

SEVENTH EDITION

# ADVANCED ENGINEERING MATHEMATICS

**ERWIN KREYSZIG**

*Kreyszig: Maple Supplement*



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