

A New Dimension to Quantum Chemistry

Analytic Derivative Methods in *Ab Initio*
Molecular Electronic Structure Theory

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