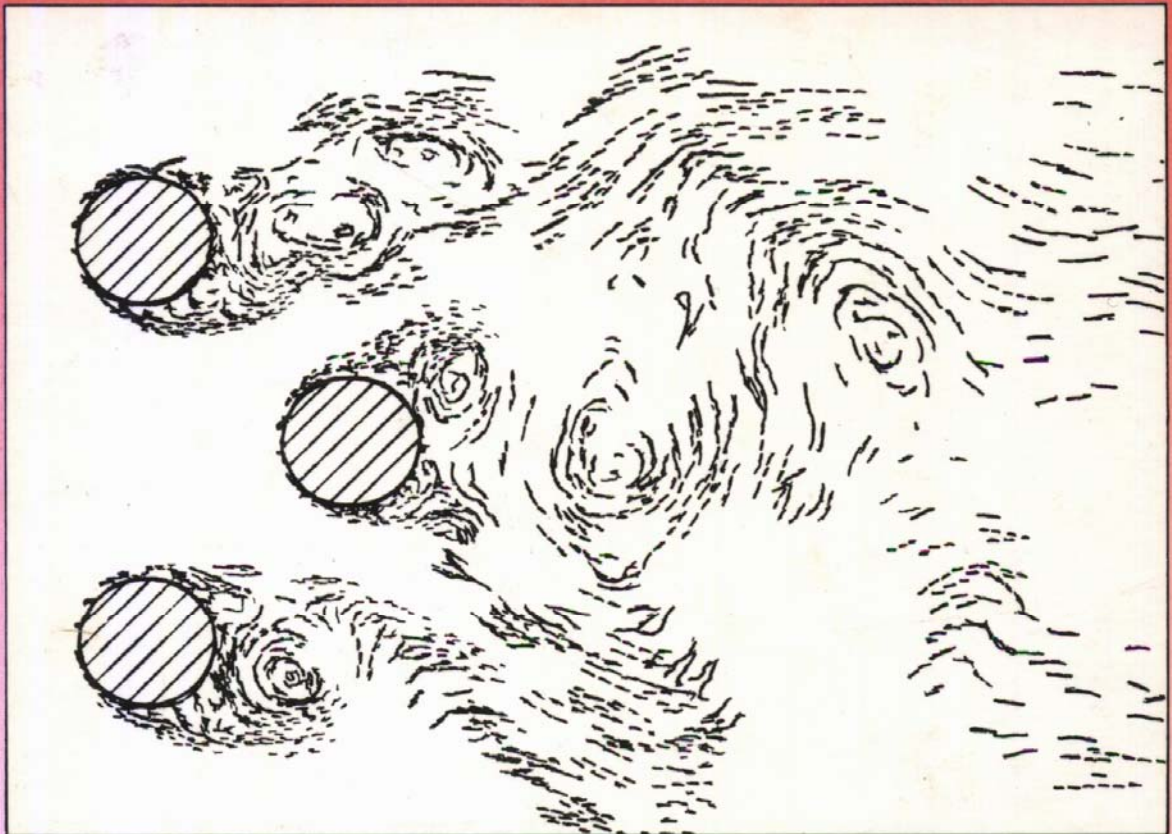

CAMBRIDGE
**ENGINE
TECHNOLOGY**
SERIES

**VORTEX ELEMENT
METHODS
FOR FLUID DYNAMIC
ANALYSIS
OF ENGINEERING
SYSTEMS**



R. I. Lewis

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